

# Recommendation Systems

Deepak Venugopal

4745

Some Slides Courtesy Vibhav Gogate,  
Padhraic Smyth

# Recommendation Systems

- Some references
- Chapter by Melville and Sindhvani , encyclopedia of ML, 2010
- Chapter on recommendation systems, Mining massive datasets, Rajaraman, Leskovec and Ullman

# Recommendation Systems

- Present a user with items he/she may like
- Useful in several practical cases
  - Movies
  - Products
  - Services
  - Etc.
- [https://en.wikipedia.org/wiki/Collaborative\\_filtering#/media/File:Collaborative\\_filtering.gif](https://en.wikipedia.org/wiki/Collaborative_filtering#/media/File:Collaborative_filtering.gif)

# Ratings data

- Data with users  $N$  and  $M$  items
- Represent as a  $N \times M$  matrix
- Entries of the matrix
  - Ratings
  - Binary value (did user like/dislike, watch/did-not-watch, listen/did-not-listen,...)

# Examples

- Amazon
- Ebay
- Netflix
- Youtube
- Pandora
- Goodreads
- ...

# Netflix Challenge

- Netflix held a 1Million\$ challenge to improve their recommendation system
- Privacy issues??
- Currently the Netflix dataset has been taken down

Users

| Item-1 | Item-2 | Item-3 | Item-4 | Item-5 |
|--------|--------|--------|--------|--------|
| 2      | 2      | 1      | 3      |        |
| 1      |        | 1      | 1      |        |
| 2      |        | 1      | 3      | 3      |
| 2      |        | 1      | 2      | 1      |
| ....   |        |        |        |        |

Training data

| Item-1 | Item-2 | Item-3 | Item-4 | Item-5 |
|--------|--------|--------|--------|--------|
| 1      | ?      | 2      | 1      | ?      |

Testing data

# Evaluation Metrics

- Mean squared error across all ratings for all users
  - $r_{ui}$ : Rating of user  $u$  for item  $i$
  - $p_{ui}$ : Predicted rating of user  $u$  for item  $i$
  - $R$  set of ratings in the test set
- $$MSE = \frac{1}{|R| \sum_{u,i \in R} (r_{ui} - p_{ui})^2}$$
- Assumes that all ratings are equally important



# Evaluation Metrics

- Precision based metrics
  - Rank the predictions
  - Measure precision in the top K places of the rankings
  - More emphasis on getting important predictions right
- Real ideal evaluation: Would the user be persuaded to buy an item X that they would otherwise not buy?
  - Harder to do

# Content based recommendations

- Recommend an item to a user if the user previously liked similar item
- Ignore the preferences of other users
- How to measure similarity?
  - Movies with certain actors, genre, director, etc.
  - Music genre, year, etc.

# Content based recommendation

- Represent each item as a feature vector
  - $x_1, \dots, x_M$
- Map a user into the same feature space
  - $x_u$
- For each item  $i$ , compute  $Sim(x_u, x_i)$

# Example:Text Content

- Items=documents
- Features=words or phrases
  - Represent a document as “bag of words”
  - Count the frequency of words in the document
- Use TF-IDF for vectorizing a document from the frequency counts

# Example:Text

- $TF_{ij}$ : Frequency of term (feature)  $j$  in document (item)  $i$
- $n_i$ : Number of documents that mention  $i$
- $N$ : total number of documents
- $IDF_i = \log \left( \frac{N}{n_i} \right)$
- TF-IDF  $w_{ij} = TF_{ij} IDF_i$

# Example:Text

- Feature vector for items
  - $x_i$  is the vector of words represented by the TF-IDF scores
- Feature vector for user
  - Average TF-IDF for items rated by the user weighted by the user rating
- Prediction
  - Cosine similarity  $\cos(x_u, x_i) = x_u \cdot x_i / (||x_u|| \cdot ||x_i||)$

# Tf-idf

Document 1

| TERM   | TERM-COUNT |
|--------|------------|
| This   | 1          |
| Is     | 1          |
| a      | 2          |
| Sample | 1          |

Document 2

| TERM    | TERM-COUNT |
|---------|------------|
| This    | 1          |
| Is      | 1          |
| a       | 2          |
| example | 3          |

# Tf-idf

Document 1

| TERM   | TF  |
|--------|-----|
| This   | 1/5 |
| Is     | 1/5 |
| a      | 2/5 |
| Sample | 1/5 |

Document 2

| TERM    | TERM-COUNT |
|---------|------------|
| This    | 1/7        |
| Is      | 1/7        |
| a       | 2/7        |
| example | 3/7        |



# Tf-idf

Document 1

| TERM   | idf                 |
|--------|---------------------|
| This   | $\text{Log}(2/2)=0$ |
| Is     | 0                   |
| a      | 0                   |
| Sample | $\text{Log}(2/1)$   |

Document 2

| TERM    | idf               |
|---------|-------------------|
| This    | 0                 |
| Is      | 0                 |
| a       | 0                 |
| example | $\text{Log}(2/1)$ |

# Content based recommendation

- Pros
  - Easy to implement
  - Only need data for a user
  - New items with no ratings can be included as long as we can compute its feature vector
- Cons
  - Items must have similar features which is not always the case
  - Finding features might be hard
  - How do we perform predictions for new users?

# User-User Collaborative Filtering

- Make recommendations for a given user based on similar users
- Let  $K_a$  be the nearest-neighbors of  $a$
- Generate predictions for user  $a$  based on a weighted combination of  $K_a$ 's ratings

# User-User Collaborative Filtering

- Define a similarity weight between user  $a$  and user  $u$  as correlation coefficient (Pearson's coefficient)

$$w_{a,u} = \frac{\sum_{i \in I} (r_{a,i} - \bar{r}_a)(r_{u,i} - \bar{r}_u)}{\sqrt{\sum_{i \in I} (r_{a,i} - \bar{r}_a)^2 \sum_{i \in I} (r_{u,i} - \bar{r}_u)^2}}$$

- $I$  is the set of items rated by both  $a$  and  $u$ .  $r_{ui}$  is the rating given by user  $u$  to  $i$ .  $\bar{r}_u$  is the mean rating of  $u$  across items in  $I$ .

# User-User Collaborative Filtering

- Prediction of a's rating to item i
  - $p_{ai} = \bar{r}_a + \sum_{u \in K} (r_{ui} - \bar{r}_u) w_{au} / \sum_{u \in K} w_{au}$
  - $\bar{r}_u$  is the average rating for u
  - $w_{au}$  is the similarity weight between a and u
  - K is the set of nearest neighbors for a based on  $w_{au}$
- We are computing deltas for each user in a's neighborhood weighting each delta by similarity of the user with a

# User-User Collaborative Filtering

|   | P1 | P2 | P3 | P4 | P5 | P6 |      |
|---|----|----|----|----|----|----|------|
|   | 1  |    | 3  | ?  | 1  |    |      |
|   | 1  |    | 3  | 2  | 4  |    |      |
| 0 | 1  | 2  | 3  | 3  | 5  | 4  | 0.18 |

$$p_{13} = 1.6 + (2 - 2.5) * \frac{0.18}{0.18} = 1.1$$

# User-User Algorithm

- Steps
  - For a user  $a$ , find the  $K$  nearest-neighbors using  $W_{au}$
  - Predict  $a$ 's ratings for each item using  $K$
- Problems
  - Naïve complexity =  $O(NM)$ , for  $N$  items and  $M$  users
  - Amazon has 100 million customers and 10 million items
  - A user may have neighbors with very few co-rated items

# Heuristics

- Remove customers with too few purchases
- Randomly sample customers
- Remove rarely purchased events
- Cluster customers/items etc.



# Item based Collaborative Filtering

- Match a user's rated item to similar items
  - More scalable (Fewer items)

$$w_{i,j} = \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_i)(r_{u,j} - \bar{r}_j)}{\sqrt{\sum_{u \in U} (r_{u,i} - \bar{r}_i)^2} \sqrt{\sum_{u \in U} (r_{u,j} - \bar{r}_j)^2}}$$

- $U$ : users who have rated both items  $i$  and  $j$
- $\bar{r}_i$ : Average rating for item  $i$  across all users in  $U$
- Can compute the above weights offline

# Item based Collaborative Filtering

- Predict the rating for item  $i$  by user  $a$ 
  - $p_{ai} = \sum_{j \in K} r_{aj} w_{ij} / \sum_{j \in K} |w_{ij}|$
  - $K$  is the neighborhood set most similar to item  $i$  among all items rated by user  $a$

## Toy Example of Item-Item Collaborative Filtering

users

movies

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|
| 1 | 1 |   | 3 |   |   | 5 |   |   | 5 |    | 4  |    |
| 2 |   |   | 5 | 4 |   |   | 4 |   |   | 2  | 1  | 3  |
| 3 | 2 | 4 |   | 1 | 2 |   | 3 |   | 4 | 3  | 5  |    |
| 4 |   | 2 | 4 |   | 5 |   |   | 4 |   |    | 2  |    |
| 5 |   |   | 4 | 3 | 4 | 2 |   |   |   |    | 2  | 5  |
| 6 | 1 |   | 3 |   | 3 |   |   | 2 |   |    | 4  |    |

- unknown rating- rating between 1 to 5

Figures and example courtesy of Jure Leskovec, Stanford

## Toy Example of Item-Item Collaborative Filtering

users

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|
| 1 | 1 |   | 3 |   | ? | 5 |   |   | 5 |    | 4  |    |
| 2 |   |   | 5 | 4 |   |   | 4 |   |   | 2  | 1  | 3  |
| 3 | 2 | 4 |   | 1 | 2 |   | 3 |   | 4 | 3  | 5  |    |
| 4 |   | 2 | 4 |   | 5 |   |   | 4 |   |    | 2  |    |
| 5 |   |   | 4 | 3 | 4 | 2 |   |   |   |    | 2  | 5  |
| 6 | 1 |   | 3 |   | 3 |   |   | 2 |   |    | 4  |    |

movies



- estimate rating of movie **1** by user **5**

Figures and example courtesy of Jure Leskovec, Stanford

## Toy Example of Item-Item Collaborative Filtering

|        |          | users |   |   |   |   |   |   |   |   |    |    |    | $w_{1,j}$   |
|--------|----------|-------|---|---|---|---|---|---|---|---|----|----|----|-------------|
|        |          | 1     | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |             |
| movies | 1        | 1     |   | 3 |   | ? | 5 |   |   | 5 |    | 4  |    | 1.00        |
|        | 2        |       |   |   |   |   |   |   |   |   |    |    | 3  | -0.18       |
|        | 3        |       |   |   |   |   |   |   |   |   |    |    |    | <u>0.41</u> |
|        | 4        |       |   |   |   |   |   |   |   |   |    |    |    | -0.10       |
|        | 5        |       |   | 4 | 3 | 4 | 2 |   |   |   |    | 2  | 5  | -0.31       |
|        | <u>6</u> | 1     |   | 3 |   | 3 |   |   | 2 |   |    | 4  |    | <u>0.59</u> |

Using Pearson correlation as similarity:

1) Subtract mean rating  $r_i$  from each movie  $i$

$$r_1 = (1+3+5+5+4)/5 = 3.6$$

row 1:  $[-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]$

2) Compute correlation between rows

**Neighbor selection:**

Identify movies similar to  
movie 1, rated by user 5

Figures and example courtesy of Jure Leskovec, Stanford

## Toy Example of Item-Item Collaborative Filtering

|        |          | users |   |   |   |   |   |   |   |   |    |    |    | $w_{1,j}$   |
|--------|----------|-------|---|---|---|---|---|---|---|---|----|----|----|-------------|
|        |          | 1     | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |             |
| movies | 1        | 1     |   | 3 |   | ? | 5 |   |   | 5 |    | 4  |    | 1.00        |
|        | 2        |       |   | 5 | 4 |   |   | 4 |   |   | 2  | 1  | 3  | -0.18       |
|        | <u>3</u> | 2     | 4 |   | 1 | 2 |   | 3 |   | 4 | 3  | 5  |    | <u>0.41</u> |
|        | 4        |       | 2 | 4 |   | 5 |   |   | 4 |   |    | 2  |    | -0.10       |
|        | 5        |       |   | 4 | 3 | 4 | 2 |   |   |   |    | 2  | 5  | -0.31       |
|        | <u>6</u> | 1     |   | 3 |   | 3 |   |   | 2 |   |    | 4  |    | <u>0.59</u> |

Find the 2 nearest neighbors, with similarity weights

$w_{13}=0.41$ ,  $w_{16}=0.59$

Figures and example courtesy of Jure Leskovec, Stanford

## Toy Example of Item-Item Collaborative Filtering

|        |          | users |   |   |   |     |   |   |   |   |    |    |    |
|--------|----------|-------|---|---|---|-----|---|---|---|---|----|----|----|
|        |          | 1     | 2 | 3 | 4 | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| movies | 1        | 1     |   | 3 |   | 2.6 | 5 |   |   | 5 |    | 4  |    |
|        | 2        |       |   | 5 | 4 |     |   | 4 |   |   | 2  | 1  | 3  |
|        | <u>3</u> | 2     | 4 |   | 1 | 2   |   | 3 |   | 4 | 3  | 5  |    |
|        | 4        |       | 2 | 4 |   | 5   |   |   | 4 |   |    | 2  |    |
|        | 5        |       |   | 4 | 3 | 4   | 2 |   |   |   |    | 2  | 5  |
|        | <u>6</u> | 1     |   | 3 |   | 3   |   |   | 2 |   |    | 4  |    |

Predict by taking weighted average:

$$P_{1,5} = (0.41 * 2 + 0.59 * 3) / (0.41 + 0.59) = 2.6$$

$$p_{a,i} = \frac{\sum_{j \in K} r_{a,j} w_{i,j}}{\sum_{j \in K} |w_{i,j}|}$$

Figures and example courtesy of Jure Leskovec, Stanford

# User-User Vs Item

- Item-Item appears to be more stable
- Item dimensionality is smaller. More data per item than per user
- Algorithmically Item-Item can be implemented faster than User-User (Linden,Smith and York, Amazon Recommender System)



# Pros/cons of collaborative filtering

- Pros
  - Works for any kind of item
  - No feature engineering since we don't need features (in content based we needed features)
- Cons
  - Cannot recommend items that have not been rated
  - Need to have users in the system (“cold start”: what to do when there are no users as yet)
  - Popularity bias

# Hybrid methods

- Implement both content based recommendation and collaborative filtering
- Combine the recommender results