

April 28, 2024

HOMEWORK 2 — Report

1 Experimental Work

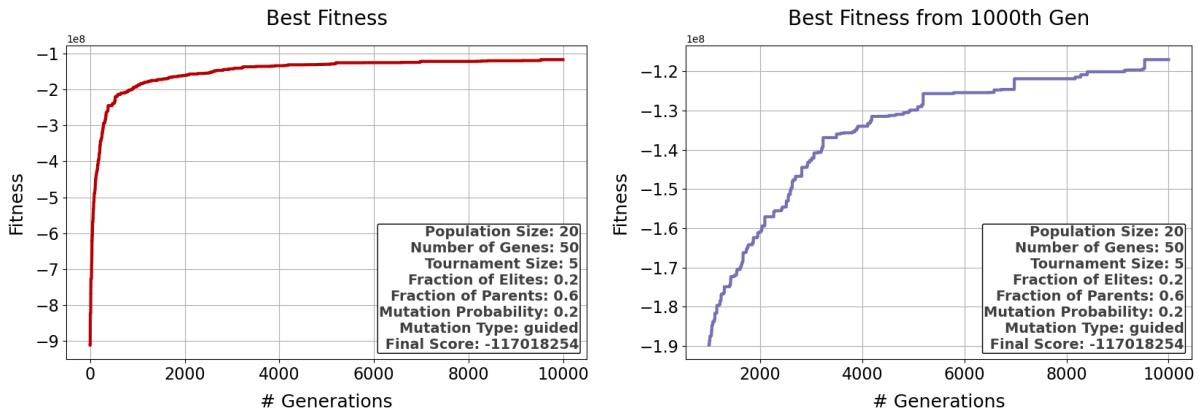
The parameter set provided in Table 1 is used to perform qualitative analysis of different evolutionary algorithm parameters.

Table 1: Parameter set used in this experiment.

Parameter	Values				
<code><num_inds></code>	5	10	20	40	60
<code><num_genes></code>	15	30	50	80	120
<code><tm_size></code>	2	5	8	16	
<code><frac_elites></code>	0.04	0.2	0.35		
<code><frac_parents></code>	0.15	0.3	0.6	0.75	
<code><mutation_prob></code>	0.1	0.2	0.4	0.75	
<code><mutation_type></code>	unguided	guided			

1.1 Default Parameter Set

Let us first start with the results of default parameter set provided in the homework description to be used as a baseline throughout the experiment. Figure 1 shows the plots associated with the fitness value of the best individual. The default parameter set is indicated as bold in Table 1.



(a) From first generation to 10000th generation. (b) From 1000th generation to 10000th generation.

Figure 1: Fitness curves.

The figures tell us that in the first 1000 generations, the fitness value of the best individual increases rapidly. However, after the 1000th generation, the increase in fitness value slows down. However, the fitness value of the best individual is still increasing and we can see this from the figure on the right. That means the algorithm continues to improve even after rapid jump.

The Figure 2 shows the evolution of the best individual quantitatively. The first image is the image in 1000th generation and the last image is the final image. The images in between are the images of the best individual in every 1000 generation. The images are generated by overlaying the circle represented by each gene one by one on the plain white image as instructed.

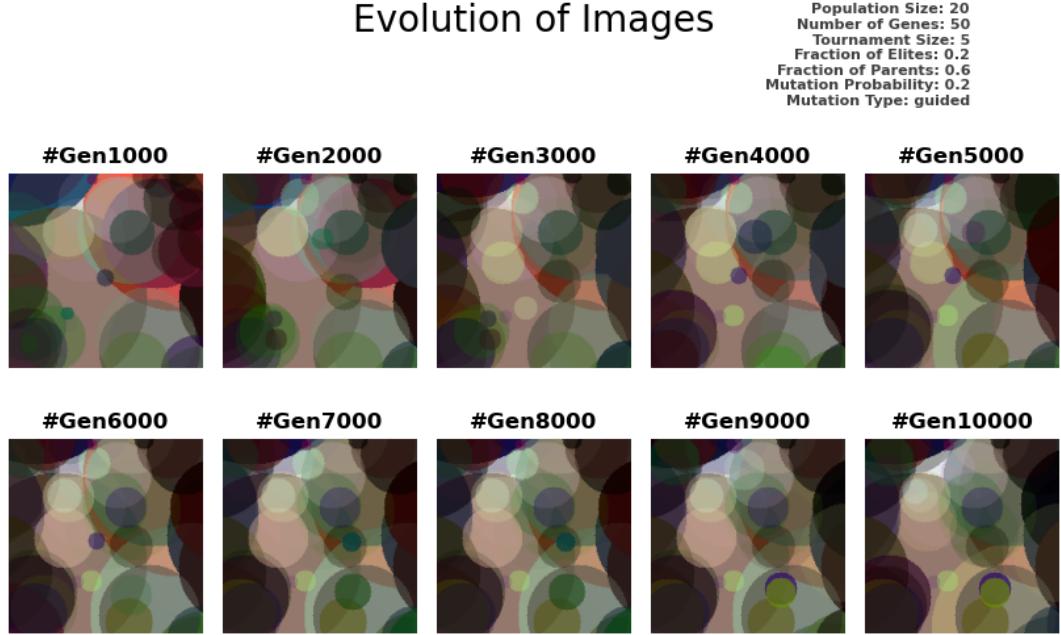


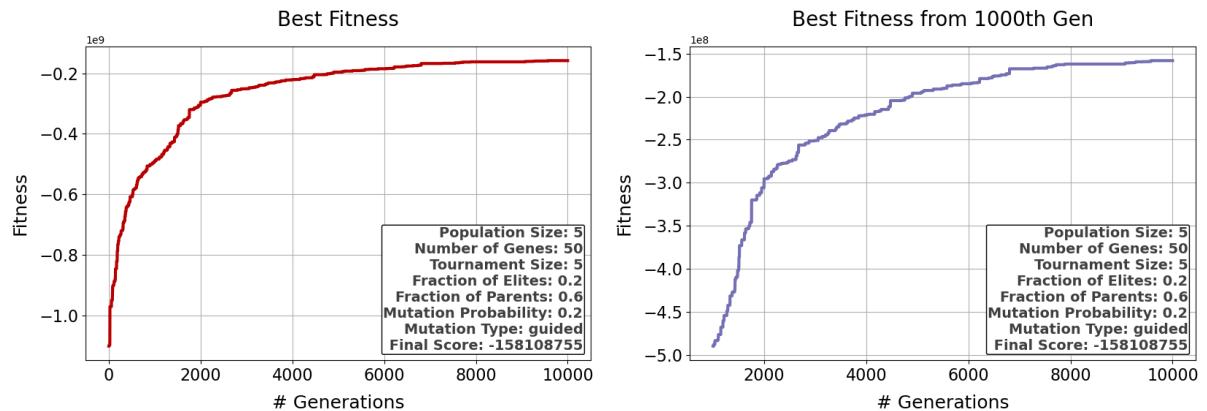
Figure 2: Quantitative evolution of the best individual in the population.

1.2 Number of Individuals

Let us first provide necessary plots for the parameter `<num_inds>`.

1.2.1 5 Individuals

Figure 3 shows the plots associated with the fitness value of the best individual for 5 individuals.



(a) From first generation to 10000th generation.

(b) From 1000th generation to 10000th generation.

Figure 3: Fitness curves for 5 individuals.

The Figure 4 shows the evolution of the best individual quantitatively for 5 individuals.

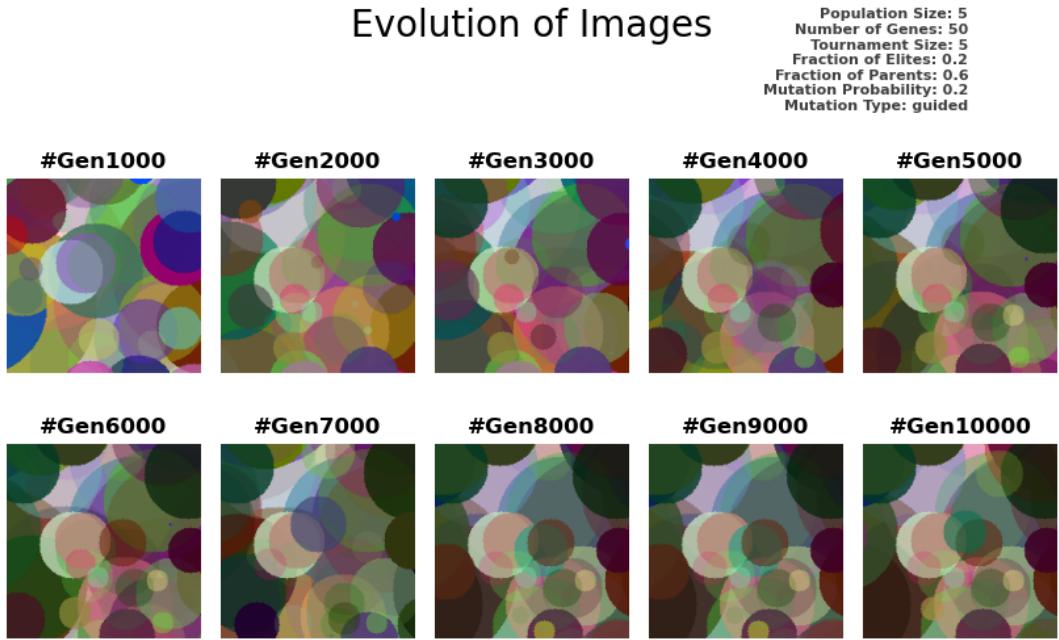


Figure 4: Quantitative evolution of the best individual in the population for 5 individuals.

1.2.2 10 Individuals

Figure 5 illustrates the plots associated with the fitness value of the best individual for 10 individuals.

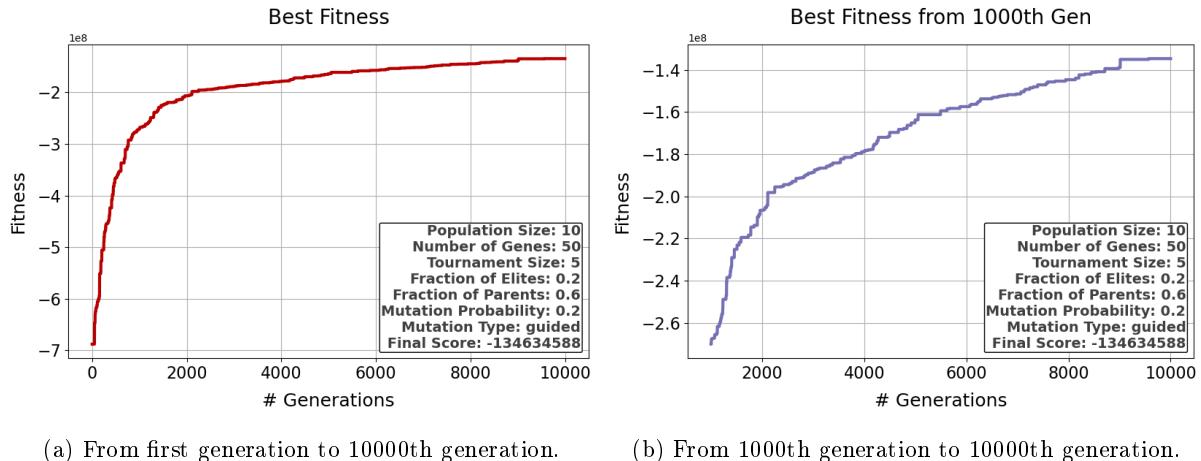


Figure 5: Fitness curves for 10 individuals.

The Figure 6 shows the evolution of the best individual quantitatively for 10 individuals.

1.2.3 40 Individuals

Figure 7 shows the plots associated with the fitness value of the best individual for 40 individuals.

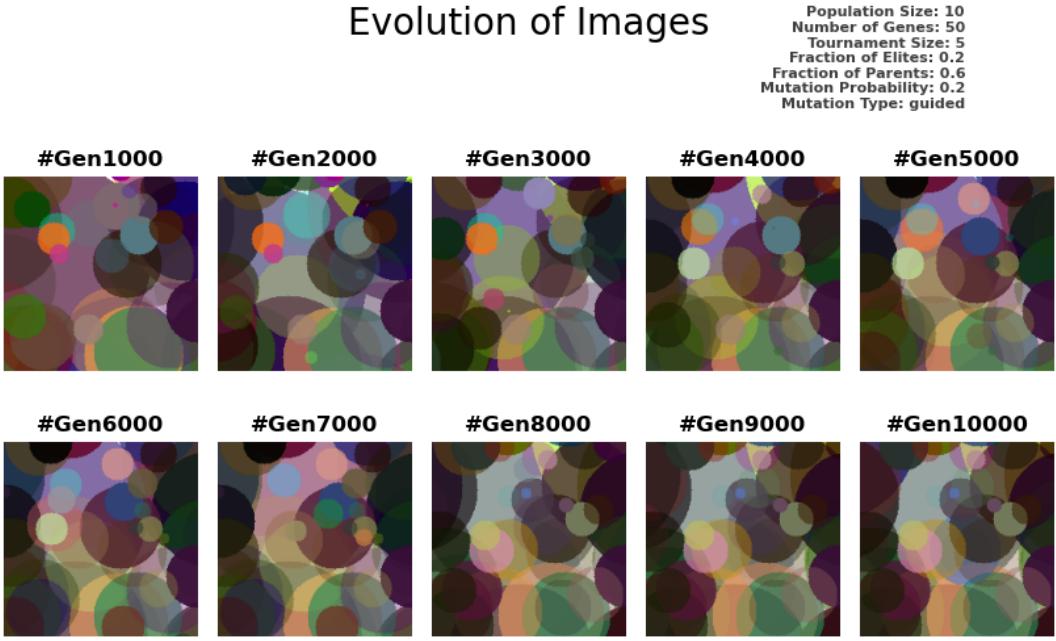


Figure 6: Quantitative evolution of the best individual in the population for 10 individuals.

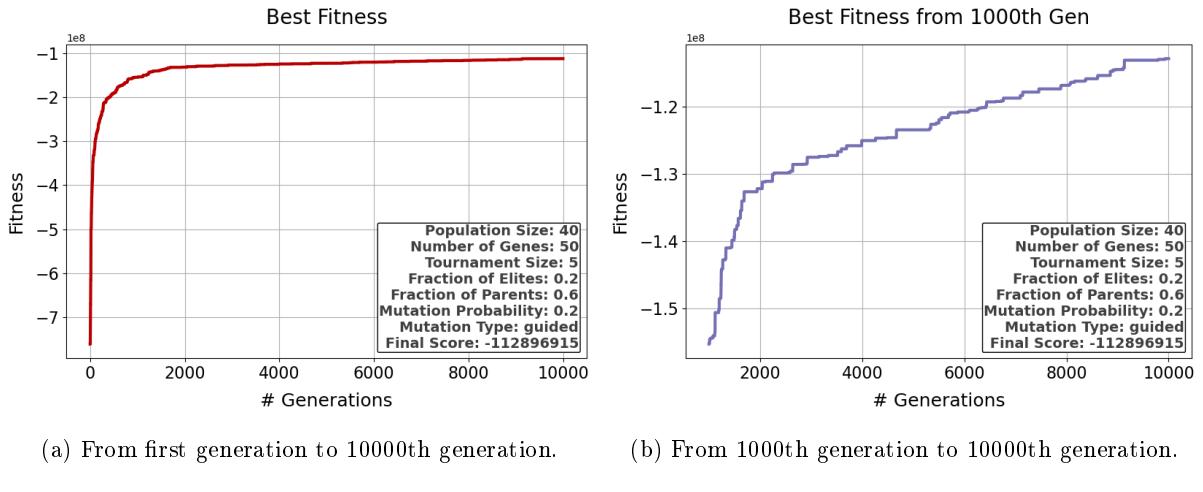


Figure 7: Fitness curves for 40 individuals.

The Figure 8 indicates the evolution of the best individual quantitatively for 40 individuals.

1.2.4 60 Individuals

Figure 9 shows the plots associated with the fitness value of the best individual for 60 individuals.

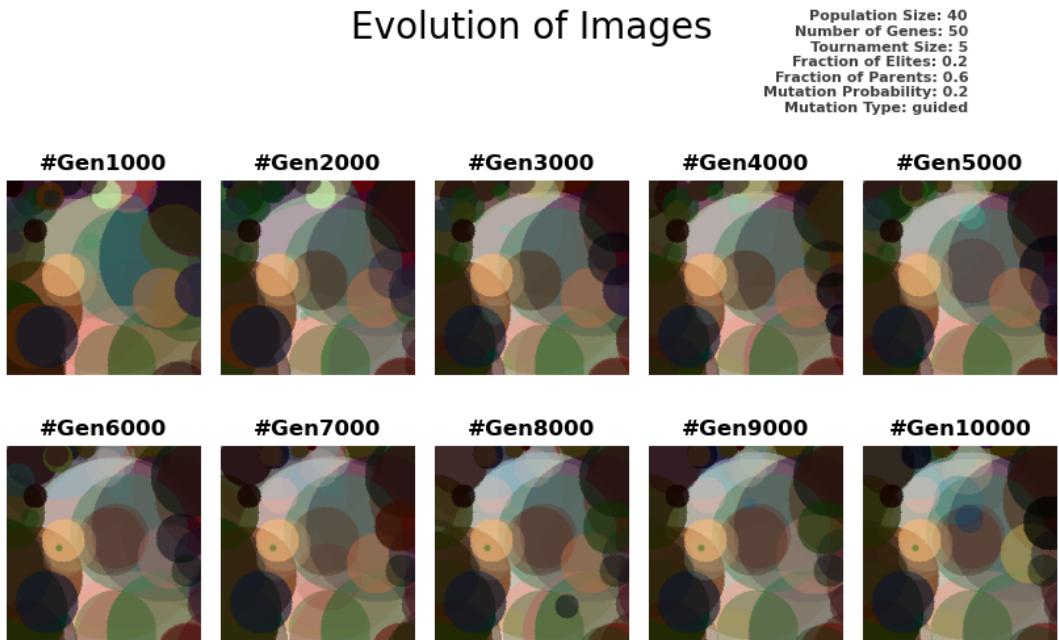


Figure 8: Quantitative evolution of the best individual in the population for 40 individuals.

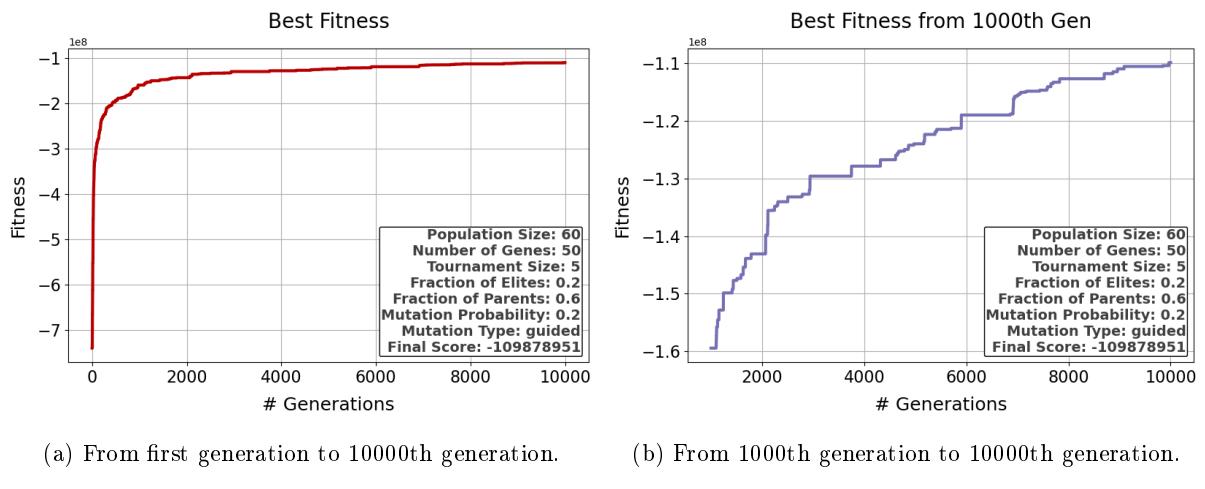


Figure 9: Fitness curves for 60 individuals.

The Figure 10 shows the evolution of the best individual quantitatively for 60 individuals.

Discussion: The results show that the number of individuals in the population has a significant effect on the performance of the algorithm. It can be deduced that as the number of individuals in the population increases, the algorithm converges to a better solution. This is because the diversity in the population increases as the number of individuals increases.

1.3 Number of Genes

Let us first provide necessary plots for the parameter `<num_genes>`.

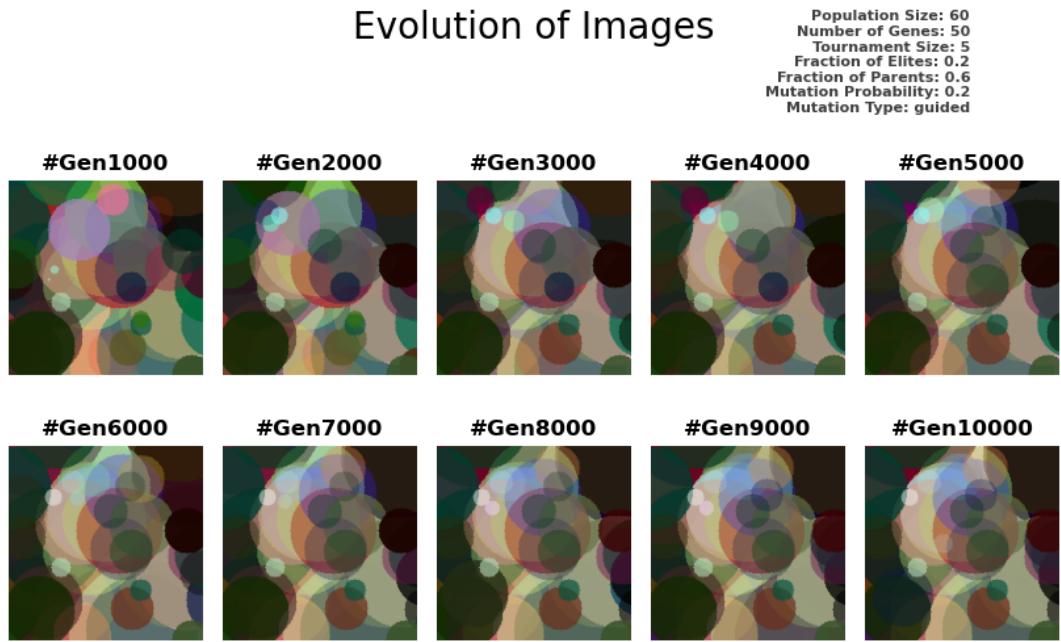


Figure 10: Quantitative evolution of the best individual in the population for 60 individuals.

1.3.1 15 Genes

Figure 11 shows the plots related to the fitness value of the best individual for 15 genes.

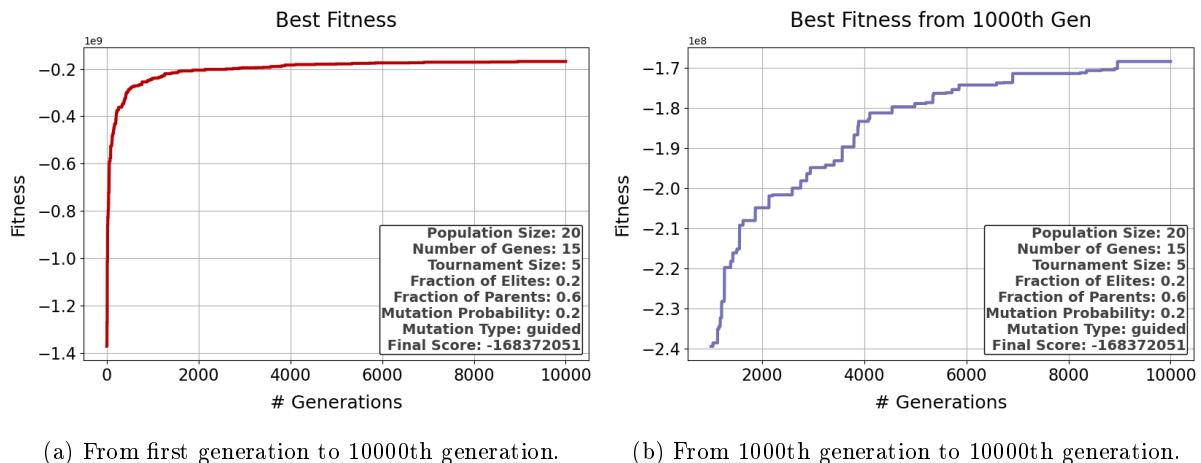


Figure 11: Fitness curves for 15 genes.

The Figure 12 illustrates the evolution of the best individual figuratively for 15 genes.

1.3.2 30 Genes

Figure 13 shows the plots related to the fitness value of the best individual for 30 genes.

Evolution of Images

Population Size: 20
 Number of Genes: 15
 Tournament Size: 5
 Fraction of Elites: 0.2
 Fraction of Parents: 0.6
 Mutation Probability: 0.2
 Mutation Type: guided

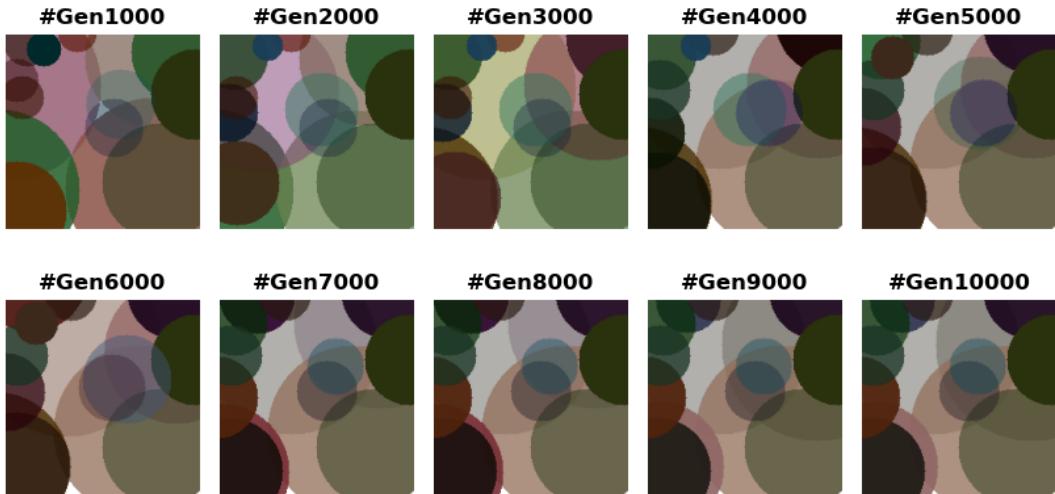


Figure 12: Quantitative evolution of the best individual in the population for 15 genes.

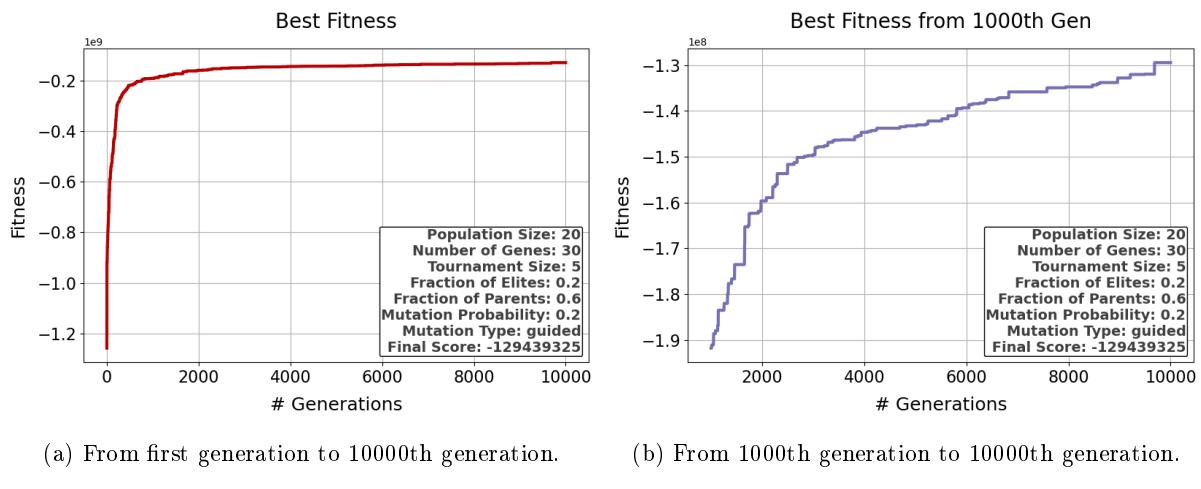


Figure 13: Fitness curves for 30 genes.

The Figure 14 shows the evolution of the best individual figuratively for 30 genes.

1.3.3 80 Genes

Figure 15 shows the plots related to the fitness value of the best individual for 80 genes.

Evolution of Images

Population Size: 20
 Number of Genes: 30
 Tournament Size: 5
 Fraction of Elites: 0.2
 Fraction of Parents: 0.6
 Mutation Probability: 0.2
 Mutation Type: guided

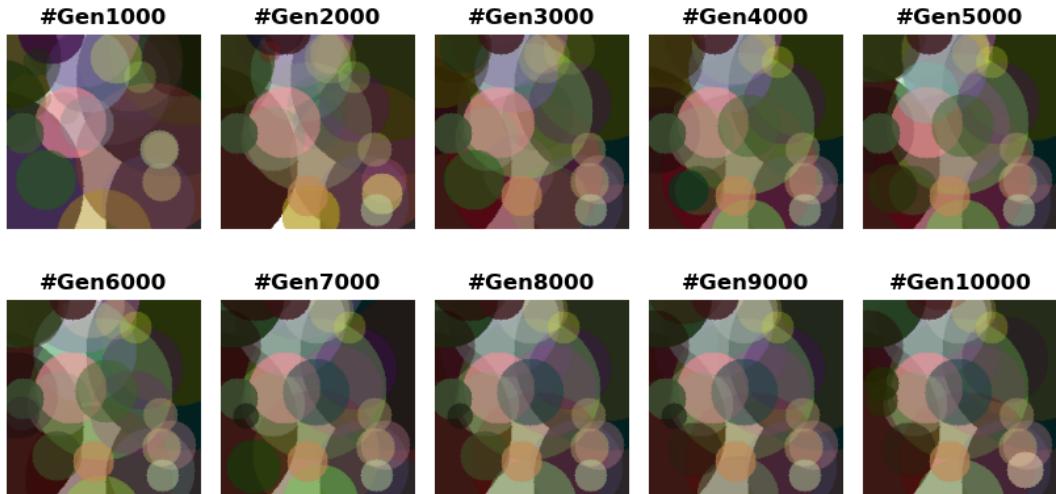


Figure 14: Quantitative evolution of the best individual in the population for 30 genes.

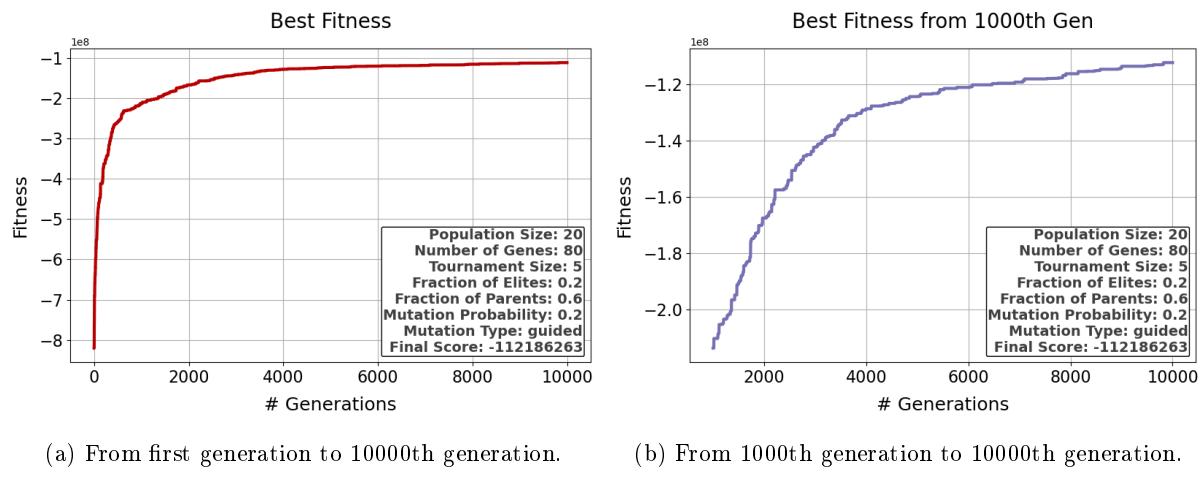


Figure 15: Fitness curves for 80 genes.

The Figure 16 shows the evolution of the best individual figuratively for 80 genes.

1.3.4 120 Genes

Figure 17 shows the plots related to the fitness value of the best individual for 120 genes.

Evolution of Images

Population Size: 20
 Number of Genes: 80
 Tournament Size: 5
 Fraction of Elites: 0.2
 Fraction of Parents: 0.6
 Mutation Probability: 0.2
 Mutation Type: guided



Figure 16: Quantitative evolution of the best individual in the population for 80 genes.

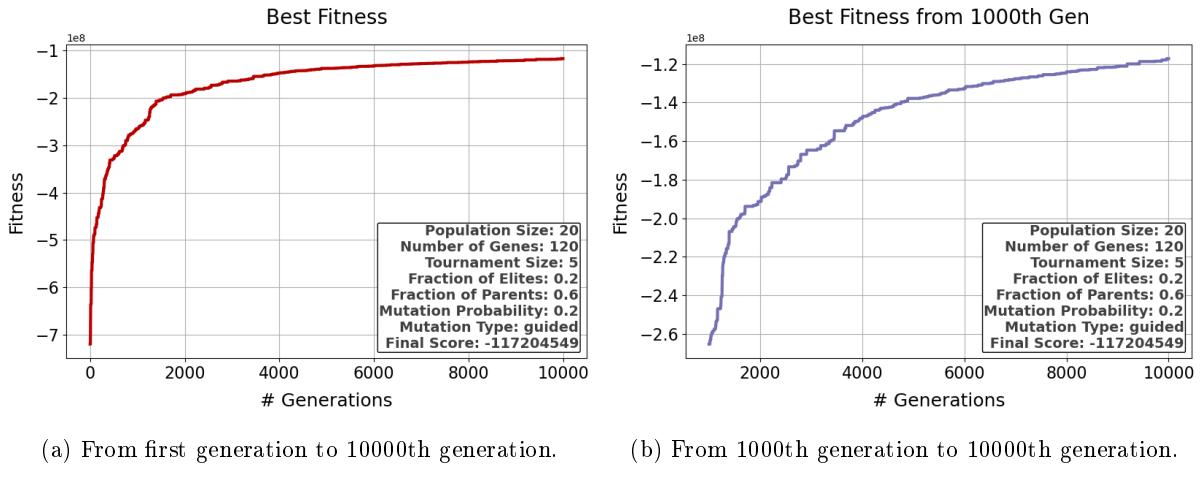


Figure 17: Fitness curves for 120 genes.

The Figure 18 is given for the evolution of the best individual for 120 genes.

Discussion: We see that, as the number of genes in the individual increases, the algorithm converges to a better solution. This is because the number of genes in the individual increases, the search space increases and the algorithm can explore more possibilities. This was somewhat expected since the number of genes in the individual is directly related to "paint brushes" we have in this problem. However, from 80 genes to 120 genes the improvement is not observable 80 genes converged to a better solution. We can interpret that for 120 genes there need to be more generations to converge to a better solution. So, as the number of genes decreases, the number of generations needed to converge is also decreased. Given we fixed the number of generations, the results are natural in this sense.

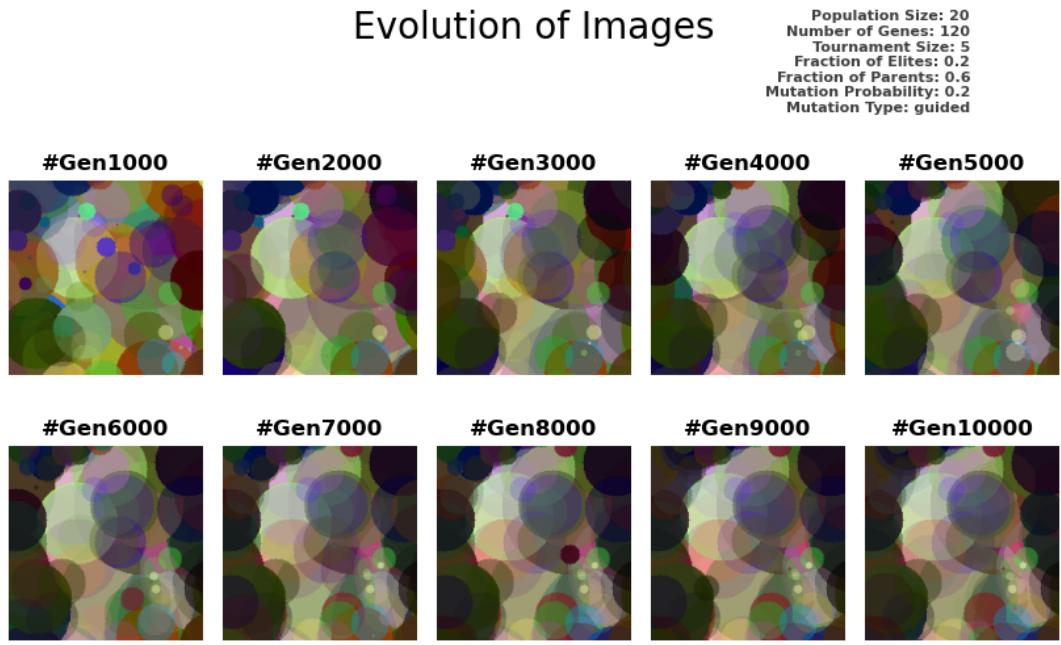


Figure 18: Quantitative evolution of the best individual in the population for 120 genes.

1.4 Tournament Size

Let us first provide necessary plots for the parameter `<tm_size>`.

1.4.1 2 Tournament Size

Figure 19 shows the plots related to the fitness value of the best individual for 2 tournament size.

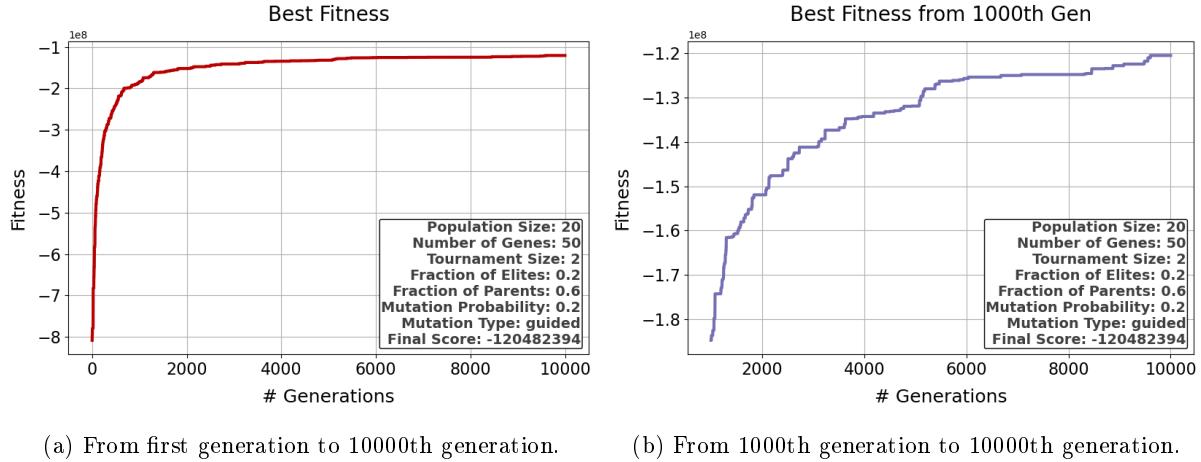


Figure 19: Fitness curves for 2 tournament size.

The Figure 20 shows the evolution of the best individual figuratively for 2 tournament size.

1.4.2 8 Tournament Size

Figure 21 shows the plots related to the fitness value of the best individual for 8 tournament size.

Evolution of Images

Population Size: 20
 Number of Genes: 50
 Tournament Size: 2
 Fraction of Elites: 0.2
 Fraction of Parents: 0.6
 Mutation Probability: 0.2
 Mutation Type: guided

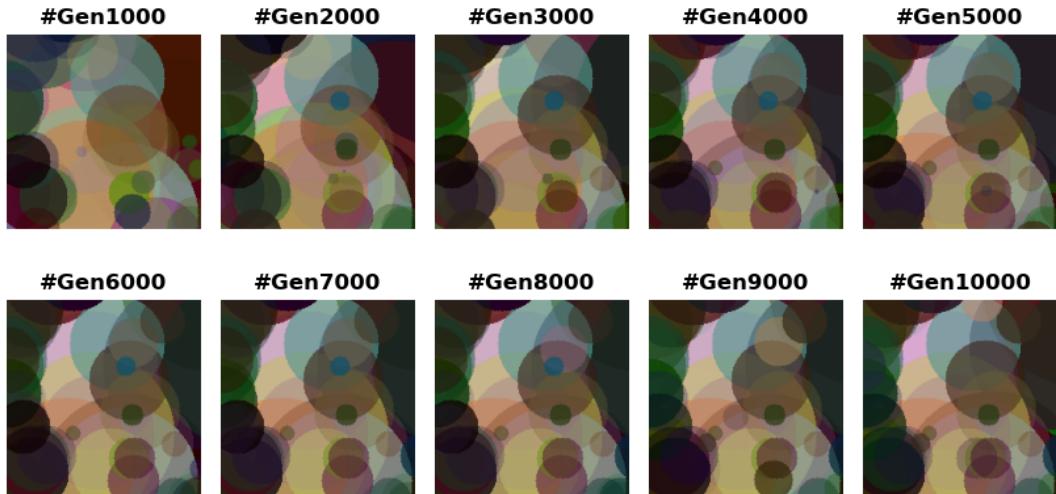


Figure 20: Quantitative evolution of the best individual in the population for 2 tournament size.

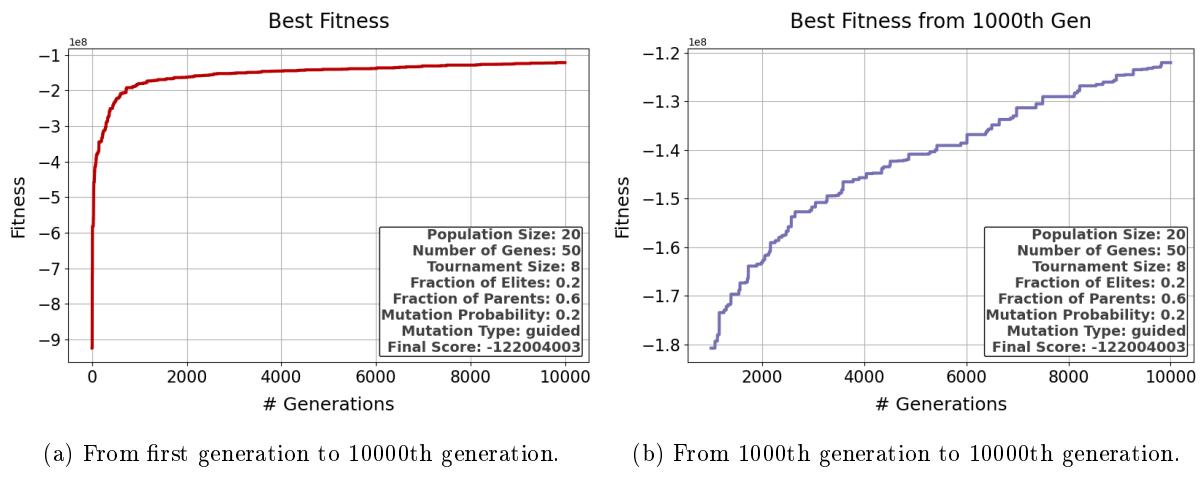


Figure 21: Fitness curves for 8 tournament size.

The Figure 22 shows the evolution of the best individual figuratively for 8 tournament size.

1.4.3 16 Tournament Size

Figure 23 shows the plots related to the fitness value of the best individual for 16 tournament size.

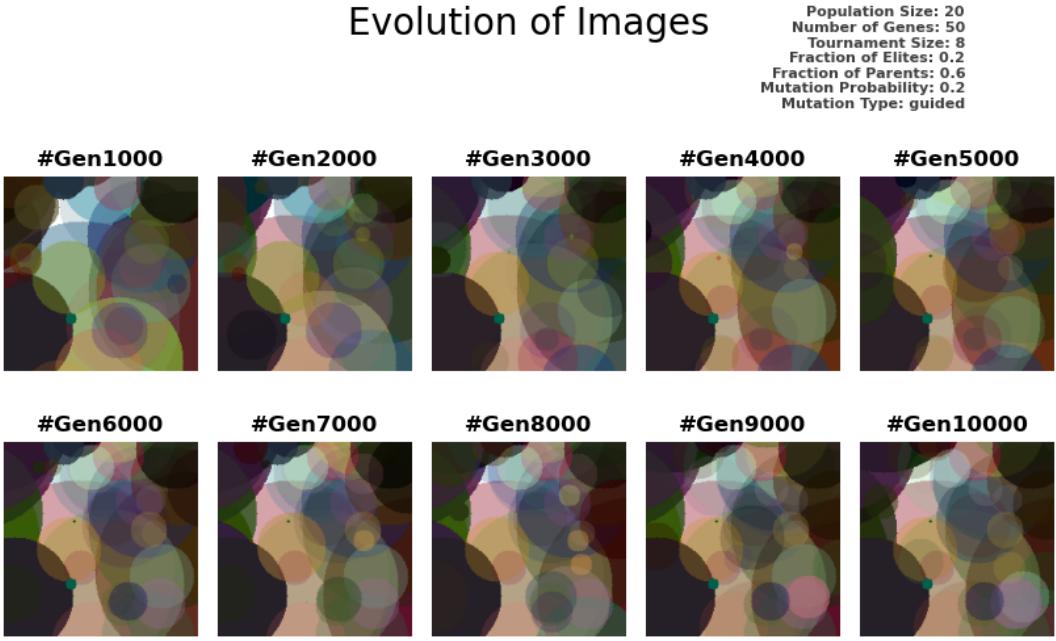


Figure 22: Quantitative evolution of the best individual in the population for 8 tournament size.

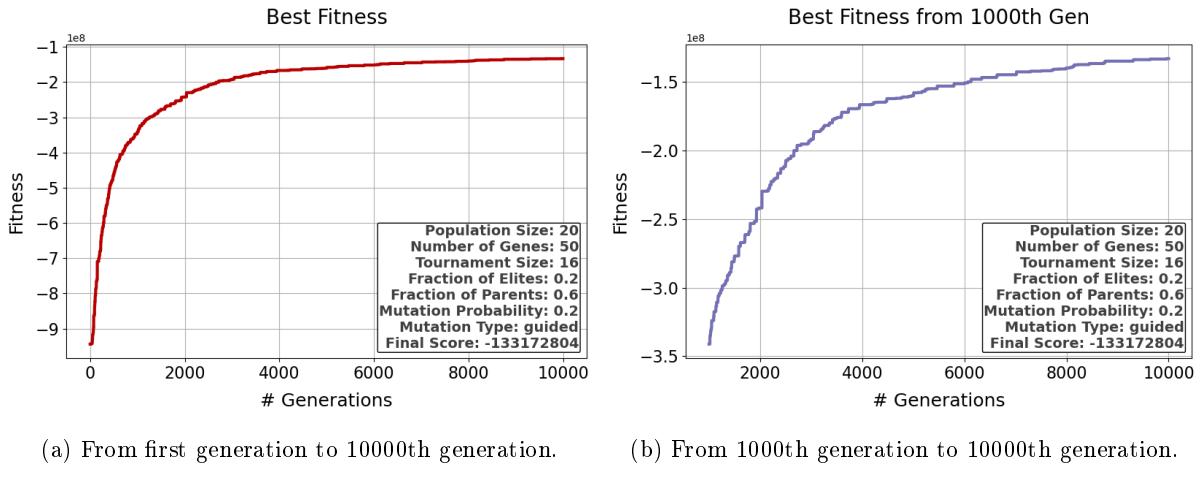


Figure 23: Fitness curves for 16 tournament size.

The Figure 24 shows the evolution of the best individual figuratively for 16 tournament size.

Discussion: From the results we see that the best tournament size selection for this parameter space was 5. This means there are no linear relationship between the tournament size and the performance of the algorithm. As the tournament size increases too much the diversity decreased. It can be said that tournament size is a hyperparameter that needs to be tuned for the specific problem. If we change the other parameters such as number of individuals, number of genes, etc. the best tournament size would change.

1.5 Number of Individuals Advancing to Next Generation

Let us first provide necessary plots for the parameter `<frac_elites>`.

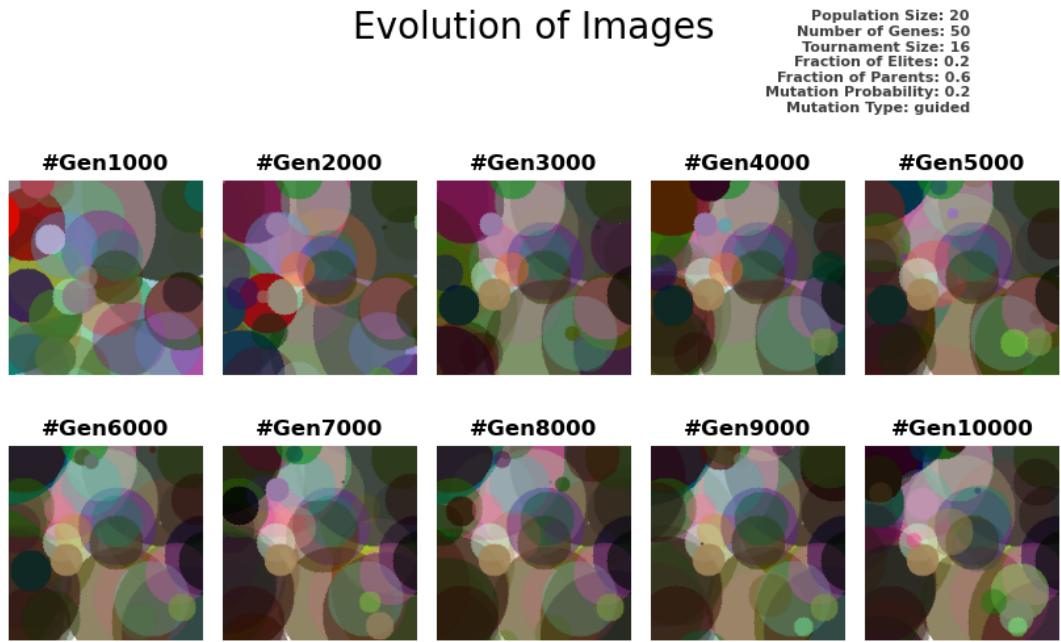


Figure 24: Quantitative evolution of the best individual in the population for 16 tournament size.

1.5.1 0.04 Fraction of Elites

Figure 25 shows the plots related to the fitness value of the best individual for 0.04 fraction of elites.

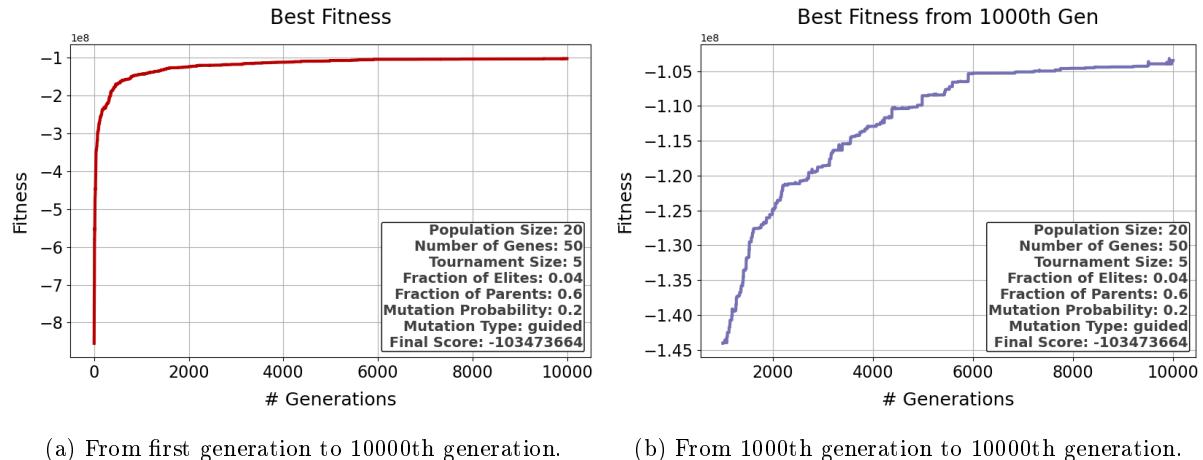


Figure 25: Fitness curves for 0.04 fraction of elites.

The Figure 26 shows the evolution of the best individual figuratively for 0.04 fraction of elites.

1.5.2 0.35 Fraction of Elites

Figure 27 shows the plots related to the fitness value of the best individual for 0.35 fraction of elites.

Evolution of Images

Population Size: 20
 Number of Genes: 50
 Tournament Size: 5
 Fraction of Elites: 0.04
 Fraction of Parents: 0.6
 Mutation Probability: 0.2
 Mutation Type: guided

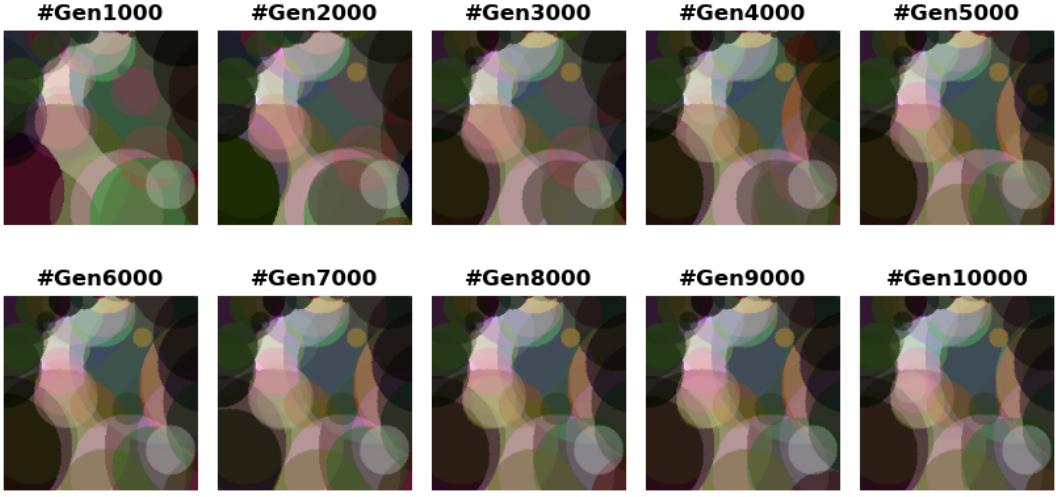


Figure 26: Quantitative evolution of the best individual in the population for 0.04 fraction of elites.

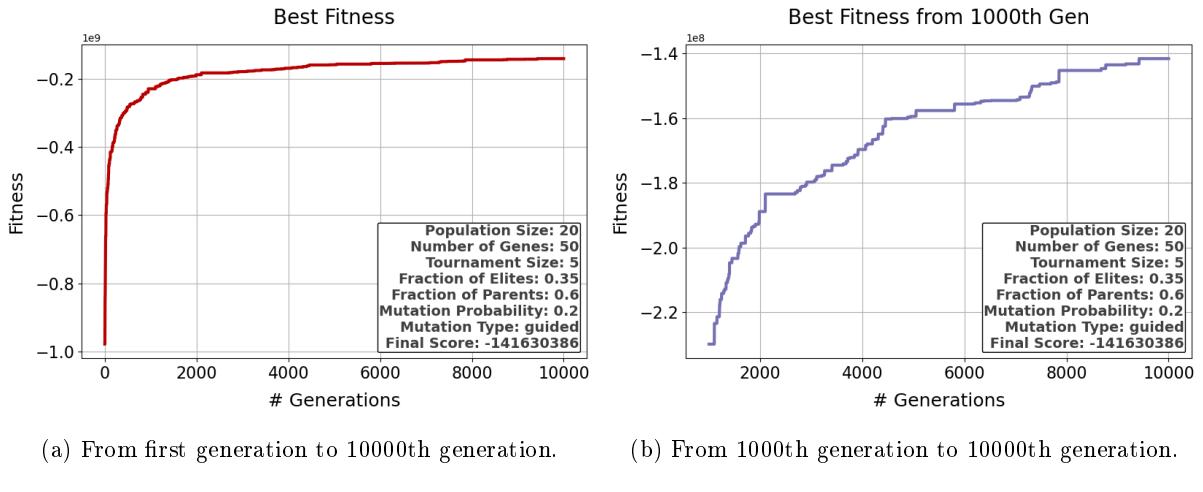


Figure 27: Fitness curves for 0.35 fraction of elites.

The Figure 28 shows the evolution of the best individual figuratively for 0.35 fraction of elites.

Discussion: The results indicates that the best selection for the fraction of elites is 0.04. We may interpret this as the algorithm needs more diversity in the population to converge to a better solution. On the other hand this implied follows, as the number of elites decreases the parents selected are shifts to a better group of individuals. So, it is hard to tell the number of elites is decoupled from the other parameters. If we select parents first from the best individuals and then from the rest of the population, the best fraction of elites would change.

1.6 Number of Parents Used In Crossover

Let us first provide necessary plots for the parameter `<frac_parents>`.

Evolution of Images

Population Size: 20
 Number of Genes: 50
 Tournament Size: 5
 Fraction of Elites: 0.35
 Fraction of Parents: 0.6
 Mutation Probability: 0.2
 Mutation Type: guided



Figure 28: Quantitative evolution of the best individual in the population for 0.35 fraction of elites.

1.6.1 0.15 Fraction of Parents

The plots about to the fitness metric of the best individual for 0.15 fraction of parents in Figure 29.

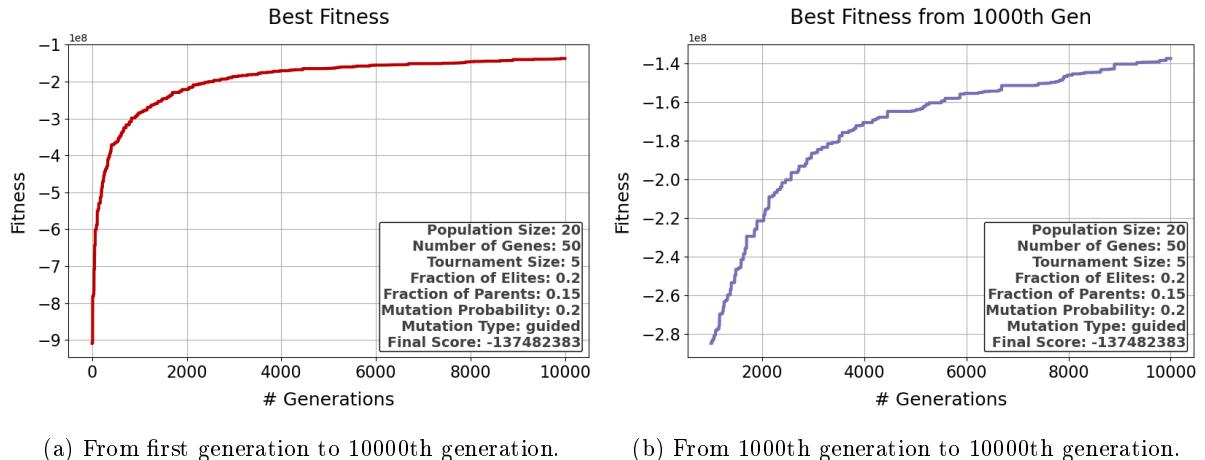


Figure 29: Fitness curves for 0.15 fraction of parents.

The evolution of the best individual for 0.15 fraction of parents is given in Figure 30.

1.6.2 0.3 Fraction of Parents

The plots about to the fitness metric of the best individual for 0.3 fraction of parents in Figure 31.

Evolution of Images

Population Size: 20
 Number of Genes: 50
 Tournament Size: 5
 Fraction of Elites: 0.2
 Fraction of Parents: 0.15
 Mutation Probability: 0.2
 Mutation Type: guided

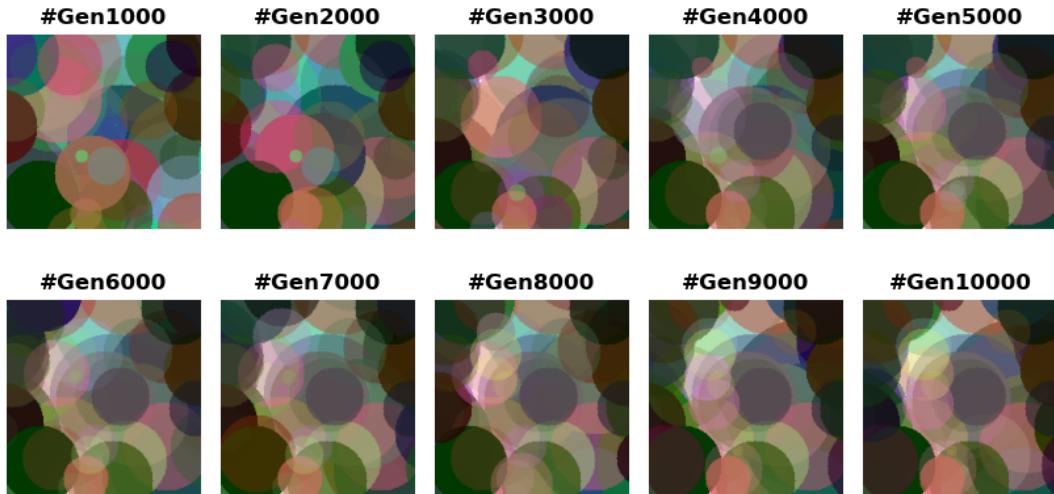


Figure 30: Quantitative evolution of the best individual in the population for 0.15 fraction of parents.

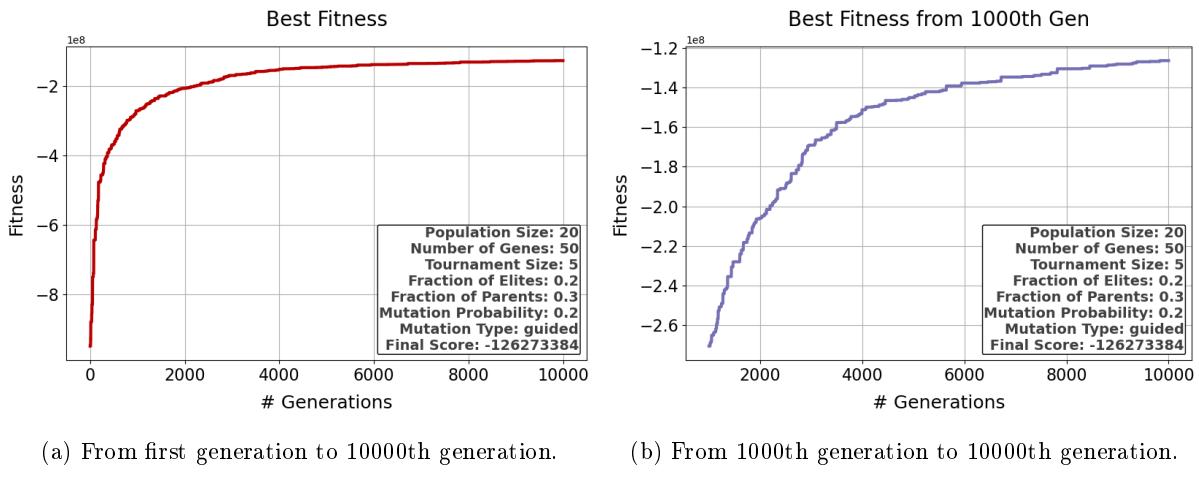


Figure 31: Fitness curves for 0.3 fraction of parents.

The evolution of the best individual for 0.3 fraction of parents is given in Figure 32.

1.6.3 0.75 Fraction of Parents

The plots about to the fitness metric of the best individual for 0.75 fraction of parents in Figure 33.

Evolution of Images

Population Size: 20
 Number of Genes: 50
 Tournament Size: 5
 Fraction of Elites: 0.2
 Fraction of Parents: 0.3
 Mutation Probability: 0.2
 Mutation Type: guided



Figure 32: Quantitative evolution of the best individual in the population for 0.3 fraction of parents.

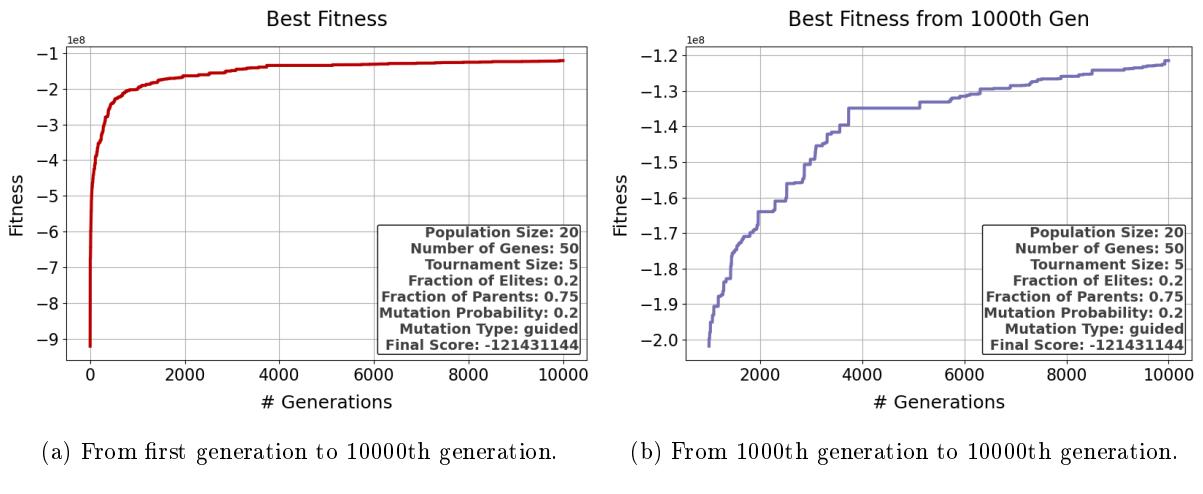


Figure 33: Fitness curves for 0.75 fraction of parents.

The evolution of the best individual for 0.75 fraction of parents is given in Figure 34.

Discussion: The results show that the best selection for the fraction of parents is 0.6. This means that the algorithm needs to select parents from a wide range of individuals to converge to a better solution. This is because the diversity in the population is important for the algorithm to explore more possibilities. If the algorithm selects parents from a small group of individuals, the algorithm may converge to a local minimum. On the other hand for the case of 0.75 fraction of parents, we see that increasing the fraction of parents did not help. We may interpret this result as the algorithm needs to select parents from a wide range of individuals but not from the whole population.

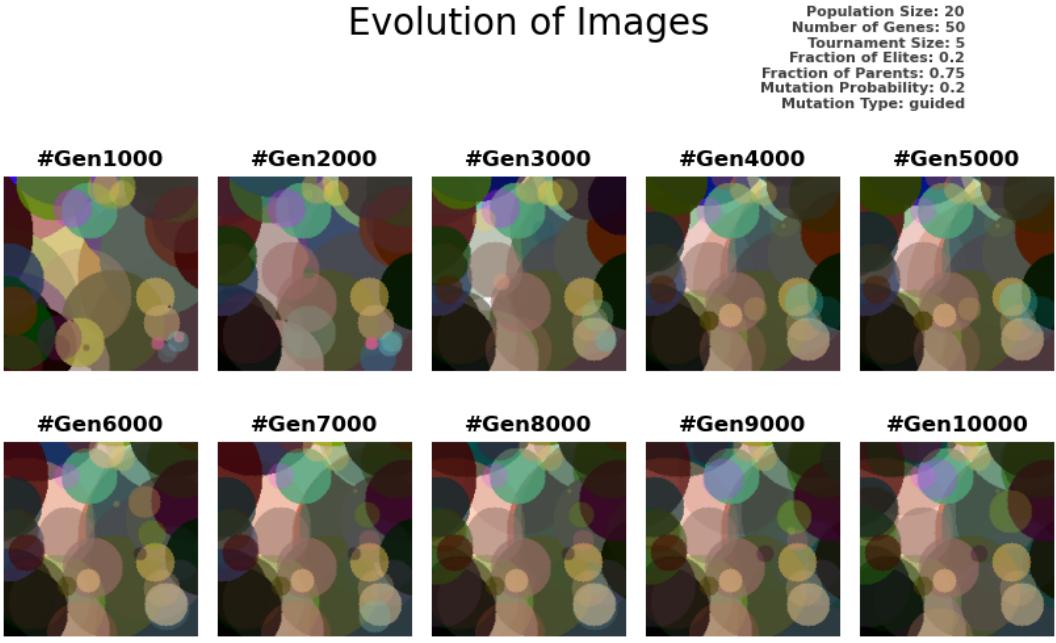


Figure 34: Quantitative evolution of the best individual in the population for 0.75 fraction of parents.

1.7 Mutation Probability

Let us first provide necessary plots for the parameter <mutation_prob>.

1.7.1 0.1 Mutation Probability

The plots about to the fitness metric of the best individual for 0.1 mutation probability in Figure 35.

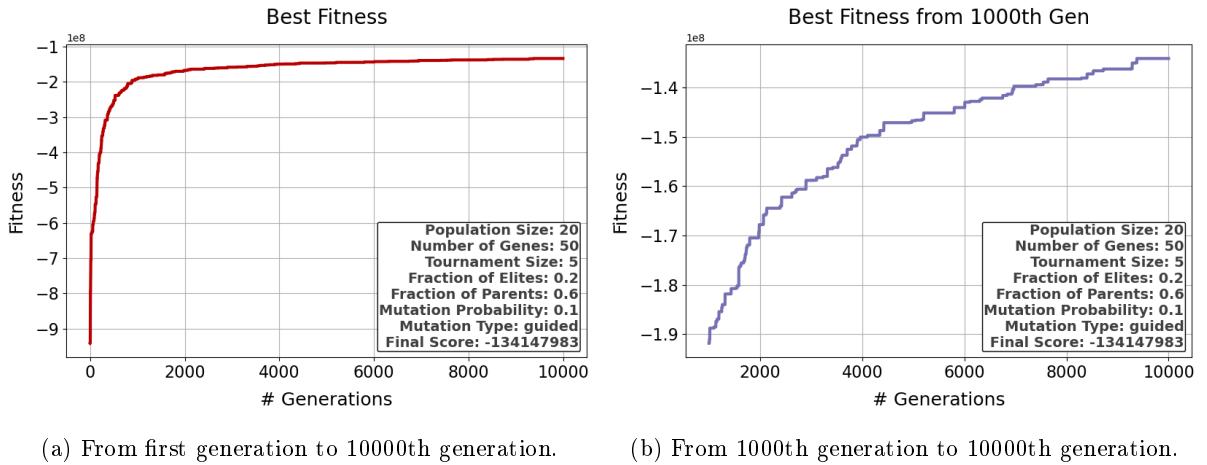


Figure 35: Fitness curves for 0.1 mutation probability.

The evolution of the best individual for 0.1 mutation probability is given in Figure 36.

1.7.2 0.4 Mutation Probability

The plots about to the fitness metric of the best individual for 0.4 mutation probability in Figure 37.

Evolution of Images

Population Size: 20
 Number of Genes: 50
 Tournament Size: 5
 Fraction of Elites: 0.2
 Fraction of Parents: 0.6
 Mutation Probability: 0.1
 Mutation Type: guided

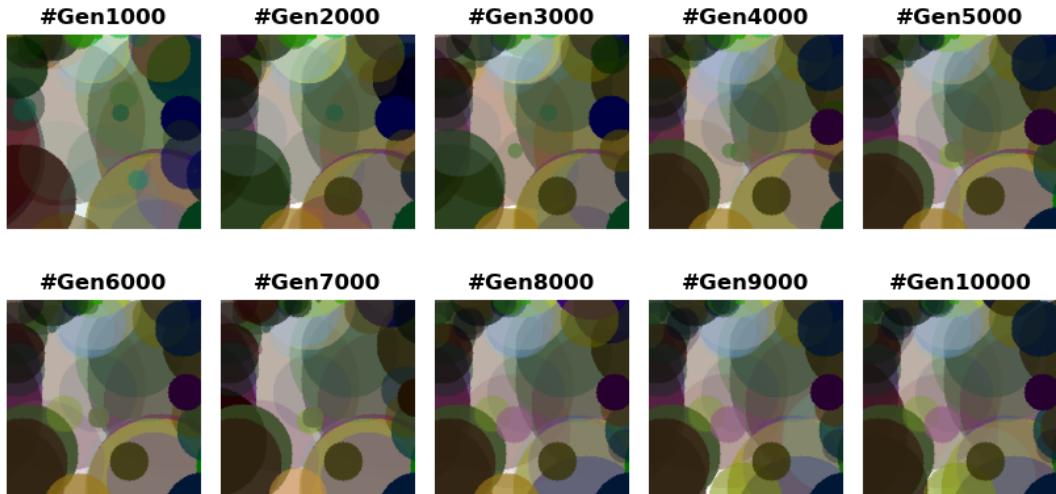


Figure 36: Quantitative evolution of the best individual in the population for 0.1 mutation probability.

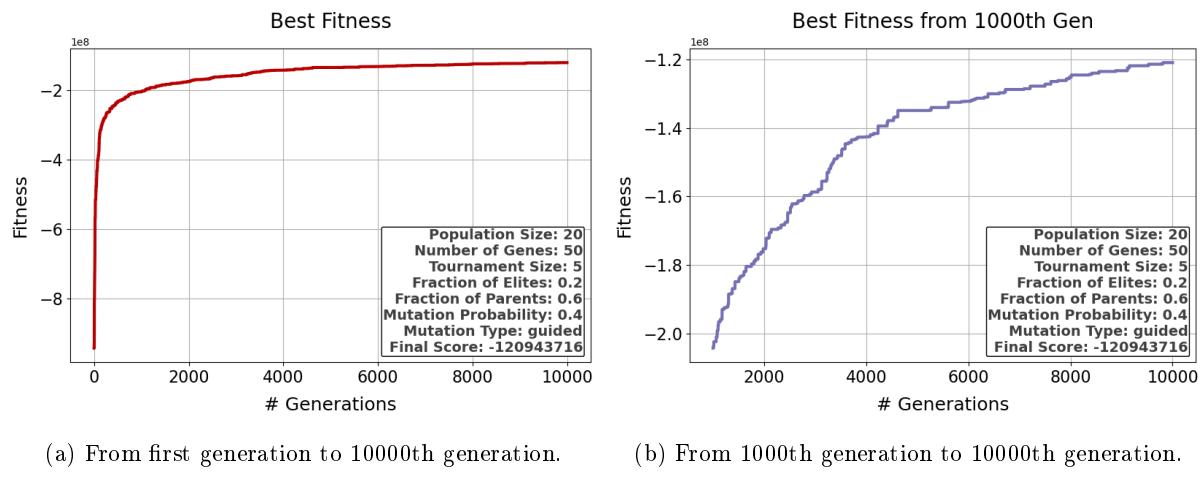


Figure 37: Fitness curves for 0.4 mutation probability.

The evolution of the best individual for 0.4 mutation probability is given in Figure 38.

1.7.3 0.75 Mutation Probability

The plots about to the fitness metric of the best individual for 0.75 mutation probability in Figure 39.

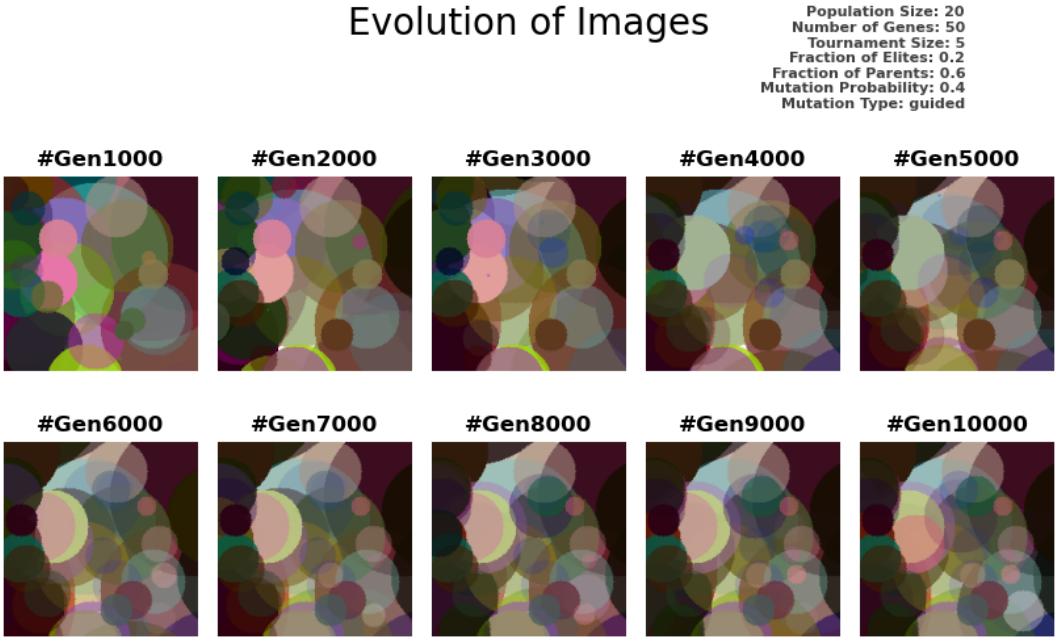


Figure 38: Quantitative evolution of the best individual in the population for 0.4 mutation probability.

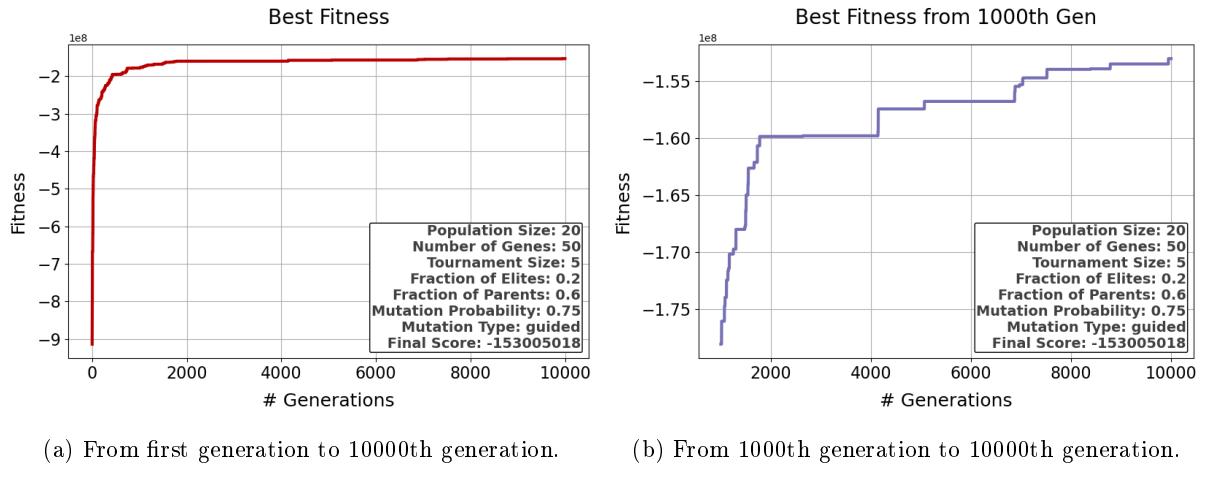


Figure 39: Fitness curves for 0.75 mutation probability.

The evolution of the best individual for 0.75 mutation probability is given in Figure 40.

Discussion: The results show that the best selection for the mutation probability is 0.2. This tells us two things. First, if the mutation probability is too low, the algorithm may not be able to explore the search space effectively. Second, if the mutation probability is too high, the algorithm may not be able to converge to a better solution. This is because if the mutation probability is too high, the algorithm may not be able to preserve the good genes in the population. So, the mutation probability is a hyperparameter that needs to be tuned for the specific problem.

1.8 Mutation Type

Let us first provide necessary plots for the parameter <mutation_type>.

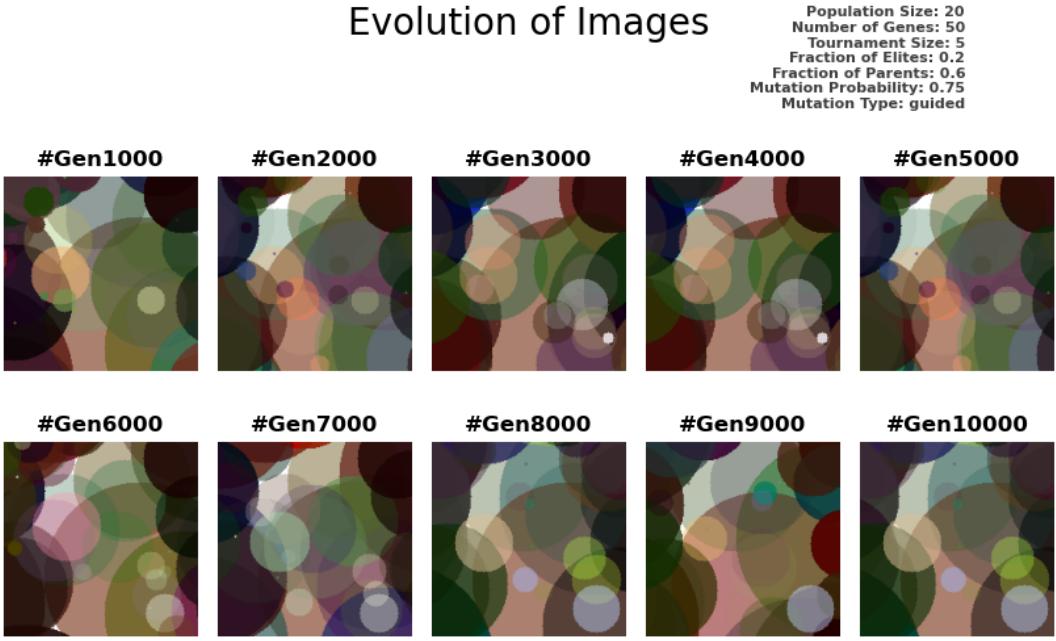


Figure 40: Quantitative evolution of the best individual in the population for 0.75 mutation probability.

1.8.1 Unguided Mutation

The plots about to the fitness metric of the best individual for unguided mutation in Figure 41.

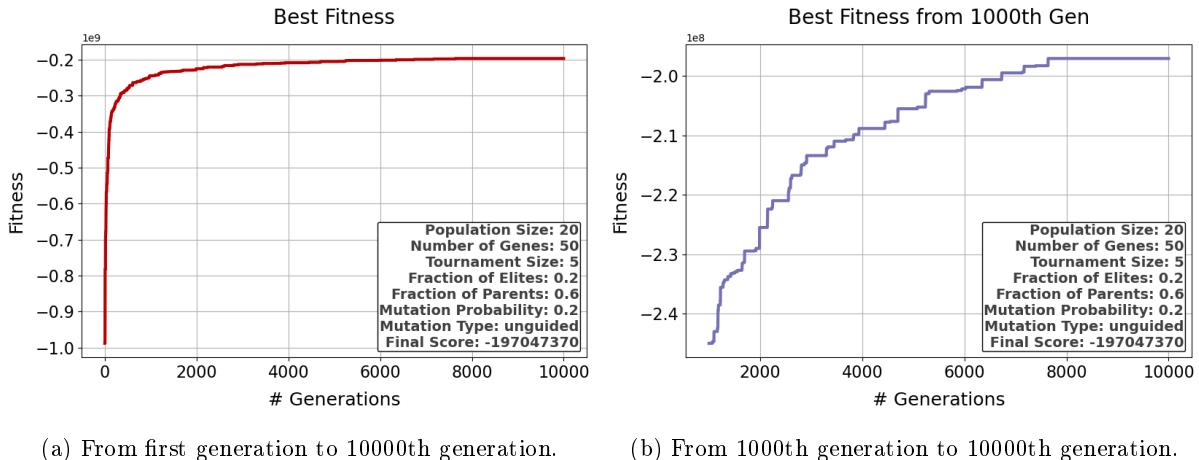


Figure 41: Fitness curves for unguided mutation.

The evolution of the best individual for unguided mutation is given in Figure 42.

Discussion: It is quite obvious that if the mutation is unguided, the algorithm cannot converge to a better solution. This is because the mutation become reinitialization of the gene. So, the mutation loses its evolutionary meaning.

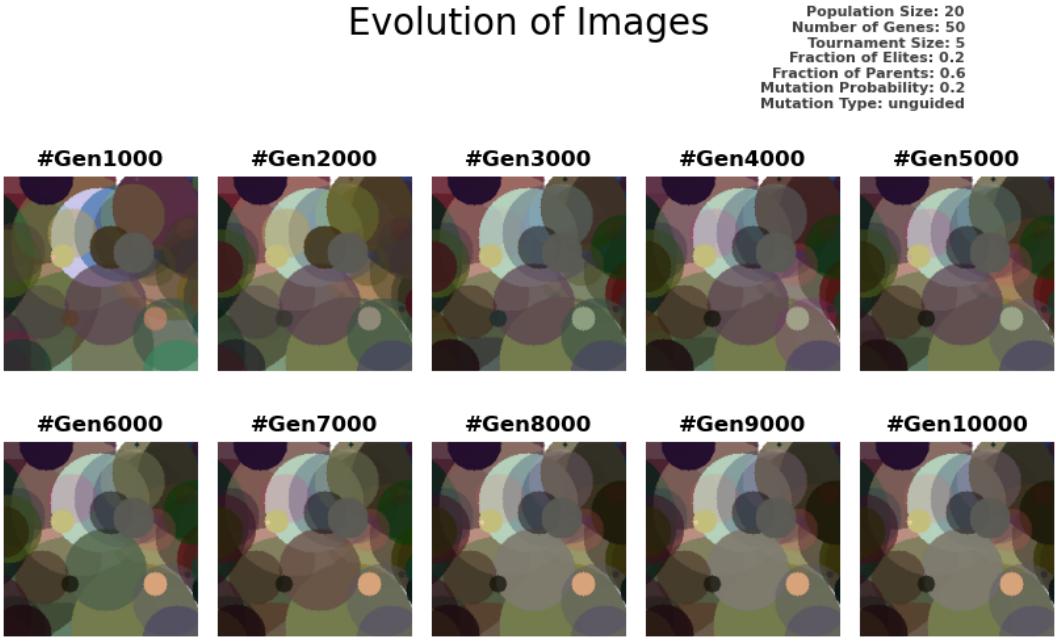


Figure 42: Quantitative evolution of the best individual in the population for unguided mutation.

2 Discussion - Suggestions

There are three suggestions, based on our observations and results, that can be made to improve the performance of the algorithm.

2.1 Probabilistic Mutation Probability

First suggestion is, the mutation probability can be made stochastic. This means that the mutation probability can be changed in each generation. This can be done by using a probability distribution. For example, the mutation probability can be sampled from a beta distribution in each generation. This can help the algorithm to explore the search space more effectively. Figure 44 shows the fitness curves for the best individual when the mutation probability is stochastic. An example pdf for beta distribution is shown in Figure 43.

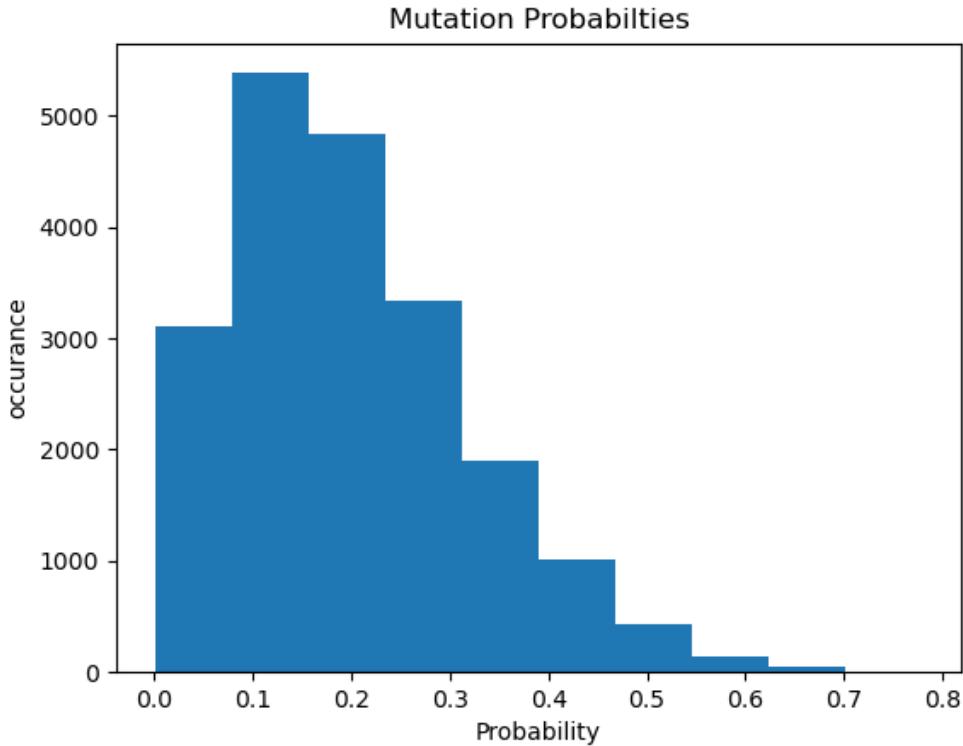
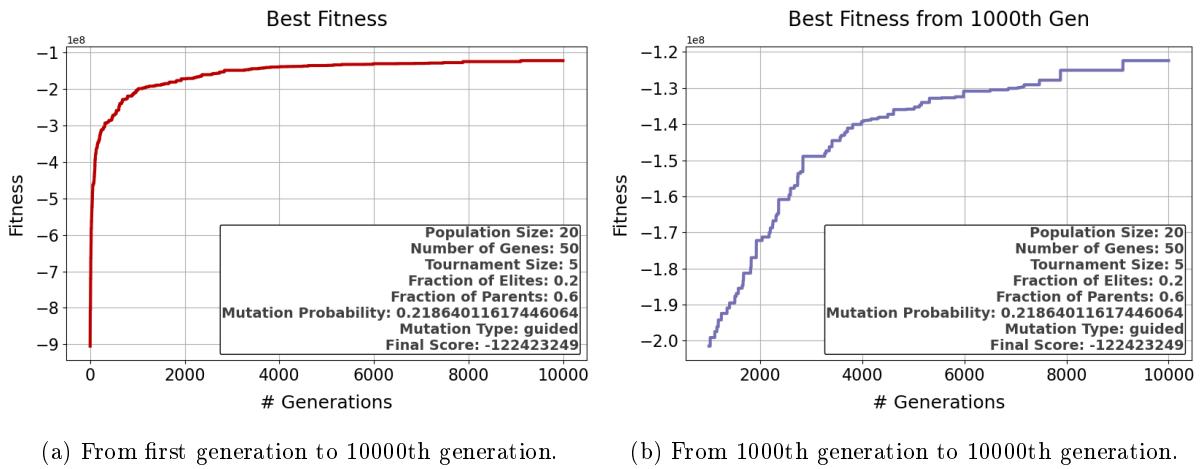


Figure 43: A sample histogram for beta distributed samples when value is around 0.2.



(a) From first generation to 10000th generation.

(b) From 1000th generation to 10000th generation.

Figure 44: Fitness curves for stochastic mutation probability.

The evolution of the best individual for stochastic mutation probability is given in Figure 45.

Even though the results are not conclusive, the stochastic mutation probability can be a good idea to explore the search space more effectively by deviating around the mutation probability in each generation.

2.2 Mutation Probability Scheduling

Similarly, the mutation probability can be made dynamic. This means that the mutation probability can be changed based on the number of generations. This can help the algorithm to escape from local minima. Figure 46 shows the fitness curves for the best individual when the mutation probability is dynamic. The mutation probability is decreased in each thousand generation starting from 0.8.

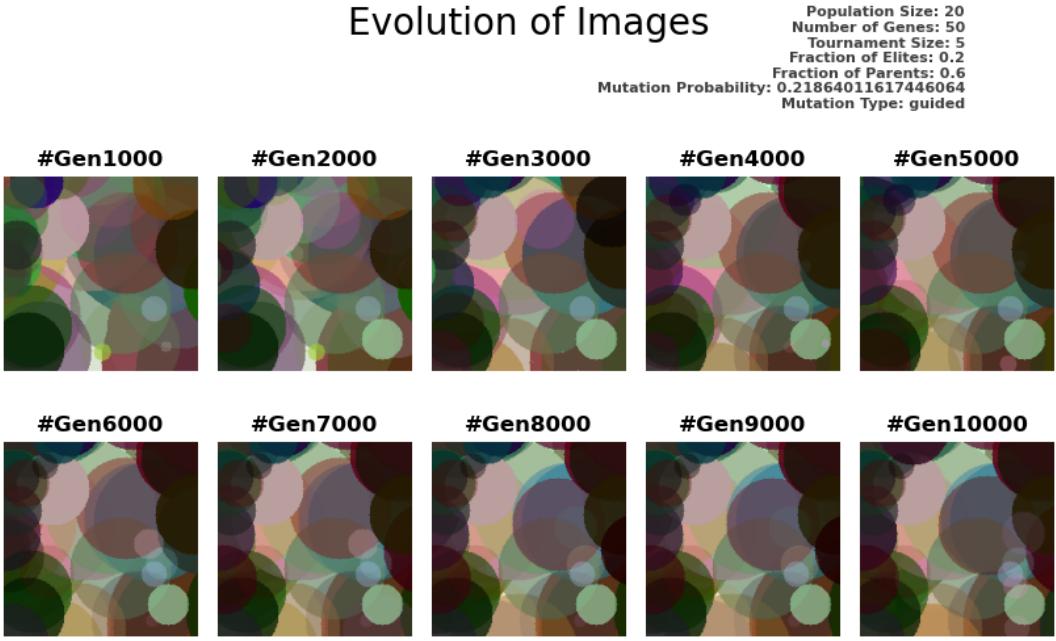


Figure 45: Quantitative evolution of the best individual in the population for stochastic mutation probability.

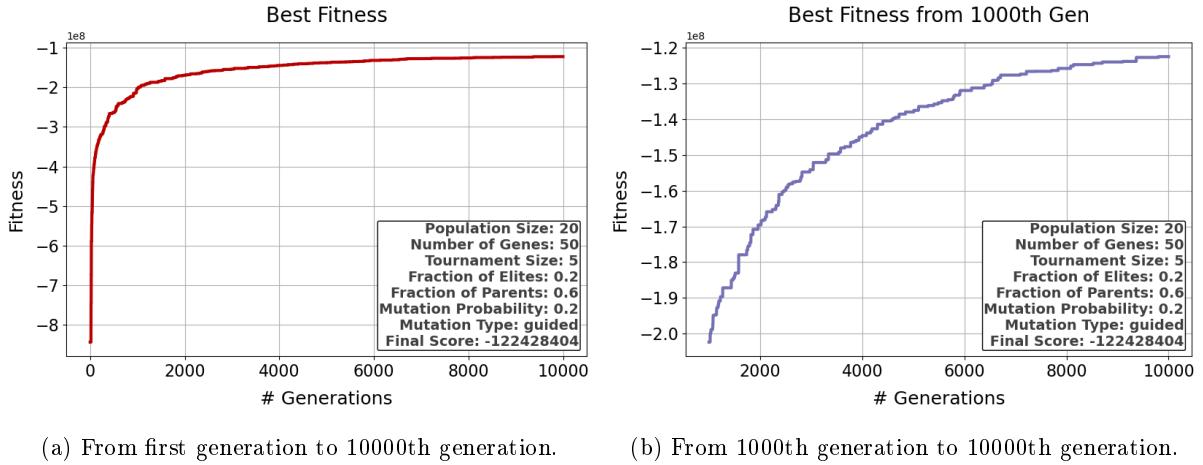


Figure 46: Fitness curves for dynamic mutation probability.

The evolution of the best individual for dynamic mutation probability is given in Figure 47.

2.3 Elite/Parents Size Scheduling

The family size can be made scheduled. This means that the family size can be changed based on the number of generations. This would help the algorithm to favor elites later in the generations. Figure 48 shows the fitness curves for the best individual when the number of elites and number of parents are dynamic. The number of parents are decreased in each thousand generation while number of elites is increased. That is fraction of parents is decreased from 0.8 to 0.35 gradually where the fraction of elites increased from 0.03 to 0.25.

Evolution of Images

Population Size: 20
 Number of Genes: 50
 Tournament Size: 5
 Fraction of Elites: 0.2
 Fraction of Parents: 0.6
 Mutation Probability: 0.2
 Mutation Type: guided

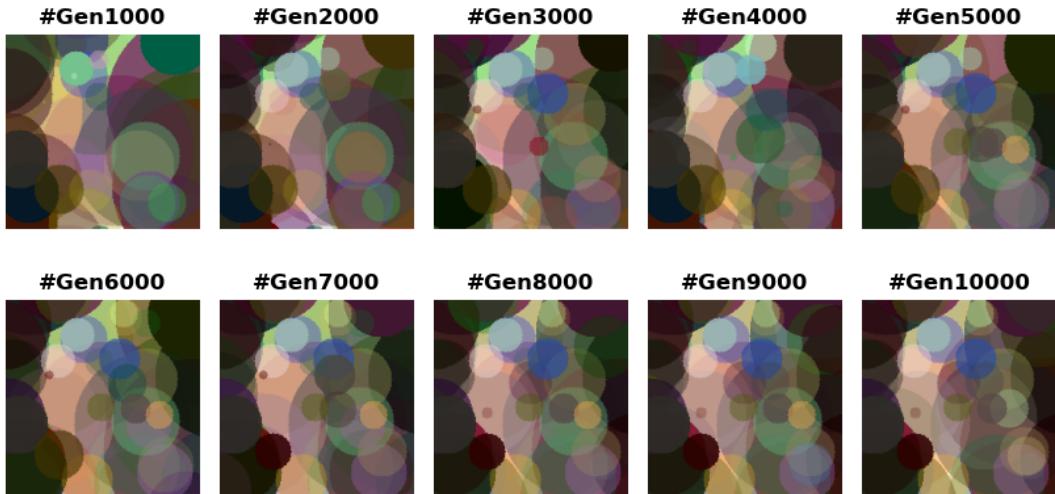


Figure 47: Quantitative evolution of the best individual in the population for dynamic mutation probability.

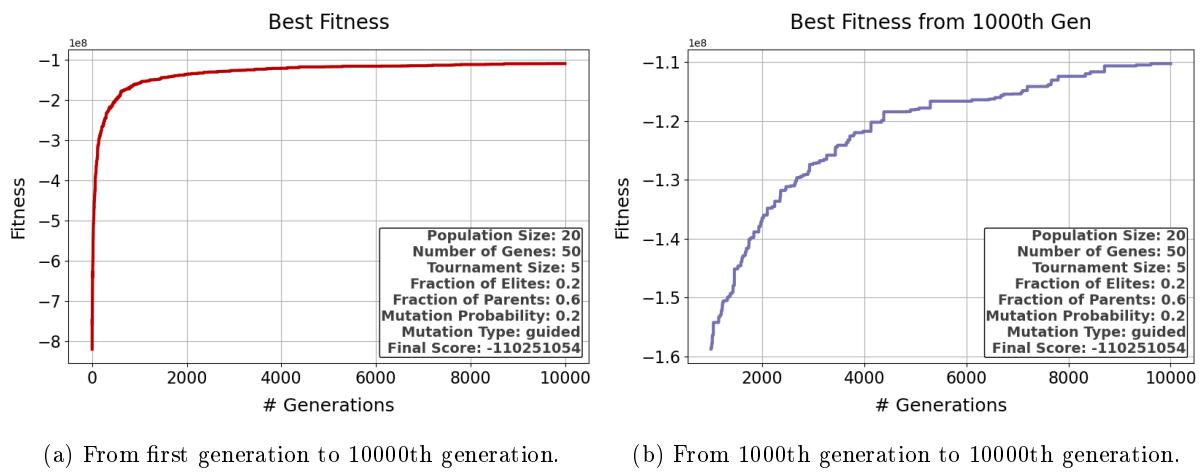


Figure 48: Fitness curves for elite/parents size scheduling.

The evolution of the best individual for dynamic family size is given in Figure 49.

Appendix

The code set used throughout this homework is provided as follows.

Submitted by Ahmet Akman 2442366 on April 28, 2024.

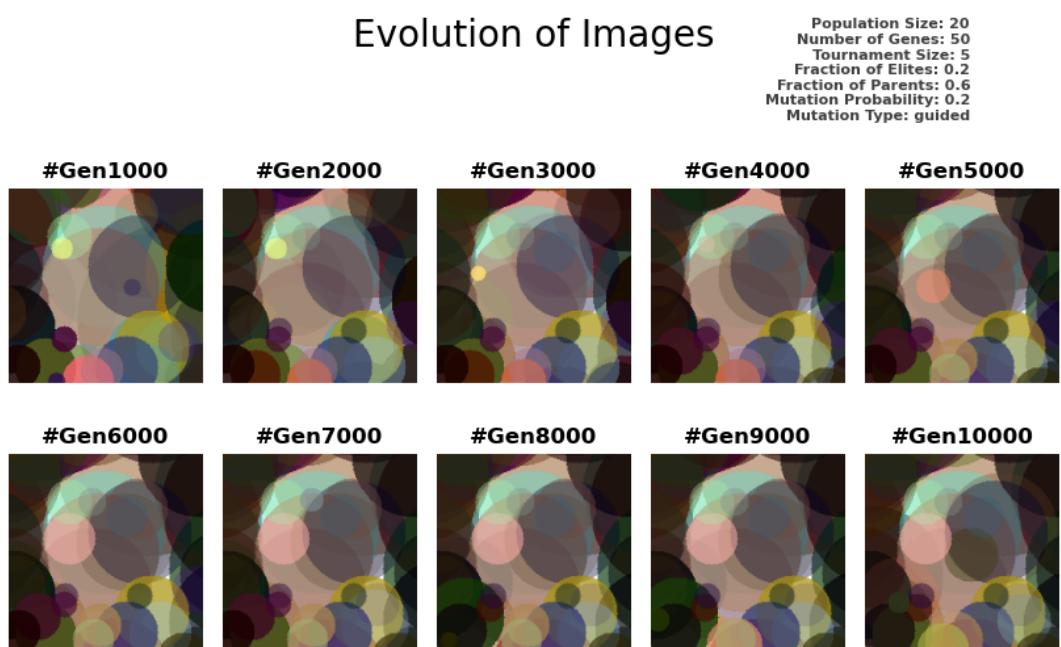


Figure 49: Quantitative evolution of the best individual in the population for elite/parents size scheduling.