

1) def min-time-to-cut(N, a):

count

if(N ≤ 1) → T₂(n)

return count

N = N - a → T₁(n)

a = 2 → T₁(n)

count += 1 T₁(n)

return min-time-to-cut(N/a) → T₁(n)

The aim of algorithm that I design is to decrease a number that is given and minimize it while doing this decreasing approach I try to find the cutting number.

a decrease the N

$$T(n) = T_1(n) + T_2(n)$$

$$T(n) = T(n/2) + 1$$

↓
Because of a is increasing 2 times of every rec part. using Master Theorem I found the complexity

$$T(n) = T(n/2) + n^0 \cdot 1 \rightarrow a=1 \quad b=2 \quad d=0$$

$$a = b^d$$

$$1 = 2^0$$

$$1 = 1 \Rightarrow T(n) = O(n^0 \cdot \log n)$$

$$= O(\log n)$$

2) Because of the code is long I did not write here. The aim of algorithm is to find worst and best case if the list contains only one element worst and best case are same and I return it. If list contains two element only I find which one is best then I return them. But if has more than two element I find middle element and using recursion I found best and worst case.

T₁(n) = First rec calls.

T₂(n) = Second rec calls.

$$T(n) = T_1(n) + T_2(n) + 1 \rightarrow T(n/2) \rightarrow \text{because of dividing 2 the array.}$$

$$T(n) = 2T(n/2) + 1$$

$$a=2, b=2, d=0 \Rightarrow T(n) = O(n^{\log_2 2})$$

$$T(n) = O(n)$$

3) Question is wanted an algorithm that find k th meaning f.l. experiment. So it occurred to me to use the quick select algorithm directly it is decrease and conquer also, so it makes our work easy, so I implement quick select to find k th meaningful experiment.

```

function Meaningful (list, left, right, k)
    if left == right
        return list[left]
    pivotIndex := partition (list, left, right, pivotIndex)
    if k == pivotIndex
        return list[k]
    else if k < pivotIndex
        right = pivotIndex - 1
    else
        left = pivotIndex + 1
    
```

Best case: 1st element is the solution. $T(n) = n + 1 = \Theta(n)$

Avg of case: $T(n) = T(n/2) + n + 1 = \Theta(n)$

Worst case: $\Theta(n^2)$

A more sophisticated choice of the pivot leads to a complicated algorithm with $\Theta(n)$ worst-case efficiency.

4) To find easily the number of reversed-ordered pairs I design merge and count algorithm because it is divide and conquer also. the pairs that need to be counted during the merge step. Therefore to get the total number of inversions that needs to be added on the numbers of inversions in the left subarray right subarray and merge.

Time complexity: $T(n) = T_1(n/2) + T_2(n/2) + n$

$$T(n) = 2T(n/2) + n \rightarrow \begin{matrix} a=2 \\ b=2 \\ d=1 \end{matrix}$$

$$T(n) = \Theta(n \log n)$$

$T_1(n)$ = first rec call.

$T_2(n)$ = second rec call.

Space Complexity: $O(n)$, Temporary array

5) function power

if. $y = 1$

return $x \rightarrow T_1()$

if. $y = 0$

return 1 $\rightarrow T_1()$

if($y \% 2 = 0$)

temp = power($x, y/2$) $\rightarrow T_1()$

return temp * temp

else

temp = power($x, (y-1)/2$) $\rightarrow T_2()$

return x * temp * temp

$$T(n) = T_1(n) + T_2(n) + 1$$

$$T(n) = 2T(n/2) + 1$$

$$a = 2 \quad b = 2 \quad d = 0$$

$$T(n) = \Theta(n^2)$$