Data Analytics
EEE 4774 & 6777

Module 4 - Classification

Anomaly Detection

Spring 2022

Not a standard classification problem

- Classification is a supervised learning problem, i.e., there are samples from each class to train on.
- Although anomaly vs. no-anomaly (nominal) decision looks like a binary classification problem, anomaly detection is inherently not a supervised learning problem.
- The “anomaly class” is, by definition, an open-ended set – we want to detect any pattern that doesn’t look nominal,
  - e.g., cyberattack, traffic accident, robbery, device failure, fraudulent transaction, tumor, etc.
- Even if we may have some training data from a type of anomaly, we still want to detect unseen anomaly types.
- Hence, in training we typically see only nominal data (semi-supervised learning) or a dataset which may contain some anomalies (unsupervised learning)

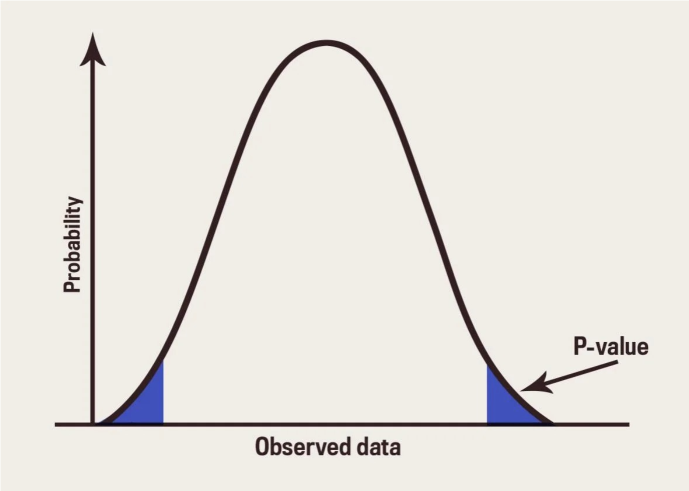
Generative Models

- p-value: probability of seeing a data instance as extreme as or more extreme than the observed one (the sum of tail probabilities)

  e.g., Gaussian generative (likelihood) model

- A test instance can be declared an anomaly/outlier if

its p-value is less than a critical threshold, e.g., 0.05, 0.01, 0.001, etc.



Assumption: Training data is clean of anomalies

Training: fit a suitable likelihood model to (nominal) training data

Testing: compute the likelihood of the test instance under the trained model and compare it with the threshold to decide.

Example: Biased coin detection

- Fitting binomial distribution to historical nominal data from unbiased coin give the heads (success) probability as $\theta = 0.5$
- Test data: 9 heads and 3 tails in 12 tosses

$$\text{One-sided p-value: } p = \sum_{h=9}^{12} \binom{12}{h} 0.5^h 0.5^{12-h} = \sum_{h=9}^{12} \binom{12}{h} 0.5^{12} = 0.073$$

$$\text{Two-sided p-value: } p = \sum_{h=9}^{12} \binom{12}{h} 0.5^{12} + \sum_{h=0}^{3} \binom{12}{h} 0.5^{12} = 0.073 + 0.073 = 0.146$$

- Both one-sided and two-sided p-values are higher than the commonly-used statistical significance level (threshold) of 0.05, so we can conclude that the coin used to obtain the test data is **nominal** (unbiased)
- However, there is a fundamental issue with p-value tests! See the following formulation of the same problem.

Example: Biased coin detection

- Consider a different generative model for the observed data: keep tossing the coin until 3 tails are observed.
- In this case, instead of binomial distribution, the most appropriate generative model that explains the experiment is the negative binomial distribution with pmf $\binom{N-1}{f-1}\theta^{s}(1-\theta)^{f}$
- Test data: 9 heads and 3 tails in 12 tosses

$$p = \sum_{h=9}^{\infty}\binom{3+h-1}{3-1}0.5^{h}0.5^{3} = \sum_{h=9}^{\infty}\binom{2+h}{2}0.5^{3+h} = 0.0327$$

- Interestingly, in this case, the p-value is smaller than the commonly-used statistical significance level (threshold) of 0.05.
- For the same dataset, we can now decide for an anomaly (biased coin) using the same threshold!
- Beware of threshold selection! Fixed thresholds do not work for every model.

Threshold Selection

- In binary classification, threshold determines the fundamental trade-off between two conflicting objectives:
  - Maximizing True Positive Rate (TPR), i.e., true alarms
    $$TPR = \frac{\#\, true\ positives}{\#\, positives}$$
  - Minimizing False Positive Rate (FPR), i.e., false alarms
    $$FPR = \frac{\#\, false\ positives}{\#\, negatives}$$
- Decreasing threshold will increase TPR but also increase FPR
- Increasing threshold will decrease FPR but also decrease TPR
- While setting the threshold the goal is to set a desired balance between TPR and FPR.
- A practical way to do that is to choose the lowest threshold that satisfies a false alarm constraint (FPR), e.g., 0.01

Performance Evaluation

- Binary classification metrics TPR and FPR commonly used to evaluate the anomaly detection performance.
- ROC (receiver operating characteristic) curve plots TPR vs. FPR to show this trade-off for different threshold values
- AUC (area under the ROC curve) gives a scalar metric to compare detectors – higher AUC means better detector

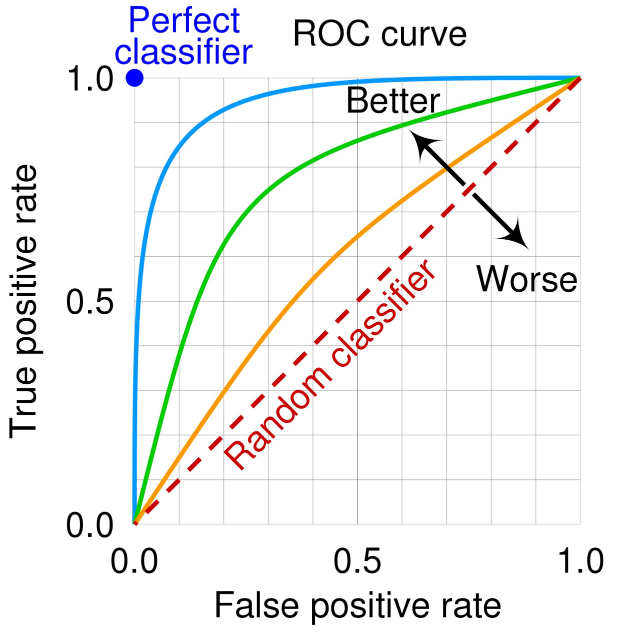
$$AUC \in (0,1)$$

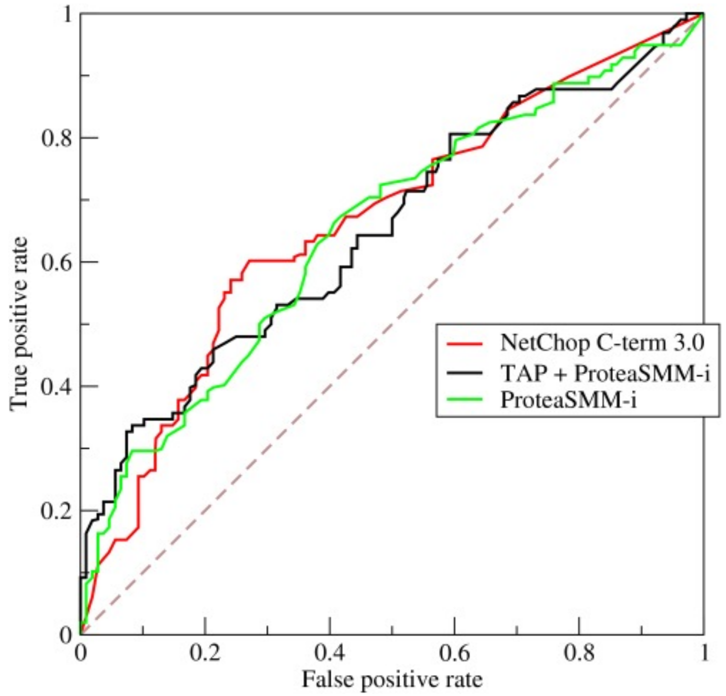
- F1 score similarly gives a scalar performance metric. It deals better with class imbalance.

$$F1 = 2 \times \frac{Precision \times Recall}{Precision + Recall}$$

$$Precision = \frac{\#\ true\ positives}{\#\ true\ positives + \#\ false\ positives} = \frac{\#\ true\ alarms}{\#\ all\ alarms}$$

$$Recall = TPR$$





Other methods for anomaly detection

- kNN, sklearn.neighbors.LocalOutlierFactor (scikit-learn)
- One-Class SVM, svm.OneClassSVM (scikit-learn)
- Isolation Forest, ensemble.IsolationForest (scikit-learn)
- For more information see:

https://scikit-learn.org/stable/modules/outlier_detection.html#outlier-detection