EE407-HW2

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Q1)

a) $p = \rho g h + p_a$, $\Delta p = p - p_a$ } $\Delta p = \rho g h$

$$\frac{dh(t)}{dt} = w_i(t) - w_o(t) = w_i(t) - k\sqrt{\rho g h}$$

$$\boxed{\dot{h} = w_i - k\sqrt{\rho g h}}$$

b) $\Delta p = \rho g h$

$$\frac{d\Delta p(t)}{dt} = \rho g \frac{dh(t)}{dt} = \rho g \left[w_i - k\sqrt{\rho g h(t)}\right]$$

$$= \rho g \left[w_i - k\sqrt{\rho g \frac{\Delta p(t)}{\rho g}}\right] = \rho g \left[w_i - k\sqrt{\Delta p(t)}\right]$$

$$\frac{d\Delta p(t)}{dt} = \rho g w_i - \rho g k\sqrt{\Delta p(t)}$$

$$\boxed{\dot{\Delta p} = \rho g w_i - \rho g k\sqrt{\Delta p}}$$

c) $\frac{dh(t)}{dt} = 0$, to make $h(t)$ constant.

$$w_i - k\sqrt{\rho g h} = 0$$

$$\boxed{w_i = k\sqrt{\rho g r}} = u_{ss}$$

Q2

a)

$$V \cdot \frac{dx_1}{dt} = 0 - V d x_1 + V \cdot \underset{\mu x_1}{\underset{\Downarrow}{r_1}} \Rightarrow \boxed{\frac{dx_1}{dt} = \mu x_1 - d x_1}$$

$$V \frac{dx_2}{dt} = V d (x_{2f} - x_2) - V r_2$$

$$\frac{dx_2}{dt} = d(x_{2f}) - d x_2 - r_2 \qquad \boxed{\frac{dx_2}{dt} = d x_{2f} - d x_2 - \frac{\mu x_1}{Y}}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \mu - d & 0 \\ -\frac{\mu}{Y} & -d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ d \end{bmatrix} x_{2f}$$

b)

$$\frac{dx_1}{dt} = 0 \ , \quad \mu = \mu_m x_2 / (k_m + x_2)$$

$$\frac{\mu_m \cancel{x_1} x_2}{k_m + x_2} = d \cancel{x_1} \Rightarrow \boxed{x_1 = 0}$$

$$\mu_m x_2 = d k_m + d x_2$$

$$\boxed{x_2 = \frac{d k_m}{\mu_m - d}}$$

$$\frac{dx_2}{dt} = 0 \ /$$

$$d x_{2f} - d x_2 = \frac{\mu_m x_1 x_2}{Y (k_m + x_2)}$$

$$dY \left[-x_2^2 + (x_{2f} - k_m) x_2 + k_m x_{2f} \right] = \mu_m x_1 x_2$$

$$x_2^2 + (x_1 + k_m - x_{2f}) x_2 - k_m x_{2f} = 0$$

$x_1 = 0$;

$$x_2^2 + (k_m - x_{2f}) x_2 - k_m x_{2f} = 0$$

$$\begin{matrix} x_2 & & k_m \\ x_2 & & -x_{2f} \end{matrix}$$

$$\boxed{x_2 = x_{2f}}$$

$$x_2 = \frac{d k_m}{\mu_m - d} \ ;$$

$$d x_{2f} - \frac{d^2 k_m}{\mu_m - d} - x_1 \left(\frac{\frac{\mu_m d k_m}{\mu_m - d}}{Y \left(k_m + \frac{d k_m}{\mu_m - d} \right)} \right)$$

$$x_1 = Y \left(x_{2f} - \frac{d k_m}{\mu_m - d} \right)$$

$$x_{eq1} = \left[Y \left(x_{2f} - \frac{d k_m}{\mu_m - d} \right) , \ \frac{d k_m}{\mu_m - d} \right]$$

$$x_{eq2} = \left[0, \ x_{2f} \right]$$

Q2
c) for eq1;

$$\frac{d x_1(t)}{dt} = \left(-d + \frac{\mu_m x_2(t)}{k_m + x_2(t)}\right) x_1(t)$$

$$\frac{dx_1}{dt} \cong \left(-d + \frac{\mu_m x_2(t)}{k_m + x_2(t)}\right)\left(x_1(t) - Y\left(x_{2f} - \frac{d k_m}{\mu_m - d}\right)\right) + \frac{x_1(t)\,\mu_m k_m}{(k_m + x_2(t))^2}\Bigg|_{\substack{x_1(t) = x_1^{op} \\ x_2(t) = x_2^{op}}}\left(x_2(t) - \frac{d k_m}{\mu_m - d}\right)$$

$$= \underbrace{\left(-d + \frac{\mu_m d k_m}{\mu_m k_m}\right)}_{0}\left(x_1(t) - Y\left(x_{2f} - \frac{d k_m}{\mu_m - d}\right)\right) + \frac{(x_{2f}\mu_m - x_{2f} d - d k_m)(\mu_m - d)}{\mu_m k_m}\left[x_2(t) - \frac{d k_m}{\mu_m - d}\right]$$

$x_2(t)\mu_m - d x_2 - d k_m$

$$= \frac{(x_{2f}\mu_m - x_{2f} d - d k_m)(x_2(t)\mu_m - d x_2(t) - d k_m)}{\mu_m k_m}$$

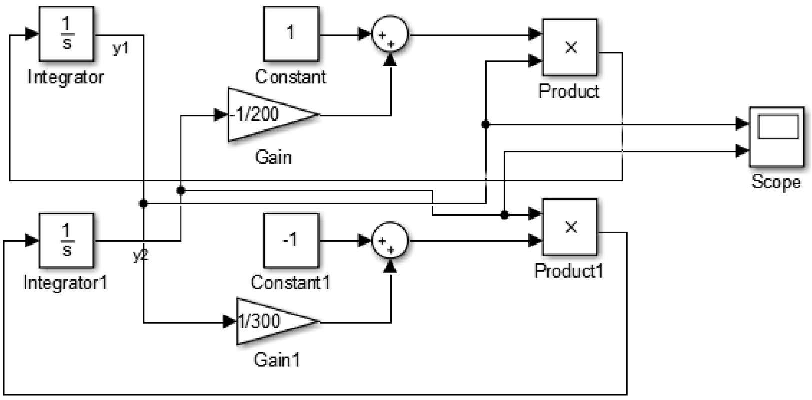
for eq2;

$$\frac{d x_1(t)}{dt} = \left(-d + \frac{\mu_m x_2(t)}{k_m + x_2(t)}\right) x_1(t)$$

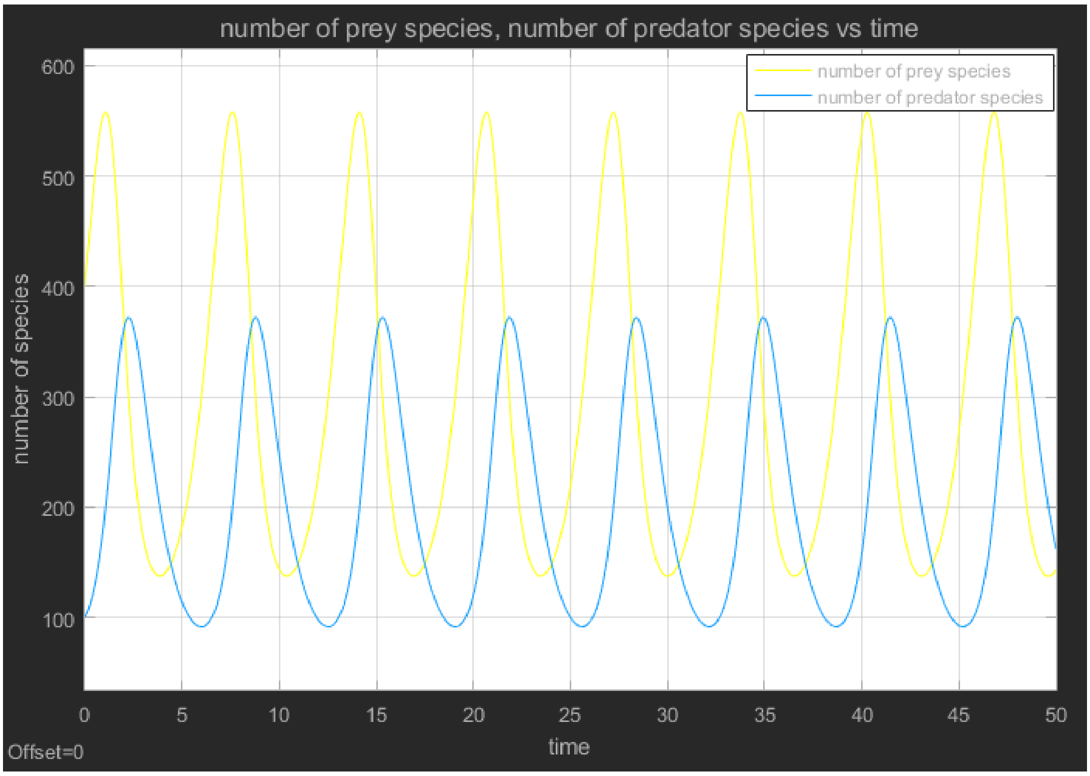
$$\frac{d x_1(t)}{dt} \simeq \left[-d + \frac{\mu_m x_2(t)}{k_m + x_2(t)}\right]\Bigg|_{\substack{x_1(t) = x_1^{op} \\ x_2(t) = x_2^{op}}}(x_1(t) - 0) + \left(\frac{x_1(t)\,\mu_m k_m}{(k_m + x_2(t))^2}\right)\Bigg|_{\substack{x_1(t) = x_1^{op} \\ x_2(t) = x_2^{op}}}(x_2(t) - x_{2f})$$

$$\frac{d x_1(t)}{dt} \cong \left[-d + \frac{\mu_m x_{2f}}{k_m + x_{2f}}\right] x_1(t)$$

3a



3b



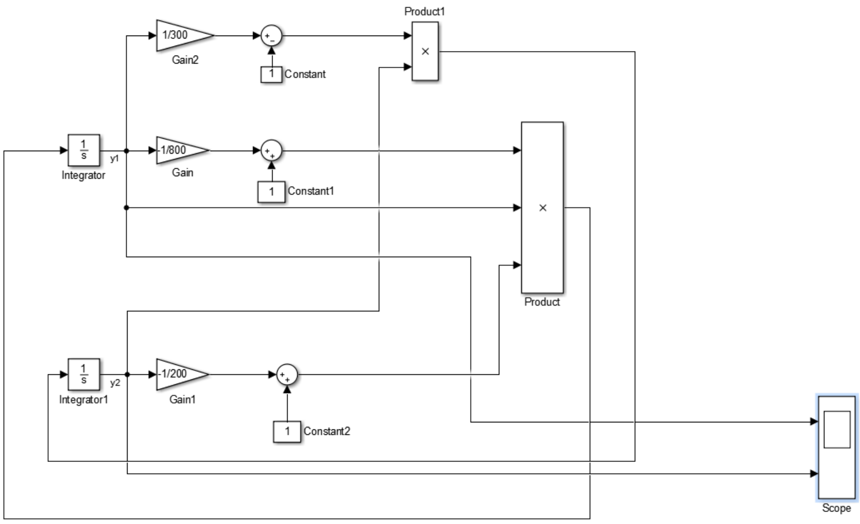
3c

It is remarkable that the number predator species lags number of prey species which is realistic response. If the number of preys is bigger than number of predators, it creates an increase in the number of predators. Moreover, when the number of predators increased, it causes a decrease in the number of prey species and this situation results in a cyclic behaviour. The number of each species affects the number of other one. Finally, a periodic response is observed.

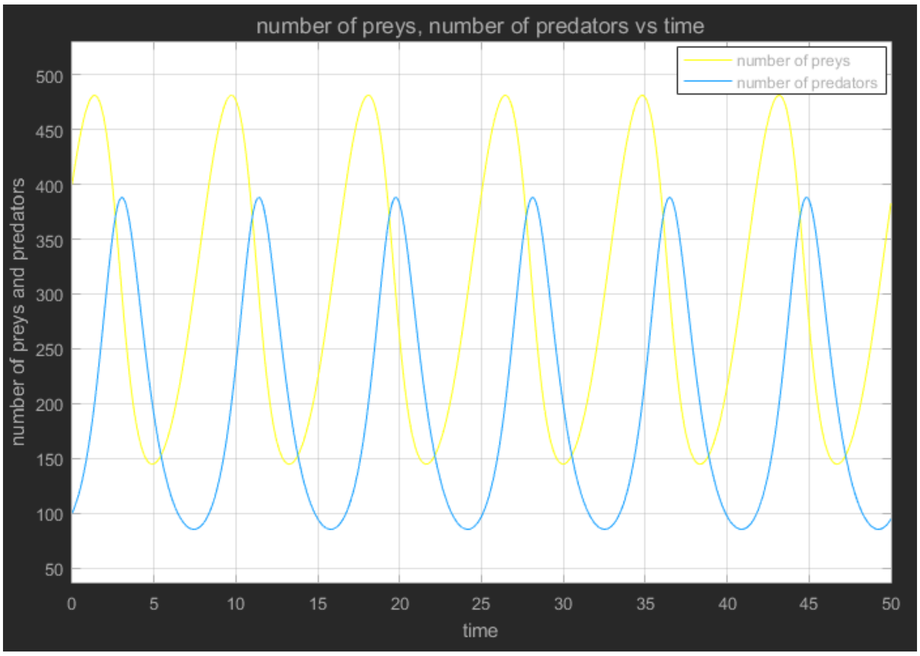
3e

The derivative terms for both y1 and y2 will be 0 at equilibrium points and which makes y1 = n1, y2 = n2. When the initial value of populations is far from the equilibrium point, the period did not change, however the signal started to lose its sinusoidal shape.

3f



3g



The solution curves are still periodic; however, the period has increased as seen in figure above.