EE407-HW3

Ahmet Asım Bektaş

2231405

Q1)

a) $$G_{ol} = G_p G_v G_m G_c = \frac{3K_c e^{-2s}}{\underbrace{(60s+1)}_{\text{highest time constant}}(5s+1)(3s+1)(2s+1)} \simeq \frac{K e^{-\theta s}}{\tau s+1}$$

$$K = 3K_c$$

$$\tau = \tau_{largest} = 60$$

$$\theta = \tau_2 + \tau_3 + \tau_4 = 10 \text{ sec}$$

$$\boxed{G_{ol}(s) \simeq \frac{3K_c e^{-12s}}{60s+1}}$$

b) $$q(s) = 1 + G_{ol}(s) = 1 + \frac{3K_c e^{-12s}}{60s+1} = 1 + \frac{3K_c}{(60s+1)} \cdot \frac{(1-6s)}{(1+6s)}$$

$$(60s+1)(1+6s) + 3K_c(1-6s) = 0$$

$$360s^2 + (66-18K_c)s + 3K_c + 1 = 0$$

s^2	360	$3K_c+1$
s^1	$66-18K_c$	0
s	$3K_c+1$	

$66 - 18K_c < 0$

$K_c < \frac{11}{3}$

$3K_c + 1 > 0$

$K_c > -\frac{1}{3}$

for stability; $66-18K_c < 0$ and $3K_c + 1 > 0$

$K_c < \frac{11}{3}$ and $K_c > -\frac{1}{3}$

$$\boxed{-\frac{1}{3} < K_c < \frac{11}{3}}$$

c) Replace $s = j\omega$

$$\underbrace{-360\omega^2 + 1 + 3K_c}_{=0} + j\omega\underbrace{(66-18K_c)}_{=0} = 0$$

$$K_c = \frac{11}{3} \rightarrow 360\omega^2 = 1 + 3 \times \frac{11}{3}$$

$$\boxed{\omega \simeq 0.18}$$

Q2)

dye concent. → output

photodetector → sensor

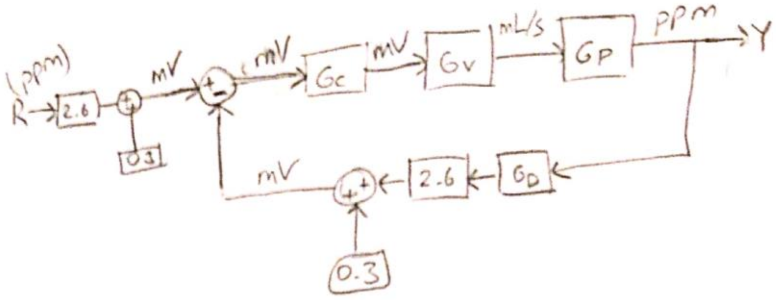
valve(regulating) → actuator

flow rate $= \frac{2L}{5}$

cross sec Area (pipe) $= 5cm^2$

valve eq $= \frac{0.6}{(0.2s+1)}$

mixing eq $= \frac{0.8}{2s+1}$



b) $$G_V = \frac{0.6}{0.2s+1}$$

$$G_P = \frac{0.8}{\tau s+1} \quad \text{where } \tau = \frac{20}{5} = 4$$

$$G_D = e^{-\theta s} \quad \text{where } \theta = \frac{290}{2000/5} = 0.725$$

c) Simulink

d) $$G_a(s) = \cancel{G_c}^{1}(s)\, G_V(s)\, G_P(s)\, G_D(s)\,(2.6)$$

$$= \frac{0.6}{(0.2s+1)} \cdot \frac{0.8}{(4s+1)} \cdot e^{-0.725s} * 2.6$$

$$= \frac{1.25\, e^{-0.725s}}{(4s+1)(0.2s+1)} \simeq \frac{Ke^{-\theta s}}{(\tau s+1)}$$

$K = 1.25$

$\tau = \tau_{largest} = 4$

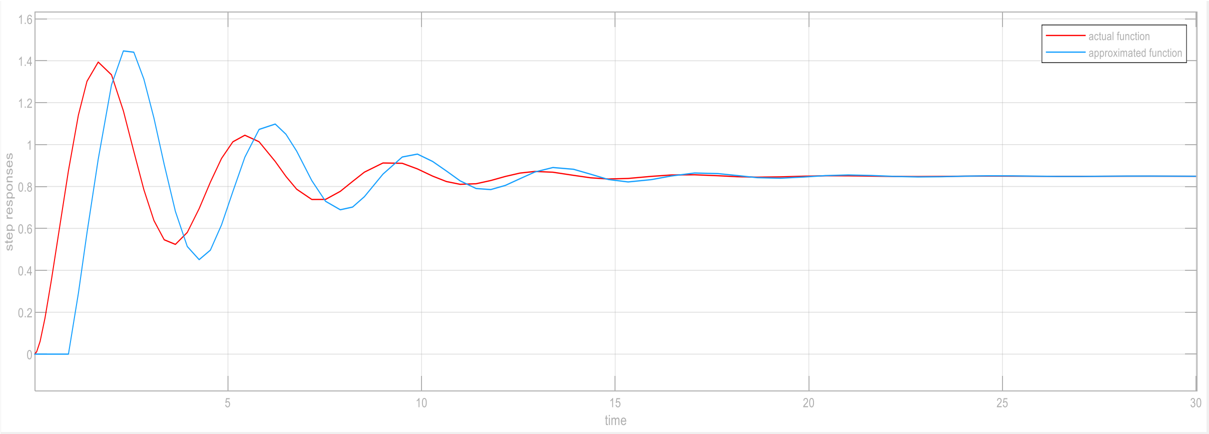
$\theta - \tau_2 = 0.2$

$$\boxed{G_a(s) = \frac{1.25e^{-0.925s}}{4s+1}}$$

+ simulink

Q2)

Sımulink part c,d;



Q2

e) $q(s) = 1 + K_u G_{OL}(s) = 1 + \dfrac{K_u 1.25\, e^{-0.925s}}{4s+1} = 1 + \dfrac{K_u 1.25(1-0.463s)}{(4s+1)(1+0.463s)}$

replace $s = j\omega$

$\boxed{K_u \simeq 7.72 \ , \ \omega \simeq 2.40}$ (found in computational tool)

$\dfrac{2\pi}{P_u} = \omega \Rightarrow P_u = 2.62$

$K_p = \dfrac{K_u}{1.7} = 4.54$

$K_I = P_u/2 = 1.31$

$K_D = P_u/8 = 0.33$

Q3)

Q3)

a) $\dot{h}_1 A_1 = q_i - q_1$

$\dot{h}_1 A_1 = q_i - \frac{h_1}{R_1}$

$\dot{h}_2 A_2 = \frac{h_1}{R_1} - \frac{h_2}{R_2}$

$\dot{h}_3 A_3 = \frac{h_2}{R_2} - \frac{h_3}{R_3}$

$\vdots \qquad \vdots$

$\dot{h}_n A_n = \frac{h_{n-1}}{R_{n-1}} - \frac{h_n}{R_n}$

b)

$s H_1(s) A_1 = Q_i(s) - \frac{H_1(s)}{R_1}$

$\frac{H_1(s)}{Q_i(s)} = \frac{1}{sA_1 + \frac{1}{R_1}}$

$sH_2(s) A_2 = \frac{H_1(s)}{R_1} - \frac{H_2(s)}{R_2}$

$\frac{H_2(s)}{H_1(s)} = \frac{1/R_1}{sA_2 + 1/R_2}$

$\frac{H_n(s)}{Q_i(s)} = \frac{H_1(s)}{Q_i(s)} \cdot \frac{H_2(s)}{H_1(s)} \cdots \cdots \frac{H_n(s)}{H_{n-1}(s)} = \frac{\frac{1}{R_1 R_2 R_3 \ldots R_{n-1}}}{(sA_1 + 1/R_1)(sA_2 + 1/R_2) \ldots (sA_n + \frac{1}{R_n})}$

$$\boxed{= \frac{R_n}{(1+sA_1R_1)(1+sA_2R_2)\ldots(1+sA_nR_n)}}$$

c) $G'(s) = \frac{Q_n(s)}{Q_i(s)} = \frac{H_n(s)}{Q_i(s)} \cdot \frac{Q_n(s)}{H_n(s)}$

$q_k = \frac{h_k}{R_k} \qquad \frac{H_n(s)}{Q_i(s)} \cdot \frac{1}{R_n}$

$\lim_{s \to 0} \left(\frac{1}{(1+sA_1R_1)(1+ \ldots} \right) = 1$: DC gain

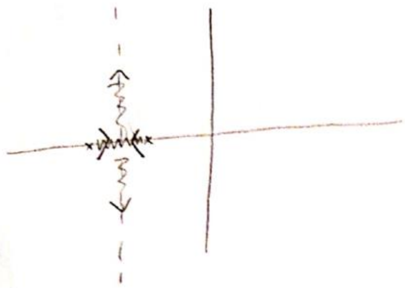
This makes sense. Tanks do not accumulate in the steady state. (self regulating)

Q3)

d) if $n=2$;

$$G(s) = \frac{R_2}{(1+sA_1R_1)(1+sA_2R_2)}$$

$$s_1 = \frac{-1}{A_1R_1} \quad , \quad s_2 = \frac{-1}{A_2R_2}$$



From RL, it is seen that poles never cross $j\omega$ axis which means that sustained oscillations cannot be obtained

From Bode Plot; Phase response of the system reaches -180° at infinity which again shows us that sustained oscillations cannot be obtained

e) $$G(s) = \frac{R_3}{(1+sA_1R_1)(1+sA_2R_2)(1+sA_3R_3)}$$

$$p_1 = \frac{-1}{A_1R_1} \quad p_2 = \frac{-1}{A_2R_2} \quad p_3 = \frac{-1}{A_3R_3}$$

$$1 + K_c \frac{R_3}{(1+sA_1R_1)(1+sA_2R_2)(1+sA_3R_3)} = 0 \Rightarrow s^3+3s^2+3s+1+K_c=0$$

Routh-Hurwitz

s^3	1	3
s^2	3	K_c+1
s^1	$\frac{8-K_c}{3}$	
s^0	K_c+1	

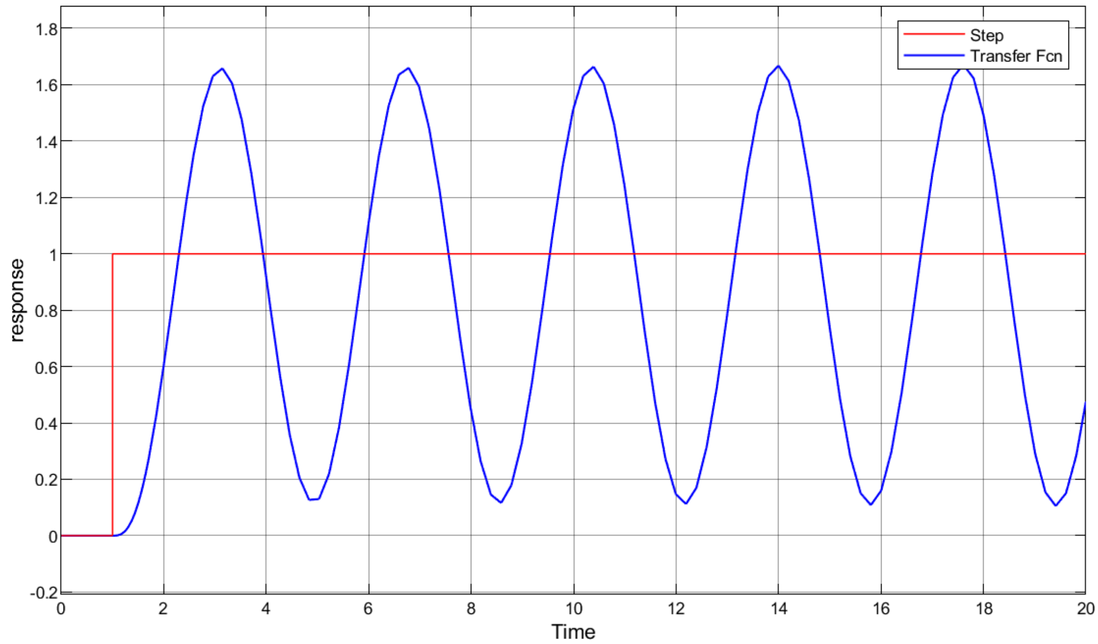
$K_c=8$ critical gain

$3s^2+K_c+1=0$

$s=j\omega$

$\omega=\sqrt{3}$ rad/min

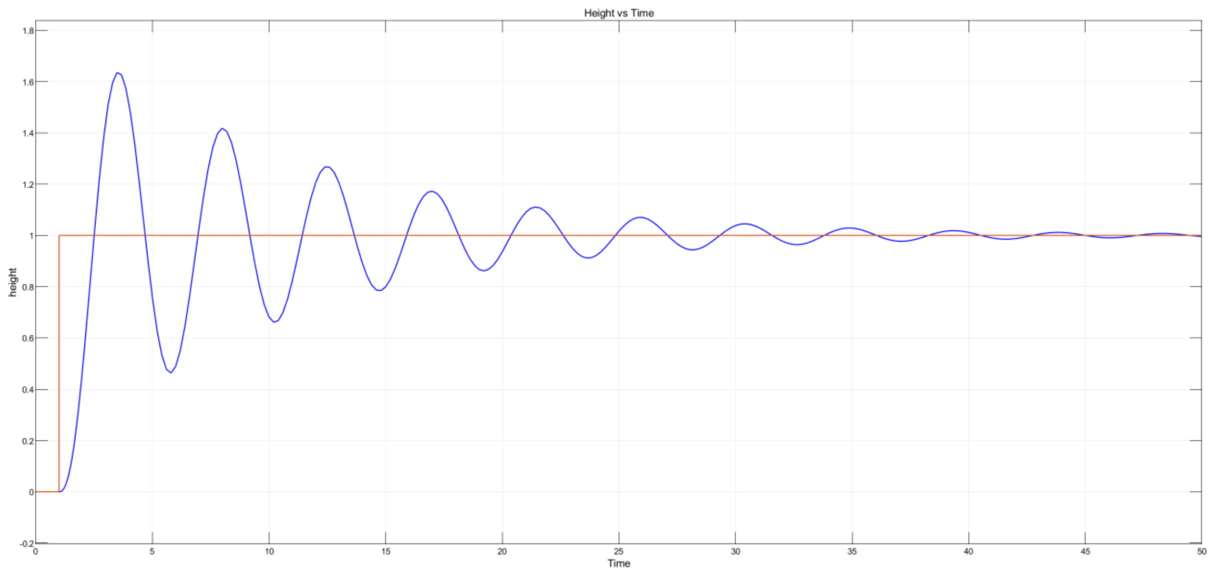
Simulink part e



f) $P_u = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}} \simeq 3.63$

$K_c = \frac{K_u}{1.7} = 4.71$ $\quad \tau_I = \frac{P_u}{2} = 1.81$ $\quad \tau_D = \frac{P_u}{8} = 0.45$

Simulink part f



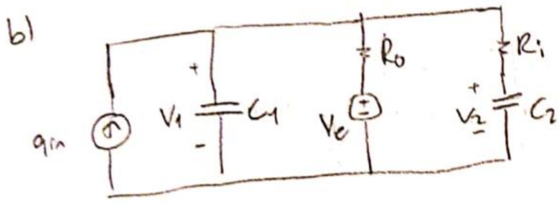
Q4

Q4)

a) $$m_1 c_1 \frac{\partial T_1}{dt} = q_{in} + h_o A_o (T_e - T_1) + h_i A_i (T_2 - T_1)$$

$$m_2 c_2 \frac{dT_2}{dt} = h_i A_i (T_1 - T_2)$$

b)



$V_1 = T_1$ | $R_o = \frac{1}{h_o A_o}$ | $mc_1 = C_1$

$V_e = T_e$ | $R_i = \frac{1}{h_i A_i}$ | $m_2 c_2 = C_2$

$V_2 = T_2$

c) $$Q_{in}(s) = T_1(s)\left[sC_1 + \frac{1}{R_o} + \frac{sC_2}{R_i C_2 s + 1} \right]$$

$$G(s) = \frac{T_1(s)}{Q_{in}(s)} = \frac{R_o(R_i C_2 s + 1)}{sC_1 R_o (R_i C_2 s + 1) + R_i C_2 s + 1 + sR_o C_2}$$

Q5

b)

i) It a good idea to have a dead band in an on-off controller since dead band forms a region to prevent controller to change its state(on-off) so frequently and therefore it protects actuator. Air conditioner can be an example.

ii) Integrating system. The integral term adds a 1/s term which makes ess=0. On the other hand, self-regulating system cannot make ess=0. It can only decrease its value with changing gain parameters.

iv)

a-IAE

b-Integral Square Error

c-ITAE

vii) If the actuator saturates before error term diminishes to zero, error term will stay non-zero.Then when the desired output changes and e(t) will change sign. The actuator will remain saturated and will not respond until “I” term fall below a certain point. One solution can be Anti Windup.

viii) Decay ratio and settling time are generally behaves in a similar way. It is often can be said that bigger the decay ratio creates a bigger the settling time. Rise and peak time are correlated similarly. Often smaller rise time cause a smaller peak time.

ix) Derivative kick is due to a sudden change in e(t). General solution is to use derivative of the output instead of e(t) for the derivative term.

x) When the high frequency noise is considerably high, derivative term will amplify it and will make it much more larger and this will cause many problems on the controller. A low pass filter can be a solution.