

S_1 S_2

$$r_{x1}, r_{x2} \xleftarrow{R} \mathbb{Z}_N$$

$$[[x_1'^m]] = [[x_1']] \cdot [[r_{x1}]], [[x_2'^m]] = [[x_2']] \cdot [[r_{x2}]]$$

$$[x_1'^m]_1 = \text{PartDec}_{sk_1}([x_1'^m]), [x_2'^m]_1 = \text{PartDec}_{sk_1}([x_2'^m])$$

$$[x_1'^m]_2 = \text{PartDec}_{sk_2}([x_1'^m]), [x_2'^m]_2 = \text{PartDec}_{sk_2}([x_2'^m])$$

$$x_1'^m = \text{FullDec}([x_1'^m]_1, [x_1'^m]_2), x_2'^m = \text{FullDec}([x_2'^m]_1, [x_2'^m]_2)$$

$$h = x_1'^m \cdot x_2'^m = (x_1' + r_{x1}) \cdot (x_2' + r_{x2}) = x_1' \cdot x_2' + \textcolor{red}{x_1' \cdot r_{x2} + r_{x1} \cdot x_2' + r_{x1} \cdot r_{x2}}$$

$$[[h]] = \text{Enc}_{pk}(h)$$

$$s1 = [[r_{x1} \cdot r_{x2}]]^{N-1}, s2 = [[x_1']]^{N-r_{x2}}, s3 = [[x_2']]^{N-r_{x1}}$$

$$[[h]].s1.s2.s3 = [[h - r_{x2} \cdot x_1' - r_{x1} \cdot x_2' - r_{x1} \cdot r_{x2}]] = [[x_1' \cdot x_2']]$$