4.A. DERIVATION OF AREA PRODUCT EQUATIONS FOR TRANSFORMER DESIGN

4.A.1. *DERIVATION OF TRANSFORMER AREA PRODUCT (AP)*

The transformer input power P_{in} depends on the output power P_{out} and efficiency η.

Hence

$$P_{in} = \frac{P_{out}}{\eta}$$

(4.A.1)

The average DC input current to the converter transformer I_{dc} depends on the input power P_{in} and the DC input voltage V_{in}. Hence

$$I_{dc} = \frac{P_{in}}{V_{in}}$$

(4.A.2)

The maximum rms primary current I_{pm} occurs when the input voltage is minimum $V_{in(min)}$ and the pulse width is maximum. Factor K_t relates the DC input to the rms primary current dependent on the converter topology. $K_t = I_{dc}/I_{pm}$. Hence

$$I_{pm} = \frac{I_{dc}}{K_t}$$

(4.A.3)

Substituting Eq. (4.A.2) for I_{dc} at minimum input voltage,

$$I_{pm} = \frac{P_{in}}{V_{in(min)}K_t}$$

(4.A.4)

The usable window area A_p (available for the primary winding) depends on the total window A_w; the window area reserved for the primary, given by the "primary area factor" K_p; and the primary area "utilization factor" K_u. Hence

$$A_p = A_w K_p K_u$$

(4.A.5)

The number of primary turns N_p that will just fill the primary window space A_p at a wire current density of J depends on the primary current; hence

$$N_p = \frac{A_p J}{I_{pm}}$$

(4.A.6)

Substituting Eq. (4.A.5) for A_p and Eq. (4.A.4) for I_{pm},

$$N_p = \frac{A_w K_p K_u J V_{\text{in(min)}} K_t}{P_{\text{in}}}$$

(4.A.7)

or

$$A_w = \frac{N_p P_{\text{in}}}{K_p K_u J V_{\text{in(min)}} K_t}$$

From Faraday's law,

$$E_{dt} = Nd\,\Phi$$

(4.A.8)

Hence

$$V_{\text{in(min)}}\, t_{\text{on(max)}} = N_p \Delta B A_e$$

or

$$A_e = \frac{V_{\text{in(min)}} t_{\text{on(max)}}}{N_p \Delta B}$$

where t_{on} = "on" period

ΔB = flux density change during "on" period

A_e = effective core area

The maximum "on" time is one half period at the operating frequency f; hence

$$t_{\text{on(max)}} = \frac{1}{2f}$$

(4.A.9)

Substituting Eq. (4.A.9) into Eq. (4.A.8),

$$A_e = \frac{V_{\text{in(min)}}}{N_p \Delta B 2f}$$

(4.A.10)

Now

$$AP = A_w A_e$$

Combining Eqs. (4.A.7) and (4.A.10),

$$AP = \frac{P_{\text{in}}}{K_t K_u K_p J \Delta B 2f} \qquad \text{m}^4$$

(4.A.11)

If the transformer is limited to a 30°C temperature rise under convection-cooled conditions, the wire current density *J* is given by the empirical relation[1,2]

$$J = 450 \times 10^4 \times AP^{-0.125} \qquad \text{A/m}^2$$

(4.A.12)

(For a constant temperature rise, the current density must fall as the transformer size increases, because the ratio of volume to surface area falls with increasing size.)

Substituting Eq. (4.A.12) into Eq. (4.A.11) and converting *AP* to centimeters,

$$AP = \frac{P_{\text{in}} \times 10^8}{K_t K_u K_p \times 450 \times 10^4 \times AP^{0.125} \times \Delta B \times 2f} \qquad \text{cm}^4$$

$$AP^{(1-0.125)} = \frac{P_{\text{in}} \times 10^4}{K_t K_u K_p \times 450 \times \Delta B \times 2f} \qquad \text{cm}^4$$

Therefore

$$AP = \left(\frac{P_{\text{in}} \times 10^4}{K_t K_u K_p \times 450 \times \Delta B \times 2f} \right)^{1.143} \qquad \text{cm}^4$$

(4.A.13)

Since $K' = K_t K_u K_p$ (see Table 3.4.1). Substituting K' in Eq. (4.A.13) and simplifying gives

$$AP = \left(\frac{11.1\, P_{\text{in}}}{K' \Delta B f} \right)^{1.143} \qquad \text{cm}^4$$

(4.A.14)

Hence the size of the transformer, in terms of area product *AP*, can be found knowing the input power P_{in}, flux density swing ΔB, frequency *f*, and a constant of topology K' for a free air temperature rise of 30°C.

4.A.2. *TOPOLOGY FACTORS K'*

The topology factor *K'* depends on the type of converter, the type of secondary winding and rectificatioin, insulation and screening requirements, and current waveforms.

K' is made up of three subfactors as follows:

$$K' = K_p K_u K_t$$

(4.A.15)

4.A.3. *PRIMARY AREA FACTOR K_P*

This is the ratio of the winding area provided for the primary to the total window area (A_p/A_w). Although the window area is normally split equally between primary and secondary, the primary area is not always fully used for the main primary winding. For example, in the forward converter, an energy recovery winding is usually bifilar-wound, with the main primary taking up part of the winding area. Further, in the center-tapped push-pull topology only half the primary is active at any time, reducing the effective primary to 25% of the total window area. In the same way, a center-tapped secondary winding has the same 25% utility factor.

4.A.4. *WINDOW UTILIZATION FACTOR K_U*

This is the ratio of the area of window occupied by copper to the total available window area. With round wires and normal insulation, this factor is typically 0.4 (40%). When bobbin windings are used, this may be as low as 30%.

4.A.5. *CURRENT FACTOR K_T*

This is the ratio of the DC input current to the maximum primary current (I_{dc}/I_p). It depends on the topology of the converter and the shape of the primary current waveform. For simplicity, rectangular waveforms are assumed; this introduces little error in practice.

The winding form of the primary is defined by the type of converter. However, a choice exists for the secondary winding form depending on the rectification circuit. Bridge rectifiers require a single winding and biphase rectifiers a center-tapped winding with a lower copper utilization factor.

4.A.6. *TEMPERATURE RISE*

The temperature rise of the transformer under free air convection-cooled conditions depends on the total internal losses (core loss plus copper losses) and the effective surface area. Figure 3.1.7 is developed from measured results and information published in References 1, 2, and 15. It shows the relationship of surface area to area product for typical switchmode ferrite cores. It also predicts the temperature rise above ambient as a function of total internal dissipation with area product or surface area as a parameter. The temperature rise predictions assume free air cooling and an ambient air temperature of 25°C. The predicted temperature rise (in the range 20 to 70°C) may be obtained directly from the nomogram by entering with the surface area of the transformer and internal dissipation. If the area product is known, an indication of the surface area may be obtained from the same nomogram using the "*AP* line" area product intersect.

Alternatively, for a small temperature rise in the range 20 to 50°C, the following approximate formula developed from Fig. 3.1.7 may be used:

$$\Delta T = \frac{800 P_t}{A_s} \qquad ^\circ\mathrm{C}$$

(4.A.16)

where ΔT = temperature rise, °C

P_t = total internal loss, W

A_s = surface area of transformer, cm^2

The surface area A_s is related to the area product AP as follows:

$$A_s = 34 AP^{0.51} \qquad \mathrm{cm}^2$$

(4.A.17)

Substituting for A_s in Eq. (4.A.16),

$$\Delta T = \frac{23.5 P_t}{\sqrt{AP}} \qquad ^\circ\mathrm{C}$$

from which the thermal resistance R_t (normalized for a 30°C temperature rise) will be

$$R_t = 23.5 AP^{0.5} \qquad ^\circ\mathrm{C/W}$$

(4.A.18)