

Question 1

(b) for the naïve iterative Method involves looping n times performing multiplication n times to compute a^n . so the time complexity is $\Theta(n)$

In divide and conquer we can use recursion as

$$T(n) = T(n/2) + O(1)$$

as the problem divides each step and a constant amount of work is performed at each step that represent $O(1)$ as $a=1$ as we call function once with the same parameter but we multiply it twice with the result

If we want to solve recurrence relation, we can use recurrence tree method:

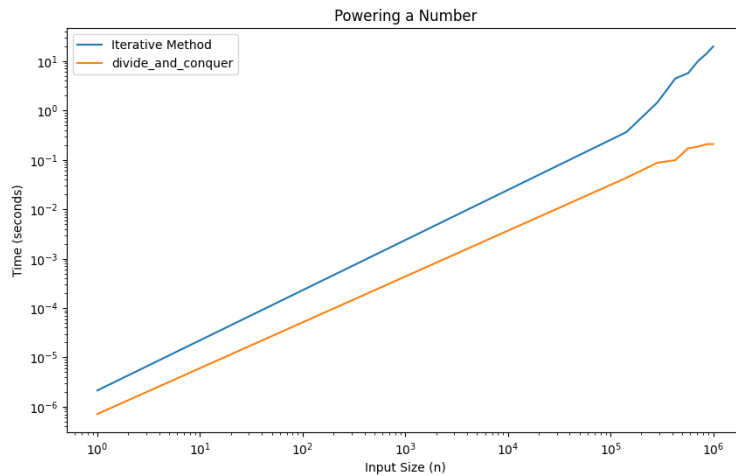
Level	Tree	nodes	cost at level
0	c	1	c
1	c/2	1	c/2
2	c/4	4	c/4
h	$O(1) O(1) O(1) O(1) O(1) O(1)$	n	$c/2^h$

Number of level $h = \log_2(n)$

$$\text{Cost of all leaves} = \sum_{i=0}^{\log_2(n)-1} a^i f(n/b) + \log_2(n) = \sum_{i=0}^{\log_2(n)-1} c/2^i + \log_2(n) =$$

By using geometric series $= 2c(1 - (1/n)) + \log_2(n)$ as $\log_2(n)$ is higher than $(-1/n)$ as decay of $(-1/n)$ faster

So the $\Theta(\log(n))$



I ran the graph function on Colab

As you can see the the effect on time when we use divide and conquer approach as we increase size of n iterative Method is $O(n)$ and divide and conquer is $O(\log(n))$

(d) as illustrated in the graph the shape of in divide and conquer the shape of function tends to be logarithmic ($\log(n)$) as we and it is less than the iterative Method as it appears to be $O(n)$

Question 2

Merge sort $T(n) = 2T(n/2) + O(n)$

$O(n)$ for merge

Level	Tree	nodes	cost at level
0	n	1	n
1	n/2 n/2	2	$2(n/2)$
2	n/4 n/4 n/4 n/4	4	
h	$O(1) O(1) O(1) O(1) O(1) O(1)$	n	$2^i(n/2^i)$

Number of level $h = \log_2(n)$

$$\text{Cost of all leaves} = \sum_{i=0}^{\log_2(n)-1} 2^i f(n/2^i) + \log_2(n) = \sum_{i=0}^{\log_2(n)-1} 2^i (n/2^i) + n \log_2(n) =$$

By using geometric series it will end by $\Theta(n \log_2(n))$

And for binary search $T(n) = T(n/2) + O(1)$

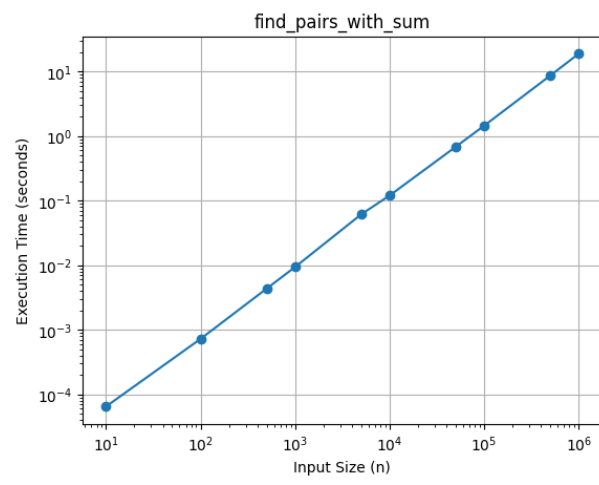
Number of level $h = \log_2(n)$

$$\text{Cost of all leaves} = \sum_{i=0}^{\log_2(n)-1} c/2^i + \log_2(n) =$$

By using geometric series $= 2c(1 - (1/2)^{\log_2(n)}) + \log_2(n)$ as $\log_2(n)$ is higher than $(-1/2)^{\log_2(n)}$ as decay of $(-1/2)^{\log_2(n)}$ is faster

So the $\Theta(\log(n))$

So overall $\Theta(n \log(n))$



(c) as illustrated in the graph the shape of in divide and conquer the shape of function tends to be $n \log(n)$ as the calculated before