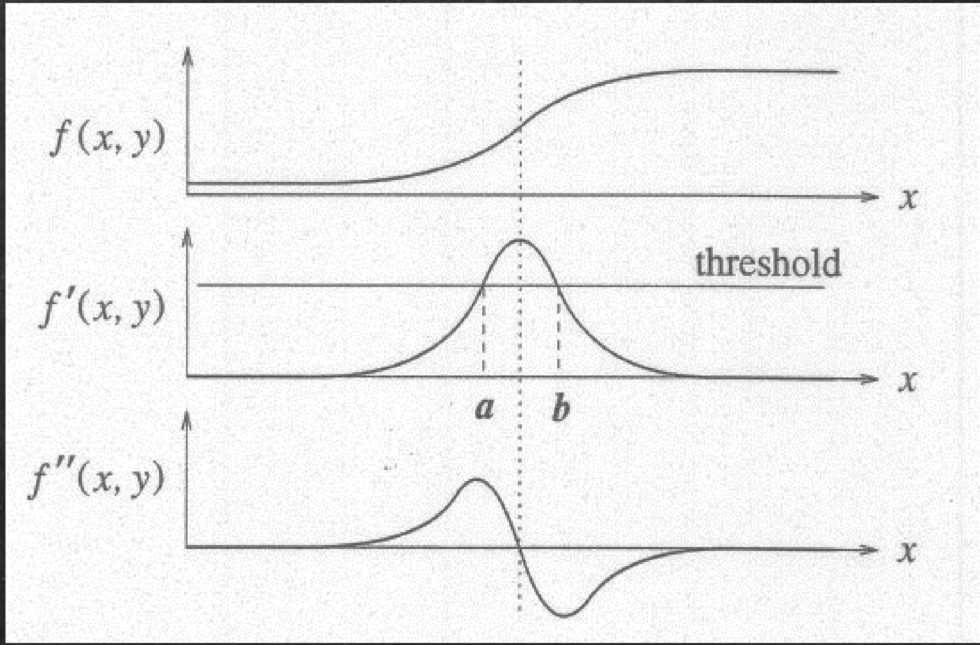
Lecture-07

Feature Extraction

Laplacian/Laplacian of Gaussian

- Second Derivative Operators



Laplacian/Laplacian of Gaussian

- The Laplacian $L(x, y)$ of an image with pixel intensity values $f(x, y)$ is given by:

$$\nabla^2 f = L(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- The gradient of f in both x and y direction are:

$$G_x \cong f[i, j+1] - f[i, j]$$

$$G_y \cong f[i, j] - f[i+1, j]$$

Laplacian/Laplacian of Gaussian

- The second derivatives along the x and y directions are

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial G_x}{\partial x} = \frac{\partial (f[i, j+1] - f[i, j])}{\partial x}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial (f[i, j+1])}{\partial x} - \frac{\partial (f[i, j])}{\partial x}$$

Laplacian/Laplacian of Gaussian

$$\frac{\partial^2 f}{\partial x^2} = f[i, j+2] - f[i, j+1] - (f[i, j+1] - f[i, j])$$

$$\frac{\partial^2 f}{\partial x^2} = f[i, j+2] - 2f[i, j+1] + f[i, j]$$

- However, this approximation is centered about the pixel $[i, j + 1]$. Therefore, by replacing j with $j - 1$, we obtain

$$\frac{\partial^2 f}{\partial x^2} = f[i, j+1] - 2f[i, j] + f[i, j-1]$$

Laplacian/Laplacian of Gaussian

- which is the desired approximation to the second partial derivative centered about $[i, j]$. Similarly,

$$\frac{\partial^2 f}{\partial y^2} = f[i+1, j] - 2f[i, j] + f[i-1, j]$$

- By combining these two equations into a single operator, the following mask can be used to approximate the Laplacian:

$$\nabla^2 f = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Laplacian/Laplacian of Gaussian

- Using one of these kernels, the Laplacian can be calculated using standard convolution methods.

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

LoG (`Laplacian of Gaussian')

- The 2-D LoG function centered on zero and with Gaussian standard deviation σ has the form:

$$LoG(x, y) = -\frac{1}{\pi\sigma^4}\left[1 - \frac{x^2 + y^2}{2\sigma^2}\right]e^{-\frac{x^2+y^2}{2\sigma^2}}$$

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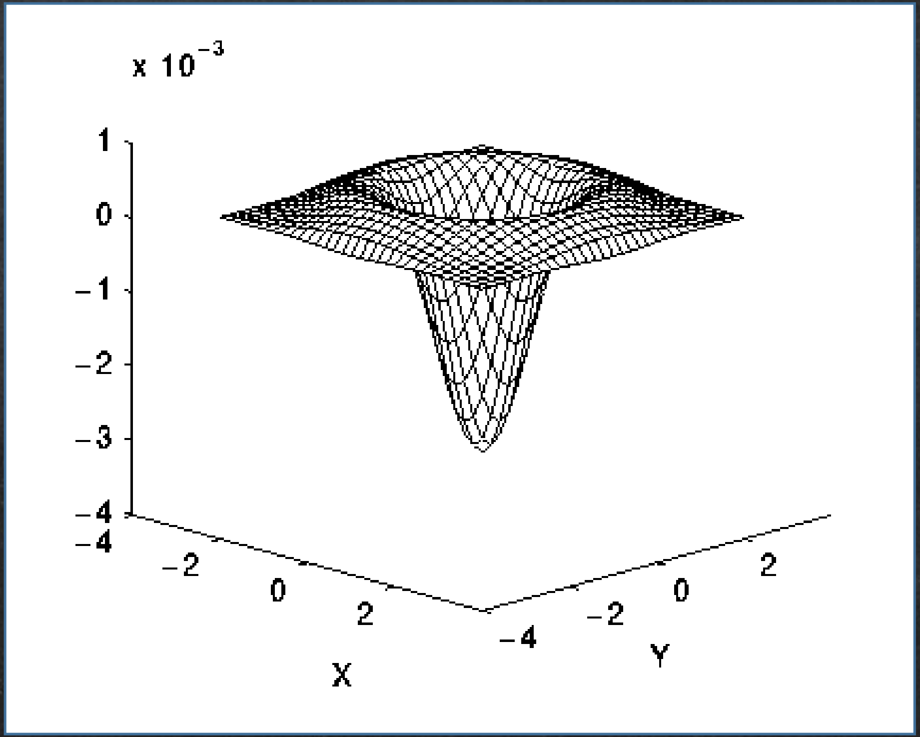

LoG (`Laplacian of Gaussian')

- A discrete kernel that approximates this function (for a Gaussian $\sigma = 1.4$)

0	1	1	2	2	2	1	1	0
1	2	4	5	5	5	4	2	1
1	4	5	3	0	3	5	4	1
2	5	3	-12	-24	-12	3	5	2
2	5	0	-24	-40	-24	0	5	2
2	5	3	-12	-24	-12	3	5	2
1	4	5	3	0	3	5	4	1
1	2	4	5	5	5	4	2	1
0	1	1	2	2	2	1	1	0