

HOUGH TRANSFORM

Lines

Image and parameter space

variables
 $y = mx + b$
parameters

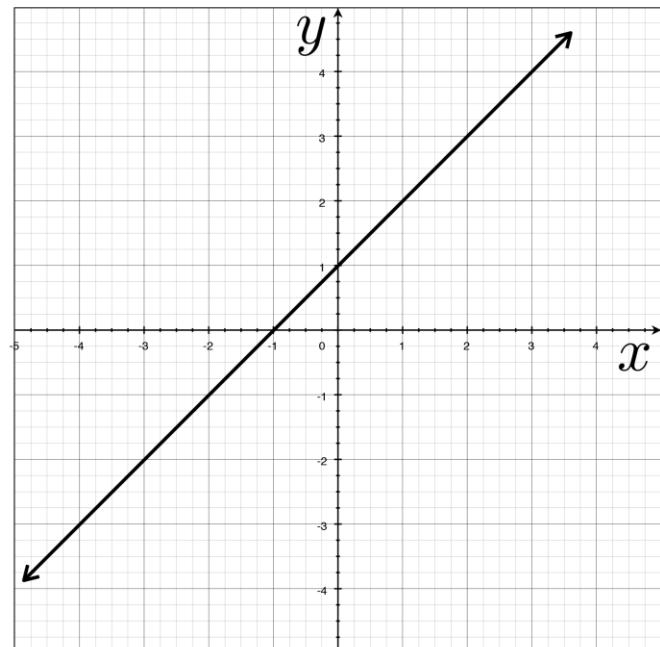
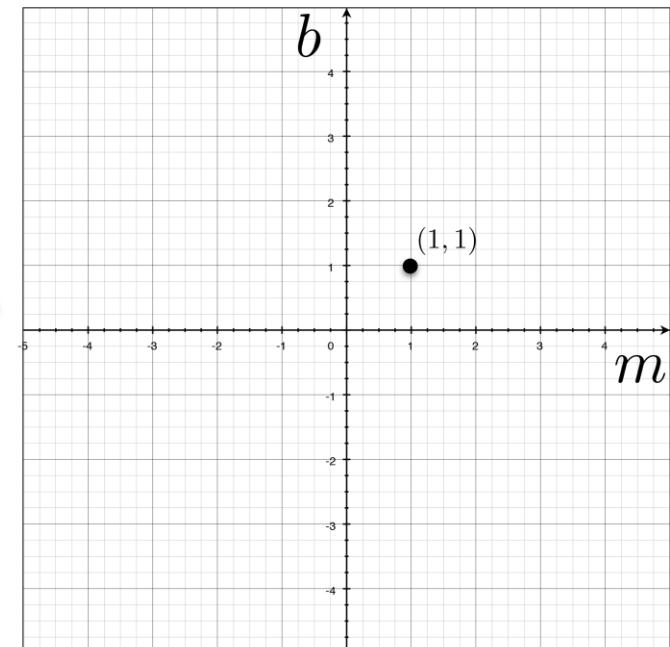


Image space

variables
 $y - mx = b$
parameters



Parameter space

a line
becomes
a point



Image and parameter space

$$y = mx + b$$

variables
parameters

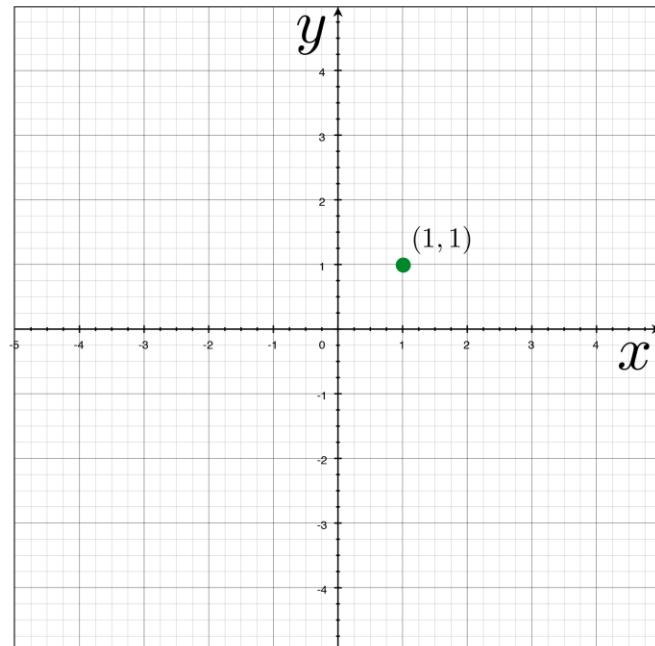
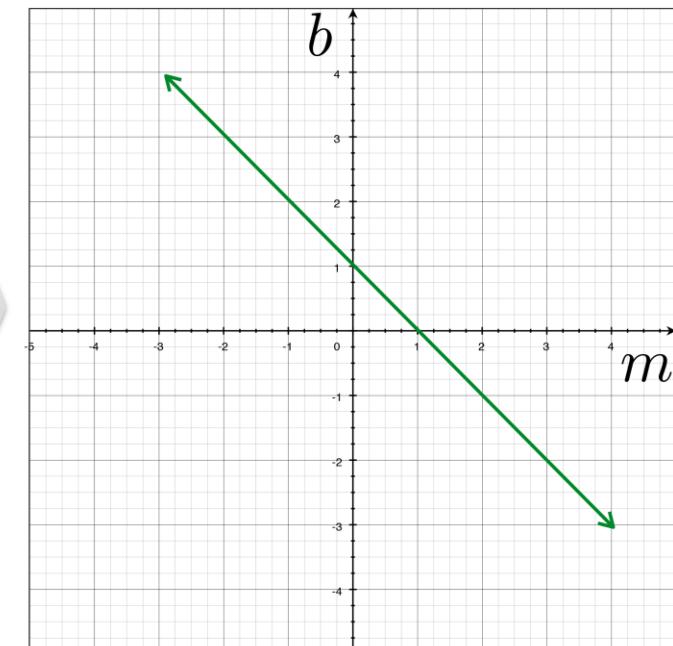


Image space

$$y - mx = b$$

variables
parameters



Parameter space

a point
becomes
a line



Image and parameter space

$$y = mx + b$$

variables
parameters

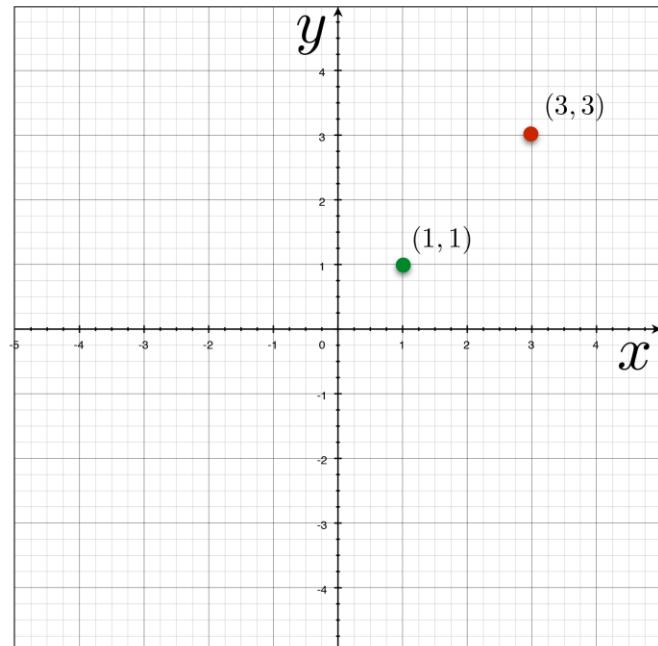
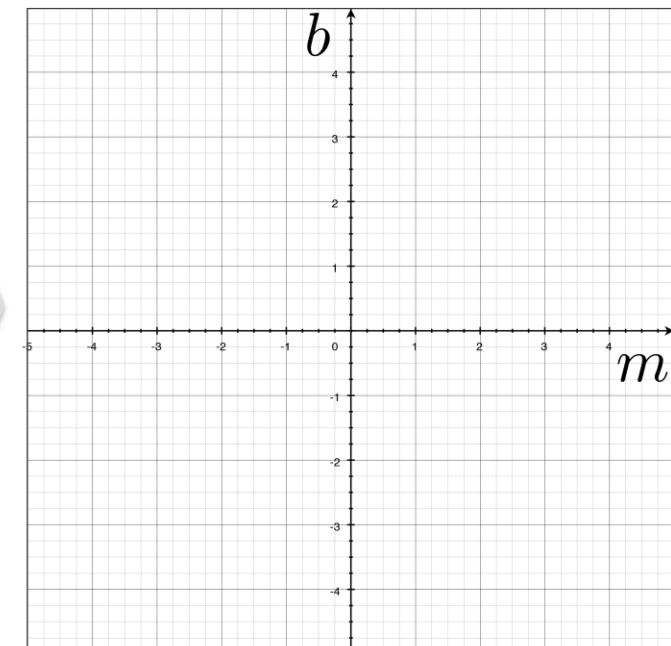


Image space

$$y - mx = b$$

variables
parameters



Parameter space

two points
become
?



Image and parameter space

$$y = mx + b$$

variables
parameters

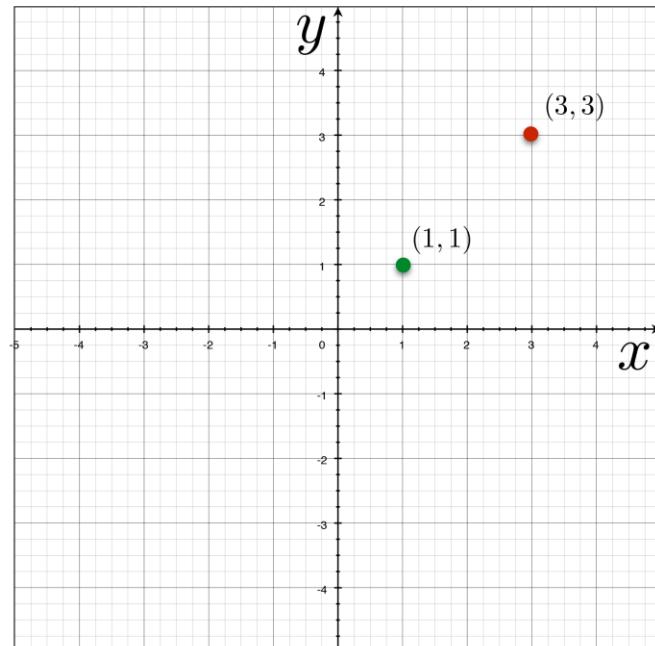
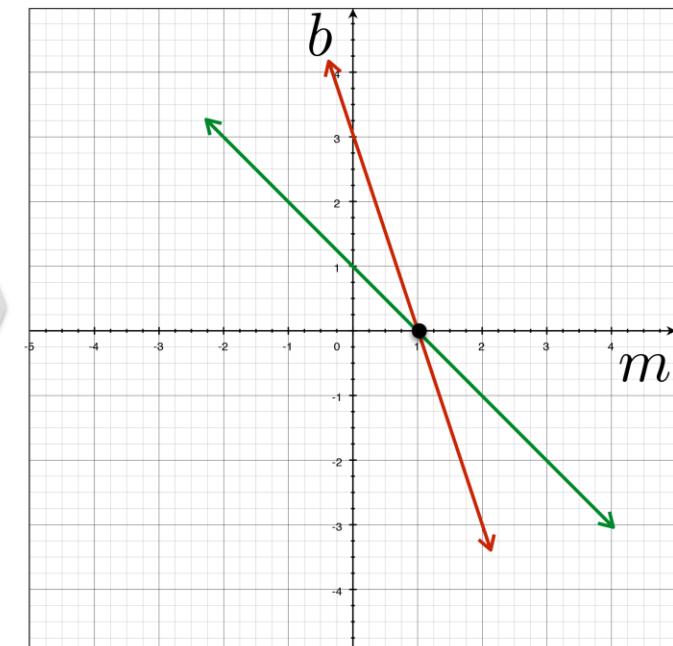


Image space

$$y - mx = b$$

variables
parameters



Parameter space

two points
become
?



Image and parameter space

$$y = mx + b$$

variables
parameters

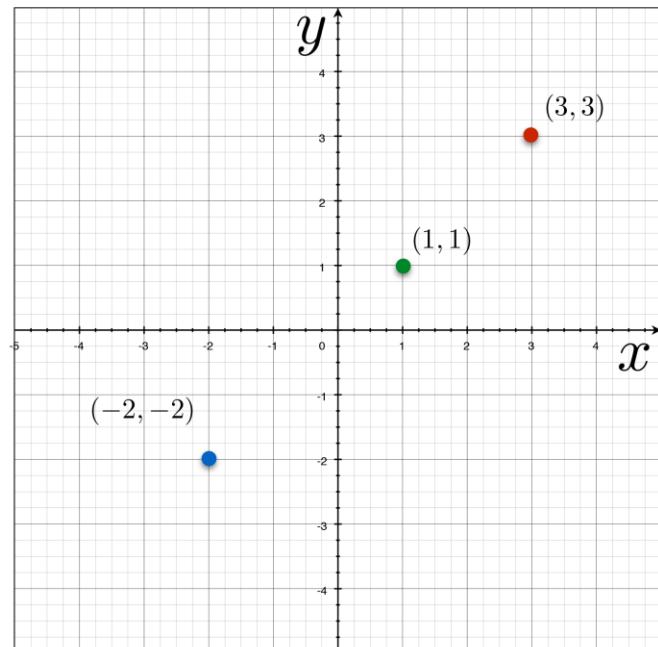
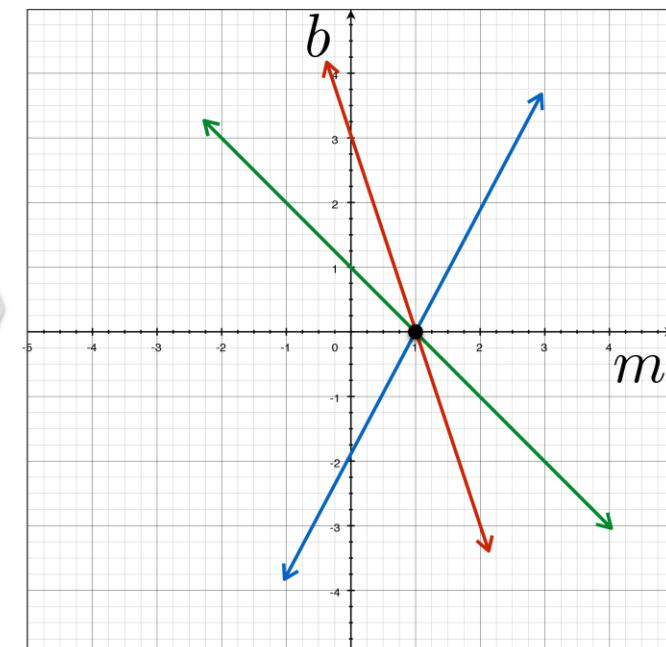


Image space

$$y - mx = b$$

variables
parameters



Parameter space

three points
become
?



Image and parameter space

variables
 $y = mx + b$
parameters

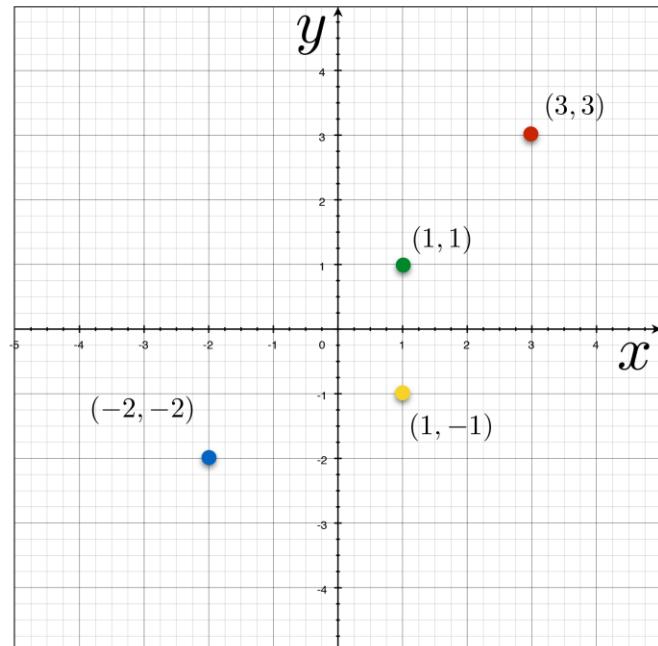
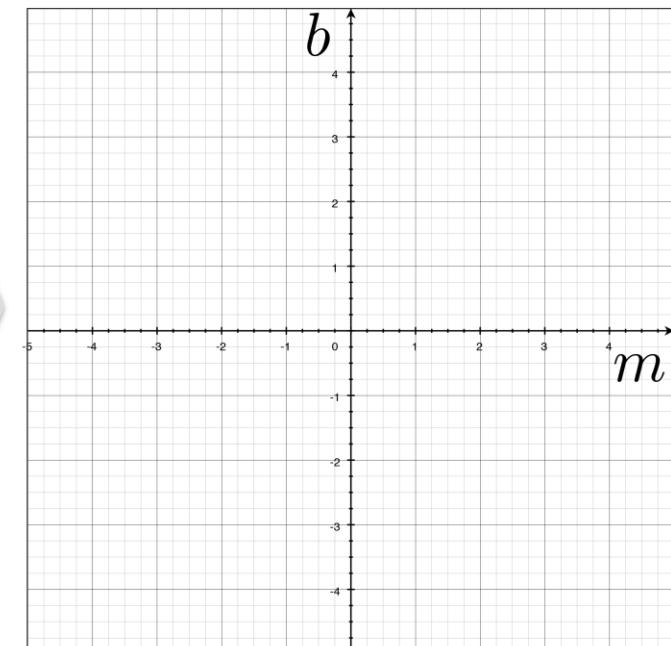


Image space

variables
 $y - mx = b$
parameters



Parameter space

four points
become
?



Image and parameter space

$$y = mx + b$$

variables
parameters

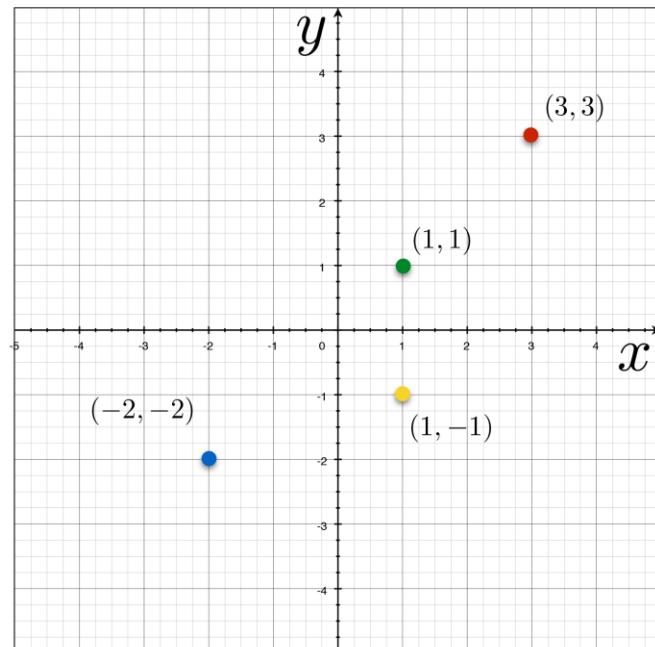
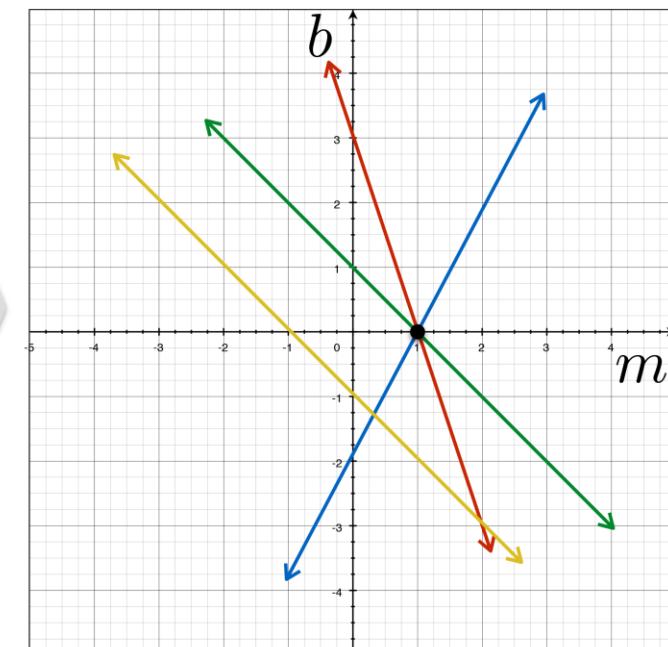


Image space

$$y - mx = b$$

variables
parameters



Parameter space

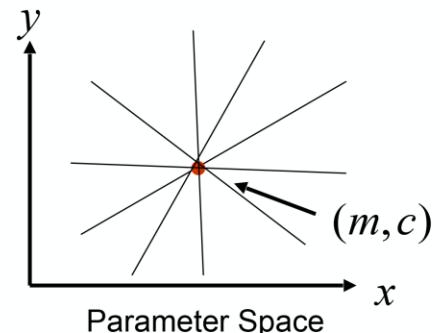
four points
become
?



Line Detection by Hough Transform

Algorithm:

1. Quantize Parameter Space (m, c)
2. Create Accumulator Array $A(m, c)$
3. Set $A(m, c) = 0 \quad \forall m, c$
4. For each image edge (x_i, y_i)
For each element in $A(m, c)$
If (m, c) lies on the line: $c = -x_i m + y_i$
Increment $A(m, c) = A(m, c) + 1$
5. Find local maxima in $A(m, c)$



A(m, c)								
1								1
	1						1	
		1			1			
			2					
		1		1				
	1					1		
1							1	



PARAMETER SPACE PROBLEMS

1. What is M and B ranges?
 1. M
 1. $-\infty \leq M \leq \infty$
 2. The space is infinite !
 3. 90 degrees angle slope.



Better Parameterization

Use normal form:

$$x \cos \theta + y \sin \theta = \rho$$

Given points (x_i, y_i) find (ρ, θ)

Hough Space Sinusoid

$$0 \leq \theta \leq 2\pi$$

$$0 \leq \rho \leq \rho_{\max}$$

(Finite Accumulator Array Size)

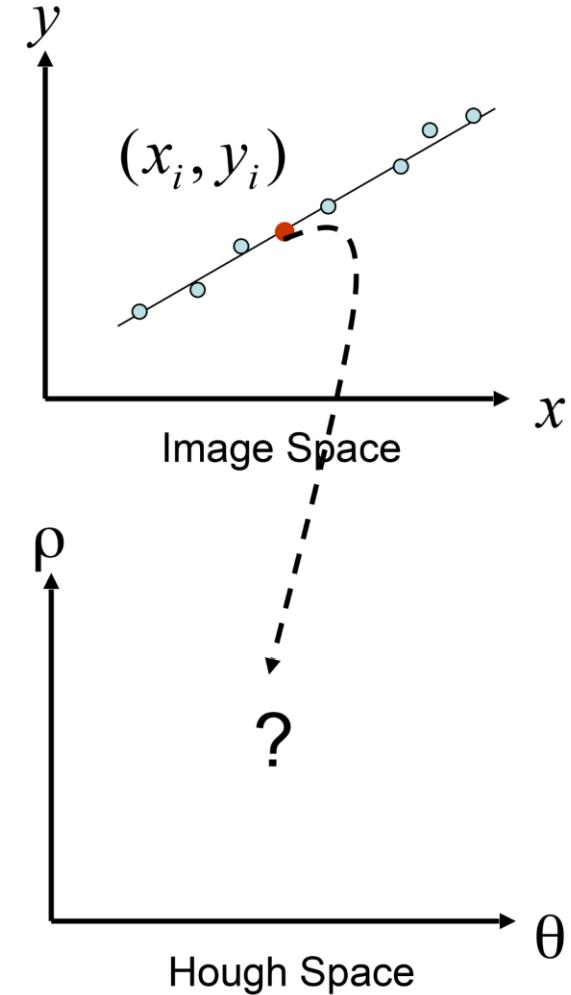


Image and parameter space

$$y = mx + b$$

variables
parameters

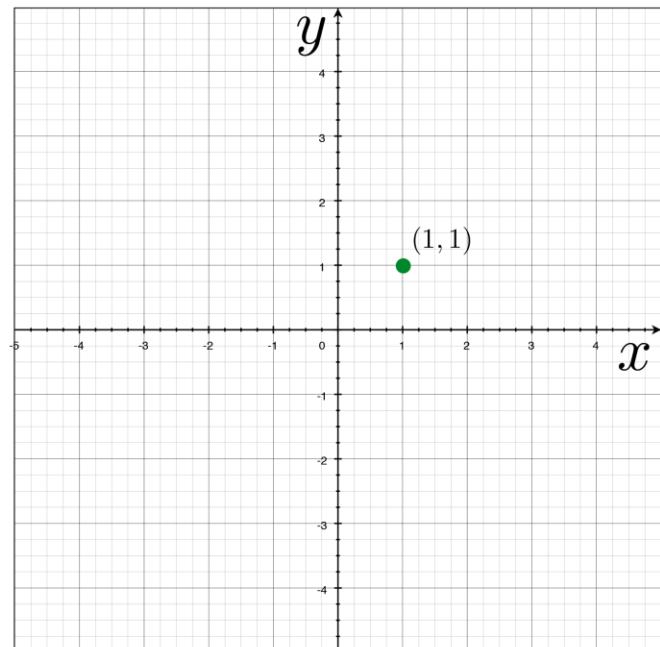
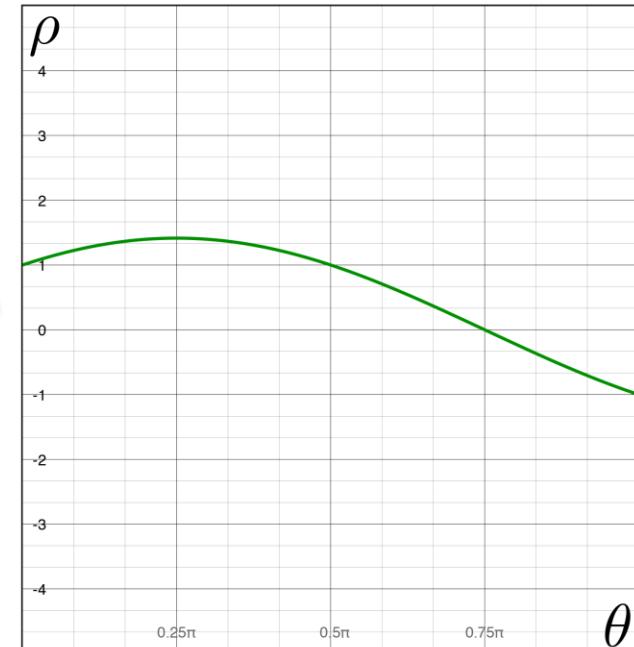


Image space

$$x \cos \theta + y \sin \theta = \rho$$

parameters
variables



Parameter space



Image and parameter space

$$y = mx + b$$

variables
parameters

$$x \cos \theta + y \sin \theta = \rho$$

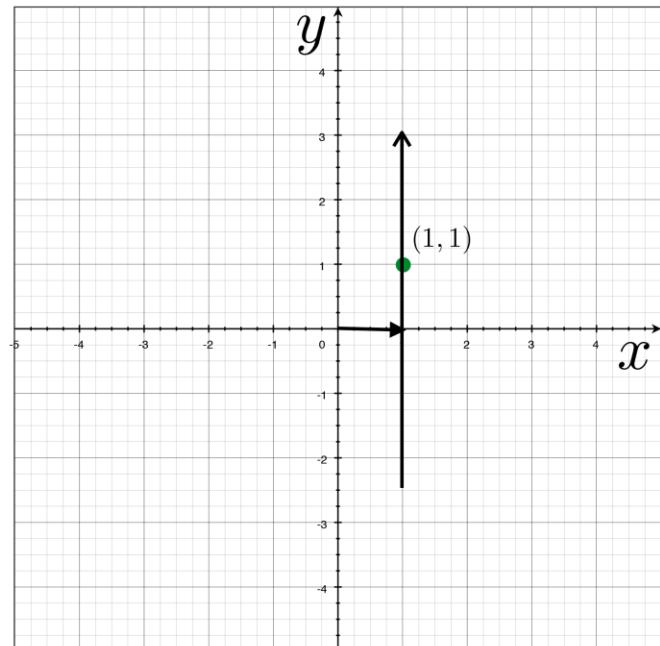
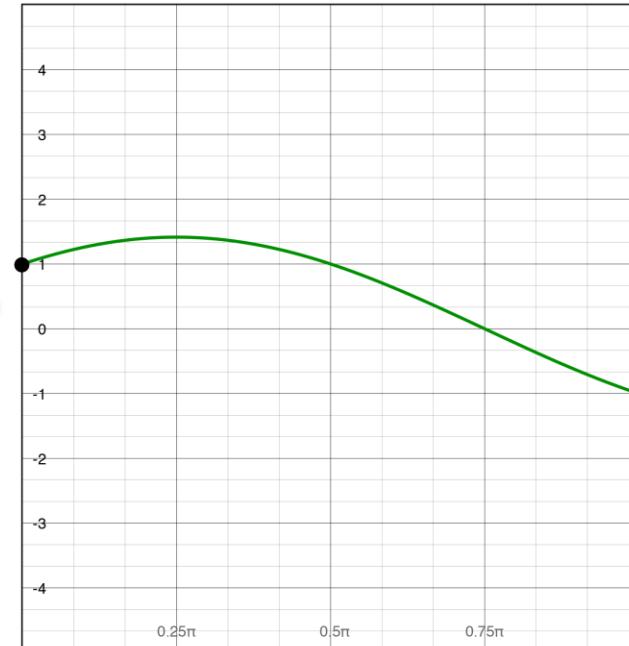


Image space

a line
becomes
a point

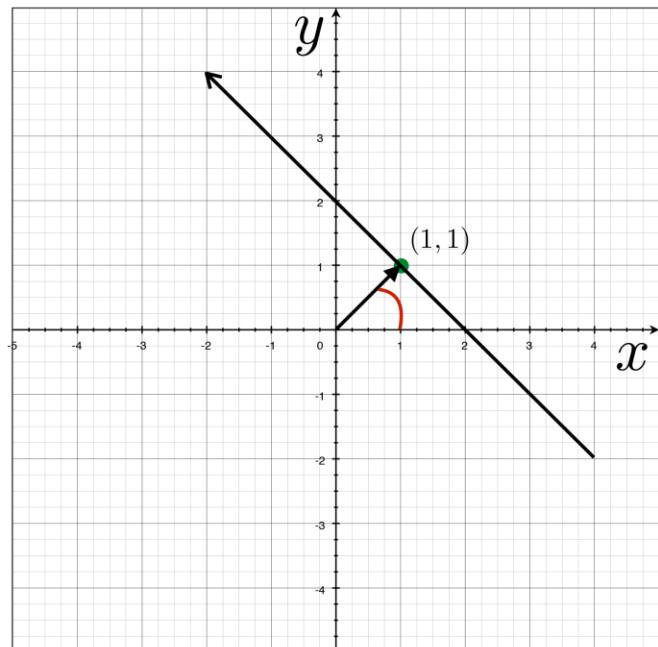


Parameter space



Image and parameter space

variables
 $y = mx + b$
parameters



a line
becomes
a point

$$x \cos \theta + y \sin \theta = \rho$$

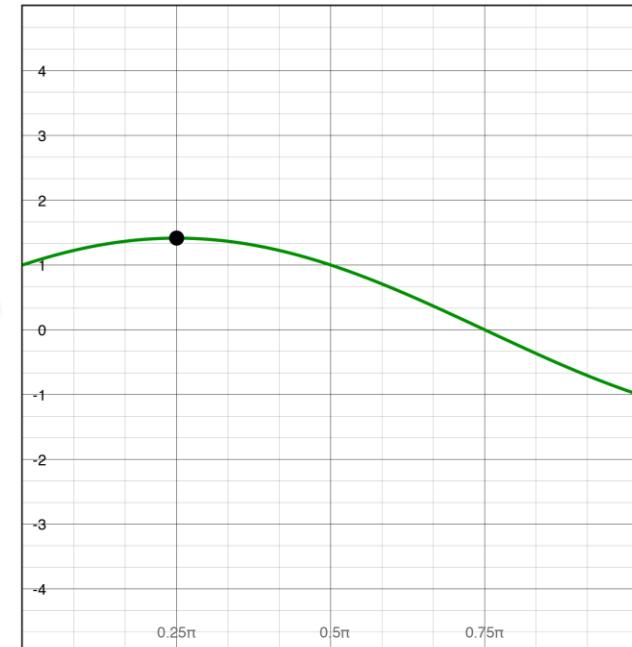


Image space

Parameter space



Image and parameter space

$$y = mx + b$$

variables
parameters

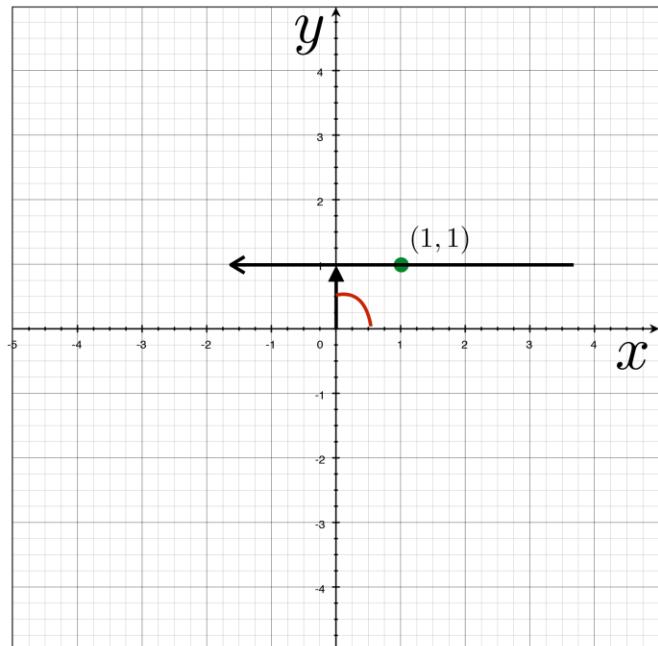
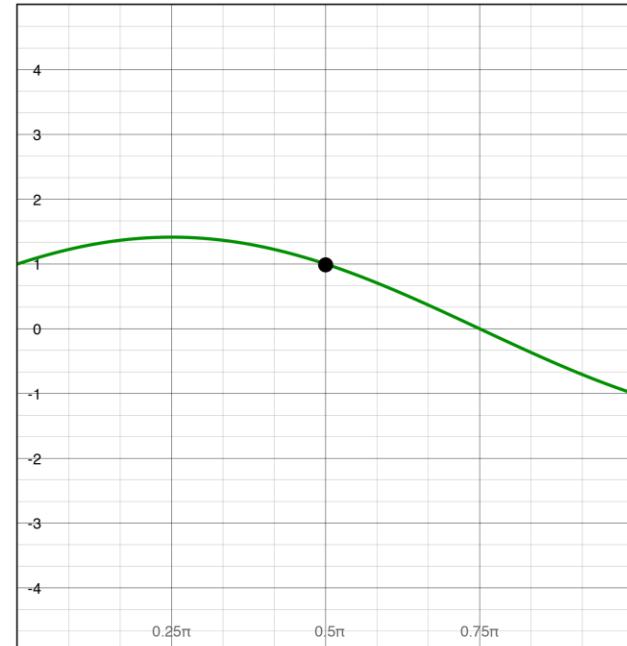


Image space

$$x \cos \theta + y \sin \theta = \rho$$

a line
becomes
a point



Parameter space



Image and parameter space

$$y = mx + b$$

variables
parameters

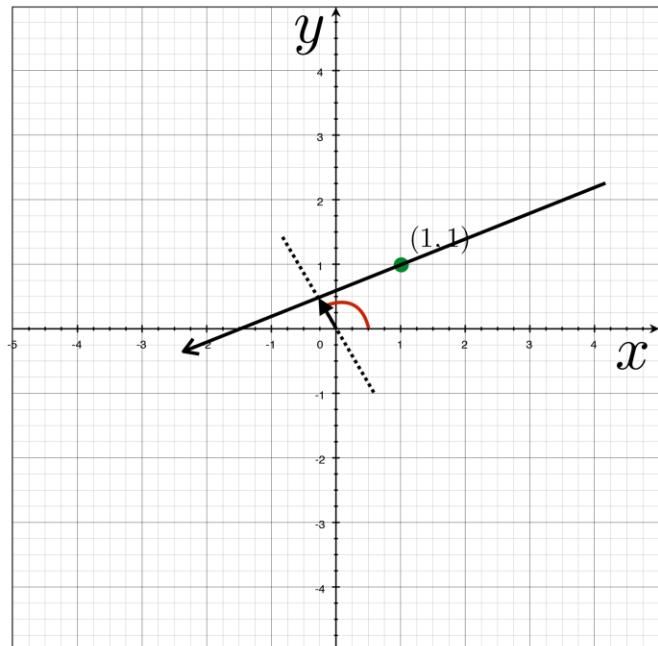
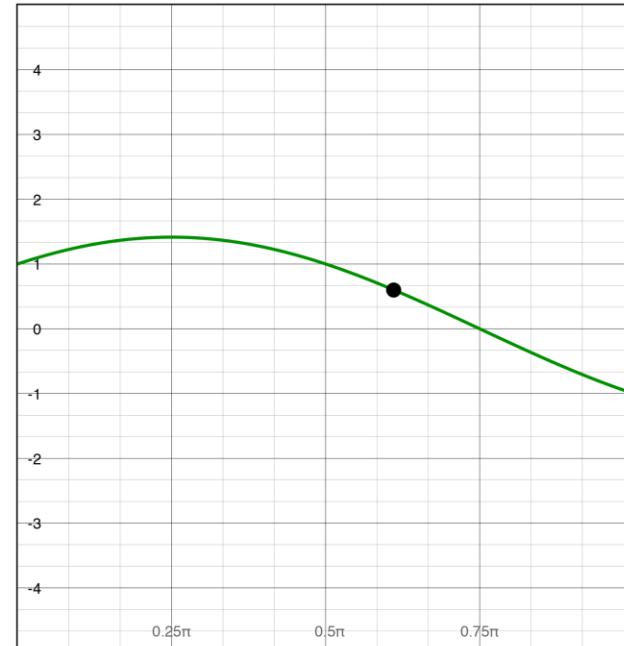


Image space

$$x \cos \theta + y \sin \theta = \rho$$

a line
becomes
a point



Parameter space



Image and parameter space

variables
 $y = mx + b$
parameters

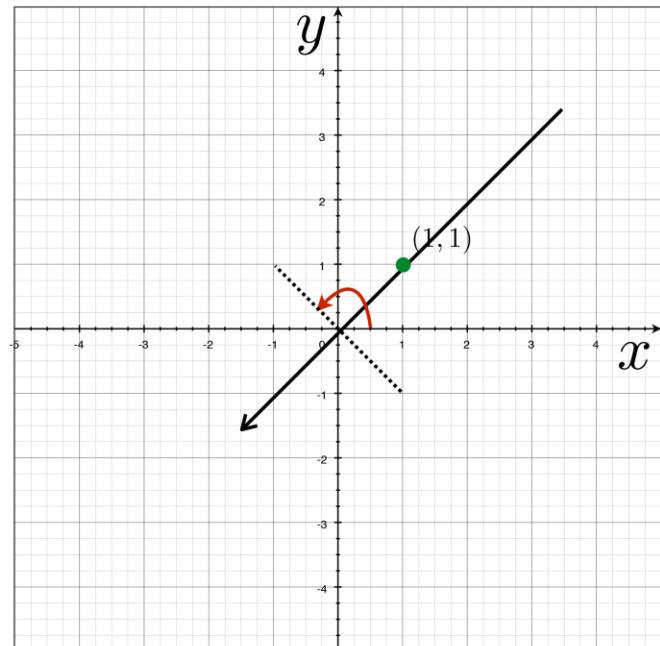
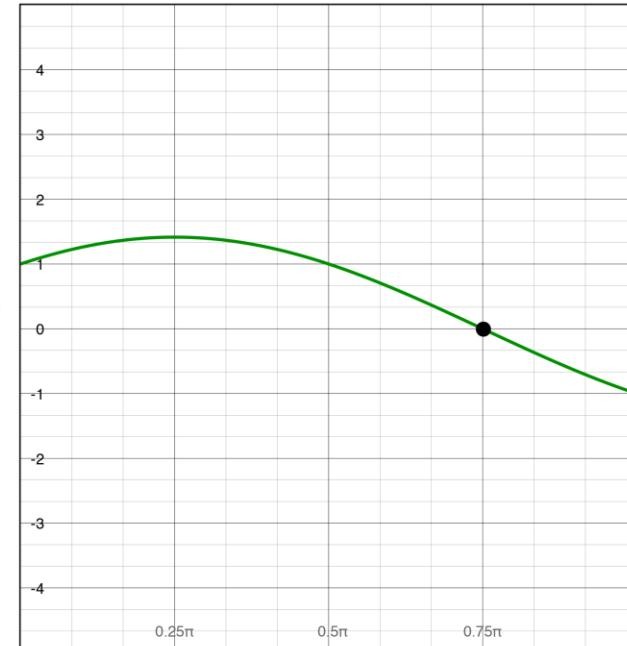


Image space

$$x \cos \theta + y \sin \theta = \rho$$

a line
becomes
a point



Parameter space



Image and parameter space

variables
 $y = mx + b$
parameters

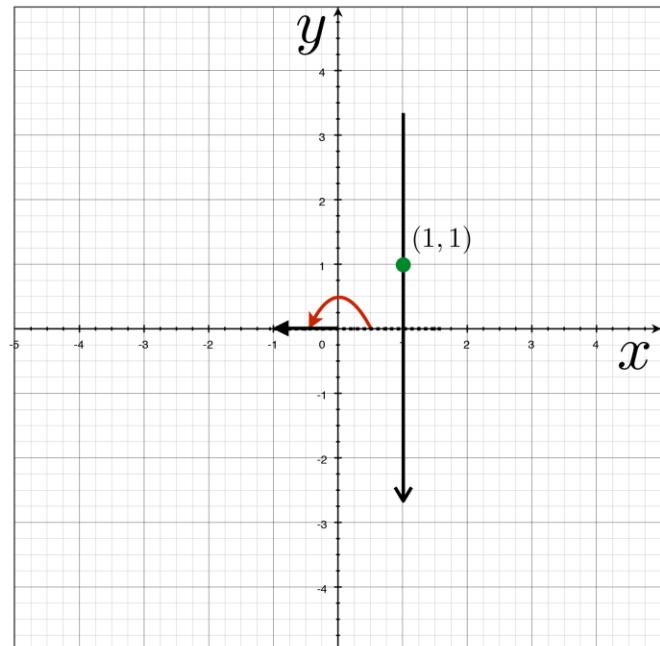
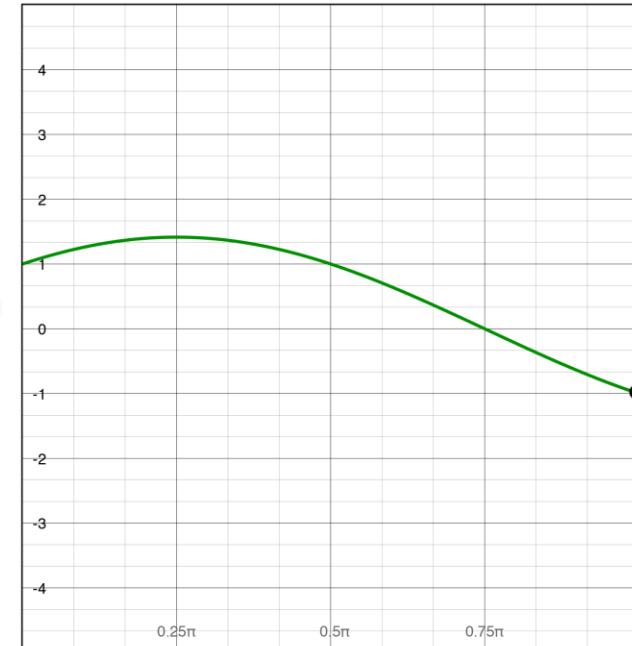


Image space

$$x \cos \theta + y \sin \theta = \rho$$

a line
becomes
a point



Parameter space



Image and parameter space

variables
 $y = mx + b$
parameters

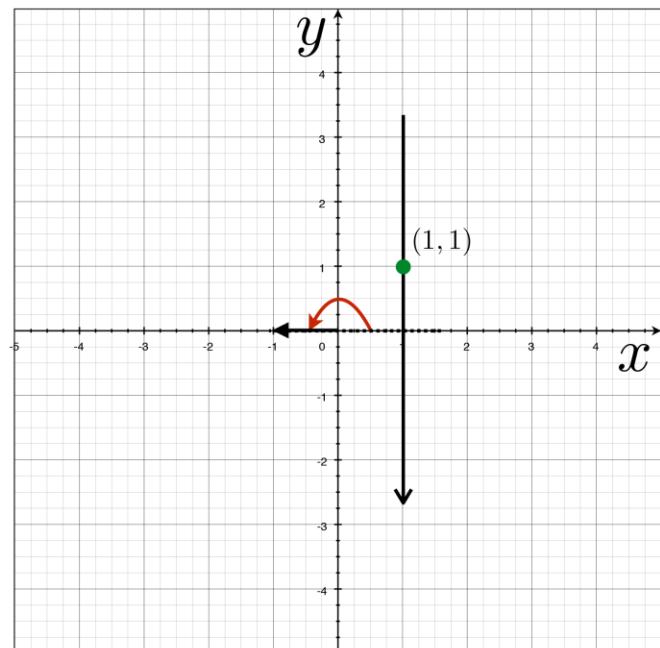
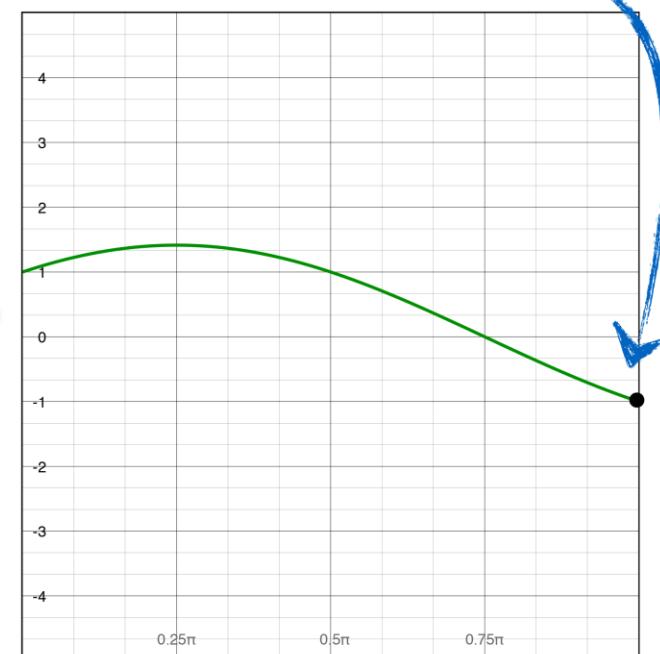


Image space

$$x \cos \theta + y \sin \theta = \rho$$

Wait ...why is rho negative?



Parameter space



Image and parameter space

variables
 $y = mx + b$
parameters

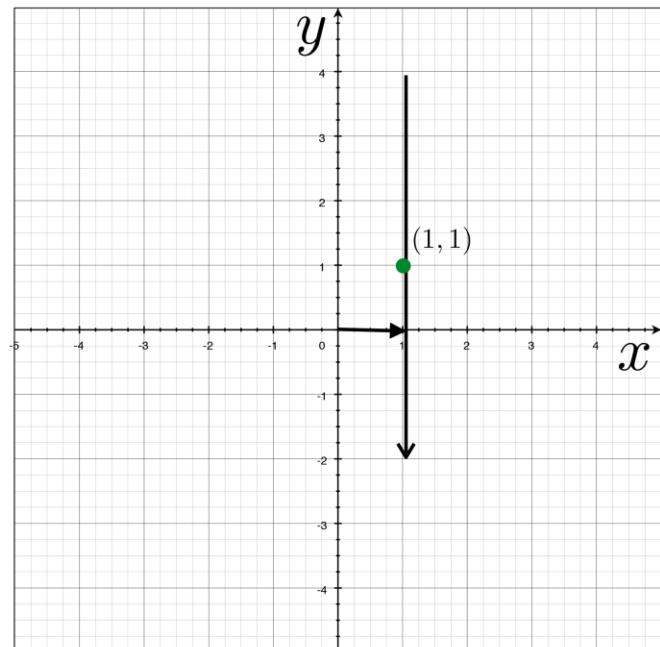
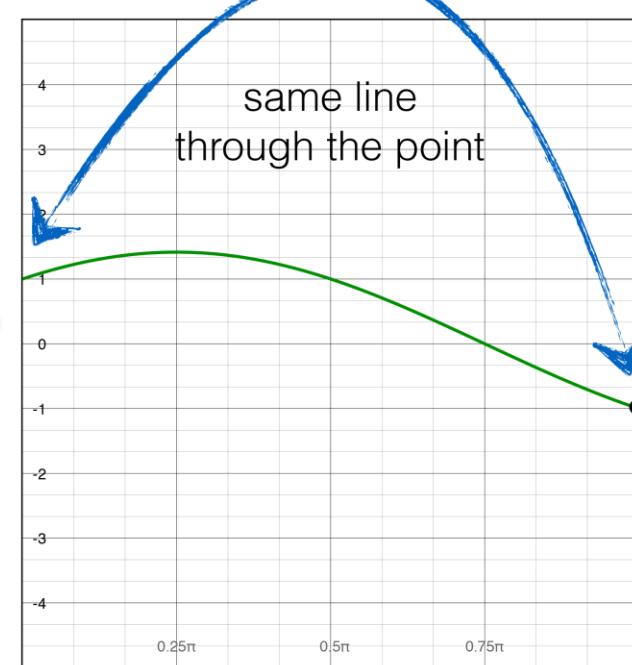


Image space

$$x \cos \theta + y \sin \theta = \rho$$

a line
becomes
a point



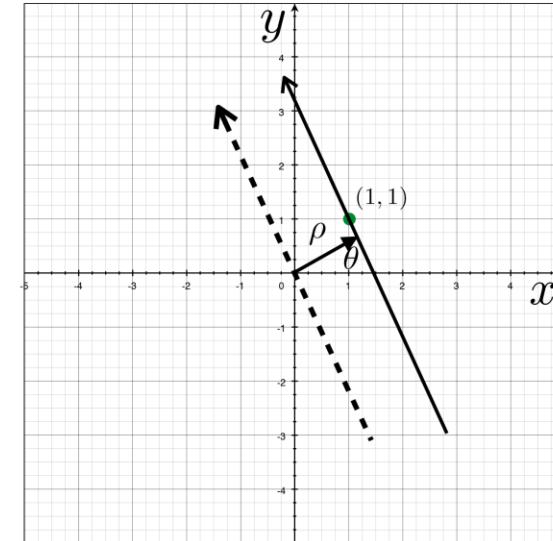
Parameter space



There are two ways to write the same line:

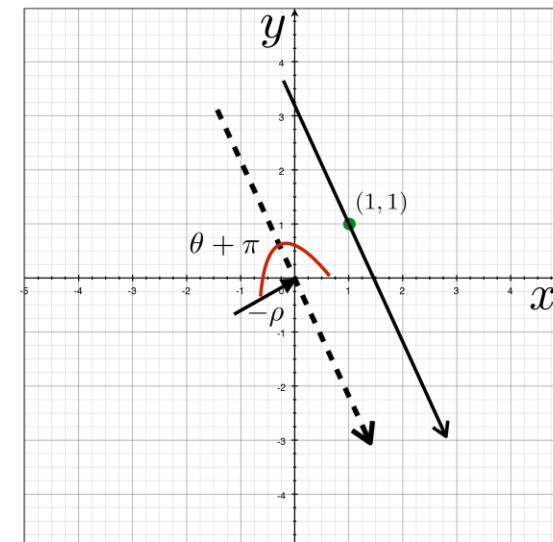
Positive rho version:

$$x \cos \theta + y \sin \theta = \rho$$



Negative rho version:

$$x \cos(\theta + \pi) + y \sin(\theta + \pi) = -\rho$$



Recall:

$$\sin(\theta) = -\sin(\theta + \pi)$$

$$\cos(\theta) = -\cos(\theta + \pi)$$



Image and parameter space

variables
 $y = mx + b$
parameters

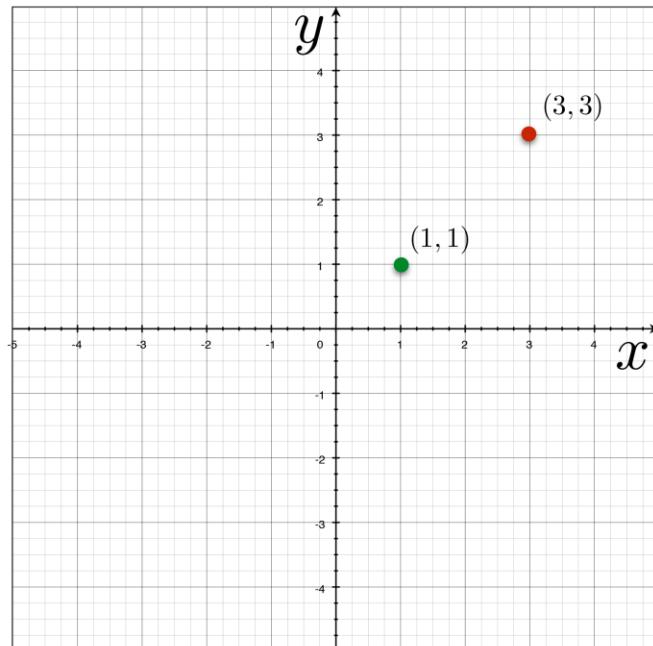
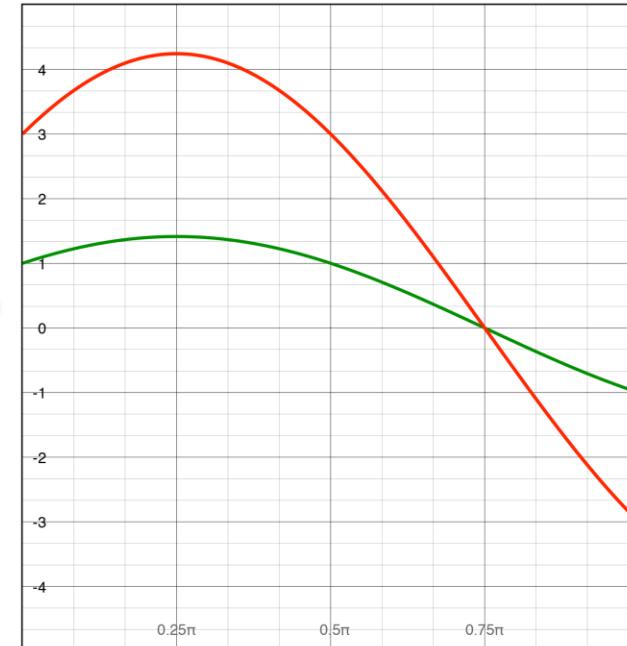


Image space

two points
become
?



Parameter space



Image and parameter space

variables
 $y = mx + b$
parameters

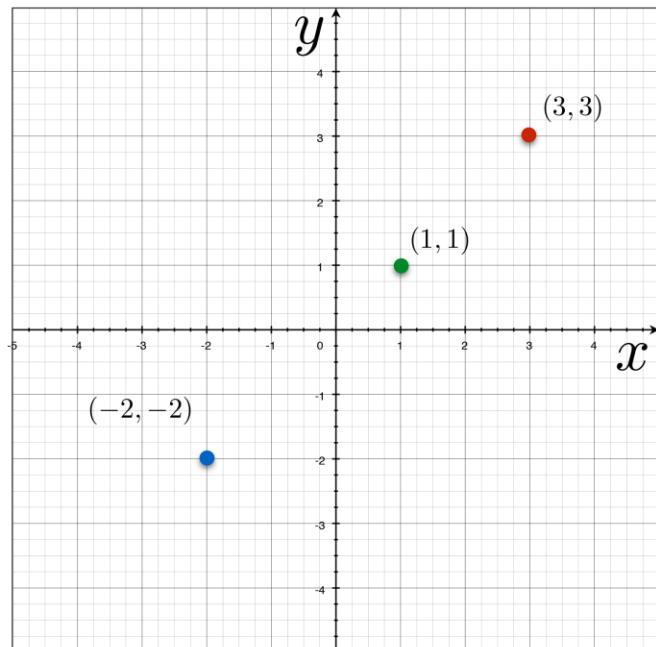
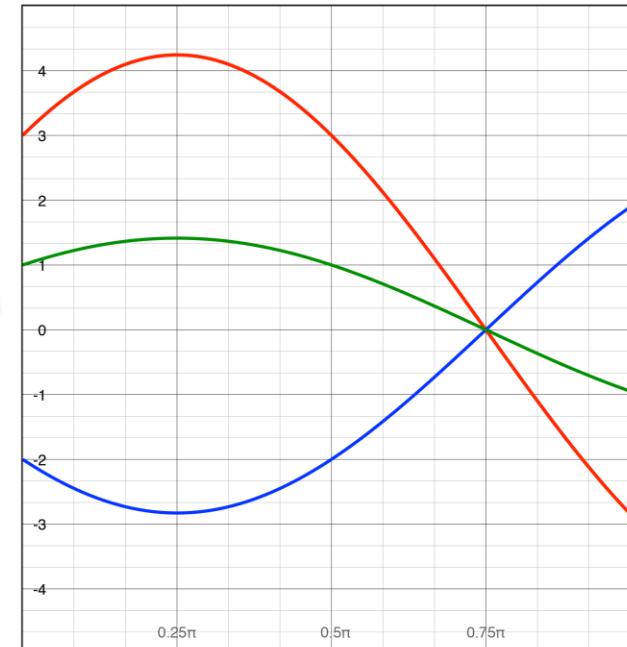


Image space

three points
become
?



Parameter space



Image and parameter space

variables
 $y = mx + b$
parameters

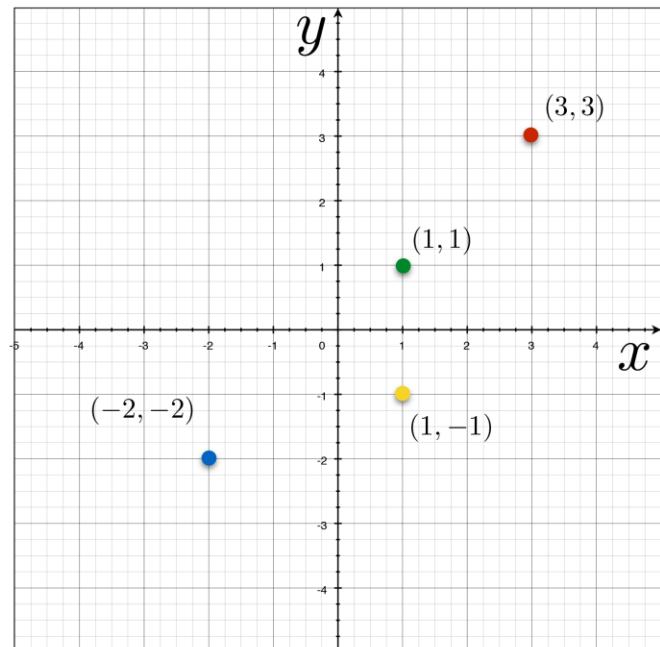


Image space

four points
become
?



Parameter space



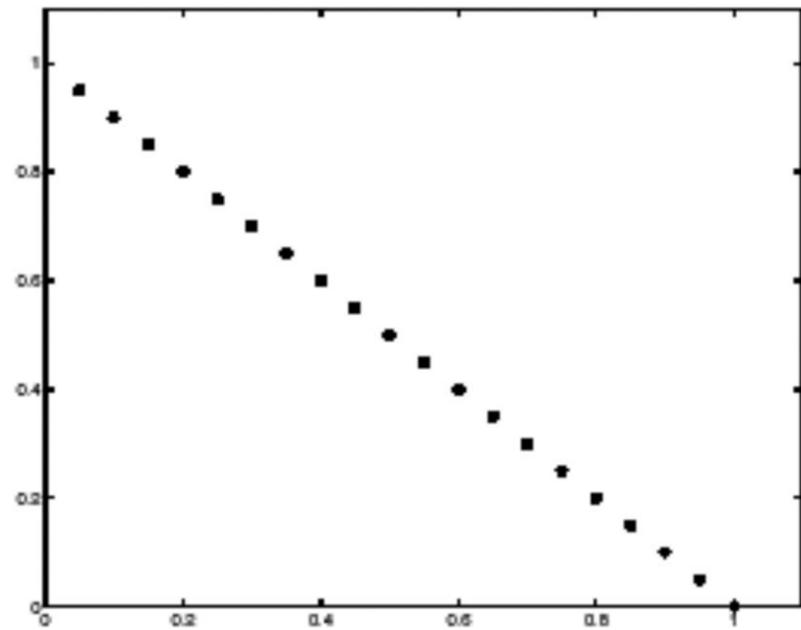
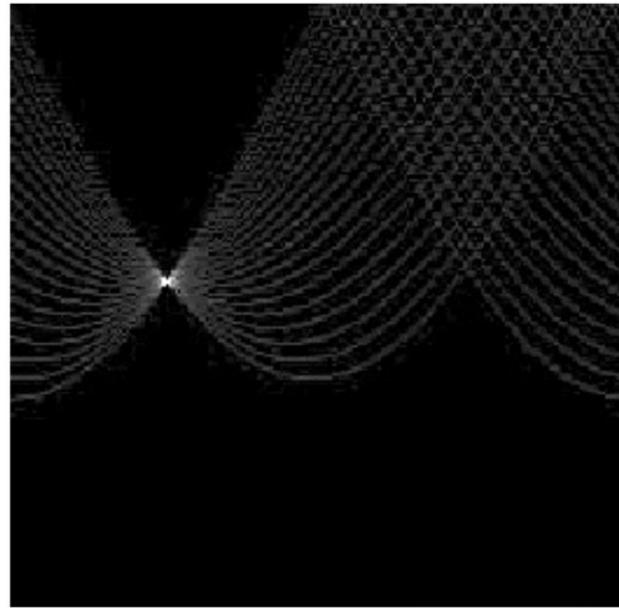


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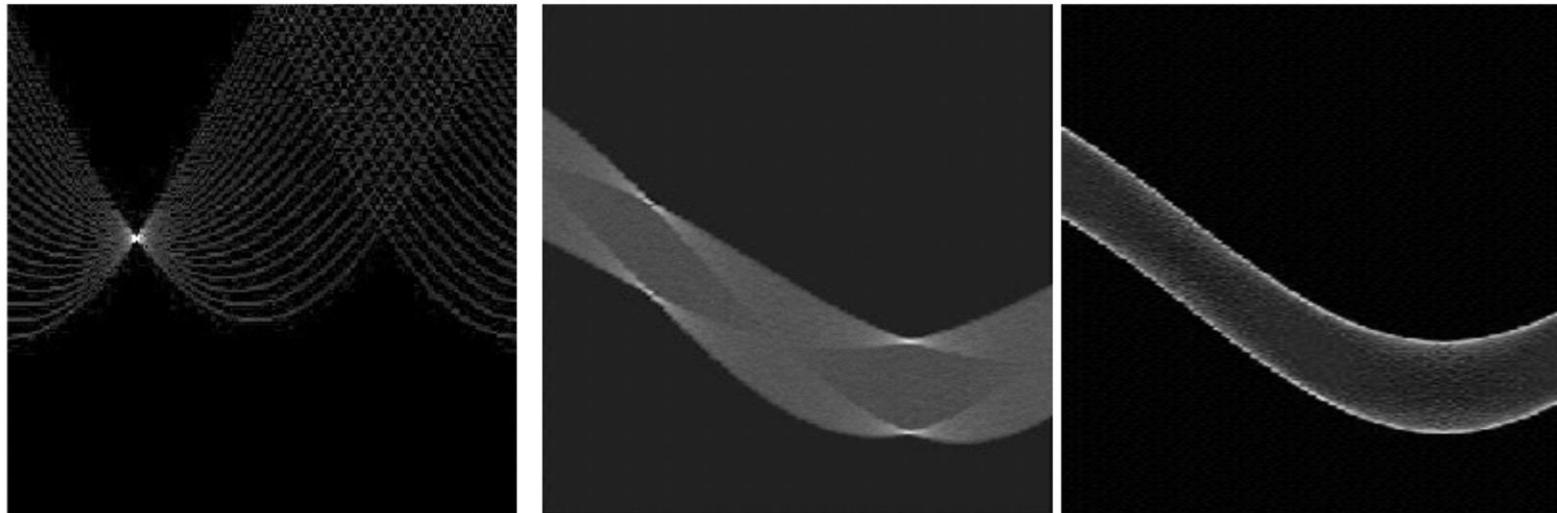


Votes



Basic shapes

(in polar parameter space)



can you guess the shape?



Thank You

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