FA21-BEE-017

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Q1 (a)

$$\lim_{(x,y)\to(1,2)} \frac{x^2-2xy}{x^2-4y^2}$$

$\cancel{x^2 = 4y^2}$ $x = 4y$

$y = \frac{x}{4}$

$$\frac{x^2 - 2(x(x/4))}{x^2 - 4\left(\frac{x^2}{16}\right)}$$

$$\frac{x^2 - x^2/2}{x^2 - \frac{x^2}{4}}$$

$$\frac{\frac{2x^2 - x^2}{2}}{\frac{4x^2 - x^2}{4}} = \frac{(2x^2 - x^2)4^2}{2(4x^2 - x^2)}$$

$$\frac{2x^2}{3x^2} = \boxed{\frac{2}{3}}$$

(b)

$$\lim_{(x,y)\to(6,0)} \frac{x-4y}{6y+7x}$$

$6y = 7x$

$y = 7x/6$

$$\frac{x - 28x/6}{14x}$$

$$= \frac{6x - 28x}{6(14x}$$

$$= \frac{-22x}{6(14x)} = \frac{-11}{52}$$

(c) $$\lim_{x,y\to 0,0} \frac{x^2 - y^6}{xy^3}$$

let $y = mx$

$$\frac{x^2 - m^6x^6}{x m^3 x^3} = \frac{x^2(1 - m^6x^3)}{x^4 m^3}$$

$$\frac{1 - m^6 x^3}{m^3 x^2}$$

(d) $\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3 - 2e^{2y}}{6x+2y-3z}$

$\frac{(-1)^3 - 4(0)e^7}{6(-1)+2(0)-3(4)} = -1/-16$

$= 1/16$

Question no. 2

(a) $f(x,y) = \cos(x/y)$ in $v = (3,-4)$

$f\nabla = \frac{\partial}{\partial x}(\cos(x/y))i + \frac{\partial}{\partial y}(\cos x/y)\,j$

$= -1/y\,(\sin x/y)i + x(-\sin x/y)(-1/y^2)j$

$= -1/y\,\sin(x/y)\,i + \frac{x}{y^2}\sin(x/y)\,j$

unit vector

$\frac{3i-4j}{\sqrt{9+16}} = \frac{3i}{25} - 4/25\,j$

$D_{\vec{u}}f = \frac{-3}{5y}\sin(x/y) + \frac{4x}{5y^2}\sin(x/y)$

$= 1/5y\,\sin(x/y)\,(4x/y - 3)$

b) $f(x,y,z)\ x^2y^3-4xz \quad , v=(-1,2,0)$

$\nabla f = (2y^3xi + (-4z)i) + 3y^2x^2j - 4xk$

$\hat{v} = \dfrac{-i+2j+0k}{\sqrt{1+4}} = \dfrac{-1}{\sqrt{15}}i + \dfrac{2}{\sqrt{5}}j$

$D\vec{u}f \quad \dfrac{-1}{\sqrt{5}}(2y^3x-4z) + \dfrac{2}{\sqrt{5}}(3y^2x^2) + 0$

Question no. 3

$f(x,y,z) = 4x - y^2e^{3xz}$

$\nabla f = 4-y^2e^{3xz}(3z)i - 2ye^{3xz}j + y^2e^{3xz}3xk$

$\nabla f_{(3,-1,0)} = 4-(-1)^2e^{3(3)(0)}(3)i - 2(-1)e^{3(3)(0)}j$

$-(-1)^2e^{3(3)(0)}\cdot 3(3)\ k$

$= (4-0)i - 2j - 9k$

$U = (-1,4,2)$

$U = \dfrac{-i+4j+2k}{\sqrt{1+16+4}}$

$-\dfrac{1}{\sqrt{21}}(4) - \dfrac{2(4)}{\sqrt{21}} \quad \dfrac{-9(2)}{\sqrt{21}}$

$= \dfrac{-32}{\sqrt{28}}$

Question 4

(a) $f(x,y) = \sqrt{x^2+y^2}$ at $(-2,3)$

$\nabla f = \frac{1}{2}(x^2+y^3)^{-1/2}(2x)\,i + \frac{1}{2}(x^2+y^3)^{-1/2}(3y^2)\,j$

$\nabla f(-2,3) = (4+9)^{-1/2}(-2)\,i + \frac{1}{2}(4+9)^{-1/2}\,3(9)\,j$

$= \frac{-2}{\sqrt{13}}\,i + \frac{27}{2\sqrt{13}}\,j$

(b) $f(x,y,z) = e^{2x}(\cos y - 2z)$ at $(4,-2,0)$

$\nabla f = (e^{2x}2(\cos y - 2z))\,i + e^{2x}(-\sin(y-2z))(1)\,j$

$+ e^{2x}(-\sin(y-2z))(-2)\,k$

$\nabla f(4,-2,0) = e^{4x}\cdot 2\cos(-2-0)\,i + e^{4x}(-\sin(-2$
$+ e^{4x}(-\sin(-2)(-2)$

$\nabla f(4,-2,0)\ e^{4x}(-2\cos\cdot 2i - \sin(-2)j + 2\sin -2$

Question 5

(a) $F = x^2 y\,i - 2(z^3 - 3x)\,j + 4y^2 k$

$\nabla = 2xyi - 0j + 0$

$\text{Div} f = \nabla f \cdot f$

$= (2xy\,i) \cdot (x^2 y i - z^3 - 3x) j + xyz k)$

$\text{Div} f = 2x^3 \cdot y^2 + 0$

$3) - 1/2 (3y^2) j$

$\text{curve} = \nabla f \times f$

$2\ 3(4) j$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (z^3 - x^3) & y z \end{vmatrix}$$

$= 2,0)$

$= i(8y + 3z^2) - j(0-0) + k(+3x^2 - x^2)$

$y - 2z)(0) j$

$(8y + 3z^2) i - 0 j + (2x^2) k$

(b) $F = (2x + 2z^2) i + \frac{x^3 y^2}{2} j - (z \cdot 7x) k$

$z^4 x - \sin(-$

$\text{div} = \nabla f \cdot f$

$-\frac{1}{2} j + 2 \sin - 2$

$\left(\left(\frac{\partial}{\partial x}\right) i + \left(\frac{\partial}{\partial y}\right) j + \left(\frac{\partial}{\partial z}\right) k \right) (2x + 2z^2 i) + \left(\frac{x^3 y^2}{2}\right) j - (z \cdot 7x) k$

$|^2 k$

$2 + \frac{2x^2}{2} y - (1)$

$= 2 + \frac{2x^3}{2} y^{-1}$

$\text{curve} = \nabla f \times f$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x+2z^2 & \frac{x^3y^2}{z} & (-z-7x) \end{vmatrix}$$

$$i\left(0+\frac{x^3}{z^2}y^2\right)-j\left(7-4x\right)+k\left(\frac{3x^2}{z}y^2-0\right)$$

$$\frac{x^3y^2}{z^2}i-(7-4z)j+\left(\frac{3x^2}{z}y^2\right)k$$

Question 6

(a) $F=\left(4y^2+\frac{3x^2}{z^2}y\right)i+\left(8xy+\frac{x^3}{z^2}\right)j+\left(11-\frac{2x^3y}{z^3}\right)$

The vector field is conservative if and only if

$$\frac{dM}{dy}=\frac{\partial N}{\partial x},\ \frac{\partial N}{\partial z}=\frac{\partial P}{\partial y},\ \frac{\partial M}{\partial z}=\frac{\partial P}{\partial x}$$

$$F=\left(4y^2+\frac{3x^2}{z^2}y\right)i+\left(8xy+\frac{x^3}{z^2}\right)$$
$$+\left(11-\frac{2x^3y}{z^3}\right)k \Rightarrow P$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}, \quad \frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$$

$$\frac{\partial N}{\partial z} = x^3 z^{-2} = x^3(-2)z^{-3} = -\frac{2x^3}{z^3}$$

$$\frac{\partial P}{\partial y} = -\frac{2x^3}{z^3}$$

$$\frac{\partial M}{\partial z} = 4x^2 + \frac{3x^2 y}{z^2} = 3x^2 y(-2)z^{-3}$$

$$= \frac{6x^2 y}{z^3}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}\left(11 - \frac{2x^2 y}{z^2}\right) = \frac{-6x^2 y}{z^3}$$

Since given conditions are satisfied vector field is conservative.

(b) $F = 6xi + (2x - y^2)j + (6z - x^3)k$

$$F = \underset{M}{6xi} + \underset{N}{(2x - y^2)}j + \underset{P}{(6z - x^3)}k$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} 6x = 0 , \quad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2x - yz)$$

$$= 2$$

$$\frac{\partial N}{\partial z} = \frac{\partial}{z}(2x - yz) = 0 , \quad \frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(6z - x^3) = 0$$

$$\frac{\partial M}{\partial z} = \frac{\partial}{\partial z} 6x = 0 , \quad \frac{\partial P}{\partial x} = \frac{\partial}{\partial x} 6z - x^3$$

$$= -3x^2$$

Hence vector field ~~fie~~ is not conservative.

Question 7

(a) $z = \frac{x^2 - w}{y^4}$, $x = t^3 + 7$

$y = \cos 2t$. $w = 4t$

$$\frac{dz}{dr} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial z}{y} \cdot \frac{\partial y}{\partial r} + \frac{\partial z}{\partial w} \cdot \frac{dw}{dr}$$

$$\frac{dz}{dr} = \frac{2x}{y^4} , \quad \frac{dz}{dy} = (x^2 - w) y^{-4}$$

$$\frac{dx}{dt} = 3t^2 , \quad \frac{dz}{dy} = (x^2 - w) - 4y^{-5}$$

$$\frac{dz}{dy} = -4 \frac{x^2 - w}{y^5}$$

$$\frac{dy}{dt} = \frac{d}{dt} \cos 2t = -\sin 2t \cdot 2$$
$$= -2\sin 2t$$

$$\frac{\partial z}{\partial w} = \frac{-1}{y^4} \qquad \frac{dw}{dt} = 4$$

$$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt} + \frac{dz}{\partial w} \cdot \frac{dw}{dt}$$

$$\frac{2x^2}{y^4} \cdot 3t^2 + \left(-4 \frac{(x^2 - w)}{y^5}\right) - 2\sin 2t$$

$$- \frac{1}{y^4} \cdot 4$$

$$= \frac{6x^2 t^2}{y^4} + \frac{8(x^2 - w)\sin 2t}{y^5} - \frac{4w}{y^4}$$

(b) $z = x^2 y^4 - 2y \quad , \quad y = \sin x^2$

$$\frac{dz}{dx} = ?$$

$$\frac{dz}{dy} = \frac{\partial}{\partial y}(x^2y^4 - 2y)$$
$$= 4y^3x^2 - 2$$

$$\frac{dy}{dx} = \frac{d}{dx}\sin(x^2) = \cos x^2 \cdot 2x$$
$$= 2x\cos x^2$$

$$\frac{dz}{dx} = \frac{\partial g}{\partial y} \cdot \frac{dy}{dx}$$
$$= (4x^2y^3 - 2)(2x\cos x^2)$$
$$= 8x^3y^3\cos x^2 - 4x\cos x^2$$

(c)

$$x^2y^4 - 3 = \sin(xy)$$
$$\frac{d}{dx}(x^2y^4 - 3) = \frac{d}{dx}\sin xy)$$

$$2xy^4 + x^2 4y^3\frac{dy}{dx} = y\cos(xy)$$
$$2xy^4 + 4x^2y^3\frac{dy}{dx} = 4\cos(xy)$$
$$4x^2y^3\frac{dy}{dx} = y\cos(xy) - 2xy^4$$
$$\frac{dy}{dx} = y\left(\frac{\cos(xy) - 2xy^3}{4x^2y^3}\right)$$

$$\frac{dy}{dx} = \frac{\left[\cos(xy) - 2xy^3\right]}{2x^2y^2}$$