Regression Models in R

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Introduction

Workshop description

- This is an intermediate/advanced R course
- Appropriate for those with basic knowledge of R
- This is not a statistics course!
- Learning objectives:
  - Learn the R formula interface
  - Specify factor contrasts to test specific hypotheses
  - Perform model comparisons
  - Run and interpret variety of regression models in R

Materials and Setup

labsetup

Lab computer users: Log in using the user name and password on the board to your left.

Laptop users:

- you should have R installed–if not, open a web browser and go to http://cran.r-project.org/ and download and install it
- also helpful to install RStudo (download from http://rstudio.com/)

Everyone:

- Download materials from http://tutorials.iq.harvard.edu/R/Rstatistics.zip
- Extract materials from RStatistics.zip (on lab machines *right-click -> WinZip -> Extract to here*) and move to your desktop.

Launch RStudio

labsetup

- Open the RStudio program from the Windows start menu
- Open up today's R script
  - In RStudio, Go to **File => Open Script**
  - Locate and open the `Rstatistics.R` script in the Rstatistics folder on your desktop

- Go to **Tools => Set working directory => To source file location** (more on the working directory later)
- I encourage you to add your own notes to this file!

Set working directory

It is often helpful to start your R session by setting your working directory so you don't have to type the full path names to your data and other files

```
# set the working directory
# setwd("~/Desktop/Rstatistics")
# setwd("C:/Users/dataclass/Desktop/Rstatistics")
```

```
> # set the working directory
> # setwd("~/Desktop/Rstatistics")
> # setwd("C:/Users/dataclass/Desktop/Rstatistics")
>
```

You might also start by listing the files in your working directory

```
getwd() # where am I?
list.files("dataSets") # files in the dataSets folder
```

```
> getwd() # where am I?
[1] "/home/izahn/Documents/Work/IQSS/Classes/IQSS_Stats_Workshops/R/Rstatistics"
> list.files("dataSets") # files in the dataSets folder
[1] "Exam.rds"          "NatHealth2008MI"   "NatHealth2011.rds"
[4] "states.dta"        "states.rds"
>
```

Load the states data

The *states.dta* data comes from http://anawida.de/teach/SS14/anawida/4.linReg/data/states.dta.txt and appears to have originally appeared in *Statistics with Stata* by Lawrence C. Hamilton.

```r
# read the states data
states.data <- readRDS("dataSets/states.rds")
#get labels
states.info <- data.frame(attributes(states.data)[c("names", "var.labels")])
#look at last few labels
tail(states.info, 8)
```

```
> # read the states data
> states.data <- readRDS("dataSets/states.rds")
> #get labels
> states.info <- data.frame(attributes(states.data)[c("names", "var.labels")])
> #look at last few labels
> tail(states.info, 8)
     names                      var.labels
14    csat        Mean composite SAT score
15    vsat           Mean verbal SAT score
16    msat             Mean math SAT score
17 percent       % HS graduates taking SAT
18 expense Per pupil expenditures prim&sec
19  income Median household income, $1,000
20    high             % adults HS diploma
21 college         % adults college degree
>
```

Linear regression

Examine the data before fitting models

Start by examining the data to check for problems.

```r
# summary of expense and csat columns, all rows
sts.ex.sat <- subset(states.data, select = c("expense", "csat"))
summary(sts.ex.sat)
# correlation between expense and csat
cor(sts.ex.sat)
```

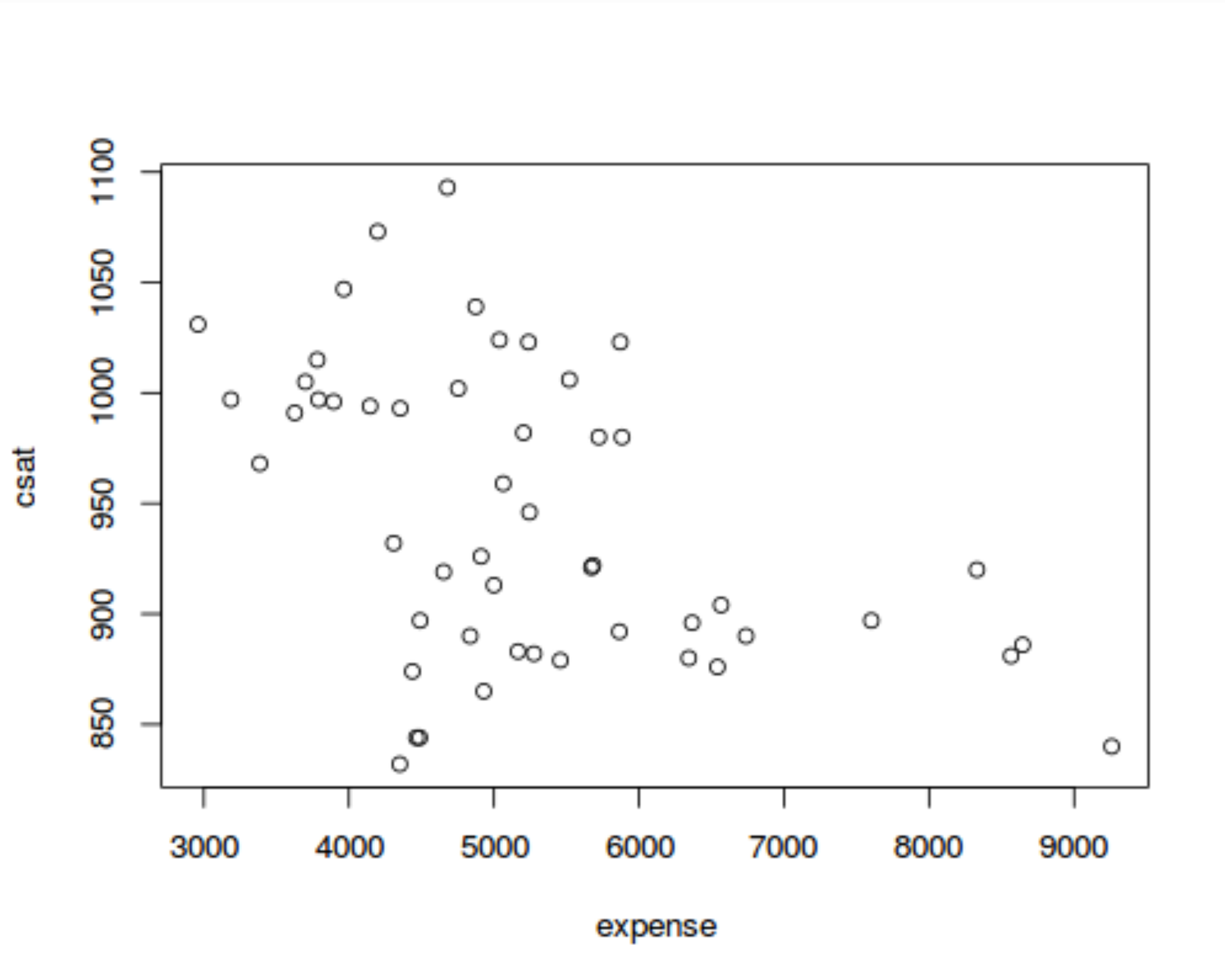
```
> # summary of expense and csat columns, all rows
> sts.ex.sat <- subset(states.data, select = c("expense", "csat"))
> summary(sts.ex.sat)
    expense          csat      
 Min.   :2960   Min.   : 832  
 1st Qu.:4352   1st Qu.: 888  
 Median :5000   Median : 926  
 Mean   :5236   Mean   : 944  
```

```
 Mean   :5236   Mean   : 944
 3rd Qu.:5794   3rd Qu.: 997
 Max.   :9259   Max.   :1093
> # correlation between expense and csat
> cor(sts.ex.sat)
         expense   csat
expense   1.000 -0.466
csat     -0.466  1.000
>
```

Plot the data before fitting models

Plot the data to look for multivariate outliers, non-linear relationships etc.

```
# scatter plot of expense vs csat
plot(sts.ex.sat)
```



Linear regression example

- Linear regression models can be fit with the `lm()` function
- For example, we can use `lm` to predict SAT scores based on per-pupal expenditures:

```r
# Fit our regression model
sat.mod <- lm(csat ~ expense, # regression formula
              data=states.data) # data set
# Summarize and print the results
summary(sat.mod) # show regression coefficients table
```

```
> # Fit our regression model
> sat.mod <- lm(csat ~ expense, # regression formula
+               data=states.data) # data set
> # Summarize and print the results
> summary(sat.mod) # show regression coefficients table

Call:
lm(formula = csat ~ expense, data = states.data)

Residuals:
    Min      1Q  Median      3Q     Max 
-131.81  -38.08    5.61   37.85  136.50 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 1060.73244   32.70090   32.44  < 2e-16
expense       -0.02228    0.00604   -3.69  0.00056

Residual standard error: 59.8 on 49 degrees of freedom
Multiple R-squared:  0.217,	Adjusted R-squared:  0.201 
F-statistic: 13.6 on 1 and 49 DF,  p-value: 0.000563

>
```

Why is the association between expense and SAT scores *negative*?

Many people find it surprising that the per-capita expenditure on students is negatively related to SAT scores. The beauty of multiple regression is that we can try to pull these apart. What would the association between expense and SAT scores be if there were no difference among the states in the percentage of students taking the SAT?

```r
summary(lm(csat ~ expense + percent, data = states.data))
```

```
> summary(lm(csat ~ expense + percent, data = states.data))
```

```
> summary(lm(csat ~ expense + percent, data = states.data))

Call:
lm(formula = csat ~ expense + percent, data = states.data)

Residuals:
   Min     1Q Median     3Q    Max 
-62.92 -24.32   1.74  15.50  75.62 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 989.8074    18.3958   53.81  < 2e-16
expense       0.0086     0.0042    2.05    0.046
percent      -2.5377     0.2249  -11.28  4.2e-15

Residual standard error: 31.6 on 48 degrees of freedom
Multiple R-squared:  0.786,	Adjusted R-squared:  0.777 
F-statistic:   88 on 2 and 48 DF,  p-value: <2e-16

>
```

The lm class and methods

OK, we fit our model. Now what?

- Examine the model object:

```
class(sat.mod)
names(sat.mod)
methods(class = class(sat.mod))[1:9]
```

```
> class(sat.mod)
[1] "lm"
> names(sat.mod)
 [1] "coefficients"  "residuals"     "effects"       "rank"         
 [5] "fitted.values" "assign"        "qr"            "df.residual"  
 [9] "xlevels"       "call"          "terms"         "model"        
> methods(class = class(sat.mod))[1:9]
[1] "add1.lm"                   "alias.lm"                 
[3] "anova.lm"                  "case.names.lm"            
[5] "coerce,oldClass,S3-method" "confint.lm"               
[7] "cooks.distance.lm"         "deviance.lm"              
[9] "dfbeta.lm"                
>
```

- Use function methods to get more information about the fit

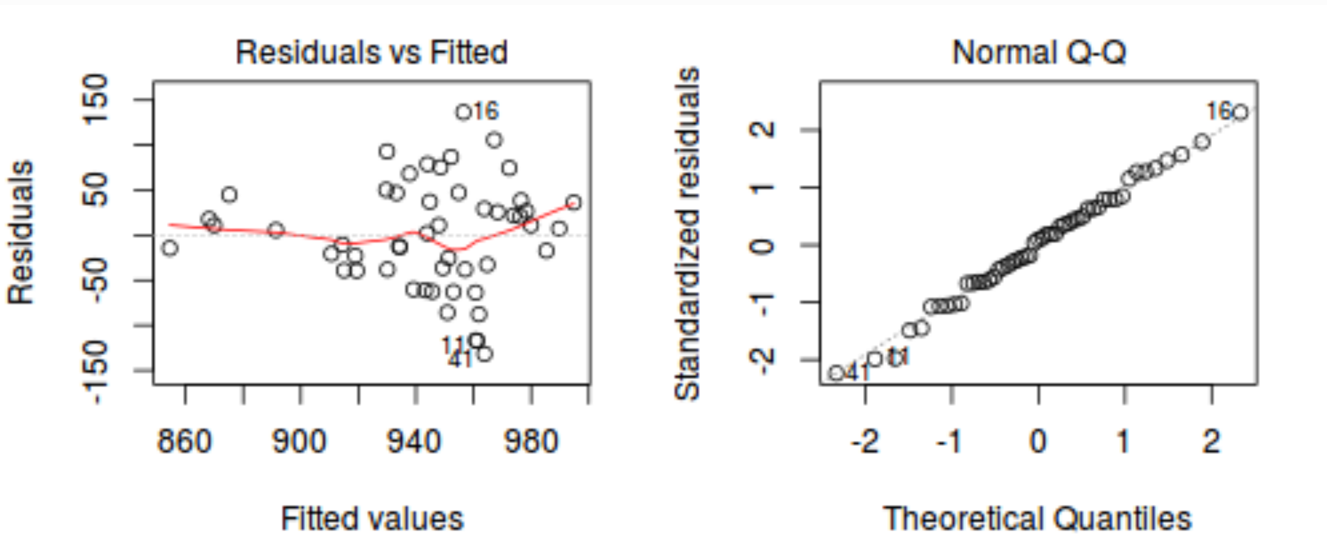
```
confint(sat.mod)
# hist(residuals(sat.mod))
```

```
> confint(sat.mod)
                2.5 %    97.5 %
(Intercept) 995.0175 1126.4474
expense      -0.0344   -0.0101
> # hist(residuals(sat.mod))
>
```

Linear Regression Assumptions

- Ordinary least squares regression relies on several assumptions, including that the residuals are normally distributed and homoscedastic, the errors are independent and the relationships are linear.
- Investigate these assumptions visually by plotting your model:

```
par(mar = c(4, 4, 2, 2), mfrow = c(1, 2)) #optional
plot(sat.mod, which = c(1, 2)) # "which" argument optional
```



Comparing models

Do congressional voting patterns predict SAT scores over and above expense? Fit two models and compare them:

```r
# fit another model, adding house and senate as predictors
sat.voting.mod <-  lm(csat ~ expense + house + senate,
                      data = na.omit(states.data))
sat.mod <- update(sat.mod, data=na.omit(states.data))
# compare using the anova() function
anova(sat.mod, sat.voting.mod)
coef(summary(sat.voting.mod))
```

```
> # fit another model, adding house and senate as predictors
> sat.voting.mod <-  lm(csat ~ expense + house + senate,
+                       data = na.omit(states.data))
> sat.mod <- update(sat.mod, data=na.omit(states.data))
> # compare using the anova() function
> anova(sat.mod, sat.voting.mod)
Analysis of Variance Table

Model 1: csat ~ expense
Model 2: csat ~ expense + house + senate
  Res.Df    RSS Df Sum of Sq    F Pr(>F)
1     46 169050
2     44 149284  2     19766 2.91  0.065
> coef(summary(sat.voting.mod))
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 1082.9344   38.63381   28.03 1.07e-29
expense       -0.0187    0.00969   -1.93 6.00e-02
house         -1.4424    0.60048   -2.40 2.06e-02
senate         0.4982    0.51356    0.97 3.37e-01
>
```

Exercise 0: least squares regression

Use the *states.rds* data set. Fit a model predicting energy consumed per capita (energy) from the percentage of residents living in metropolitan areas (metro). Be sure to

1. Examine/plot the data before fitting the model
2. Print and interpret the model `summary`
3. `plot` the model to look for deviations from modeling assumptions

Select one or more additional predictors to add to your model and repeat steps 1-3. Is this model significantly better than the model with *metro* as the only predictor?

Interactions and factors

Modeling interactions

Interactions allow us assess the extent to which the association between one predictor and the outcome depends on a second predictor. For example: Does the association between expense and SAT scores depend on the median income in the state?

```
  #Add the interaction to the model
sat.expense.by.percent <- lm(csat ~ expense*income,
                             data=states.data)
#Show the results
  coef(summary(sat.expense.by.percent)) # show regression coefficients table
```

```
>   #Add the interaction to the model
> sat.expense.by.percent <- lm(csat ~ expense*income,
+                              data=states.data)
> #Show the results
>   coef(summary(sat.expense.by.percent)) # show regression coefficients table
                 Estimate Std. Error t value       Pr(>|t|)
(Intercept)    1380.36423 172.086252    8.02 0.000000000237
expense          -0.06384   0.032701   -1.95 0.056878369245
income          -10.49785   4.991463   -2.10 0.040832525071
expense:income    0.00138   0.000864    1.60 0.115539488253
>
```

Regression with categorical predictors

Let's try to predict SAT scores from region, a categorical variable. Note that you must make sure R does not think your categorical variable is numeric.

```
# make sure R knows region is categorical
str(states.data$region)
states.data$region <- factor(states.data$region)
#Add region to the model
sat.region <- lm(csat ~ region,
                 data=states.data)
#Show the results
coef(summary(sat.region)) # show regression coefficients table
anova(sat.region) # show ANOVA table
```

```
> # make sure R knows region is categorical
> str(states.data$region)
```

```
 Factor w/ 4 levels "West","N. East",..: 3 1 1 3 1 1 2 3 NA 3 ...
> states.data$region <- factor(states.data$region)
> #Add region to the model
> sat.region <- lm(csat ~ region,
+                  data=states.data)
> #Show the results
> coef(summary(sat.region)) # show regression coefficients table
              Estimate Std. Error t value Pr(>|t|)
(Intercept)      946.3       14.8  63.958 1.35e-46
regionN. East    -56.8       23.1  -2.453 1.80e-02
regionSouth      -16.3       19.9  -0.819 4.17e-01
regionMidwest     63.8       21.4   2.986 4.51e-03
> anova(sat.region) # show ANOVA table
Analysis of Variance Table

Response: csat
          Df Sum Sq Mean Sq F value   Pr(>F)
region     3  82049   27350    9.61 0.000049
Residuals 46 130912    2846
>
```

Again, **make sure to tell R which variables are categorical by converting them to factors!**

Setting factor reference groups and contrasts

In the previous example we use the default contrasts for region. The default in R is treatment contrasts, with the first level as the reference. We can change the reference group or use another coding scheme using the `C` function.

```
# print default contrasts
contrasts(states.data$region)
# change the reference group
coef(summary(lm(csat ~ C(region, base=4),
                data=states.data)))
# change the coding scheme
coef(summary(lm(csat ~ C(region, contr.helmert),
                data=states.data)))
```

```
> # print default contrasts
> contrasts(states.data$region)
        N. East South Midwest
West          0     0       0
N. East       1     0       0
```

```
South          0      1        0
Midwest        0      0        1
> # change the reference group
> coef(summary(lm(csat ~ C(region, base=4),
+                 data=states.data)))
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)            1010.1       15.4   65.59 4.30e-47
C(region, base = 4)1    -63.8       21.4   -2.99 4.51e-03
C(region, base = 4)2   -120.5       23.5   -5.12 5.80e-06
C(region, base = 4)3    -80.1       20.4   -3.93 2.83e-04
> # change the coding scheme
> coef(summary(lm(csat ~ C(region, contr.helmert),
+                 data=states.data)))
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                 943.99       7.71 122.498 1.69e-59
C(region, contr.helmert)1   -28.38      11.57  -2.453 1.80e-02
C(region, contr.helmert)2     4.02       5.88   0.684 4.98e-01
C(region, contr.helmert)3    22.03       4.45   4.955 1.02e-05
>
```

See also `?contrasts`, `?contr.treatment`, and `?relevel`.

Exercise 1: interactions and factors

Use the states data set.

1. Add on to the regression equation that you created in exercise 1 by generating an interaction term and testing the interaction.
2. Try adding region to the model. Are there significant differences across the four regions?

Regression with binary outcomes

Logistic regression

This far we have used the `lm` function to fit our regression models. `lm` is great, but limited–in particular it only fits models for continuous dependent variables. For categorical dependent variables we can use the `glm()` function.

For these models we will use a different dataset, drawn from the National Health Interview Survey. From the CDC website:

> *The National Health Interview Survey (NHIS) has monitored the health of the nation since 1957. NHIS data on a broad range of health topics are collected through personal household interviews. For over 50 years, the U.S. Census Bureau has been the data collection agent for the National Health Interview Survey. Survey results have been instrumental in providing data to track health status, health care access, and progress toward achieving national health objectives.*

Load the National Health Interview Survey data:

```
NH11 <- readRDS("dataSets/NatHealth2011.rds")
labs <- attributes(NH11)$labels
```

```
> NH11 <- readRDS("dataSets/NatHealth2011.rds")
> labs <- attributes(NH11)$labels
>
```

Logistic regression example

Let's predict the probability of being diagnosed with hypertension based on age, sex, sleep, and bmi

```
str(NH11$hypev) # check stucture of hypev
levels(NH11$hypev) # check levels of hypev
# collapse all missing values to NA
NH11$hypev <- factor(NH11$hypev, levels=c("2 No", "1 Yes"))
# run our regression model
hyp.out <- glm(hypev~age_p+sex+sleep+bmi,
              data=NH11, family="binomial")
coef(summary(hyp.out))
```

```
> str(NH11$hypev) # check stucture of hypev
 Factor w/ 5 levels "1 Yes","2 No",..: 2 2 1 2 2 1 2 2 1 2 ...
> levels(NH11$hypev) # check levels of hypev
[1] "1 Yes"             "2 No"              "7 Refused"
[4] "8 Not ascertained" "9 Don't know"
> # collapse all missing values to NA
> NH11$hypev <- factor(NH11$hypev, levels=c("2 No", "1 Yes"))
> # run our regression model
> hyp.out <- glm(hypev~age_p+sex+sleep+bmi,
```

```
+                 data=NH11, family="binomial")
> coef(summary(hyp.out))
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -4.26947   0.056495  -75.57 0.00e+00
age_p        0.06070   0.000823   73.78 0.00e+00
sex2 Female -0.14403   0.026798   -5.37 7.68e-08
sleep       -0.00704   0.001640   -4.29 1.78e-05
bmi          0.01857   0.000951   19.53 6.49e-85
>
```

Logistic regression coefficients

Generalized linear models use link functions, so raw coefficients are difficult to interpret. For example, the age coefficient of .06 in the previous model tells us that for every one unit increase in age, the log odds of hypertension diagnosis increases by 0.06. Since most of us are not used to thinking in log odds this is not too helpful!

One solution is to transform the coefficients to make them easier to interpret

```
hyp.out.tab <- coef(summary(hyp.out))
hyp.out.tab[, "Estimate"] <- exp(coef(hyp.out))
hyp.out.tab
```

```
> hyp.out.tab <- coef(summary(hyp.out))
> hyp.out.tab[, "Estimate"] <- exp(coef(hyp.out))
> hyp.out.tab
             Estimate Std. Error z value Pr(>|z|)
(Intercept)     0.014   0.056495  -75.57 0.00e+00
age_p           1.063   0.000823   73.78 0.00e+00
sex2 Female     0.866   0.026798   -5.37 7.68e-08
sleep           0.993   0.001640   -4.29 1.78e-05
bmi             1.019   0.000951   19.53 6.49e-85
>
```

Generating predicted values

In addition to transforming the log-odds produced by `glm` to odds, we can use the `predict()` function to make direct statements about the predictors in our model. For example, we can ask "How much more likely is a 63 year old female to have hypertension compared to a 33 year old female?".

```r
# Create a dataset with predictors set at desired levels
predDat <- with(NH11,
                expand.grid(age_p = c(33, 63),
                            sex = "2 Female",
                            bmi = mean(bmi, na.rm = TRUE),
                            sleep = mean(sleep, na.rm = TRUE)))
# predict hypertension at those levels
cbind(predDat, predict(hyp.out, type = "response",
                       se.fit = TRUE, interval="confidence",
                       newdata = predDat))
```

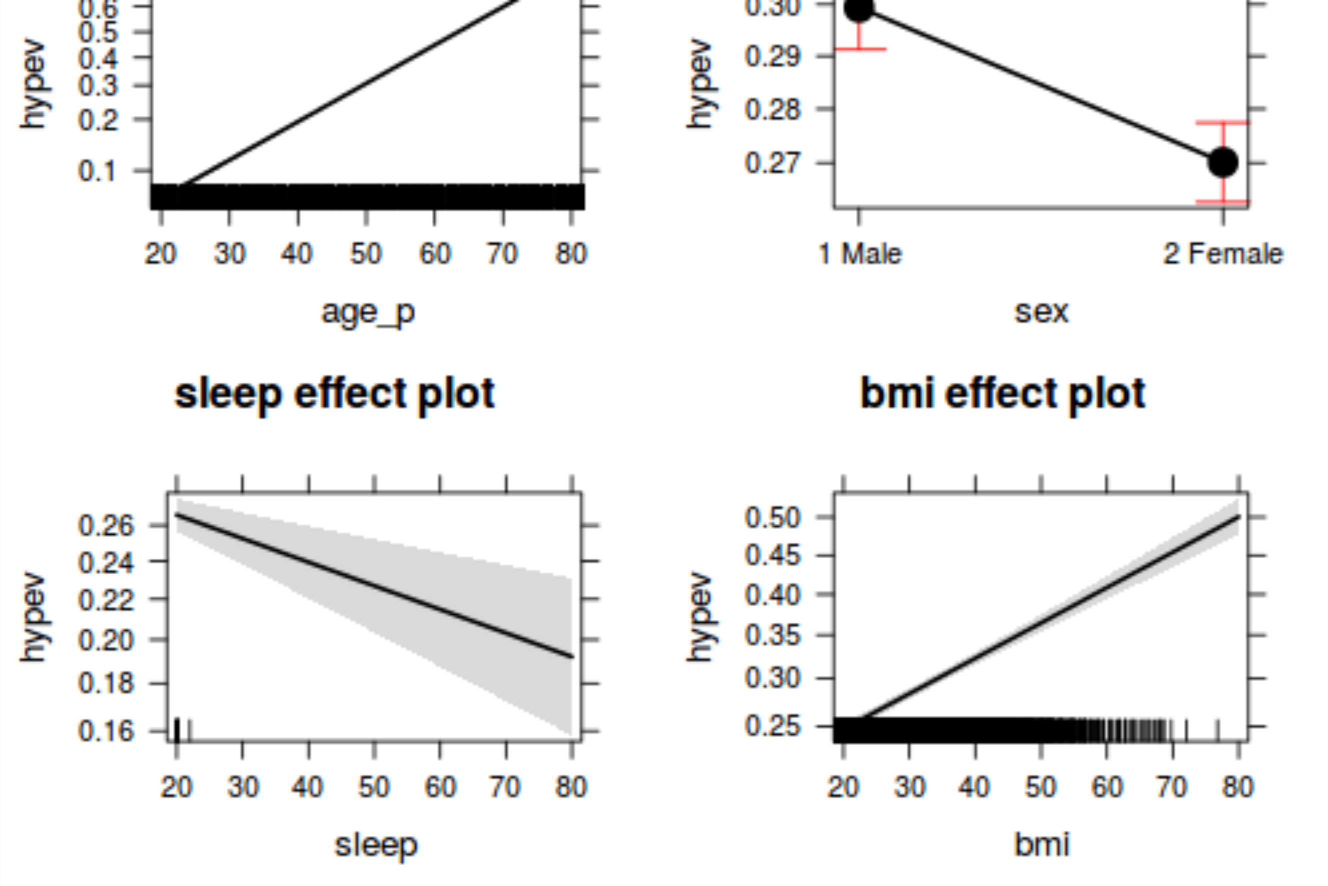
```
> # Create a dataset with predictors set at desired levels
> predDat <- with(NH11,
+                 expand.grid(age_p = c(33, 63),
+                             sex = "2 Female",
+                             bmi = mean(bmi, na.rm = TRUE),
+                             sleep = mean(sleep, na.rm = TRUE)))
> # predict hypertension at those levels
> cbind(predDat, predict(hyp.out, type = "response",
+                        se.fit = TRUE, interval="confidence",
+                        newdata = predDat))
  age_p      sex  bmi sleep   fit  se.fit residual.scale
1    33 2 Female 29.9  7.86 0.129 0.00285              1
2    63 2 Female 29.9  7.86 0.478 0.00482              1
>
```

This tells us that a 33 year old female has a 13% probability of having been diagnosed with hypertension, while and 63 year old female has a 48% probability of having been diagnosed.

Packages for computing and graphing predicted values

Instead of doing all this ourselves, we can use the effects package to compute quantities of interest for us (cf. the Zelig package).

```r
library(effects)
plot(allEffects(hyp.out))
```



Exercise 2: logistic regression

Use the NH11 data set that we loaded earlier.

1. Use glm to conduct a logistic regression to predict ever worked (everwrk) using age (age$_{p}$) and marital status (r$_{maritl}$).
2. Predict the probability of working for each level of marital status.

Note that the data is not perfectly clean and ready to be modeled. You will need to clean up at least some of the variables before fitting the model.

Multilevel Modeling

Multilevel modeling overview

- Multi-level (AKA hierarchical) models are a type of mixed-effects models
- Used to model variation due to group membership where the goal is to generalize to a population of groups
- Can model different intercepts and/or slopes for each group
- Mixed-effecs models include two types of predictors: fixed-effects and random effects
  - Fixed-effects – observed levels are of direct interest (.e.g, sex, political party…)
  - Random-effects – observed levels not of direct interest; goal is to make inferences

to a population represented by observed levels

- In R the lme4 package is the most popular for mixed effects models
  - Use the `lmer` function for liner mixed models, `glmer` for generalized mixed models

```
library(lme4)
```

```
> library(lme4)
>
```

The Exam data

The Exam data set contans exam scores of 4,059 students from 65 schools in Inner London. The variable names are as follows:

variable	Description
school	School ID - a factor.
normexam	Normalized exam score.
schgend	School gender - a factor. Levels are 'mixed', 'boys', and 'girls'.
schavg	School average of intake score.
vr	Student level Verbal Reasoning (VR) score band at intake - 'bottom 25%', 'mid 50%', and 'top 25%'.
intake	Band of student's intake score - a factor. Levels are 'bottom 25%', 'mid 50%' and 'top 25%'./
standLRT	Standardised LR test score.
sex	Sex of the student - levels are 'F' and 'M'.
type	School type - levels are 'Mxd' and 'Sngl'.
student	Student id (within school) - a factor

```
Exam <- readRDS("dataSets/Exam.rds")
```

```
> Exam <- readRDS("dataSets/Exam.rds")
>
```

The null model and ICC

As a preliminary step it is often useful to partition the variance in the dependent variable into the various levels. This can be accomplished by running a null model (i.e., a model with a random effects grouping structure, but no fixed-effects predictors).

```
# null model, grouping by school but not fixed effects.
Norm1 <-lmer(normexam ~ 1 + (1|school),
              data=Exam, REML = FALSE)
summary(Norm1)
```

```
> # null model, grouping by school but not fixed effects.
> Norm1 <-lmer(normexam ~ 1 + (1|school),
+               data=Exam, REML = FALSE)
> summary(Norm1)
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: normexam ~ 1 + (1 | school)
   Data: Exam

     AIC      BIC   logLik deviance df.resid
   10826    10844    -5410    10820     3984

Scaled residuals:
   Min     1Q Median     3Q    Max
-3.902 -0.646  0.003  0.698  3.636

Random effects:
 Groups   Name        Variance Std.Dev.
 school   (Intercept) 0.169    0.412
 Residual             0.848    0.921
Number of obs: 3987, groups:  school, 65

Fixed effects:
            Estimate Std. Error t value
(Intercept)  -0.0141     0.0538   -0.26
>
```

The is .169/(.169 + .848) = .17: 17% of the variance is at the school level.

Adding fixed-effects predictors

Predict exam scores from student's standardized tests scores

```
Norm2 <-lmer(normexam~standLRT + (1|school),
             data=Exam,
             REML = FALSE)
summary(Norm2)
```

```
> Norm2 <-lmer(normexam~standLRT + (1|school),
+               data=Exam,
+               REML = FALSE)
> summary(Norm2)
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: normexam ~ standLRT + (1 | school)
   Data: Exam

     AIC      BIC   logLik deviance df.resid
    9143     9168    -4568     9135     3954

Scaled residuals:
   Min     1Q Median     3Q    Max
-3.700 -0.625  0.024  0.678  3.262

Random effects:
 Groups   Name        Variance Std.Dev.
 school   (Intercept) 0.0919   0.303
 Residual             0.5670   0.753
Number of obs: 3958, groups:  school, 65

Fixed effects:
            Estimate Std. Error t value
(Intercept)  0.00121    0.04004     0.0
standLRT     0.56559    0.01265    44.7

Correlation of Fixed Effects:
         (Intr)
standLRT 0.007
>
```

Multiple degree of freedom comparisons

As with `lm` and `glm` models, you can compare the two `lmer` models using the `anova` function.

```
anova(Norm1, Norm2)
```

Random slopes

Add a random effect of students' standardized test scores as well. Now in addition to estimating the distribution of intercepts across schools, we also estimate the distribution of the slope of exam on standardized test.

```
Norm3 <- lmer(normexam~standLRT + (standLRT|school), data=Exam,
              REML = FALSE)
summary(Norm3)
```

```
> Norm3 <- lmer(normexam~standLRT + (standLRT|school), data=Exam,
+                 REML = FALSE)
> summary(Norm3)
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: normexam ~ standLRT + (standLRT | school)
   Data: Exam

     AIC      BIC   logLik deviance df.resid
    9108     9146    -4548     9096     3952

Scaled residuals:
   Min      1Q Median      3Q     Max
-3.813 -0.634  0.033  0.673  3.452

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 school   (Intercept) 0.0899   0.300
          standLRT    0.0141   0.119    0.51
 Residual             0.5552   0.745
Number of obs: 3958, groups:  school, 65

Fixed effects:
            Estimate Std. Error t value
(Intercept)  -0.0122     0.0397   -0.31
standLRT      0.5586     0.0199   28.08

Correlation of Fixed Effects:
         (Intr)
standLRT 0.371
>
```

Test the significance of the random slope

To test the significance of a random slope just compare models with and without the random slope term

```
anova(Norm2, Norm3)
```

```
> anova(Norm2, Norm3)
Data: Exam
Models:
Norm2: normexam ~ standLRT + (1 | school)
Norm3: normexam ~ standLRT + (standLRT | school)
      Df  AIC  BIC logLik deviance Chisq Chi Df   Pr(>Chisq)
Norm2  4 9143 9169  -4568     9135
Norm3  6 9108 9146  -4548     9096    39      2 0.0000000035
>
```

Exercise 3: multilevel modeling

Use the dataset, bh1996: `data(bh1996, package="multilevel")`

From the data documentation:

> *Variables are Cohesion (COHES), Leadership Climate (LEAD), Well-Being (WBEING) and Work Hours (HRS). Each of these variables has two variants - a group mean version that replicates each group mean for every individual, and a within-group version where the group mean is subtracted from each individual response. The group mean version is designated with a G. (e.g., G.HRS), and the within-group version is designated with a W. (e.g., W.HRS).*

1. Create a null model predicting wellbeing ("WBEING")
2. Calculate the ICC for your null model
3. Run a second multi-level model that adds two individual-level predictors, average number of hours worked ("HRS") and leadership skills ("LEAD") to the model and interpret your output.
4. Now, add a random effect of average number of hours worked ("HRS") to the model and interpret your output. Test the significance of this random term.

Exercise solutions

prototype

Exercise 0 prototype

Use the *states.rds* data set.

```
states <- readRDS("dataSets/states.rds")
```

Fit a model predicting energy consumed per capita (energy) from the percentage of residents living in metropolitan areas (metro). Be sure to

1. Examine/plot the data before fitting the model

```
states.en.met <- subset(states, select = c("metro", "energy"))
summary(states.en.met)
plot(states.en.met)
cor(states.en.met, use="pairwise")
```

2. Print and interpret the model `summary`

```
mod.en.met <- lm(energy ~ metro, data = states)
summary(mod.en.met)
```

3. `plot` the model to look for deviations from modeling assumptions

```
plot(mod.en.met)
```

Select one or more additional predictors to add to your model and repeat steps 1-3. Is this model significantly better than the model with *metro* as the only predictor?

```
states.en.met.pop.wst <- subset(states, select = c("energy", "metro", "pop", "waste"))
summary(states.en.met.pop.wst)
plot(states.en.met.pop.wst)
cor(states.en.met.pop.wst, use = "pairwise")
mod.en.met.pop.waste <- lm(energy ~ metro + pop + waste, data = states)
summary(mod.en.met.pop.waste)
```

```
anova(mod.en.met, mod.en.met.pop.waste)
```

Exercise 1: prototype

Use the states data set.

1. Add on to the regression equation that you created in exercise 1 by generating an interaction term and testing the interaction.

```
mod.en.metro.by.waste <- lm(energy ~ metro * waste, data = states)
```

1. Try adding a region to the model. Are there significant differences across the four regions?

```
mod.en.region <- lm(energy ~ metro * waste + region, data = states)
anova(mod.en.region)
```

Exercise 2 prototype

Use the NH11 data set that we loaded earlier. Note that the data is not perfectly clean and ready to be modeled. You will need to clean up at least some of the variables before fitting the model.

1. Use glm to conduct a logistic regression to predict ever worked (everwrk) using age (age$_p$) and marital status (r$_{maritl}$).

```
nh11.wrk.age.mar <- subset(NH11, select = c("everwrk", "age_p", "r_maritl"))
summary(nh11.wrk.age.mar)
NH11 <- transform(NH11,
                  everwrk = factor(everwrk,
                      levels = c("1 Yes", "2 No")),
                  r_maritl = droplevels(r_maritl))

mod.wk.age.mar <- glm(everwrk ~ age_p + r_maritl, data = NH11,
                      family = "binomial")

summary(mod.wk.age.mar)
```

2. Predict the probability of working for each level of marital status.

```
library(effects)
data.frame(Effect("r_maritl", mod.wk.age.mar))
```

Exercise 3 prototype

Use the dataset, bh1996:

```
data(bh1996, package="multilevel")
```

From the data documentation:

> *Variables are Cohesion (COHES), Leadership Climate (LEAD), Well-Being (WBEING) and Work Hours (HRS). Each of these variables has two variants - a group mean version that replicates each group mean for every individual, and a within-group version where the group mean is subtracted from each individual response. The group mean version is designated with a G. (e.g., G.HRS), and the within-group version is designated with a W. (e.g., W.HRS).*

Note that the group identifier is named "GRP".

1. Create a null model predicting wellbeing ("WBEING")

```
library(lme4)
mod.grp0 <- lmer(WBEING ~ 1 + (1 | GRP), data = bh1996)
summary(mod.grp0)
```

=> library(lme4) > mod.grp0 <- lmer(WBEING ~ 1 + (1 | GRP), data = bh1996) > summary(mod.grp0) Linear mixed model fit by REML ['lmerMod'] Formula: WBEING ~ 1 + (1 | GRP) Data: bh1996

REML criterion at convergence: 19347

Scaled residuals: Min 1Q Median 3Q Max -3.322 -0.648 0.031 0.718 2.667

Random effects: Groups Name Variance Std.Dev. GRP (Intercept) 0.0358 0.189 Residual 0.7895 0.889 Number of obs: 7382, groups: GRP, 99

Fixed effects: Estimate Std. Error t value (Intercept) 2.7743 0.0222 125 > =2. [@2] Calculate the ICC for your null model `ICC = .0358/(.0358 + .7895) = .04`

3. Run a second multi-level model that adds two individual-level predictors, average number of hours worked ("HRS") and leadership skills ("LEAD") to the model and interpret your output.

```
mod.grp1 <- lmer(WBEING ~ HRS + LEAD + (1 | GRP), data = bh1996)
summary(mod.grp1)
```

4. Now, add a random effect of average number of hours worked ("HRS") to the model and interpret your output. Test the significance of this random term.

```
mod.grp2 <- lmer(WBEING ~ HRS + LEAD + (1 + HRS | GRP), data = bh1996)
anova(mod.grp1, mod.grp2)
```

Wrap-up

Help us make this workshop better!

- Please take a moment to fill out a very short

feedback form

- These workshops exist for you – tell us what you need!
- http://tinyurl.com/RstatisticsFeedback

Additional resources

- IQSS workshops: http://projects.iq.harvard.edu/rtc/filter_by/workshops
- IQSS statistical consulting: http://dss.iq.harvard.edu/
- Zelig
  - Website: http://gking.harvard.edu/zelig

- Documentation: http://r.iq.harvard.edu/docs/zelig.pdf

- Ameila
  - Website: http://gking.harvard.edu/Amelia/
  - Documetation: http://r.iq.harvard.edu/docs/amelia/amelia.pdf