Application Of Newton's Law Of Cooling

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Case Scenario:

Paul is a young Nigerian man that is a huge fan of soup. Every weekend he makes soup for the whole week and stores it in the refrigerator before he sleeps. His refrigerator can not hold anything with a temperature warmer than 20 C. Since Paul is very smart, he discovered that he could cool the pot of soup by using a sink full of water at a constant temperature of 5 C and constantly stirring. That would bring down the temperature of the soup to 60 degrees C in 10 minutes. Since Paul is very particular about his sleeping time, he would like to know how long will it take for the soup to be ready so he can put it in the fridge and sleep on time.

Introduction:

This is where the concept of Newton’s Law of cooling comes in. In the 17th Century, Isaac Newton studied the nature if cooling and found out that if there is a less than 10 degrees difference between two objects the rate of heat loss is proportional to the temperature difference. He applied this principle to estimate the temperature of a red-hot iron ball by observing the time which it took to cool from a red heat to a known temperature, and comparing this with the time taken to cool through a known range at ordinary temperature. According to this law, if the excess of temperature of a body above its surroundings is observed at an equal interval of time, the observed value will form a geometrical progression with a common ratio. We will look at the limitation of this law as we go further.

From the above case, we can use the concept of differential equations (**Newton's law of cooling**) to help Paul solve his problem. Differential equations are used to describe physical phenomena around us. Different case scenarios like population growth, the decay of a bacterial or spread of a virus can be solved using differential equations.

In the end, we get a functional equation which is a solution to the differential equations instead of the actual value. We can apply the functional equation as many times as possible to obtain different levels of differential equations. The main goal in solving a differential equation is to find the function so that we can understand the behaviour of a specific input at all times instead of just one output value.

We can say a function is a solution to a differential equation if, we plug it's derivative into the equation we get the equation is satisfied.

Note: The solution to a differential equation is always a function. This is not like a solution to an algebraic equation which is a number or a set of numbers.
An example of a differential equation is $dy/dx = f(x)$. This is the same as $f'(x) = f(x)$

More Important Note: Paul's case is just one of a million case scenarios. But Newton's law of cooling can be used whenever we want to calculate how long it will take for a body to cool down from a specific temperature. To do that we have built an application that will work for all case scenarios **(with the same material as paul i.e same fluid and same container)** where the temperature of the surroundings and the temperature of the body is different. Everyday life applications include calculation of cooling time, for example, tea, soup, hot water and so on.

Limitation and assumption of Newton's Law of Cooling:

You can’t calculate the cooling time if you only know initial and ambient temperatures. You will also have to provide a solution (temperature) at some selected moment of time.

- Radiation effect is always there together with the convection so that the response of temperature with time is not linear. Practically, we can’t achieve the ideal case due to the radiation effect.
- One of the assumptions of Newton's law of cooling is that this law holds true only if the temperature of the surroundings remains constant throughout the cooling if the body. In real life, this is rarely true because the surrounding temperature sometimes changes even by a small amount. This will lead to us getting values that do not necessarily agree with the curve so we will fix this by finding the curve of best fit.
- It is valid for small temperature only and not for the bodies at large temperature.

Going back to Paul’s problem.

Newton's law of cooling states that the rate of heat loss of a body is directly proportional to the difference in temperatures between the body and its surroundings which makes it perfect for this case scenario.

Let

$T(t)$ = Temperature of the soup at time t (in min).

$T(0) = T_o$ = Initial Temperature of the soup =100 deg.

T_a = Surrounding temperature = 5 deg.

So, The rate of change of the temperature of the soup dT/dt is proportional to the difference between the temperature of the soup T and the surrounding (ambient) temperature Ta which means:

$$dT/dt \text{ is proportional to } T - Ta$$

The previous equation can be translated to:

$$dT/dt = -k\,(T - Ta)$$

Note:

1. K is a positive constant that represents the heat transfer coefficient which is a quantitative characteristic of convective heat transfer between a fluid medium (soup) and the surface flowed over by the fluid (surface of the soup container). K will change depending on the kind of object being cooled and the container materials. This means that when we use a plastic container and a metallic container the value of K will be different. This will also depend on the fluid or the object that is being cooled.

$$k = -(dT/dt)\,/\,T - Ta$$

2. The negative sign (-) at the beginning of the equation means that the soup is **cooling down** which means that the change in the temperature of the soup is decreasing because we guarantee that it is higher than the temperature of the surrounding at the beginning of the experiment.

Objective:

Finding an equation of $T(t)$ to get the temperature of the soup at any time during the experiment knowing that k and Ta are constants.

Solution:

Using the separation technique to solve the Differential equation:

$$\int dT/(T - Ta) = -\int kdt$$

By integrating both sides:

$$ln(T - Ta) = -kt + c$$

Applying e for both sides:

$$T - Ta = e^{-kt}.c$$

$$T(t) = e^{-kt}c + Ta$$

Note:

1. C is the temperate constant of the difference between the initial temperature of the body and the temperature of the surrounding. C will change if the temperature of the surrounding changes or the initial temperature of the object is different.

Using the initial case to get the value of c and k:

At $t = 0$; $T(t) = 100$

$$T(t) = e^{0}c + 5$$

$$100 - 5 = c$$

$$c = 95$$

We also know that after 10 minutes, the soup cools to 60 degrees; $t = 10$ and $T(t) = 60$

$$60 = 95e^{-10k} + 5$$

$$55/95 = e^{-10k}$$

$$k = 0.054 \text{ per minute}$$

Note: Theoretically, people will calculate K with just two values, like what we did above but in real-world, scientists do that with multiple values to make sure that the value of K is not biased to just those two temperature values.

To do this we will go back to $T(t) = e^{-kt}c + Ta$ and introduce natural logarithm (ln) both sides. Then making K the subject. During the plotting of the graph, we will always find some points that do not necessarily agree with our assumption and sometimes the value of K. So we will correct this by drawing the graph that fits the best so taking into consideration the points outside and inside the curve.

Final Equation:

$$T(t) = 95e^{-0.054t} + 5$$

Paul is interested in the amount of time it will take him to cool the soup to 20 degrees C so he can put the soup in the refrigerator and sleep as he is very particular about his sleeping time. From the equation above $T(t) = 20\ degrees\ C$, since he will need to wait until it reaches 20 degrees C.

$$20 = 95e^{-0.054t} + 5$$

$$15/95 = e^{-0.054t}$$

When you solve, $t = 34.2\ minutes$

The following graph shows the cooling of the soup over time:

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Enter the intial temp of your object: 100
Enter the temp of the surrounding: 5
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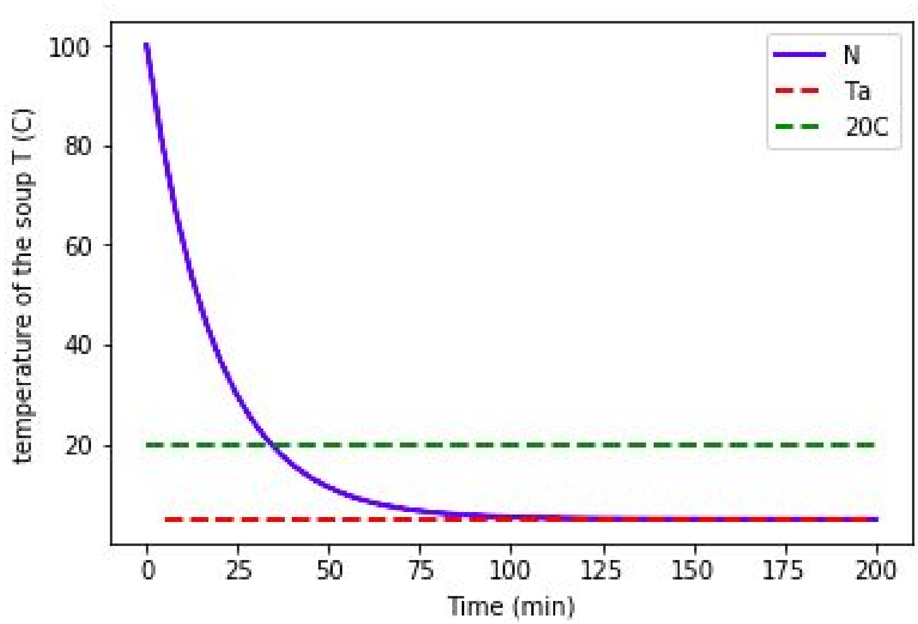

Code

As the graph shows the temperature of the soup started at 100 C and then it decreased quickly during the first 30 minutes reaching almost 24 C, then it started to decrease slowly until it reached the temperature of the surrounding after 100 minutes.

So it will take Paul 34.2 minutes to cool the soup to 20 degrees every time he boils soup if the temperature of the water used is 5 degrees and if he also boiled the soup to 100 degrees C.

Conclusion:

Now, Paul and any other person can use the above application to know how long it will take for a certain body to cool down when exposed to the surrounding temperature. This will help us when we have a different temperature of the body and the different temperature of the surrounding and still want to know how the temperature will drop.

Source:

slee1111 (2010). Newton’s Law Of Cooling. [online] Slideshare.net. Available at: https://www.slideshare.net/slee1111/newtons-law-of-cooling-4385942 [Accessed 29 Nov. 2019].

History and applications - Newton's law of cooling

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