1. We prove that the given identities are true using induction on "n" as follows:

BASIS STEP: We have that, for $n=1$,

$$\overline{\bigcup_{i=1}^{n} S_i} = \overline{\bigcup_{i=1}^{1} S_i} = \overline{S_1} = \bigcap_{i=1}^{1} \overline{S_i} = \bigcap_{i=1}^{n} \overline{S_i}$$

$$\overline{\bigcap_{i=1}^{n} S_i} = \overline{\bigcap_{i=1}^{1} S_i} = \overline{S_1} = \bigcup_{i=1}^{1} \overline{S_i} = \bigcup_{i=1}^{n} \overline{S_i}$$

Hence, the identities are true for $n=1$. Next, we show that the identities hold for $n=2$.

$$\overline{\bigcup_{i=1}^{n} S_i} = \underbrace{\overline{S_1 \cup S_2} = \overline{S_1} \cap \overline{S_2}}_{\text{using venn diagram}} = \bigcap_{i=1}^{2} \overline{S_i} = \bigcap_{i=1}^{n} \overline{S_i}$$

$$\overline{\bigcap_{i=1}^{n} S_i} = \overbrace{\overline{S_1 \cap S_2} = \overline{S_1} \cup \overline{S_2}} = \bigcup_{i=1}^{2} \overline{S_i} = \bigcup_{i=1}^{n} \overline{S_i}$$

Hence, the identities hold for $\underline{n=2}$.

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INDUCTIVE STEP: Assume that, for some $n \geqslant 1$,

$$\overline{\bigcup_{i=1}^{n} S_i} = \bigcap_{i=1}^{n} \overline{S_i} \quad \& \quad \overline{\bigcap_{i=1}^{n} S_i} = \bigcup_{i=1}^{n} \overline{S_i}$$

We show that the identities hold for $n+1$.

$$\overline{\bigcup_{i=1}^{n+1} S_i} = \overline{\bigcup_{i=1}^{n} S_i \cup S_{n+1}} = \overline{\bigcup_{i=1}^{n} S_i} \cap \overline{S_{n+1}} \quad (\because \text{BASIS})$$

$$= \bigcap_{i=1}^{n} \overline{S_i} \cap \overline{S_{n+1}} \quad (\because \text{Inductive Hypothesis})$$

$$= \bigcap_{i=1}^{n+1} \overline{S_i}$$

$$\overline{\bigcap_{i=1}^{n+1} S_i} = \overline{\bigcap_{i=1}^{n} S_i \cap S_{n+1}} = \overline{\bigcap_{i=1}^{n} S_i} \cup \overline{S_{n+1}} \quad (\because \text{BASIS})$$

$$= \bigcup_{i=1}^{n} \overline{S_i} \cup \overline{S_{n+1}} \quad (\because \text{Inductive Hypothesis})$$

$$= \bigcup_{i=1}^{n+1} \overline{S_i}$$

Hence, Proved.

2. We have that,

$$S_1 \cup S_2 - (S_1 \cap \overline{S_2}) = (S_1 \cup S_2) \cap (\overline{S_1 \cap \overline{S_2}})$$

$(\because S - T = S \cap \overline{T})$

$= (S_1 \cup S_2) \cap (\overline{S_1} \cup S_2)$ $(\because$ Demorgan's laws$)$

$= ((S_1 \cup S_2) \cap \overline{S_1}) \cup ((S_1 \cup S_2) \cap S_2)$ $(\because$ Distributive Law$)$

$= ((S_1 \cup S_2) \cap \overline{S_1}) \cup S_2$ $(\because (S_1 \cup S_2) \cap S_2 = S_2)$ using venn diagram

$= ((S_1 \cap \overline{S_1}) \cup (\overline{S_1} \cap S_2) \cup S_2$ $(\because$ Distributive Law$)$

$= (\phi \cup (\overline{S_1} \cap S_2)) \cup S_2$ $(\because S_1 \cap \overline{S_1} = \phi)$

$= (\overline{S_1} \cap S_2) \cup S_2$ $(\because \phi \cup A = A$ for any set $A)$

$= S_2$ $(\because (A \cap B) \cup B = B$ for any sets $A, B)$ using venn diagram

Hence, Proved.

3. The given grammar generates strings of the form $(aab)^*$, i.e. strings that have zero or more copies of the substring "aab" concatenated

$S \rightarrow \lambda$

$S \rightarrow aaA \rightarrow aabS \rightarrow aab$

$S \rightarrow aaA \rightarrow aabS \rightarrow aabaaA \rightarrow aabaabS \rightarrow aabaab$

$\vdots$

Hence the language generated by the given grammar is

$L = \{ w \in \{a, b\}^* \mid w$ has zero or more copies of "aab" concatenated $\}$

4. The given grammar generates an empty language

$L = \emptyset$.

This is because none of the rules in the grammar has a non-terminal on the right hand side.

Hence, none of the derivations ever complete into a sequence of terminals.

Thus, no string is generated by the grammar.

5 (a) The following grammar generates strings with atleast two a's :

$$S \to AaAaA$$
$$A \to aA \mid bA \mid \lambda$$

The grammar generates strings with two a's fixed & then any string following the first a, in the beginning, & in the end of the string.

(b)

$$S \to AXAXAXA$$
$$A \to bA \mid \lambda$$
$$X \to a \mid b \mid \lambda$$

The string consists of only 3 a's at max, as 'a' is only generated using the non-terminal X, and there are only 3 X's in the string derived from the start symbol S.