Simulation and Modeling (CS302)

Lecture 03: Example of QS: Single Server

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Agenda

- Single Server
  - Probability Mapping
  - Data Analysis
- Lab Tutorial

Example 3.1: Single-Server Queue

- A small grocery store has only one checkout counter. Customers arrive at this checkout counter at random from 1 to 8 minutes apart. Each possible value of interarrival/interval time has the same probability of occurrence, as shown below:

Interarrival (Minutes)	1	2	3	4	5	6	7	8
Probability	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

- The service times vary from 1 to 6 minutes with the probabilities shown below:

Service Time (Minutes)	1	2	3	4	5	6
Probability	0.10	0.20	0.30	0.25	0.10	0.05

- The problem is to analyze the system by simulating the arrival and service of 20 customers.

Example 1: Single-Server Queue …

- In reality, 20 customers is too small as a sample size to allow drawing any reliable conclusions. However, the purpose of the exercise is to demonstrate how simple simulations can be carried out in a table, either manually or with a spreadsheet, not to recommend changes in the grocery store.
- In order to generate the arrivals at the checkout counter, a set of random numbers, which have the following properties, is needed:
  - The set of random numbers is uniformly distributed between 0 and 1.
  - Successive random numbers are independent.
- With tabular simulations, random digits, such as those found in Table A.1 (page 572), can be converted to random numbers. If using a spreadsheet, most have a built-in random number generator such as RAND() in Excel.

Example 3.1: Single-Server Queue …

- The rightmost two columns of Tables 3.1 and 3.2 are used to generate random arrivals and random service times. The third column in each table contains the cumulative probability for the distribution. The rightmost column contains the random digit-assignment.

Table 3.1. Probability distribution of time between arrivals

Interarrival (Minutes)	Probability	Cumulative Probability	Random Digit Assignment
1	0.125		
2	0.125		
3	0.125		
4	0.125		
5	0.125		
6	0.125		
7	0.125		
8	0.125		

From the rightmost column of Table 3.1, there are 1000 3-digit values possible (001 through 000).

Example 3.1: Single-Server Queue …

- From the rightmost column of Table 3.2, there are 100 2-digit values possible (01 through 00).

Table 3.2. Probability distribution of service times

Service Time (Minutes)	Probability	Cumulative Probability	Random Digit Assignment
1	0.10		
2	0.20		
3	0.30		
4	0.25		
5	0.10		
6	0.05		

- Time between arrivals for 19 customers are generated by listing 19 three-digit values from Table A.1 and comparing them to the random-digit assignment of Table 3.1, as shown in Table 3.3.
- Similarly, service times for all 20 customers can be generated by listing 20 two-digit values from Table A.1 and comparing them to the random-digit assignment of Table 3.2, as shown in Table 3.4.

Example 3.1: Single-Server Queue …

Table 3.3. Time Between Arrivals Determination

Customer	Random Digits	Time Between Arrivals (minutes)	Customer	Random Digits	Time Between Arrivals (minutes)
1	--	--	11	109	1
2	913	8	12	093	1
3	727	6	13	607	5
4	015	1	14	738	6
5	948	8	15	359	3
6	309	3	16	888	8
7	922	8	17	106	1
8	753	7	18	212	2
9	235	2	19	493	4
10	302	3	20	535	5

Example 3.1: Single-Server Queue …

Table 3.4. Service Times Determination

Customer	Random Digits	Service Time (minutes)	Customer	Random Digits	Service Time (minutes)
1	84	4	11	32	3
2	10	1	12	94	5
3	74	4	13	79	4
4	53	3	14	05	1
5	17	2	15	89	5
6	79	4	16	84	4
7	91	5	17	52	3
8	67	4	18	55	3
9	89	5	19	30	2
10	38	3	20	50	3

Example 3.1: Single-Server Queue …

- The essence of a manual simulation is the *simulation table*. These tables are designed for the problem at hand, with columns added to answer the questions posed. The simulation table for the single-server queue consists of the following columns:

	A	B	C	D	E	F	G	H	I
1	Customer No.	Interarrival (minutes)	Arrival Time	Service Time (minutes)	Time Service Begins	Time customer wait in queue	Time Service Ends	Time Customer spends in system	Idle Time of Server (minutes)

- Values in columns B and D are the inputs and can be generated as described above.
- The first step is to initialize the table by filling in cells for the first customer. Then subsequent rows in the table are based on the random numbers for interarrival time and service time and the completion time of the previous customer.

Example 3.1: Single-Server Queue …

Customer No.	Time Between Arrivals (minutes)	Arrival Time	Service Time (minutes)	Time Service Begins	Time customer wait in queue	Time Service Ends	Time Customer spends in system	Idle Time of Server (minutes)
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								

Example 3.1: Single-Server Queue …

Customer No.	Time Between Arrivals (minutes)	Arrival Time	Service Time (minutes)	Time Service Begins	Time customer wait in queue	Time Service Ends	Time Customer spends in system	Idle Time of Server (minutes)
1		0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
11	1	47	3	53	6	56	9	0
12	1	48	5	56	8	61	13	0
13	5	53	4	61	8	65	12	0
14	6	59	1	65	6	66	7	0
15	3	62	5	66	4	71	9	0
16	8	70	4	71	1	75	5	0
17	1	71	3	75	4	78	7	0
18	2	73	3	78	5	81	8	0
19	4	77	2	81	4	83	6	0
20	5	82	3	83	1	86	4	0
			68		56		124	18

Example 3.1: Single-Server Queue …

- Extra columns have been added to collect statistical measures of performance such as each customer's time in the system and the server's idle time (if any) since the previous customer departed.
- In order to compute summary statistics, totals are formed for service times, time customers spend in the system, idle time of the server, and time the customers wait in the queue.
- Some of the findings from the simulation table are as follows:
- *Average waiting time (minutes) for a customer*

$$= \frac{total\,time\,customers\,wait\,in\,queue}{total\,number\,of\,customers} = \frac{56}{20}$$ = 2.8 minutes

Example 3.1: Single-Server Queue …

2. *Probability that a customer has to wait in the queue*

$$= \frac{\text{number of customers who wait}}{\text{total number of customers}} = \frac{13}{20} = 0.65$$

3. *Probability of server being idle*

$$= \frac{\text{total idle time of server}}{\text{total run time of simulation}} = \frac{18}{86} = 0.21$$

Probability of server being busy = 1 – Probability of server being idle = 0.79

4. *Average service time* $= \dfrac{\text{total service time}}{\text{total number of customers}} = \dfrac{68}{20} = 3.4$ minutes

Expected service time = mean of service time distribution

$$= E(S) = \sum_s s\,p(s)$$

where s is a service time and $p(s)$ is its probability.

Example 3.1: Single-Server Queue …

From Table 3.2 we get:

Expected service time = 1(0.10) + 2(0.20) + 3(0.30) + 4(0.25) + 5(0.10) + 6(0.05) = 3.2 minutes

5. Average time between arrivals (minutes)

$$= \frac{\text{sum of all times between arrivals}}{\text{number of arrivals} - 1} = \frac{82}{19} = 4.3 \text{ minutes}$$

One is subtracted from denominator because the first arrival is assumed to occur at time 0.

- Expected time between arrivals =

mean of the discrete uniform distribution whose end points are a and b = E(A) $= \frac{a+b}{2} = \frac{1+8}{2} = 4.5$ minutes

Example 3.1: Single-Server Queue …

6. *Average waiting time of those who wait (minutes)*

$$= \frac{total\,time\,customers\,wait\,in\,queue}{total\,number\,of\,customers\,who\,wait} = \frac{56}{13} = 4.3 \text{ minutes}$$

7. Average time customer spends in the system (minutes)

$$= \frac{total\,time\,customers\,spend\,in\,system}{total\,number\,of\,customers} = \frac{124}{20} = 6.2 \text{ minutes}$$

= average time customer spends waiting in queue +
average time customer spends in service

= 2.8 + 3.4 = 6.2 minutes

Example 3.1: Single-Server Queue …

- Comments on the results of Example 3.1:
- A decision maker would be interested in results of this type, but a longer simulation would increase the accuracy of the findings.
- However, some subjective inferences can be drawn at this point:
  - Most customers have to wait; however, the average waiting time is not excessive.
  - The server does not have an undue amount of idle time.
- Objective statements about the results would depend on balancing the cost of waiting with the cost of additional servers.

Lab Tutorial for QS

Equations of Main simulation table:

$$average\ waitng\ time = \frac{total\ time\ customers\ wait\ in\ queue}{total\ numbers\ of\ customers} = \frac{...}{...} = ...\text{(min)}$$

$$probability\ (wait) = \frac{number\ of\ customers\ who\ wait}{total\ numbers\ of\ customers} = \frac{...}{...} = ...$$

$$probability\ of\ idle\ server = \frac{total\ idle\ time\ of\ server}{total\ run\ time\ of\ simulation} = \frac{...}{...} = ...$$

$$average\ service\ time = \frac{total\ service\ time}{total\ numbers\ of\ customers} = \frac{...}{...} = ...\text{(min)}$$

The expected service time $E(S) = \sum_{s=0}^{\infty} sp(s)$

Equations of Main simulation table:

$$average\ time\ between\ arrivals = \frac{sum\ of\ all\ times\ between\ arrivals}{numbers\ of\ arrivals - 1} = \frac{...}{...} = ...\ (\text{min})$$

This result can be compared to the expected time between arrivals by finding the mean of the discrete uniform distribution whose endpoints are a=1 and b=8.

$$E(A) = \frac{a+b}{2}$$

$$e.g., E(A) = \frac{1+8}{2} = 4.5\ (\text{min})$$

Lab Tutorial for QS

Arrival Propability				
Time Between Arrivals (Minutes)	Probability (0:1)	Cumulative Probability	Random Digit Assignment From	To
1	0.125	0.125	0	125
2	0.125	0.25	126	250
3	0.125	0.375	251	375
4	0.125	0.5	376	500
5	0.125	0.625	501	625
6	0.125	0.75	626	750
7	0.125	0.875	751	875
8	0.125	1	876	1000
# of Rows=	8	Expected time	4.5	

Services Probability				
Service Time (Minutes)	Probability (0:1)	Cumulative Probability	Random Digit From	To
1	0.1	0.1	0	10
2	0.2	0.3	11	30
3	0.3	0.6	31	60
4	0.25	0.85	61	85
5	0.1	0.95	86	95
6	0.05	1	96	100

Expected Serv. Time	3.2

Customer No	Rand. Digit	Interval Time	Arr. Clock	Start Time	Rand. Digit	Serv. Duration	End Time	Cust. Waiting Time	Server Idle Time	Cust.Spent Time
1	0	0	0	0	37	3	3	0	0	3
2	165	2	2	3	15	2	5	*1*	0	3
3	299	3	5	5	15	2	7	0	0	2
4	284	3	8	8	27	2	10	0	1	2
5	292	3	11	11	32	3	14	0	1	3
6	614	5	16	16	96	6	22	0	2	6
7	92	1	17	22	54	3	25	*5*	0	8
8	316	3	20	25	47	3	28	*5*	0	8
9	610	5	25	28	98	6	34	*3*	0	9
10	3	1	26	34	60	3	37	*8*	0	11

avg waiting 4 who wait	4.4
avg waiting	2.2
waiting probability	50%
Serv.Idle.Prob	11%
avg. serv.duraions	3.3
avg. intervals	2.888889
avg cust spent time	5.5

Dr. Mina Section_03 | Sheet2 | Sheet3

Lab Tutorial for QS

	A	B	C	D	E	F
1						
2			Arrival Propability			
3		Time Between Arrivals	Probability	Cumulative	Random Digit Assignment	
4		(Minutes)	(0:1)	Probability	From	To
5		1	0.125	0.125	0	125
6		2	0.125	0.25	126	250
7		3	0.125	0.375	251	375
8		4	0.125	0.5	376	500
9		5	0.125	0.625	501	625
10		6	0.125	0.75	626	750
11		7	0.125	0.875	751	875
12		8	0.125	1	876	1000
13		# of Rows=	8	Expected time	4.5	

Equations of Main simulation table:

- D5=C5, D6=C6+D5
- E5=0, E6=F5+1
- F5= D5 * 1000

Lab Tutorial for QS

	H	I	J	K	L
1					
2		Services Probability			
3	Service Time	Probability	Cumulative	Random Digit	
4	(Minutes)	(0:1)	Probability	From	To
5	1	0.1	0.1	0	10
6	2	0.2	0.3	11	30
7	3	0.3	0.6	31	60
8	4	0.25	0.85	61	85
9	5	0.1	0.95	86	95
10	6	0.05	1	96	100
11					
12	Expected	3.2			
13	Serv. Time				

Equations of Main simulation table:

- J5=I5, J6=I6+J5
- K5=0,K6=L5+1
- J5= J5 * 100

- I12=SUM(H5*I5,H6*I6,H7*I7,H8*I8,H9*I9,H10*I10)

Lab Tutorial for QS

	B	C	D	E	F	G	H	I	J	K	L
15	Customer No	Rand. Digit	Interval Time	Arr. Clock	Start Time	Rand. Digit	Serv. Duration	End Time	Cust. Waiting Time	Server Idle Time	Cust.Spent Time
16	1	0	0	0	0	37	3	3	0	0	3
17	2	165	2	2	3	15	2	5	1	0	3
18	3	299	3	5	5	15	2	7	0	0	2
19	4	284	3	8	8	27	2	10	0	1	2
20	5	292	3	11	11	32	3	14	0	1	3
21	6	614	5	16	16	96	6	22	0	2	6
22	7	92	1	17	22	54	3	25	5	0	8
23	8	316	3	20	25	47	3	28	5	0	8
24	9	610	5	25	28	98	6	34	3	0	9
25	10	3	1	26	34	60	3	37	8	0	11

<u>Equations of Main simulation table:</u>

- C16=0, C17=INT(RAND()*1000)
- D16=0, D17=LOOKUP(C17,$E$5:$F$12,$B$5:$B$12)
- E16=0, E17=E16+D17
- F16=0, F17=MAX(E17,I16)
- G16=INT(RAND()*100)
- H16=LOOKUP(G16,$K$5:$L$10,$H$5:$H$10)
- I16=F16+H16

Lab Tutorial for QS

	A	B	C	D	E	F	G	H	I	J	K	L
15		Customer No	Rand. Digit	Interval Time	Arr. Clock	Start Time	Rand. Digit	Serv. Duration	End Time	Cust. Waiting Time	Server Idle Time	Cust.Spent Time
16		1	0	0	0	0	37	3	3	0	0	3
17		2	165	2	2	3	15	2	5	1	0	3
18		3	299	3	5	5	15	2	7	0	0	2
19		4	284	3	8	8	27	2	10	0	1	2
20		5	292	3	11	11	32	3	14	0	1	3
21		6	614	5	16	16	96	6	22	0	2	6
22		7	92	1	17	22	54	3	25	5	0	8
23		8	316	3	20	25	47	3	28	5	0	8
24		9	610	5	25	28	98	6	34	3	0	9
25		10	3	1	26	34	60	3	37	8	0	11

Equations of Main simulation table:

- J16=0, J17=F17-E17
- K16=0, K17=F17-I16
- L16=I16-E16

Lab Tutorial for QS

	N	O
16	avg waiting 4 who wait	4.4
17	avg waiting	2.2
18	waiting probability	50%
19	Serv.Idle.Prob	11%
20	avg. serv.duraions	3.3
21	avg. intervals	2.888889
22	avg cust spent time	5.5

<u>Equations of Main simulation table:</u>

- O16=SUM(J16:J25)/COUNTIF(J16:J25,">0")
- O17=SUM(J16:J25)/COUNT(B16:B25)
- O18=COUNTIF(J16:J25,">0")/COUNT(B16:B25)
- O19=SUM(K16:K25)/I25
- O20=SUM(H16:H25)/COUNT(B16:B25)
- O21=SUM(D16:D25)/(COUNT(B16:B25)-1)
- O22=SUM(L16:L25)/COUNT(B16:B25)

Task

Probabilities of the arrival and service's duration are set using static values. Suppose that you have a table which presents daily life process in a bank, your task is to calculate probabilities of intervals and services based on available data in this table. Then use calculated data to build auxiliary and simulation tables.

ID	arrival time	cust_name	service	duration
1	8:00	a	NewAccount	10
2	8:01	b	Inquiry	3
3	8:02	c	Deposite	6
4	8:02	d	Withdrow	11
5	8:06	e	Transfer	12
6	8:06	f	Deposite	8
7	8:06	g	Withdrow	9
8	8:10	i	NewAccount	13
9	8:12	j	Deposite	7
10	8:17	k	Deposite	5

	Arrival Propability			
Time Between Arrivals (Minutes)	Probability (0:1)	Cumulative Probability	Random Digit Assignment From	To
1	0.125	0.125	0	125
2	0.125	0.25	126	250
3	0.125	0.375	251	375
4	0.125	0.5	376	500
5	0.125	0.625	501	625
6	0.125	0.75	626	750
7	0.125	0.875	751	875
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# of Rows=	8	Expected time	4.5	

	Services Probability			
Service Time (Minutes)	Probability (0:1)	Cumulative Probability	Random Digit From	To
1	0.1	0.1	0	10
2	0.2	0.3	11	30
3	0.3	0.6	31	60
4	0.25	0.85	61	85
5	0.1	0.95	86	95
6	0.05	1	96	100
Expected Serv. Time	3.2			

Task

- Calculate required data analysis equations studied in the previous lecture for this problem. Based on the extracted knowledge and calculations, your code has to suggest how to improve performance of the system to decrease waiting time and to reduce the idle time of servers.
- Draw chronological order of events.

Chronological Ordering of Events