Computer Graphics

Lab 4

Midpoint Ellipse Algorithm

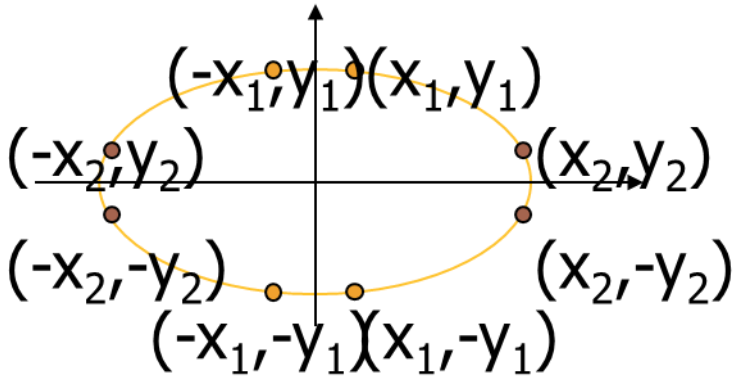
Midpoint Ellipse Algorithm

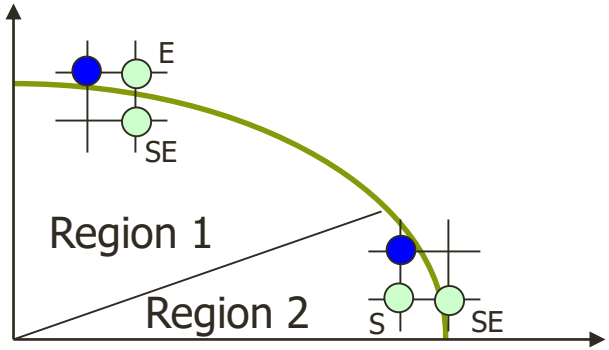
- Implicit equation is:

$$F(x,y) = b^2x^2 + a^2y^2 - a^2b^2 = 0$$

- We have only 4-way symmetry
- There exists two regions
  - In **Region 1** dx > dy
    - Increase x at each step
    - y may decrease
  - In **Region 2** dx < dy
    - Decrease y at each step
    - x may increase

Midpoint Ellipse Algorithm





Decision Parameter (Region 1)

Midpoint of the vertical line connecting E and SE is used to define the following decision parameter:

$$d_i = F(x_i + 1, y_i - \tfrac{1}{2})$$

$$= b^2(x_i + 1)^2 + a^2(y_i - \tfrac{1}{2})^2 - a^2b^2$$

if $d_i < 0$ then move to E; $(x_{i+1}, y_{i+1}) = (x_i + 1, y_i)$

$$d_{i+1} = F(x_i + 2, y_i - \tfrac{1}{2})$$

$$= b^2(x_i + 2)^2 + a^2(y_i - \tfrac{1}{2})^2 - a^2b^2$$

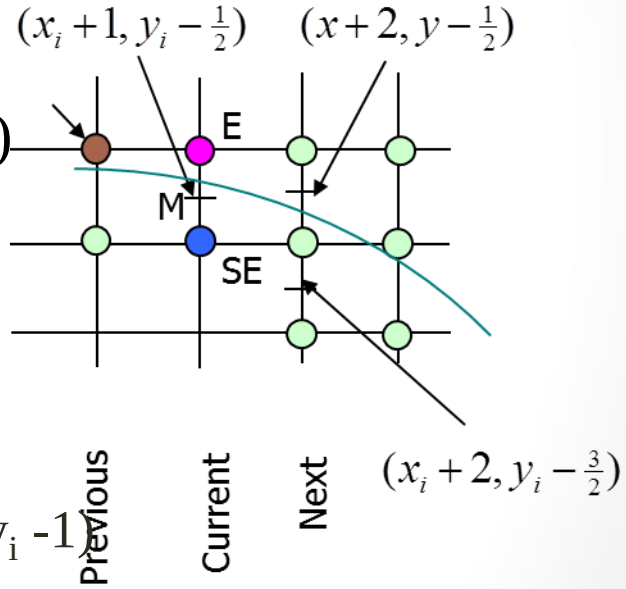
$$d_{i+1} = d_i + b^2(2x_i + 3)$$

if $d > 0$ then move to SE ; $(x_{i+1}, y_{i+1}) = (x_i + 1, y_i - 1)$

$$d_{i+1} = F(x_i + 2, y_i - \tfrac{3}{2})$$

$$= b^2(x_i + 2) + a^2(y_i - \tfrac{3}{2}) - a^2b^2$$

$$d_{i+1} = d_i + b^2(2x_i + 3) + a^2(-2y_i + 2)$$



Decision Parameter (Region 2)

$$d_j = F(x_j + \tfrac{1}{2}, y_j - 1)$$

$$= b^2(x_j + \tfrac{1}{2})^2 + a^2(y_j - 1)^2 - a^2b^2$$

if $d_j < 0$ then move to $\mathrm{SE}(x_{j+1}, y_{j+1}) = (x_j + 1, y_j - 1)$

$$d_{j+1} = F(x_j + \tfrac{3}{2}, y_j - 2)$$

$$= b^2(x_j + \tfrac{3}{2})^2 + a^2(y_j - 2)^2 - a^2b^2$$

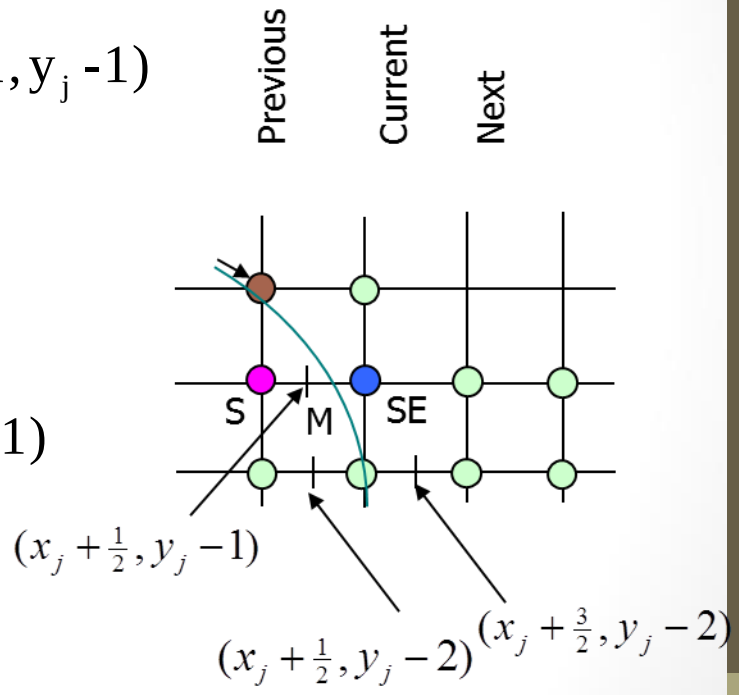
$$d_{j+1} = d_j + b^2(2x_j + 2) + a^2(-2y_j + 3)$$

if $d_j > 0$ then move to $\mathrm{S}(x_{j+1}, y_{j+1}) = (x_j, y_j - 1)$

$$d_{j+1} = F(x_j + \tfrac{1}{2}, y_j - 2)$$

$$= b^2(x_j + \tfrac{1}{2})^2 + a^2(y_j - 2)^2 - a^2b^2$$

$$d_{j+1} = d_j - a^2(2y_j - 3)$$



Example

- $r_x = 8$, $r_y = 6$
- $dx = 2r_y^2 x = 0$
- $dy = 2r_x^2 y = 2r_x^2 r_y$
- **Region 1**
- $(x_0, y_0) = (0, 6)$

$$p1_0 = r_y^2 - r_x^2 r_y + \tfrac{1}{4} r_x^2 = -332$$

i	p_i	x_{i+1}, y_{i+1}	$2r_y^2x_{i+1}$	$2r_x^2y_{i+1}$
0	-332	(1, 6)	72	768
1	-224	(2, 6)	144	768
2	-44	(3, 6)	216	768
3	208	(4, 5)	288	640
4	-108	(5, 5)	360	640
5	288	(6, 4)	432	512
6	244	(7, 3)	504	384

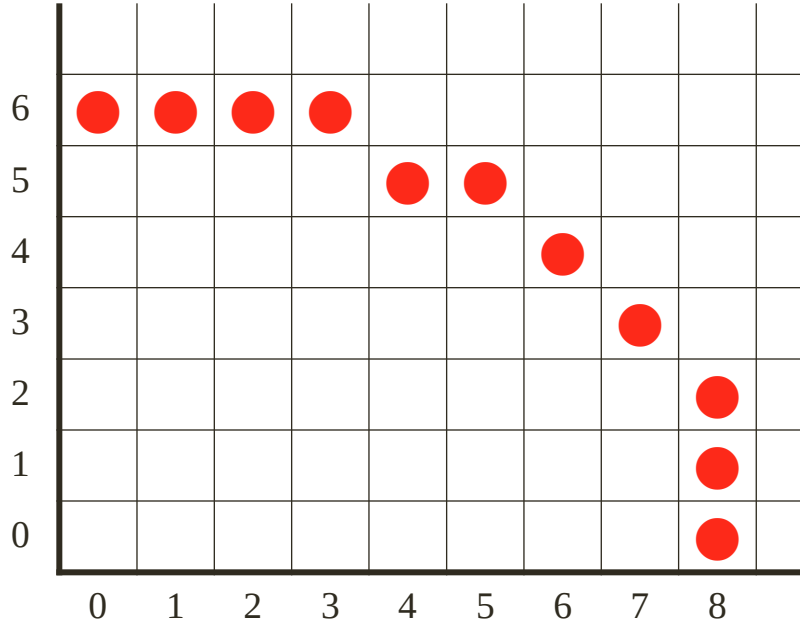
Move out of region 1 since
$\mathbf{2r_y^2x > 2r_x^2y}$

Region 2

$(x_0, y_0) = (7, 3)$ (Last position in region 1)

$$p2_0 = f_{ellipse}(7 + \tfrac{1}{2}, 2) = -151$$

i	p_i	x_{i+1}, y_{i+1}	$2r_y^2 x_{i+1}$	$2r_x^2 y_{i+1}$
0	-151	(8, 2)	576	256
1	233	(8, 1)	576	128
2	745	(8, 0)	-	-



Stop at $y = 0$

Algorithm :

Drawing Ellipse (rx,ry,xc,yc)

1- Initial point (0,ry)

2- calculate

```
p = pow(ry,2) - (pow(rx,2) * ry) + (0.25 * pow(rx,2));
dx = 2 * pow(ry,2) * x;
dy = 2 * pow(rx,2) * y;
```

3- do

```
putpixel(xc+x,yc+y);
putpixel(xc-x,yc-y);
putpixel(xc+x,yc-y);
putpixel(xc-x,yc+y);
```

If p<0 x =x+1 , dx = dx + (2 * (pow(ry,2)));

p = p + 2pow(ry,2)x+(3pow(ry,2);

Else, x=x+1, y=y-1, dx = dx + (2 * (pow(ry,2)));

dy = dy - (2 * (pow(rx,2)));

p = p + dx - dy + (pow(ry,2));

while(dx<dy);

4- calculate ,p2=pow(ry,2)*pow(x,2)+x*pow(ry,2)+pow(ry,2)/4+ pow(rx,2)*pow(y,2)-2y*pow(rx,2)-pow(rx,2)-pow(rx,2)*pow(ry,2)

5- do

putpixel(xc+x,yc+y);

putpixel(xc-x,yc-y);

putpixel(xc+x,yc-y);

putpixel(xc-x,yc+y);

if d2 >0 ,x=x , y=y-1 ,

```
dy = dy - (2 * (pow(rx,2)));
p2 = p2 - dy + pow(rx,2);
```

Else , x=x+1 , y=y-1,

```
dy = dy - (2 * (pow(rx,2)));
dx = dx + (2 * (pow(ry,2)));
 p2 = p2 +dx - dy + pow(rx,2);
while(y>0)
```

Exampes

- Draw the ellipse with $r_x = 6$, $r_y = 8$.
- Draw the ellipse with $r_x = 10$, $r_y = 14$.
- Draw the ellipse with $r_x = 14$, $r_y = 10$ and center at (15, 10).