SUMMARY

Gravitational N-body simulations in astrophysics is a tool for understanding the dynamics of systems of particles interacting through gravity. These simulations are used to study a wide range of systems, from small systems like the solar system to large scale structures like galaxies and the universe.

The n-body problem can be formulated using Newton's laws of motion. Newton's laws state that the acceleration of a body is proportional to the net force acting on it, and that the force acting on a body is equal to the mass of the body multiplied by its acceleration. Once a system is in dynamic equilibrium a long term evolution is possible, driven by two-body relaxation. Systems such as galaxies and dark matter halos have a relaxation time much longer than the life of the universe and are thus considered collisionless. On the other hand, smaller systems such as globular and open clusters have a relaxation time shorter than their age, and are considered collisional.

In a collisionless system, the motion of the particles is determined by the overall gravitational potential of the system, and the n-body problem can be used to simulate the motion of the system. However, in a collisional system, the motion of the particles is also affected by collisions between particles. In this case, the Boltzmann equation can be used to study the dynamics of the system by describing the evolution of the PDF due to collisions.

Different numerical technique to guarantee maximum performance and accuracy such as Celestial mechanics and Dense stellar systems,. There are several methods that can be used to solve the n-body problem in Newtonian gravity, including:

1. Direct N-body methods: These methods solve the equations of motion for each particle directly, This method is the most accurate at the price of the longest computation time.
2. Tree codes: These methods use a hierarchical tree structure to divide the system into smaller sub-systems. This method is faster than direct N-body methods but less accurate.
3. Fast Multipole Methods: They use a hierarchical tree structure and a multipole expansion to approximate the gravitational potential of a large number of particles, reducing the number of interactions that need to be calculated, achieving a high level of accuracy.

Mean field methods are based on the idea of replacing the interactions between individual particles with a smooth, average potential that describes the overall behavior of the system.

Grid based solvers can take advantage of standard computational methods developed to solve partial differential equations, such as successive over-relaxation and conjugate gradient methods. However it requires solving a highly dimensional (6D+time) non-linear system of partial differential equations.

Fokker-Planck and Monte Carlo methods solve the collisional Boltzmann equation starting from a given distribution function.

In the presence of a strong gravitational field, such as that in the proximity of a black hole,N-body simulations cannot be based on Newtonian physics, General Relativity (GR) should be used instead.

An alternative approach to increase the efficiency of numerical solution of the N-body problem is to optimize the hardware. GRavityPipE concept has been extremely effective.

Another recent promising hardware development is the possibility to use Graphic Cards (GPUs) to carry out the cpu intensive force evaluation.