Is SOC Intrinsically Fragile?

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Introduction

- In computational neuroscience, SOC is often hypothesized for brain driving itself towards a critical state, close to a qualitative behavior change.
- SOC is proposed as a mechanism for optimal information processing [1].
- There is an ongoing debate in neuroscience community about presence of SOC in brain.

Pros	Cons
Power law distributed avalanches are ***observed in brain*** which are compatible with theoretical framework.	It is possible to observe power law distributed avalanches ***without*** being at a critical point [2].

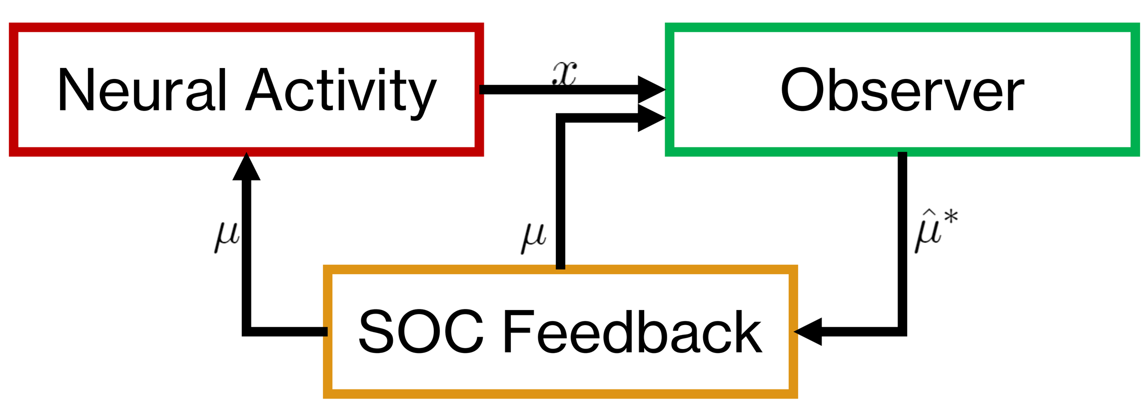
- We study a model used in SOC analysis [3] [4] [5]

$$\dot{x} = (\mu - \mu^*)x - \mu x^2 \qquad (\mathbf{1})$$

  - Criticality is achieved when $x = 0 \qquad \mu = \mu^*$
  - While ***critical value*** μ^* ***is unknown***.
- Control laws to induce SOC.
  - **Robustness issue** is observed in simulation.
- Is this robustness issue an **intrinsic** property of SOC ?

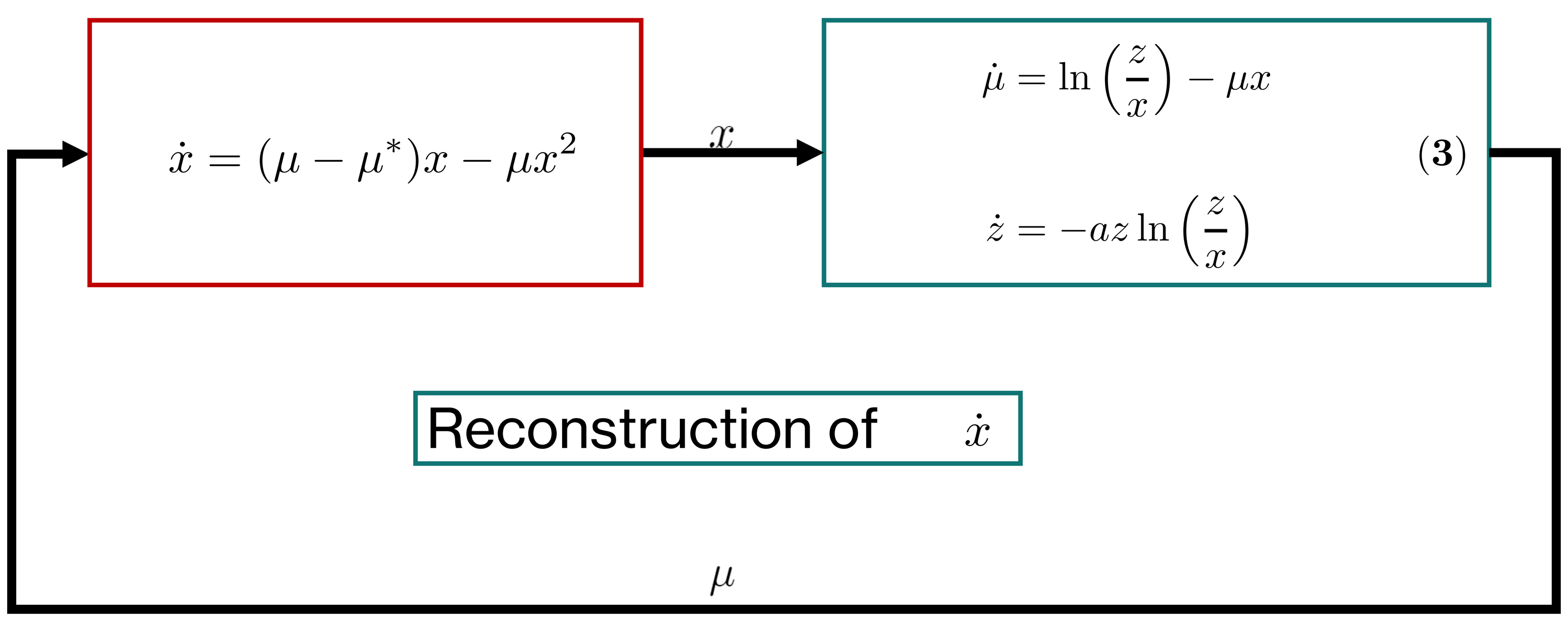
Two Different Control Strategies

- Observer based controller [6]



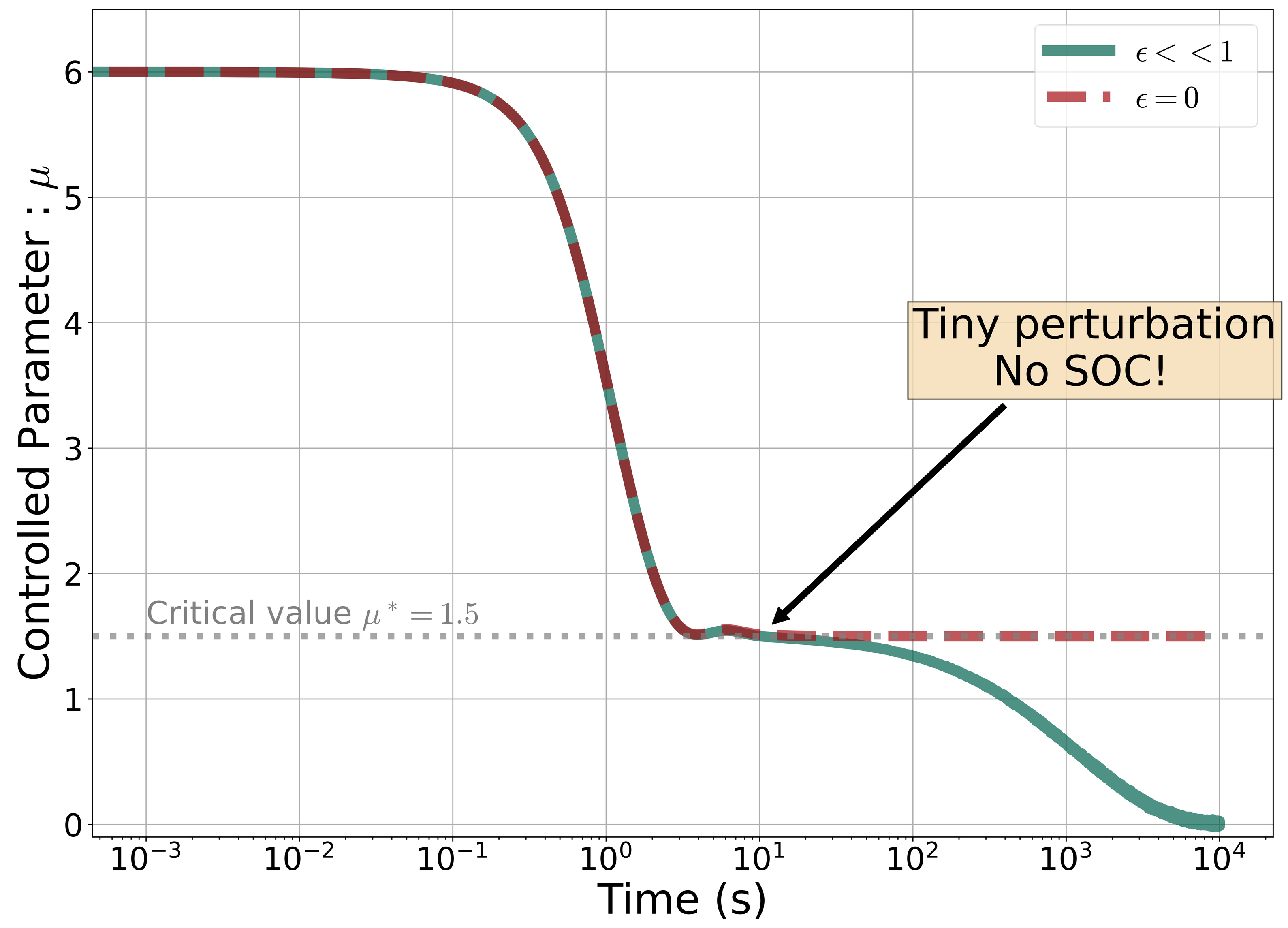
$$\begin{aligned} \dot{x} &= (\mu - \mu^*)x - \mu x^2 \\ \dot{\mu} &= -\mu + \hat{\mu}^* \\ \dot{\hat{\xi}} &= -\hat{\mu}^* + \mu(1-x) - l_1\left(\hat{\xi} - \ln(x)\right) \\ \dot{\hat{\mu}}^* &= -l_2\left(\hat{\xi} - \ln(x)\right) \end{aligned} \qquad (\mathbf{2})$$

- Derivative based controller



Simulations: Lack of SOC

$$\dot{x} = (\mu - \mu^*)x - \mu x^2 + \epsilon \qquad (\mathbf{1.1})$$



Intrinsic Obstruction to Robust SOC

Proposition 1 *Assume that the following implication holds:*

$$\begin{aligned} h_1(x,\mu,z) &= 0 \\ h_2(x,\mu,z) &= 0 \end{aligned} \quad \Rightarrow \quad x\mu = 0.$$

Then the controller:

$$\begin{aligned} \dot{\mu} &= h_1(x,\mu,z) \\ \dot{z} &= h_2(x,\mu,z) \end{aligned}$$

does ***NOT*** *ensure robustness to SOC for* (***1.1***).

Observer Based Controller (2):
Holds the implication when $z = [\hat{\mu}^*, \hat{\xi}]^T$

$$\begin{aligned} \dot{\mu} &= h_1(x,\mu,z) = 0 \\ \dot{z} &= h_2(x,\mu,z) = 0 \end{aligned}$$

Derivative Based Controller (3):
Holds the implication when

$$\begin{aligned} \dot{\mu} &= h_1(x,\mu,z) = 0 \\ \dot{z} &= h_2(x,\mu,z) = 0 \end{aligned}$$

Take Home Messages

→ There are **no robust feedback methodology for SOC** yet.
→ ***Transcritical bifurcations*** lose the criticality under non-zero perturbance.
→ Our result could be an indication of a more fundamental obstruction of SOC.
→ We studied *exact* SOC, self-organized quasi criticality (**SOqC**) can still be searched for.

References

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[3] J. Hesse, and T. Gross, "Self-organized criticality as a fundamental property of neural systems. *Frontiers in Systems Neuroscience*, vol.8 Sep. 2014
[4] G. Menesse, B. Marin, M. Girardi-Schappo ,and O. Kinouchi, "Homeostatic criticality in neuronal networks" *Chaos, Solitons & Fractals* vol.156 Mar. 2022.
[5] M. Girardi-Schappo, E.F. Galera, T.T.A. Carvalho, L. Brochini, N.L. Kamiji, A.C. Roque, and O. Kinouchi, "A unified theory pf E/I synaptic balance, quasicritical neuronal avalanches and asynchronous irregular spiking" *J. of Physics: Complexity*, vol. 2, no. 4 Oct. 2010.
[6] M.S. Perrin, L. Greco, A. Chaillet and A. Destexhe, "An observer-based approach to self-organized criticality", *M2 CNN Université Paris-Saclay* 2024.

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