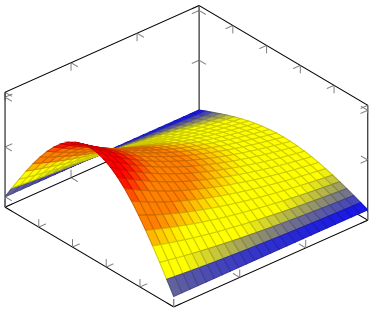
# Project 1: Solving the heat equation

The distribution of heat in a thin metal rod can be described by the "one-dimensional time dependent *heat equation* initial value problem".

$$(1)\qquad \begin{cases} \dfrac{\partial u}{\partial t}(x,t) - c\,\dfrac{\partial^2 u}{\partial x^2}(x,t) = 0\,, \\ u(0,t) = u(L,t) = 0 \qquad \text{for all } \; t \geq 0\,, \\ u(x,0) = f(x) \qquad \text{for all } \; x \in [0;L]\,. \end{cases}$$



where $u(x,t)$ is the temperature at the point $x$ in the rod at the time $t$, the real number $c > 0$ is a material constant, $L$ is the length of the rod and $f$ is the initial temperature (distribution); note that all constants and variables are written without dimension.

The goal of this project is to implement a general algorithm for numerically solving (1) and to visualize the solution for a specific example.

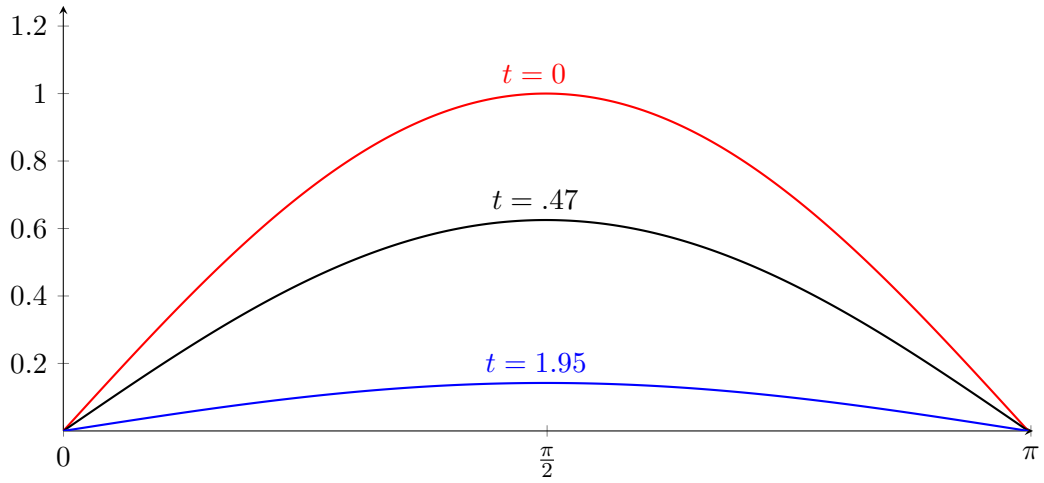



Figure 1: Temperature of a rod with length $L = \pi$, material parameter $c = 1$ and initial temperature distribution $f(x) = \sin(x)$ at the times $t = 0$, $t = 0.47$ and $t = 1.95$.

## Task 1: Computing the grid and the approximate solution

Write a Matlab function which takes the input arguments

- $f$ (the initial temperature),
- $M$ (the number of grid points in $x$-direction),
- $N$ (the number of grid points in $t$-direction),
- $c$ (the material parameter),
- $L$ (the length of the rod),
- $T$ (the endpoint of the time interval)

and returns the (approximate) solution to problem (1) for the given initial temperature $f$ on the grid given by $M$, $N$, $L$ and $T$, see Fig. 2.

Your solution must be a file called `heatsolution.m` of the following form:

```
function [ U ] = heatsolution( f, M, N, c, L, T )
    % your code
    % ...
end
```

The return value `U` must be in the form of a matrix; more specifically: `U(m,n)` must be the value

$$\texttt{U(m,n)} = u((\texttt{m}-1)\cdot\Delta x, (\texttt{n}-1)\cdot\Delta t)$$

of the solution $u$ at the $(\texttt{m},\texttt{n})$-gridpoint, see Fig. 2.

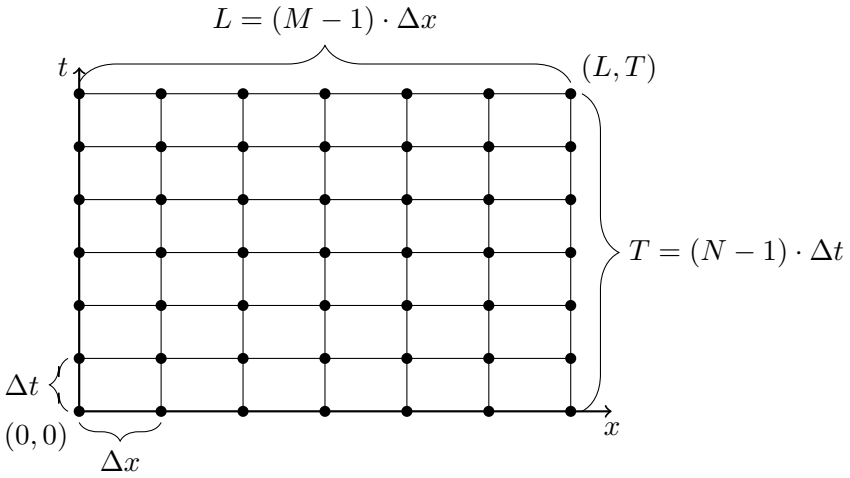



Figure 2: The grid for task 1.

## Task 2: Visualization

Find (an approximation of) the heat distribution function $u$ of a metal rod for the following parameters:

i)
- $M = 30$
- $N = 1000$
- $c = 1$
- $L = \pi$
- $T = 0.1$
- the initial temperature $f$ with $f(x) = \sin(x)$

ii)
- $M = 30$
- $N = 10000$
- $c = 1$
- $L = 1$
- $T = 0.42$
- $f(x) = x \cdot (x - 0.7) \cdot (x - 1)^2$

iii)
- $M = 20$
- $N = 10000$
- $c = 1$
- $L = 1$
- $T = 7$
- $f(x) = x \cdot (x - 0.7) \cdot (x - 1)^2$

In each of these cases, plot the (three-dimensional) graph of $u$ and the (two-dimensional) graphs of the spatial temperature distribution at times $t = 0,\ t = 0.1$ and $t = T$.

## Hints

- The output parameter `U` of your solution to Task 1 must be a matrix of size $M \times N$.
- In Task 1, the initial temperature $f$ is given to the solution function in the form of a *function handle*.
- In particular, the values `U(1,0)`, ..., `U(M,0)` must satisfy

  $$\texttt{U(m,0)} = f((\texttt{m} - 1) \cdot \Delta x)$$

  due to the initial condition.
- Of course, your solution for Task 2 can (and should) use the results obtained in Task 1.

## Solution upload

*Everybody* must upload the following

- the solution file `heatsolution.m`;
- a *presentation* of your work (containing, in particular, the results of Task 2), preferably in the `.pdf` file format, which you will use for your talk at the end of the semester;
- a list of your group colleagues.

If you prepare a presention video, it is sufficient if one group member uploads the video file. The other group members should include a comment in their solution about the video and who is responsible for the video upload.

**Groups:** 3–5 students. Please register your group as a solution to the Moodle assignmet "Assignment: Lab 1 – list of group colleagues (registration of groups)". If your group changes, please upload a new solution to the same assignment.

**Deadline:** Upload your project (.m files and pdf/presentation) by **July 19** to the Moodle assignment "Assignment: Lab 2: MATLAB code and presentation".

**Presentation:** Each group has to give a presentation (max. 10 minutes) on July 22 (in the last lecture).

**Alternative presentation:** Each group may choose to prepare a presentation video (max. 10 min) instead of giving a live presentation.