Automata

Lecture 6

Minimization

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Minimization of FSM

The minimization of FSM means reducing the number of states from given FA thus we get the FSM with redundant states after minimizing the FSM.

While minimizing FSM we first find out which two states are equivalent we can represent those two states by one representative state.

Task 1:

Minimization the given FSM by finding equivalent states.

DFA Minimization

- **DFA Minimization using Equivalence Theorem**
- If X and Y are two states in a DFA, we can combine these two states into {X, Y} if they are not distinguishable.
- Two states are distinguishable, if there is at least one string S, such that one of δ (X, S) and δ (Y, S) is accepting and another is not accepting.
- Hence, a DFA is minimal **if and only if all the states are distinguishable.**

DFA Minimization algorithm

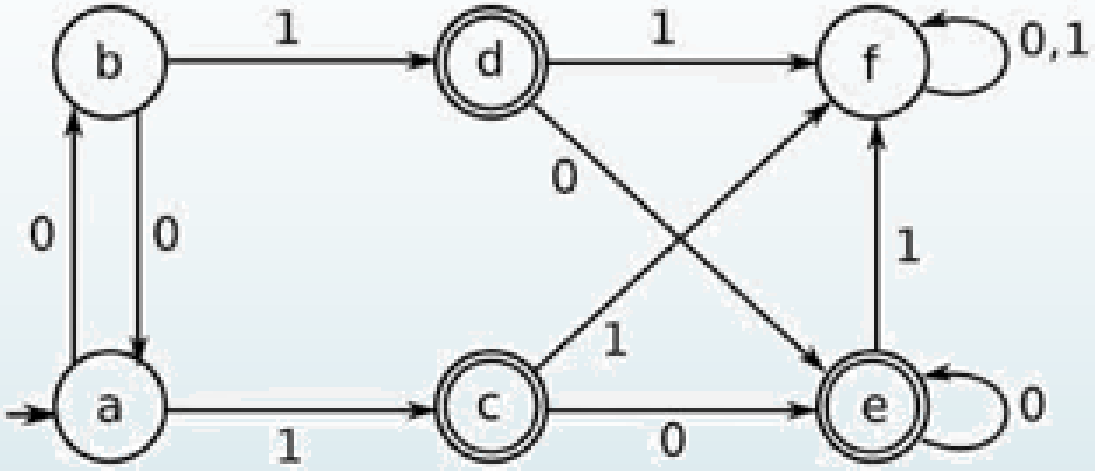
Step 1:
Partition all states into two groups, one containing **final** states whereas the other containing **non-final** states.

Step 2:
Find transitions of all states of each group for each input symbol.

Step 3:
Two states are said to be non-equivalent, if for an input symbol they have transitions to states of different groups.
Then separate the two states into two different subgroups.

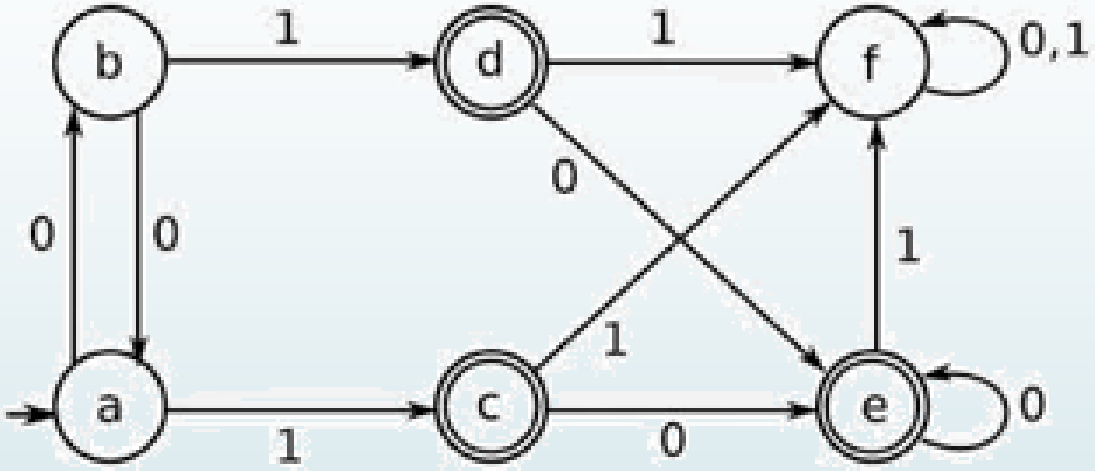
Step 4:
Repeat Step 2 and Step 3 until we get groups that no longer can be subdivided.

DFA Minimization
Example



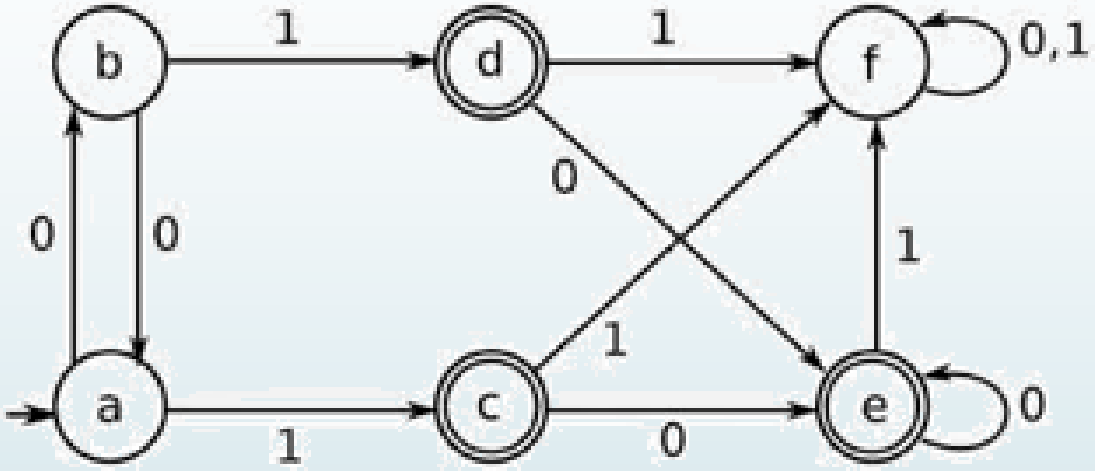
{a, b, f} {c, d, e}

Example
DFA Minimization



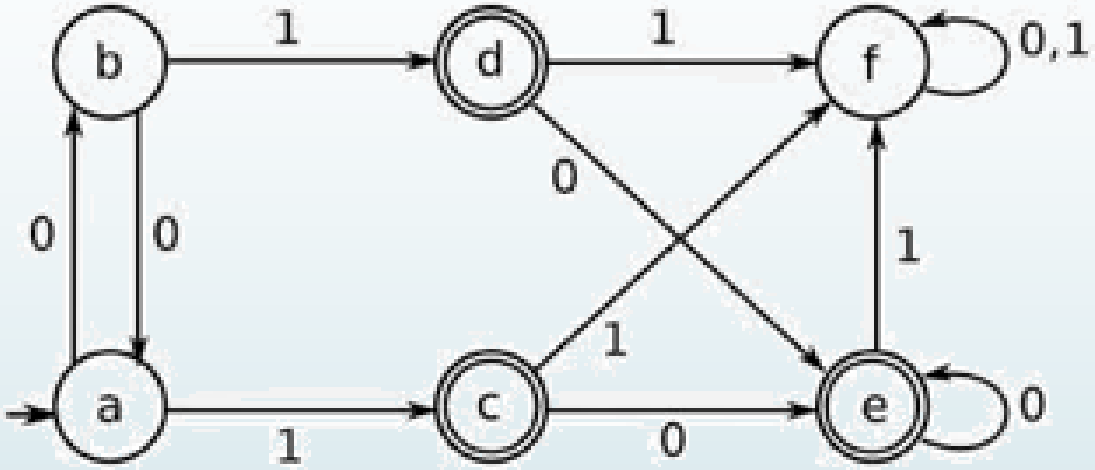
{a, b, f} {c, d, e}
{a, b} {f} {c, d, e}

Example
DFA Minimization

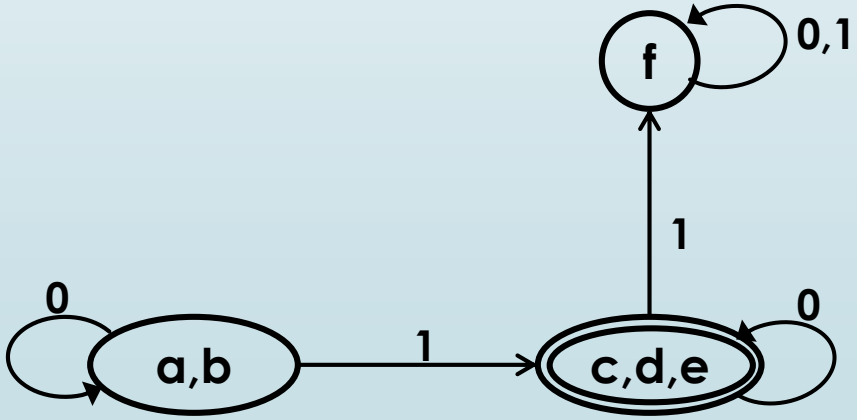


{a, b, f} {c, d, e}
{a, b} {f} {c, d, e}
{a, b} {f} {c, d, e}

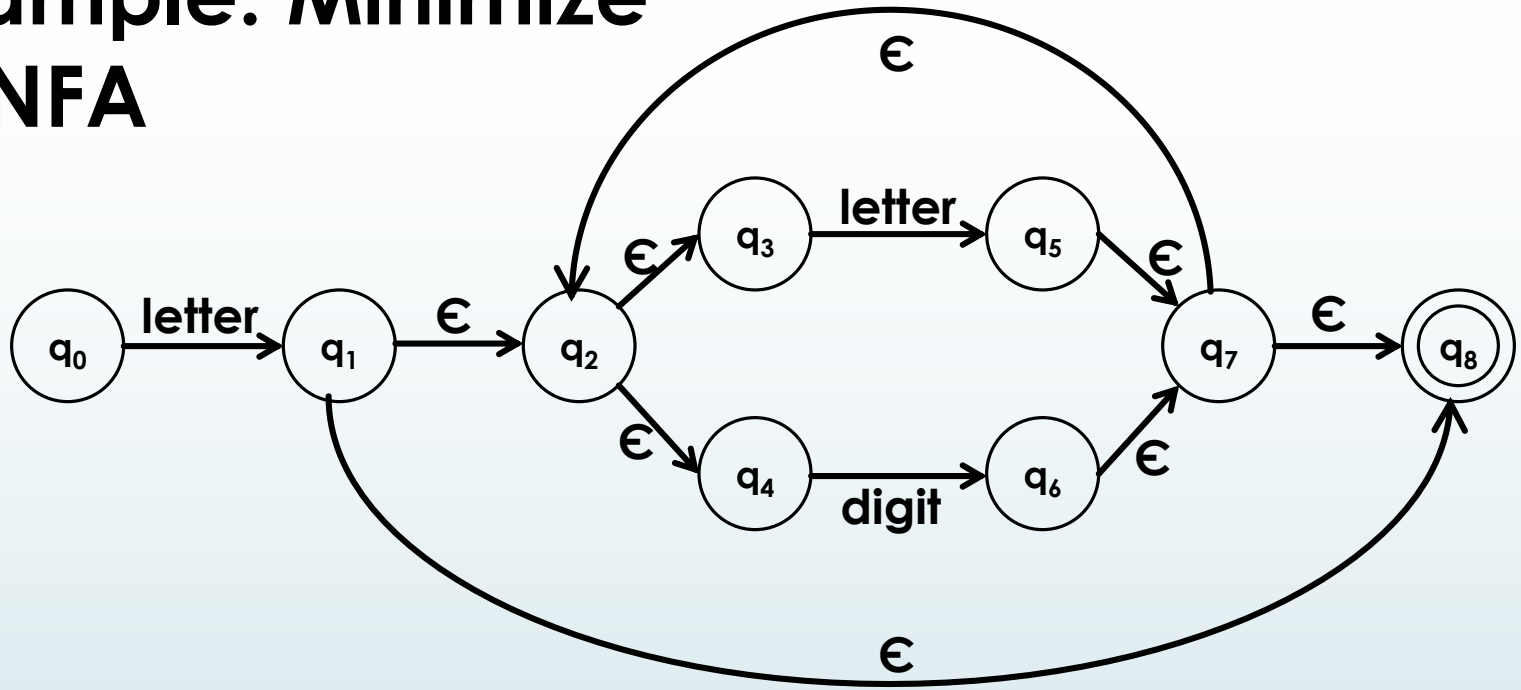
Example
DFA Minimization



{a, b, f} {c, d, e}
{a, b} {f} {c, d, e}
{a, b} {f} {c, d, e}

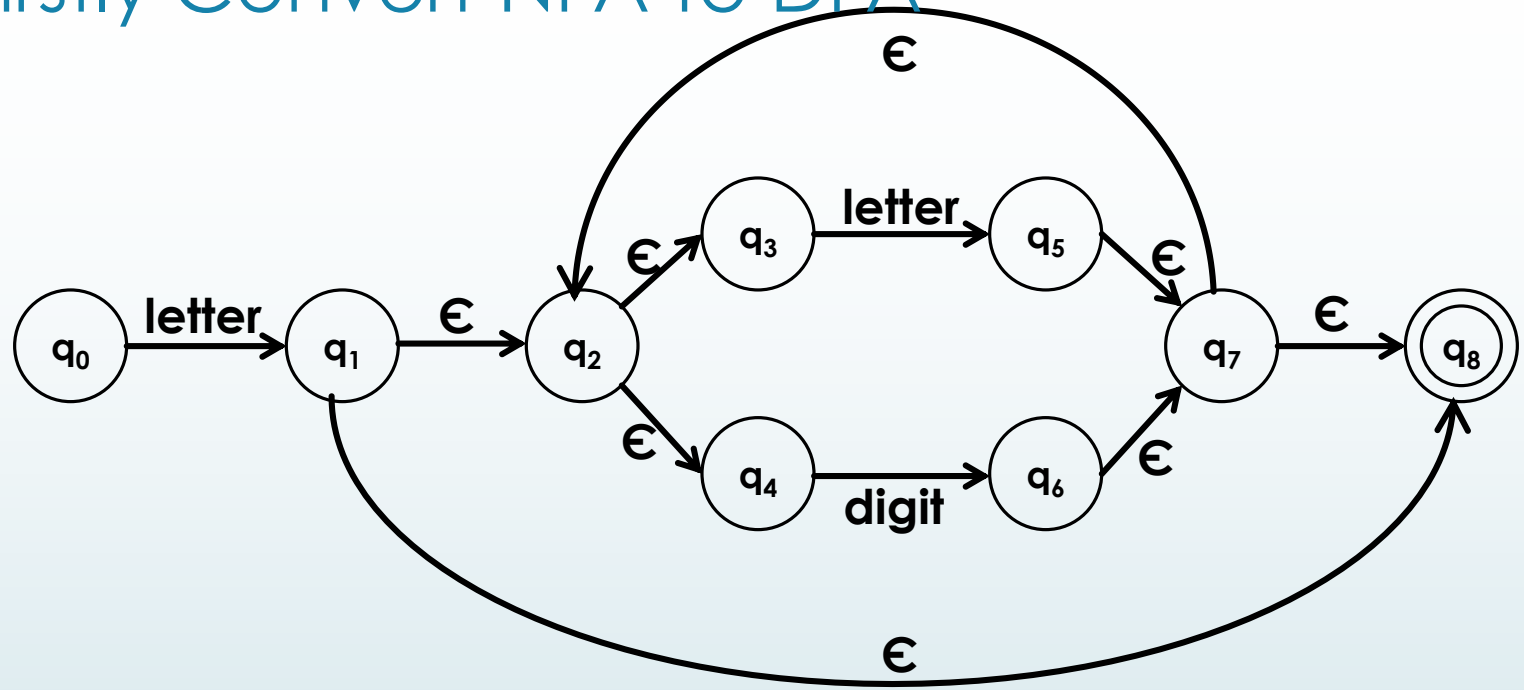


Example: Minimize Є-NFA



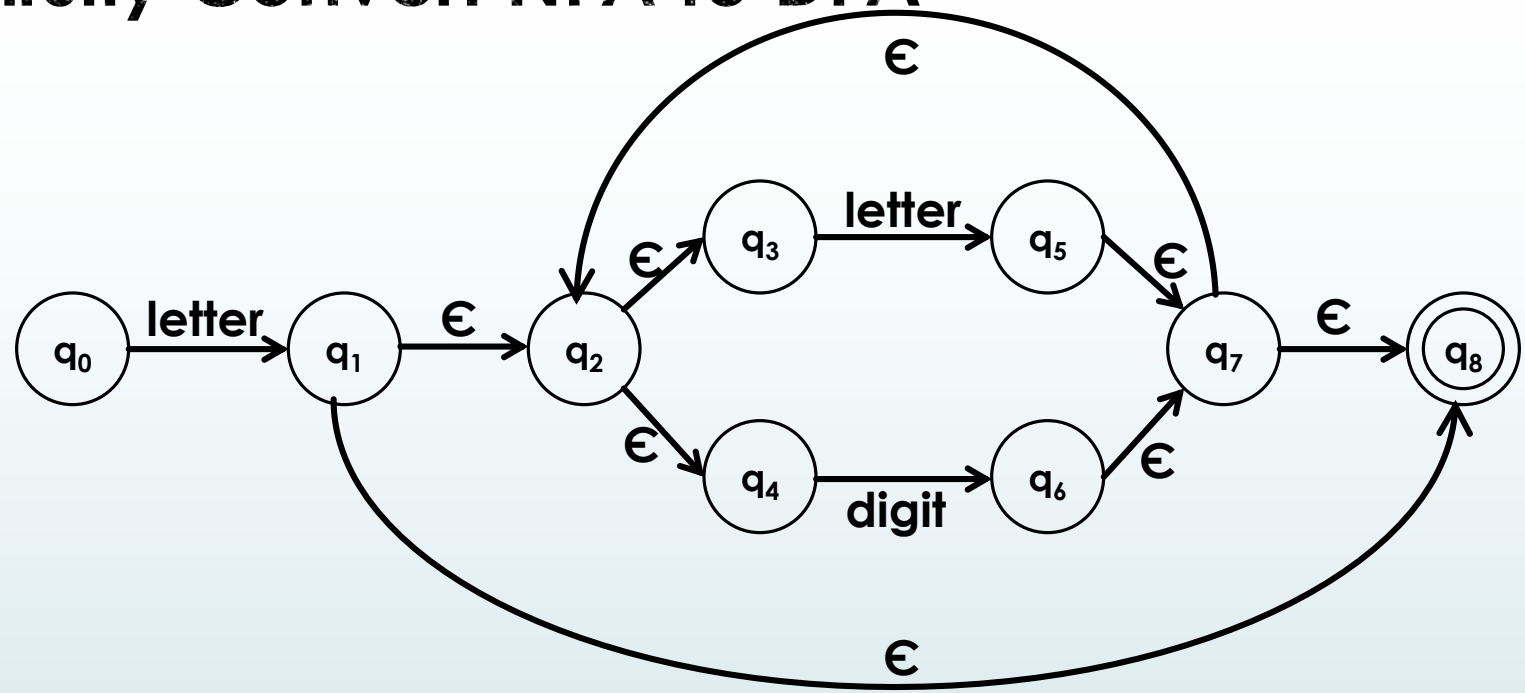
q	*δ(q,letter)*	*δ(q,digit)*
q_0	$\{q_1q_2q_3q_4q_8\}$	Ø
$\{q_1q_2q_3q_4q_8\}$	$\{q_2q_3q_4q_5q_7q_8\}$	$\{q_2q_3q_4q_6q_7q_8\}$
Ø	Ø	Ø
$\{q_2q_3q_4q_5q_7q_8\}$	$\{q_2q_3q_4q_5q_7q_8\}$	$\{q_2q_3q_4q_6q_7q_8\}$
$\{q_2q_3q_4q_6q_7q_8\}$	$\{q_2q_3q_4q_5q_7q_8\}$	$\{q_2q_3q_4q_6q_7q_8\}$

Firstly Convert NFA to DFA



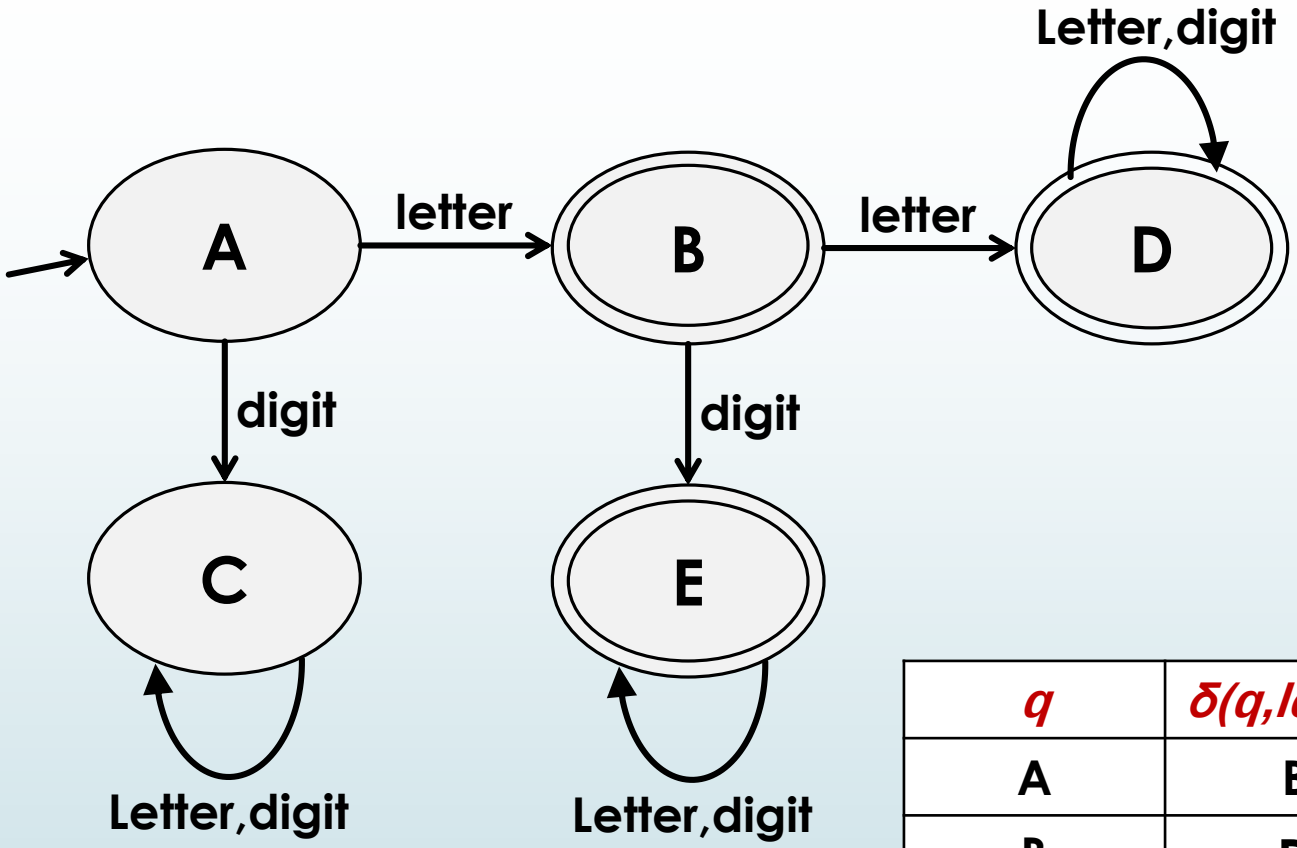
q		*δ(q,letter)*		*δ(q,digit)*	
q_0	A	$\{q_1q_2q_3q_4q_8\}$	B	∅	C
$\{q_1q_2q_3q_4q_8\}$	B	$\{q_2q_3q_4q_5q_7q_8\}$	D	$\{q_2q_3q_4q_6q_7q_8\}$	E
∅	C	∅	C	∅	C
$\{q_2q_3q_4q_5q_7q_8\}$	D	$\{q_2q_3q_4q_5q_7q_8\}$	D	$\{q_2q_3q_4q_6q_7q_8\}$	D
$\{q_2q_3q_4q_6q_7q_8\}$	E	$\{q_2q_3q_4q_5q_7q_8\}$	E	$\{q_2q_3q_4q_6q_7q_8\}$	E

Firstly Convert NFA to DFA



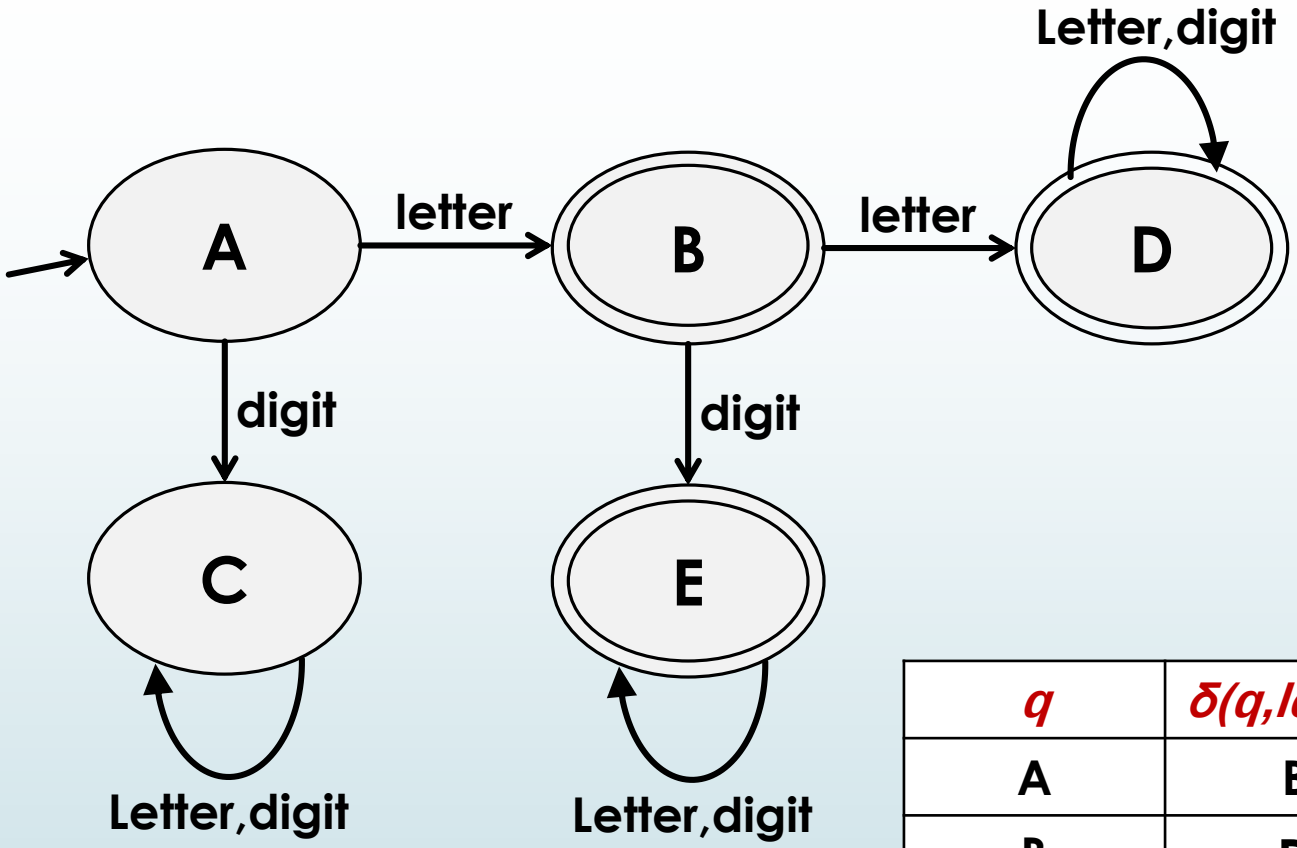
q	*δ(q,letter)*	*δ(q,digit)*
A	B	C
B	D	E
C	C	C
D	D	D
E	E	E

Firstly Convert NFA to DFA



q	*δ(q,letter)*	*δ(q,digit)*
A	B	C
B	D	E
C	C	C
D	D	D
E	E	E

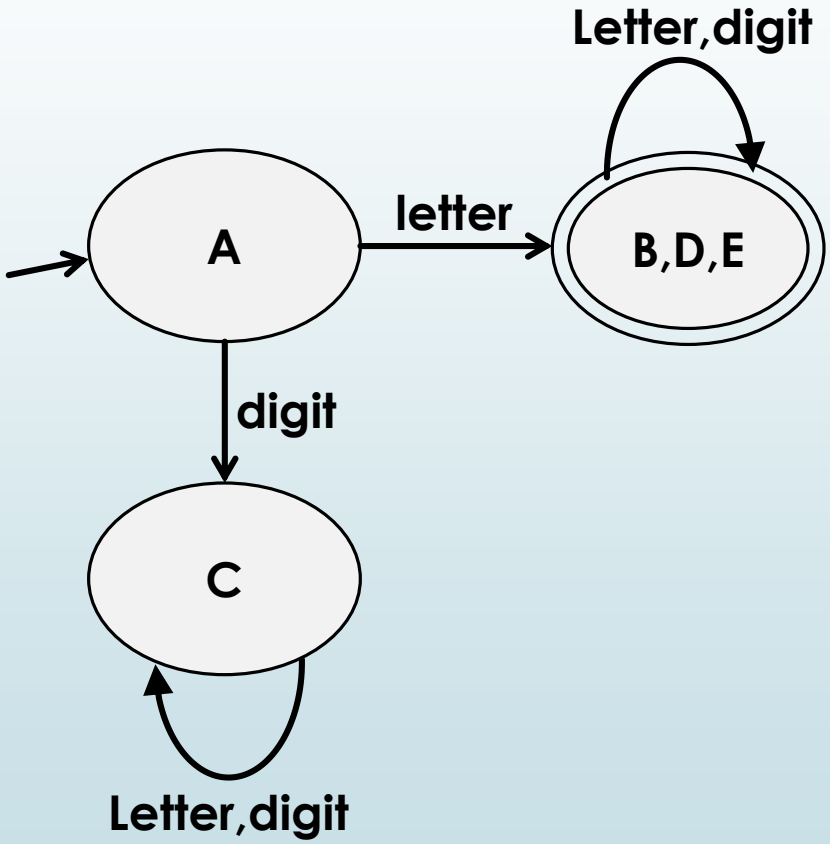
Firstly Convert NFA to DFA



q	*δ(q,letter)*	*δ(q,digit)*
A	B	C
B	D	E
C	C	C
D	D	D
E	E	E

{A, C} {B, D, E}

Secondly DFA Minimization



{A, C} {B, D, E}
{A} {C} {B, D, E}
{A} {C} {B, D, E}

q	*δ(q,letter)*	*δ(q,digit)*
A	B	C
B	D	E
C	C	C
D	D	D
E	E	E

Assignment

- **Book Problems of Ch2**