Simulation and Modeling (CS302)

Lecture 04: Example of QS: Parallel Server

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Agenda

- Parallel Server
  - Probability Mapping
  - Data Analysis
- Lab Tutorial

Example 3.2: The Able-Baker Carhop Problem

- This example illustrates the simulation procedure when there are more than one server.
- Consider a drive-in restaurant where carhops take orders and bring food to the car. Cars arrive in the manner shown in Table 3.5.

Table 3.5. Interarrival Distribution of Cars

Interarrival (Minutes)	Probability	Cumulative Probability	Random Digit Assignment
1	0.25	0.25	01 – 25
2	0.40	0.65	26 – 65
3	0.20	0.85	66 – 85
4	0.15	1.00	86 – 00

- There are two carhops – Able and Baker. Able works a bit faster than baker. The distribution of their service times is shown in Tables 3.6 and 3.7.

Example 3.2: The Able-Baker Carhop Problem

The problem is to find how well the current arrangement is working. To estimate the system measures of performance, a simulation of 1 hour of operation is made.

Table 3.6. Service Time Distribution of Able

Service Time (Minutes)	Probability	Cumulative Probability	Random Digit Assignment
2	0.30	0.30	01 – 30
3	0.28	0.58	31 – 58
4	0.25	0.83	59 – 83
5	0.17	1.00	84 – 00

Table 3.7. Service Time Distribution of Baker

Service Time (Minutes)	Probability	Cumulative Probability	Random Digit Assignment
3	0.35	0.35	01 – 35
4	0.25	0.60	36 – 60
5	0.20	0.80	61 – 80
6	0.20	1.00	81 – 00

Example 3.2: The Able-Baker Carhop Problem

- The simulation proceeds in a manner similar to Example 3.1, except that it is more complex because of the two servers.
- A simplifying rule is that Able gets the customer if both carhops are idle. Perhaps, Able has seniority. (The simulation would be different if the decision were made at random or by any other rule).
- Here there are more events:
  - a customer arrives,
  - a customer completes service from Able, and
  - a customer completes service from Baker.

Example 3.2: The Able-Baker Carhop Problem

- The simulation table consists of the following columns:

	A	B	C	D	E	F	G	H
1						Able		
2	Customer No.	R. D. for Arrival	Time Between Arrivals	Clock Time of Arrival	R. D. for Service	Time Service Begins	Service Time	Time Service Ends

I	J	K	L
Baker			Time in Queue
Time Service Begins	Service Time	Time Service Ends	

- Here we provide a few hints for implementing the simulation table in Excel:
    - The row for the first customer is filled in manually, with the random-number function RAND().
    - After the first customer, the cells for other customers must be based on logic and formulas.

Customer No.	R. D. for Arrival	Time Between Arrivals	Clock Time of Arrival	R. D. for Service	Able			Baker			Time in Queue
					Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	

Customer No.	R. D. for Arrival	Time Between Arrivals	Clock Time of Arrival	R. D. for Service	Able			Baker			Time in Queue
					Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	
1			0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1
7	74	3	17	13	18	2	20				1
8	80	3	20	62	20	4	24				0
9	68	3	23	50				23	4	27	0
10	22	1	24	49	24	3	27				0
11	48	2	26	39	27	3	30				1
12	34	2	28	53				28	4	32	0
13	45	2	30	88	30	5	35				0
14	24	1	31	1				32	3	35	1
15	34	2	33	81	35	4	39				2
16	63	2	35	53				35	4	39	0
17	38	2	37	81	39	4	43				2
18	80	3	40	64				40	5	45	0
19	42	2	42	1	43	2	45				1
20	56	2	44	67	45	4	49				1
21	89	4	48	1				48	3	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			**43**		

Example 3.2: The Able-Baker Carhop Problem

For example, the "Clock Time of Arrival" (column D) in the row for the second customer is computed as follows: D4 = D3 + C4, where C4 is the time between arrivals 1 and 2. This formula is easy generalized for any customer. (<u>Note that Customer 1 is in Row 3</u>).

- The logic to compute who gets a given customer, and when that service begins, goes as follows:

  When a customer arrives:

  - *if Able is idle then* // $D_i >= MAX(H_3 : H_{i-1})$
    - *the customer begins service immediately with Able* // $F_i = D_i$
  - *else if Baker is idle then* // $D_i >= MAX(K_3 : K_{i-1})$
    - *the customer begins service immediately with Baker* // $I_i = D_i$
  - *else if both are busy then*
    - *the customer begins service with first server to become free*

Example 3.2: The Able-Baker Carhop Problem

- To compute when Able and Baker will become free, the Excel functions IF() and MAX() are used.

  For example, for customer 10, Able will become free at MAX(H3:H11), since service completion time is in column H and we need to look at customers 1 – 9. The resulting formula to compute whether and when Able serves customer 10 is as follows:

  F12 = IF(D12 >= MAX(H3:H11); D12;

  IF(MAX(H3:H11) <= MAX(K3:K11); MAX(H3:H11); ""))

  Based on this logic, the time Able begins service customer i (column F) is computed as follows:

  F_{i+2} = IF(D_{i+2} >= MAX(H_3 : H_{i+1}); D_{i+2};

  IF(MAX(H_3:H_{i+1}) <= MAX(K_3:K_{i+1}); MAX(H_3:H_{i+1}); ""))

Example 3.2: The Able-Baker Carhop Problem

- Similarly, the time Baker begins service customer i (column I) is computed as follows:

  $$I_{i+2} = \text{IF}(F_{i+2} <> \text{""}; \text{""}; \text{IF}(D_{i+2} >= \text{MAX}(K_3:K_{i+1}); D_{i+2}; \text{MAX}(K_3:K_{i+1})))$$

- The service times for Able are computed as follows:

  $$G_{i+2} = \text{IF}(F_{i+2} > 0; \text{new\_service\_time}; \text{""})$$

  where *new_service_time* is computed by using the service time distribution of Able (Table 3.6).

  The time service ends is computed as follows:

  $$H_{i+2} = \text{IF}(F_{i+2} > 0; F_{i+2} + G_{i+2}; \text{""})$$

- Similarly, the service times and the time service ends for Baker are computed.

Example 3.2: The Able-Baker Carhop Problem

- The interarrival and service times distribution tables as they appear in the excel sheet.

Interarrival Distribution of Cars

	M	N	O
1 2	Interarrival (Minutes)	Probability	Cumulative Probability
3	1	0.25	0.25
4	2	0.40	0.65
5	3	0.20	0.85
6	4	0.15	1.00

Service Time Distribution of Able

	P	Q	R
1 2	Service Time (Minutes)	Probability	Cumulative Probability
3	2	0.30	0.30
4	3	0.28	0.58
5	4	0.25	0.83
6	5	0.17	1.00

Service Time Distribution of Baker

	S	T	U
1 2	Service Time (Minutes)	Probability	Cumulative Probability
3	3	0.35	0.35
4	4	0.25	0.60
5	5	0.20	0.80
6	6	0.20	1.00

Example 3.2: The Able-Baker Carhop Problem

- Formulas for calculating the time between arrivals and service times of Able and Baker for Customer **i** in Row **i+2** in the Excel sheet.
  - ***Time Between Arrivals:***

$$C_{i+2}=\mathrm{IF}(B_{i+2}/100>O5;M6;\mathrm{IF}(B_{i+2}/100>O4;M5;\mathrm{IF}(B_{i+2}/100>O3;M4;M3)))$$

  - ***Service Time of Able:***

$$G_{i+2}=\mathrm{IF}(F_{i+2}<>\text{""};\mathrm{IF}(E_{i+2}/100>R5;P6;\mathrm{IF}(E_{i+2}/100>R4;P5;\mathrm{IF}(E_{i+2}/100>R3;P4;P3)));\text{""})$$

  - ***Service Time of Baker:***

$$J_{i+2}=\mathrm{IF}(I_{i+2}<>\text{""};\mathrm{IF}(E_{i+2}/100>U5;S6;\mathrm{IF}(E_{i+2}/100>U4;S5;\mathrm{IF}(E_{i+2}/100>U3;S4;S3)));\text{""})$$

Example 3.2: The Able-Baker Carhop Problem

<u>The analysis of the simulation table results in the following:</u>

- Over the 62-minute period Able was busy 90% of the time, while Baker was busy only 69% of the time.
- Nine of the 26 arrivals (about 35%) had to wait. The average waiting time for all customers was only about 0.42 minutes (25 seconds), which is very small.
- These 9 who had to wait, only waited an average of 1.22 minutes, which is quite low.
- In summary, this system seems well balanced. One server cannot handle all the diners, and three servers would probably be too many. Adding and additional server would surely reduce the waiting time to nearly 0. However, the cost of waiting would have to be quite high to justify an additional server.

Lab Tutorial for QS

Equations for data analysis using main simulation table:

$$average\ waitng\ time = \frac{total\ time\ customers\ wait\ in\ queue}{total\ numbers\ of\ customers} = \frac{...}{...} = ...\ \text{(min)}$$

$$probability\ (wait) = \frac{number\ of\ customers\ who\ wait}{total\ numbers\ of\ customers} = \frac{...}{...} = ...$$

$$probability\ of\ idle\ server = \frac{total\ idle\ time\ of\ server}{total\ run\ time\ of\ simulation} = \frac{...}{...} = ...$$

$$average\ service\ time = \frac{total\ service\ time}{total\ numbers\ of\ customers} = \frac{...}{...} = ...\ \text{(min)}$$

The expected service time $E(S) = \sum_{s=0}^{\infty} sp(s)$

Lab Tutorial for QS

Equations for data analysis using main simulation table:

$$average\ time\ between\ arrivals = \frac{sum\ of\ all\ times\ between\ arrivals}{numbers\ of\ arrivals - 1} = \frac{\ldots}{\ldots} = \ldots\ (\text{min})$$

This result can be compared to the expected time between arrivals by finding the mean of the discrete uniform distribution whose endpoints are a=1 and b=8.

$$E(A) = \frac{a+b}{2}$$

$$e.g., E(A) = \frac{1+8}{2} = 4.5\ (\text{min})$$

Lab Tutorial for QS

Arrival Probability

Time Between Arrivals (Minutes)	Probability (0:1)	Accumulative Probability	Random Digit Assignment From	To
1	0.25	0.25	1	25
2	0.4	0.65	26	65
3	0.2	0.85	66	85
4	0.15	1	86	100

Server_01: Dr Mina

Service Time (Minutes)	Probability (0:1)	Accumulative Probability	Random Digit Assignment From	To
2	0.3	0.3	1	30
3	0.28	0.58	31	58
4	0.25	0.83	59	83
5	0.17	1	84	100

Server_02: Dr Younan

Service Time (Minutes)	Probability (0:1)	Accumulative Probability	Random Digit Assignment From	To
3	0.35	0.35	1	35
4	0.25	0.6	36	60
5	0.2	0.8	61	80
6	0.2	1	81	100

Openning 8:00:00 AM

					Dr Mina			Dr Younan				
cust_id	interval.rand	interval.time	Arrival.Clock	Ser.rand	start	duration	end	start	duration	end	Cust.Waiting	Ser.Ideal
1	73	3	8:03:00 AM	29	8:03:00 AM	2	8:05:00 AM				0	Dr Younan
2	71	3	8:06:00 AM	54				8:06:00 AM	4	8:10:00 AM	0	Dr Mina
3	99	4	8:10:00 AM	85	8:10:00 AM	5	8:15:00 AM				0	Dr Younan
4	8	1	8:11:00 AM	72				8:11:00 AM	5	8:16:00 AM	0	Dr Mina
5	14	1	8:12:00 AM	52	8:15:00 AM	3	8:18:00 AM				3	Dr Younan
6	2	1	8:13:00 AM	85				8:16:00 AM	6	8:22:00 AM	3	Dr Mina
7	35	2	8:15:00 AM	78	8:18:00 AM	4	8:22:00 AM				3	Dr Younan

Dr. Mina Section_04_1 | **Dr. Mina Section_04_2** | Sheet3

Lab Tutorial for QS

	A	B	C	D	E	F
1						
2			Arrival Probability			
3		Time Between Arrivals	Probability	Accumulative	Random Digit Assignment	
4		(Minutes)	(0:1)	Probability	From	To
5		1	0.25	0.25	1	25
6		2	0.4	0.65	26	65
7		3	0.2	0.85	66	85
8		4	0.15	1	86	100
9						
10		Openning	8:00:00 AM			

Equations of auxiliary tables:

- D5=C5, D6==D5+C6
- E5=1, E6=F5+1
- F5= D5 * 100

Lab Tutorial for QS

	H	I	J	K	L
1					
2		Server_01: Dr Mina			
3	Service Time	Probability	Accumulative	Random Digit Assignment	
4	(Minutes)	(0:1)	Probability	From	To
5	2	0.3	0.3	1	30
6	3	0.28	0.58	31	58
7	4	0.25	0.83	59	83
8	5	0.17	1	84	100
9					
10					

	M	N	O	P	Q	R
1						
2			Server_02: Dr Younan			
3		Service Time	Probability	Accumulative	Random Digit Assignment	
4		(Minutes)	(0:1)	Probability	From	To
5		3	0.35	0.35	1	35
6		4	0.25	0.6	36	60
7		5	0.2	0.8	61	80
8		6	0.2	1	81	100
9						
10						

<u>Equations of auxiliary table:Server_01</u>

- J5=I5, J6==J5+I6
- K5=1, K6=L5+1
- L5= J5 * 100

<u>Equations of auxiliary table:Server_02</u>

- P5=O5, P6==P5+O6
- Q5=1, Q6=R5+1
- R5= P5 * 100

Lab Tutorial for QS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
11							Dr Mina			Dr Younan				
12		cust_id	interval.rand	interval.time	Arrival.Clock	Ser.rand	start	duration	end	start	duration	end	Cust.Waiting	Ser.Ideal
13		1	73	3	8:03:00 AM	29	8:03:00 AM	2	8:05:00 AM				0	Dr Younan
14		2	71	3	8:06:00 AM	54				8:06:00 AM	4	8:10:00 AM	0	Dr Mina
15		3	99	4	8:10:00 AM	85	8:10:00 AM	5	8:15:00 AM				0	Dr Younan
16		4	8	1	8:11:00 AM	72				8:11:00 AM	5	8:16:00 AM	0	Dr Mina
17		5	14	1	8:12:00 AM	52	8:15:00 AM	3	8:18:00 AM				3	Dr Younan
18		6	2	1	8:13:00 AM	85				8:16:00 AM	6	8:22:00 AM	3	Dr Mina
19		7	35	2	8:15:00 AM	78	8:18:00 AM	4	8:22:00 AM				3	Dr Younan

Equations of main simulation table

- B13=1, B14=sum(B13,1)
- C13=INT(RAND()*100)+1
- D13=LOOKUP(C13,$E$5:$F$8,$B$5:$B$8)
- E13=C10+TIME(0,D13,0)
- F13=INT(RAND()*100)+1
- G13=E13 // first customer starts at the first server
  - G14=IF(MAX($I$13:I13)>MAX($L$13:L13),"",MAX($I$13:I13,E14))
- H13=LOOKUP(F13,K5:L8,H5:H8)
  - H14=IF(G14<>"",LOOKUP(F14,$K$5:$L$8,$H$5:$H$8),"")
- I13=G13+TIME(0,H13,0)
  - I14=IF(H14<>"",G14+TIME(0,H14,0),"")

Lab Tutorial for QS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
11							Dr Mina			Dr Younan				
12		cust_id	interval.rand	interval.time	Arrival.Clock	Ser.rand	start	duration	end	start	duration	end	Cust.Waiting	Ser.Ideal
13		1	73	3	8:03:00 AM	29	8:03:00 AM	2	8:05:00 AM				0	Dr Younan
14		2	71	3	8:06:00 AM	54				8:06:00 AM	4	8:10:00 AM	0	Dr Mina
15		3	99	4	8:10:00 AM	85	8:10:00 AM	5	8:15:00 AM				0	Dr Younan
16		4	8	1	8:11:00 AM	72				8:11:00 AM	5	8:16:00 AM	0	Dr Mina
17		5	14	1	8:12:00 AM	52	8:15:00 AM	3	8:18:00 AM				3	Dr Younan
18		6	2	1	8:13:00 AM	85				8:16:00 AM	6	8:22:00 AM	3	Dr Mina
19		7	35	2	8:15:00 AM	78	8:18:00 AM	4	8:22:00 AM				3	Dr Younan

Equations of main simulation table

- J13=NULL, J14=IF(G14<>"","",MAX($L$13:L13,E14))
- K13=NULL, K14=IF(J14<>"",LOOKUP(F14,$Q$5:$R$8,$N$5:$N$8),"")
- L13=NULL, L14=IF(J14<>"",J14+TIME(0,K14,0),"")
- M13=MINUTE(IF(G13<>"",G13-E13,J13-E13))
    - Use conditional format to display icons as shown in the above table
- N13=IF(G13<>"","Dr Younan","Dr Mina")

Task

- Probabilities of the arrival and service's duration are set using static values. Suppose that you have a table which presents daily life process in a bank, your task is to calculate probabilities of intervals between customers and services for each server based on available data in this table. Then use calculated data to build auxiliary and simulation tables.
- **Hints**: get the average of the same type of services for each serve. In case of not exist of a service for that server, service duration of the other server could be used as a default value for that server.

ID	arrival time	cust_name	service	duration	Server
1	8:00	a	NewAccount	10	Sr01
2	8:01	b	Inquiry	3	Sr02
3	8:02	c	Deposite	6	Sr02
4	8:02	d	Withdrow	11	Sr01
5	8:06	e	Transfer	12	Sr02
6	8:06	f	Deposite	8	Sr02
7	8:06	g	Withdrow	9	Sr02
8	8:10	i	NewAccount	13	Sr01
9	8:12	j	Deposite	7	Sr01
10	8:17	k	Deposite	5	Sr01

Task

- Calculate required data analysis equations studied in the previous lecture for this problem. Based on the extracted knowledge and calculations, your code has to suggest how to improve performance of the system to decrease waiting time and to reduce the idle time of servers.
- Draw chronological order of events.

Chronological Ordering of Events