Data Analytics
EEE 4774 & 6777

Module 4 - Classification

Generative Models

Spring 2022

Classification Problem & Approaches

input $\boldsymbol{x}_n$ → Classification Algorithm → output $\hat{C}_n = [0,1,0,\dots,0]$

1. Generative Models: (Likelihood, Prior, Posterior) modeling to first obtain likelihood and prior, and then posterior

$$p(C_n|\boldsymbol{x}_n) = \frac{p(\boldsymbol{x}_n|C_n)p(C_n)}{p(\boldsymbol{x}_n)}$$

2. Discriminative Models: (Posterior) modeling to directly discriminate classes

$$p(C_n|\boldsymbol{x}_n) = y(\boldsymbol{x}_n) = f(\boldsymbol{w}^T\boldsymbol{x}_n + w_0)$$

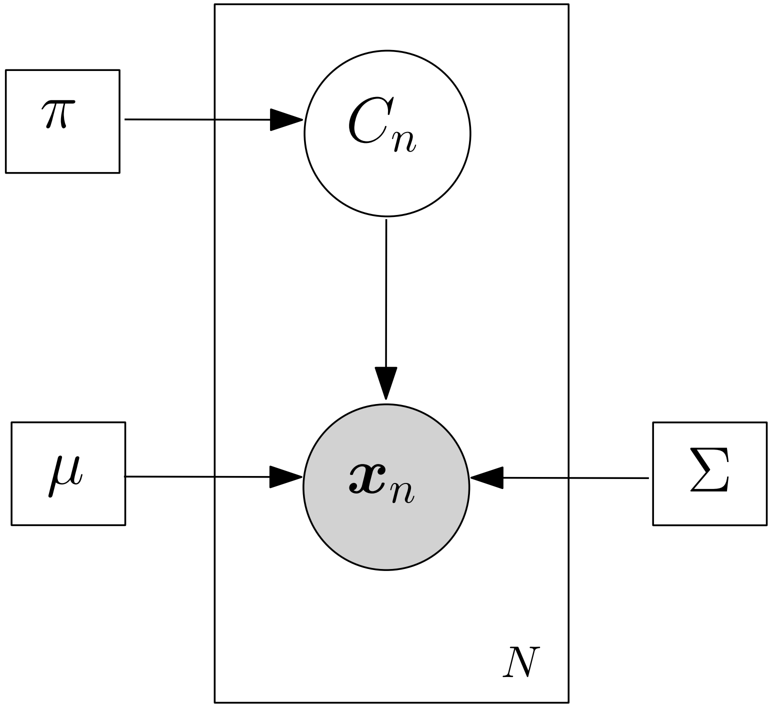
Generative Models

$$p(C_n|\boldsymbol{x}_n) = \frac{p(\boldsymbol{x}_n|C_n)p(C_n)}{p(\boldsymbol{x}_n)}$$

$$\max p(C_n|\boldsymbol{x}_n) = \max p(\boldsymbol{x}_n|C_n)\, p(C_n)$$

e.g.,
Gaussian mixture model,
Naïve Bayes,

<u>Deep Generative Models:</u>
Variational autoencoder (VAE),
Generative adversarial network (GAN)



Naïve Bayes

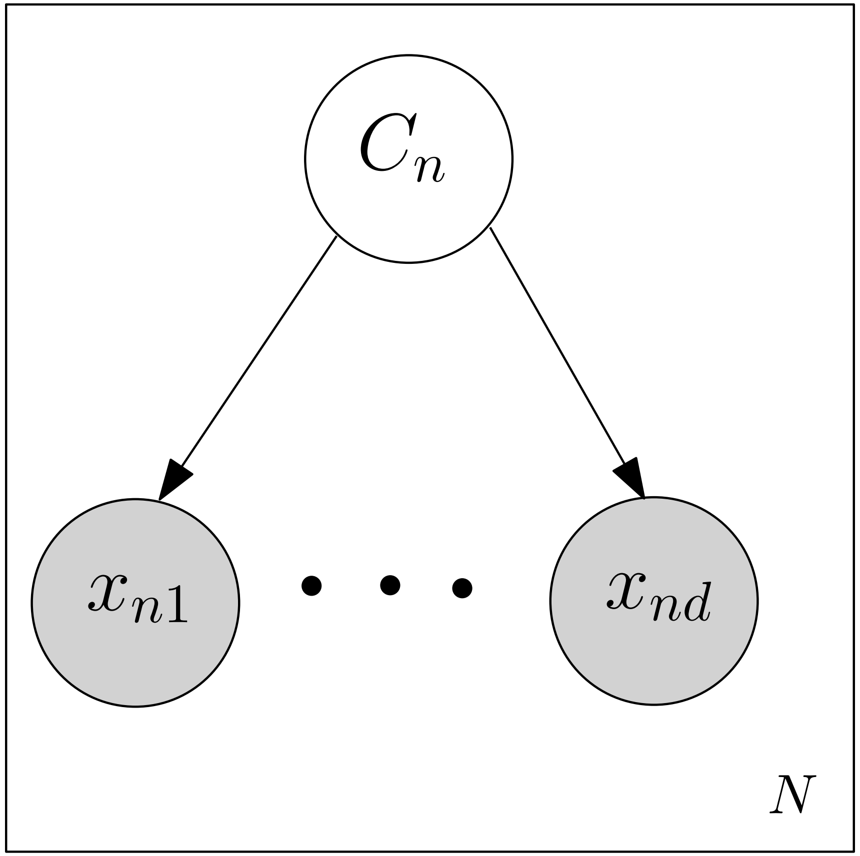
- Assumption: Conditioned on the class C_n input variables $x_{n1}, \dots, x_{nd}$ are independent

$$p(\boldsymbol{x}_n|C_n) = \prod_{i=1}^{d} p(x_{ni}|C_n)$$

$$p(\boldsymbol{x}_n) \neq \prod_{i=1}^{d} p(x_{ni})$$

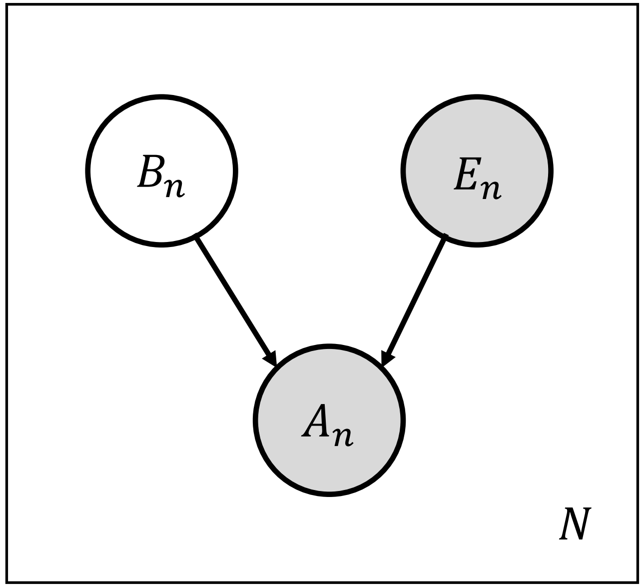
- Simplified analysis: Univariate models instead of multivariate (esp. for large d)

- Limitation: Assumption does not hold in general



Example: Burglary-Alarm-Earthquake

Sally's burglar Alarm is sounding.
Has she been Burgled, or was the alarm triggered by an Earthquake?



Training & Model Building

Fit a Bernoulli distribution for each alarm case (4 cases) using training data

$$p(A|B, E)$$

Alarm $= 1$	Burglar	Earthquake
0.9999	1	1
0.99	1	0
0.99	0	1
0.0001	0	0

Prior probabilities for Burglary and Earthquake estimated from training data

$$p(B = 1) = 0.01 \text{ and } p(E = 1) = 0.000001$$

Testing

$$p(B=1|A=1,E=0) = \frac{p(B=1,A=1,E=0)}{\sum_B p(B,A=1,E=0)} \approx 0.99$$

$$p(B=1|A=1,E=1) = \frac{p(B=1,A=1,E=1)}{\sum_B p(B,A=1,E=1)} \approx 0.01$$

$$\begin{aligned} p(B=1|A=1) = p(B=1|A=1,E=1)p(E=1) \\ +p(B=1|A=1,E=0)p(E=0) \\ \approx 0.99 \end{aligned}$$

Earthquake explains away the fact that alarm is ringing.