Using PCA with the California Housing Dataset

Objective

The aim is to apply Principal Component Analysis (PCA) to the California Housing dataset, with the goal of exploring structures and reducing dimensionality. This analysis is to identify the principal components that best capture the variability in the housing data, which can provide insights into the most influential factors affecting housing values across California.

Starting up

So let's first make sure that the required packages are installed. If they are not already installed, they will be installed.

```
if (!require("ggplot2")) install.packages("ggplot2")
```

```
Loading required package: ggplot2
```

```
library(ggplot2)
theme_set(theme_light()) # Set the default ggplot theme to the light theme
```

Loading the dataset

```
df1 = read.csv ("https://raw.githubusercontent.com/ahmedmoustafa/datasets/main/housing/hou
dim(df1)
```

```
[1] 20640    10
```

```
head(df1)
```

longitude	latitude	housing_median_age	total_rooms	total_bedrooms	population	households	median_income	median_house_value	ocean_proximity
-122.23	37.88	41	880	129	322	126	8.3252	452600	NEAR BAY
-122.22	37.86	21	7099	1106	2401	1138	8.3014	358500	NEAR BAY
-122.24	37.85	52	1467	190	496	177	7.2574	352100	NEAR BAY
-122.25	37.85	52	1274	235	558	219	5.6431	341300	NEAR BAY
-122.25	37.85	52	1627	280	565	259	3.8462	342200	NEAR BAY
-122.25	37.85	52	919	213	413	193	4.0368	269700	NEAR BAY

Cleaning the dataset

To ensure the quality of our analysis, it is essential to address missing values within our dataset. So, we implement a user-defined function named `is_any_na()`. This function will detect the presence of missing values in each row of the dataset. Identifying rows with missing data is a crucial **preprocessing** step, as incomplete information can significantly skew PCA results and lead to inaccurate interpretations.

```
is_any_na = function (x) {
  return (any(is.na(x)))
}
```

We will now utilize our user-defined function `is_any_na()` to systematically **apply** across the dataset, identifying which rows contain missing values.

```
missing = apply(df1, 1, is_any_na)
head(missing)
```

```
[1] FALSE FALSE FALSE FALSE FALSE FALSE
```

The vector `missing` is a list of Boolean values, where `TRUE` entries correspond to rows in `df1` that contain at least one missing (i.e., `NA`) entry.

Note: If we pass a list of Boolean values to the `sum()` function, it will return the **number** of `TRUE` values. So the number of rows in `df1` with missing values:

```
sum(missing)
```

```
[1] 207
```

An overview of the rows with missing values

```
head(df1[missing, ])
```

	longitude	latitude	housing_median_age	total_rooms	total_bedrooms	population	households	median_income	median_house_value	ocean_proximity
291	-122.16	37.77	47	1256	NA	570	218	4.3750	161900	NEAR BAY
342	-122.17	37.75	38	992	NA	732	259	1.6196	85100	NEAR BAY
539	-122.28	37.78	29	5154	NA	3741	1273	2.5762	173400	NEAR BAY
564	-122.24	37.75	45	891	NA	384	146	4.9489	247100	NEAR BAY
697	-122.10	37.69	41	746	NA	387	161	3.9063	178400	NEAR BAY
739	-122.14	37.67	37	3342	NA	1635	557	4.7933	186900	NEAR BAY

Now let's extract the rows without any missingness by, basically, negating (with the `!` operator) the `missing` vector i.e., switching `TRUE` to `FALSE` and switching `FALSE` to `TRUE`:

```
df2 = df1[!missing, ]
dim(df2)
```

```
[1] 20433    10
```

Dataframe `df2` has 20433 without any missing value

It is almost ready for performing the PCA analysis; however, the input matrix to the `procomp()` function must be **numeric** and our dataset contains a categorical a variable/column/feature `ocean_proximity` with the following values:

```
table(df2$ocean_proximity)
```

<1H OCEAN	INLAND	ISLAND	NEAR BAY	NEAR OCEAN
9034	6496	5	2270	2628

To be able to proceed with the PCA analysis, we must to convert `ocean_proximity` to `numeric` but first we need to convert to `factor` with the appropriate `levels` the convert the `factor` to `numeric`:

```
df2$ocean_proximity = as.numeric(factor(df2$ocean_proximity,
                                        levels = c("ISLAND", # The closest to the ocean
                                                   "NEAR BAY",
                                                   "NEAR OCEAN",
                                                   "<1H OCEAN",
                                                   "INLAND" # The farthest from the ocean
                                                   )
                                        )
                                 )
head(df2)
```

longitude	latitude	housing_median_age	total_rooms	total_bedrooms	population	households	median_income	median_house_value	ocean_proximity
-122.23	37.88	41	880	129	322	126	8.3252	452600	2
-122.22	37.86	21	7099	1106	2401	1138	8.3014	358500	2
-122.24	37.85	52	1467	190	496	177	7.2574	352100	2
-122.25	37.85	52	1274	235	558	219	5.6431	341300	2
-122.25	37.85	52	1627	280	565	259	3.8462	342200	2
-122.25	37.85	52	919	213	413	193	4.0368	269700	2

Notice the converted `numeric` values under the `ocean_proximity` column.

Calculating the PCA

Since the `median_house_value` column is the target variable in the dataset, it should not be part of the input matrix to the `prcomp()` function to perform the PCA analysis. Therefore, we will exclude it by referencing its column number (column #9) with the negative sign.

```r
pca_result = prcomp(df2[, -9], scale. = TRUE)
summary(pca_result)
```

```
Importance of components:
                          PC1    PC2    PC3    PC4     PC5     PC6     PC7
Standard deviation     1.9778 1.3968 1.1163 1.0322 0.75136 0.38516 0.25365
Proportion of Variance 0.4346 0.2168 0.1385 0.1184 0.06273 0.01648 0.00715
Cumulative Proportion  0.4346 0.6514 0.7899 0.9083 0.97100 0.98748 0.99463
                           PC8     PC9
Standard deviation     0.18325 0.12138
Proportion of Variance 0.00373 0.00164
Cumulative Proportion  0.99836 1.00000
```

The above PCA summary shows that PC1 explains about 43.5% of the total variance in the dataset, followed by PC2 which explains about 21.7%, then PC3 which explains about 13.8%. And the first three PCs (PC1, PC2, and PC3) together (cumulatively) explain about 79% of the total variance in the dataset.

Visualizing the PCA

In this section, our goal is to explore whether there's any inherent structure, clustering, or grouping in the transformed dataset, as represented by the projections (or scores) of the housing data onto the principal components (PCs). A key step in this exploration is overlaying the original features, particularly the target variable `median_house_value`, onto the transformed PCA space. This approach provides a visual representation of how the target variable relates to the principal components. To achieve this, we will construct an augmented dataframe that combines the principal components with the feature of interest, `median_house_value`.

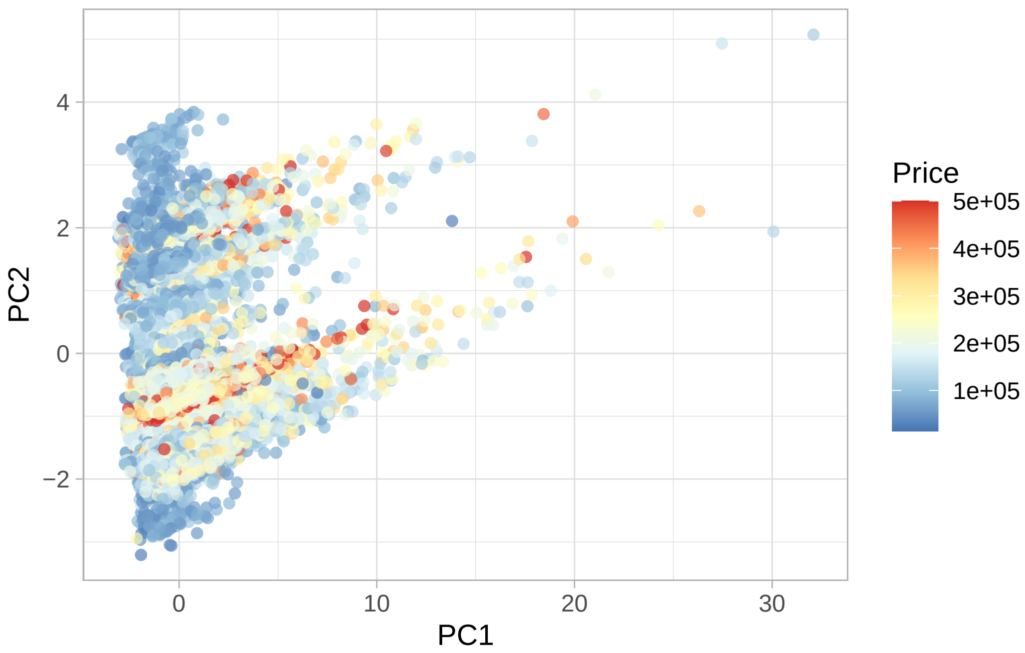
```r
pca_df = data.frame(Price = df2$median_house_value, pca_result$x)
head(pca_df)
```

Price	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9
452600	-2.185099	1.916306	-2.472818	1.5576456	-0.1891739	0.1032005	-0.1096529	0.3203179	0.0806733
358500	2.839508	2.470729	-2.221468	1.7377453	0.2792764	-0.4302054	0.1230018	0.1383711	-0.2798972
352100	-2.063872	2.014337	-2.644023	0.6088575	-0.6418161	0.0165362	0.0919113	0.2401319	0.0548983
341300	-2.011551	2.024811	-2.286231	-0.1151974	-0.3273401	-0.0131452	0.0728109	0.2043440	0.0420453
342200	-1.867736	2.050271	-1.901657	-0.8910065	-0.0088765	-0.1675948	0.2513776	0.0948725	-0.0155539
269700	-2.245589	1.994377	-1.906964	-0.7757001	0.0163533	-0.0897660	0.1179234	0.1472960	0.0249755

PC1 & PC2

Now, let's create a scatter plot to visualize the distribution of houses based on their projections onto the first two principal components, PC1 and PC2. In this plot, each house will be represented by a point, with the position determined by its scores on these principal components. To gain more insights, we will color-code these points based on the `median_house_value`

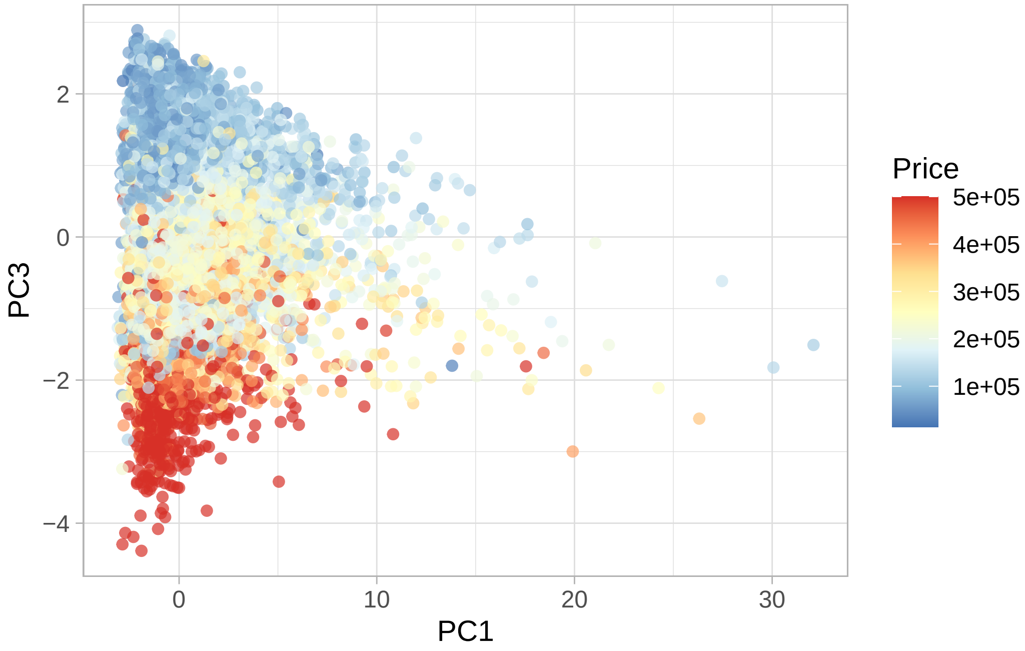
```r
ggplot(pca_df) +
  geom_point(aes(x = PC1, y = PC2, color = Price), alpha = 0.7) +
  scale_color_distiller(palette = "RdYlBu")
```



PC1 & PC3

Now that we have visualized and analyzed the distribution of house prices along PC1 and PC2, let’s proceed to explore another dimension. We will create a similar scatter plot, this time projecting the houses onto PC1 and PC3.

```
ggplot(pca_df) +
  geom_point(aes(x = PC1, y = PC3, color = Price), alpha = 0.7) +
  scale_color_distiller(palette = "RdYlBu")
```



In above visualization, we can observe a distribution where higher house prices (darker red points) are primarily concentrated at the lower end of PC3 while spread out along PC1. This suggests that PC3 captures aspects of the data that inversely relate to house prices, with lower scores on PC3 potentially associated with higher house values.

Loadings onto the Principal Components

To understand the influence of the original variables on the principal components, we will examine the loadings, which reflect how each variable contributes to, or weighs upon, each principal component. Our focus is particularly on PC3, as our visual analysis suggested it has a significant relationship with house prices. By analyzing the loadings for PC3, we can find out which features most strongly drive this component and, by extension, may have a more direct impact on the housing values in the dataset.

```
pca_result$rotation
```

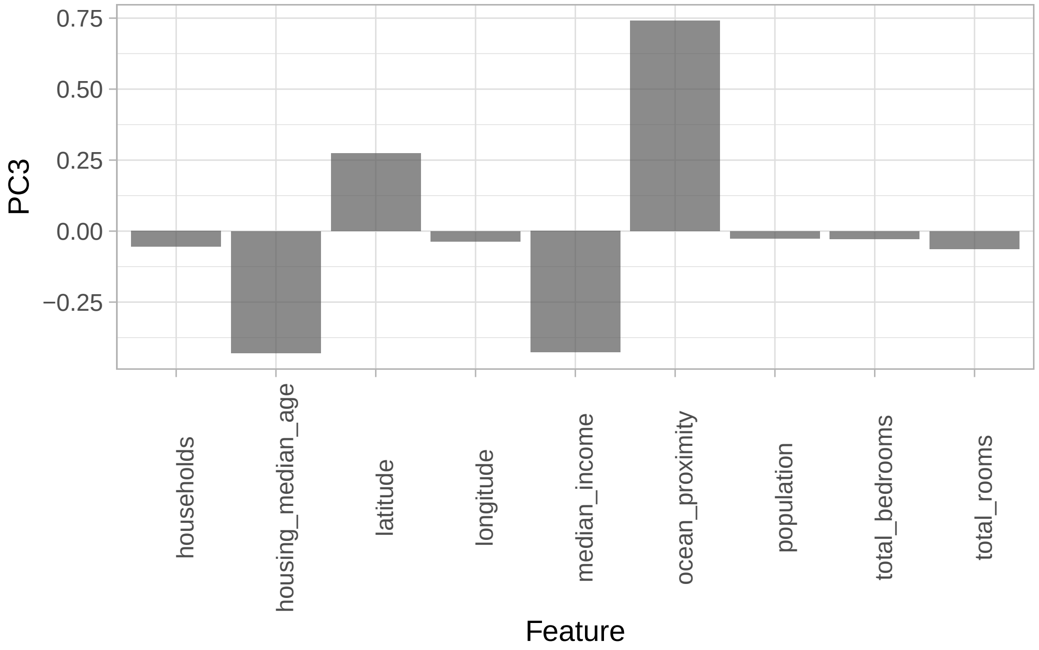
	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9
longitude	0.0805186	- 0.6982497	- 0.0361321	- 0.0502090	- 0.0049693	- 0.0954444	0.2012927	0.6669366	- 0.0876365

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9
latitude	-0.0748256	0.6630434	0.2740786	0.0602034	-0.1052334	-0.0442013	0.1655640	0.6560906	-0.0716720
housing_median_age	-0.2216216	0.0662264	-0.4296288	-0.5286401	-0.6933133	-0.0325366	0.0188148	0.0182212	0.0053527
total_rooms	0.4829436	0.0838299	-0.0623221	0.0757857	-0.1387150	-0.3133721	0.7376618	-0.2565323	-0.1507077
total_bedrooms	0.4894560	0.0706398	-0.0275453	-0.1275389	-0.0167275	-0.3794877	-0.2936749	0.1300465	0.7006926
population	0.4712478	0.0345713	-0.0256393	-0.1270844	-0.0687266	0.8500818	0.0996878	0.0764219	0.1295458
households	0.4904366	0.0769056	-0.0549193	-0.1267378	-0.0297202	-0.1406062	-0.5062419	0.0526256	-0.6743702
median_income	0.0430112	0.0041074	-0.4264794	0.8037141	-0.3659183	0.0569105	-0.1512771	0.0929070	0.0405064
ocean_proximity	0.0393394	-0.2220157	0.7408060	0.1192285	-0.5909259	0.0041157	-0.1105564	-0.1563529	0.0165306

```r
loadings = data.frame (Feature = row.names(pca_result$rotation), pca_result$rotation)
loadings
```

	Feature	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9
longitude	longitude	0.0805186	-0.6982497	-0.0361321	-0.1050209	-0.0004969	-0.0954444	0.2012927	0.6669366	-0.0876365
latitude	latitude	-0.0748256	0.6630434	0.2740786	0.0602034	-0.1052334	-0.0442013	0.1655640	0.6560906	-0.0716720
housing_median_age	housing_median_age	-0.2216216	0.0662264	-0.4296288	-0.5286401	-0.6933133	-0.0325366	0.0188148	0.0182212	0.0053527
total_rooms	total_rooms	0.4829436	0.0838299	-0.0623221	0.0757857	-0.1387150	-0.3133721	0.7376618	-0.2565323	-0.1507077
total_bedrooms	total_bedrooms	0.4894560	0.0706398	-0.0275453	-0.1275389	-0.0167275	-0.3794877	-0.2936749	0.1300465	0.7006926
population	population	0.4712478	0.0345713	-0.0256393	-0.1270844	-0.0687266	0.8500818	0.0996878	0.0764219	0.1295458
households	households	0.4904366	0.0769056	-0.0549193	-0.1267378	-0.0297202	-0.1406062	-0.5062419	0.0526256	-0.6743702
median_income	median_income	0.0430112	0.0041074	-0.4264794	0.8037141	-0.3659183	0.0569105	-0.1512771	0.0929070	0.0405064
ocean_proximity	ocean_proximity	0.0393394	-0.2220157	0.7408060	0.1192285	-0.5909259	0.0041157	-0.1105564	-0.1563529	0.0165306

```r
ggplot(loadings) +
  geom_bar(aes(x = Feature, y = PC3), stat = "identity", alpha = 0.7) +
  theme(axis.text.x = element_text(angle = 90))
```



We can see from the above plot, `ocean_proximity` holds the most positive loading among the features, suggests that `ocean_proximity` has a strong positive correlation with PC3, pointing to a significant impact of ocean proximity on the housing values as captured by this principal component. On the other hand, `housing_median_age` shows a substantial negative loading which means that areas with older houses tend to have lower scores on PC3, possibly reflecting housing market dynamics related to the age of properties.