A Simple Method for Calculating Core Temperature Rise in Power Transformers

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***Abstract*—This paper presents a new method for calculating core temperature rise in large power transformers. A general formulation of the problem is made using the electrical analogy. This formulation extends to the L- and T-joints. An expression for the thermal resistance of the core material is found using analytical methods. A comparison of the results of the new method with measured values on two power transformers is presented.**

***Index Terms*—Magnetic cores, power transformer, power transformer thermal factors, temperature.**

NOMENCLATURE

The following symbols are used in the equations which follow.

β	Cubic expansivity (of cooling liquid) [per Kelvin].
θ	Core interior temperature gradient (see below for primes) [in Kelvin].
θ_{BL}	Temperature drop in the cooling liquid boundary layer (in Kelvin).
θ_{C}	Average temperature gradient across the cross-section of the core (see text for a more detailed explanation) [K}.
θ_{E}	Plate edge temperature gradient (see below for primes) [in Kelvin].
θ_{ES}	Weighted average of plate edge and plate surface temperature gradients (see text for a more detailed explanation) [K].
θ_{S}	Plate surface temperature gradient (see below for primes) [in Kelvin].
μ	Dynamic viscosity (of cooling liquid) [kg/m/s].
ρ_{C}	Density of core material (kg/m^3).
ρ_{L}	Density of cooling liquid (kg/m^3).
a	Stacked depth between cooling ducts (in meters).
A	Cross-section between cooling ducts (m^2).
b	Average plate width between cooling ducts (m) [see below for calculation method].
c_P	Specific heat capacity at constant pressure (of cooling liquid) [J/kg/K].
f	Ratio of exposed to total surface area (see below for subscripts).
g	Acceleration due to gravity (m/s^2).
k	Thermal conductivity of cooling liquid (W/mk).
k_L	Thermal conductivity of cooling liquid (W/m/K).
k_I	Thermal conductivity of the core material in the plane of the laminations (W/m/K).
k_T	Thermal conductivity of the core material perpendicular to the plane of the laminations (W/m/K).
q	Specific loss (of core material) [see below for primes] (W/kg).
R	Thermal resistance (see below for subscripts) [km^2/W].
s	Ratio of active to nominal core cross-section.
t	Thickness (of block of generic material, see text for a more detailed explanation) [in meters].

When applied to thermal resistances (as a first subscript) the subscripts COR and OIL refer to thermal resistances of the core material and the boundary layer in the cooling liquid, respectively. When applied to thermal resistances (as a second subscript) or to ratio of total to exposed surface area, the subscripts I and T refer to the directions in the plane of the laminations and perpendicular to the plane of the laminations, respectively.

When applied to core temperature gradients or specific losses, a single prime denotes the L-joint and a double prime the T-joint. When applied to a thermal resistance for the cooling liquid, the primes denote that this was calculated using the appropriate temperature drop in the cooling liquid boundary layer.

The average plate width between ducts is given by

$$b = A/a.$$

A typical three-phase transformer core is shown in Fig. 1. There are a total of six joints between the vertical limbs and horizontal yokes. The four joints between the end limbs and the yokes are termed L-joints. The two joints between the center limb and the yokes are termed T-joints.

Losses are lowest in the limbs and yokes (linear parts of the core), higher in the L-joints, and highest in the T-joints [7].

In this paper, terms of the kind "core temperature gradient" are understood to mean the difference between the core temperature and the local cooling liquid temperature. The term "core interior temperature gradient" is understood to mean the temperature gradient of the hottest part of the core. The term "plate edge temperature gradient" is understood to mean the temperature gradient of the edges of the core laminations. The term "plate surface temperature gradient" is understood to mean the temperature gradient of the flat surfaces of the core laminations. The meaning of these last three terms is illustrated in Fig. 2.

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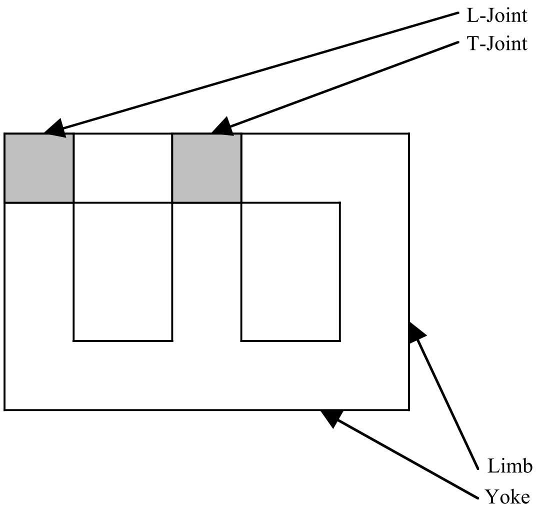

Fig. 1. Sketch of three limb, three-phase transformer core showing L- and T-joints.

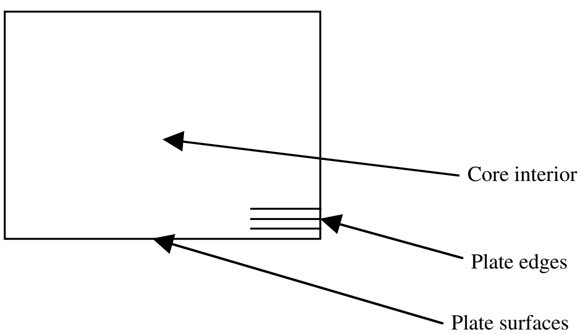

Fig. 2. Cross section of a linear part (limb or yoke) of the core.

I. Introduction

THE problem of calculating accurately the core temperature rise in large power transformers is not a particularly new one. Its importance may not be obvious since the most widely used specifications for large power transformers do not put specific limits of the core temperature rise [1], [3]. However it is obviously necessary to limit the core temperature to values that cause no damage to the core itself, adjacent materials, or the oil. It has been shown that oil may be degraded by core temperatures as low as 110 °C–120 °C [4], [5]. This has led experts in the field to suggest that 130 °C would be a reasonable limit for the core temperature [5].

The simplest method for calculating core temperature found in the literature is that proposed by Küchler [6] (and subsequently republished in its entirety by Karsai [7]). Here the temperature distribution in the core cross section is found using a simple superposition of two one-dimensional (1-D) solutions to the heat transfer equation. This method can only be applied to find the temperature rise in linear parts of the core, and not in the joints.

A more advanced method initially proposed by Cockcroft [8] and subsequently improved by Bucholz [9], Roth [10], [11], and Higgins [12] uses a solution to the two-dimensional (2-D) heat flow equation in the core cross section. As can be imagined, the algebra involved in this method is extremely complicated, as is the resulting expression for the temperature distribution in the core. Furthermore, this method can only be applied to find temperature rise in linear parts of the core and not in the joints.

An alternative advanced method, proposed by Rele and Palmer [13], uses a conducting paper analog to represent the core. Again, this type of modeling can only be used to find temperature rise in linear parts of the core and not in the joints.

A further possibility would be to use finite element modeling. This would be applied to its best advantage when coupled to the output of an advanced core loss distribution calculation. Very detailed, and potentially very accurate, results could be obtained in this way. A model of this type has been proposed by teNyenhis *et al.* [5].

While finite element modeling has undoubted merits, the detail required to construct a successful model and the time taken to do so make it difficult to apply early in the design process. It is more suitable for final design verification and or developing new designs. However, an extension of the simple method first proposed by Küchler using the electrical analogy would allow the temperature rise in the core joints to be estimated without these problems. This method could be used throughout the design and optimization process, and not merely for design verification. Thus, a reasonable approach would be to use the simple method for most design process and one of the advanced methods for final verification.

In this paper, the author describes a simple method offering three major improvements over the methods found in the literature: 1) the new method uses an improved and clarified general formulation; 2) the treatment of the cooling liquid boundary layer is more rigorous; and 3) the method can be used to find the temperature rise in the core joints and not just in the linear parts of the core.

In the explanation of the new method, which follows, the author begins with a general formulation of the heat transfer equations for a transformer core. The effects of both the core material and the cooling liquid boundary layer are then examined individually. A comparison with test results from two transformer cores is made. Finally, conclusions are drawn and recommendations for future work are made. Recommended values for the thermodynamic properties of materials and an example calculation are presented as appendices.

II. General Formulation

Using the electrical analogy, the losses distributed through the core may be replaced by a point source lying at the center of the part of the core considered. The core material and the cooling liquid boundary layer can then be represented as separate thermal resistances. This process is shown diagrammatically in Fig. 3. Expressions for the core interior, plate edge, and plate surface temperature rises may then be derived using Ohm's and Kirchoff's laws.

This gives rise to the following core interior temperature gradient in the linear parts of the core: [See (1) shown at the bottom of the next page.]

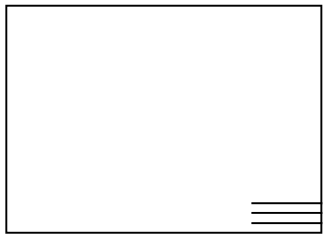

Fig. 3(a). Cross section of a linear part (limb or yoke) of the core.

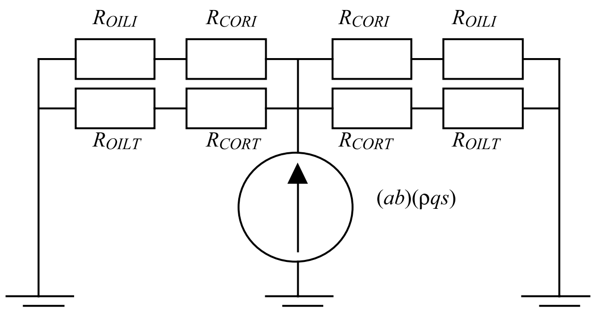

Fig. 3(b). Equivalent circuit using the electrical analogy.

Similarly for the plate edge and plate surface temperature gradients

$$\theta_E = \frac{R_{\mathrm{OIL}\,I}}{f_L R_{\mathrm{COR}\,I} + R_{\mathrm{OIL}\,I}} \theta \tag{2}$$

$$\theta_S = \frac{R_{\mathrm{OIL}\,T}}{f_T R_{\mathrm{COR}\,T} + R_{\mathrm{OIL}\,T}} \theta \tag{3}$$

The weighted average temperature gradient at the plate edges and plate surfaces is given by (4), at the bottom of the page.

Making use of the assumption that the temperature distribution in the core is approximately parabolic, it can be seen that the average temperature gradient across the cross-section of the core is given by

$$\theta_C = \left(\frac{2\theta + \theta_{\mathrm{ES}}}{3}\right). \tag{5}$$

Beginning again with the L-joints, the core interior temperature gradient is given by (6) shown at the bottom of the page.

Similarly, for the plate edge and plate surface temperature gradients

$$\theta'_E = \frac{R'_{\mathrm{OIL}\,I}}{f_I R_{\mathrm{COR}\,I} + R'_{\mathrm{OIL}\,I}} \theta' \tag{7}$$

$$\theta'_S = \frac{R'_{\mathrm{OIL}\,T}}{f_T R_{\mathrm{COR}\,T} + R'_{\mathrm{OIL}\,T}} \theta'. \tag{8}$$

Beginning again with the T-joints, the core interior temperature is given by (9) shown at the bottom of the page.

Similarly, for the plate edge and plate surface temperature gradients

$$\theta''_E = \frac{R''_{\mathrm{OIL}\,I}}{f_I R_{\mathrm{COR}\,I} + R''_{\mathrm{OIL}\,I}} \theta'' \tag{10}$$

$$\theta''_S = \frac{R'_{\mathrm{OIL}\,T}}{f_T R_{\mathrm{COR}\,T} + R'_{\mathrm{OIL}\,T}} \theta''. \tag{11}$$

III. Core Material

An expression for the thermal resistance of the core material may be derived by considering a solid slab of material in which heat is uniformly generated. Let all but two opposite edges of the block be perfectly insulated and the remaining two edges held at the same temperature.

It is a simple matter to apply Fourier's law to find the temperature distribution within this block. From this, the following

$$\theta = \frac{(\rho_C q s)(ab)}{2[a/(R_{\mathrm{COR}\,I} + R_{\mathrm{OIL}\,I}/f_I) + b/(R_{\mathrm{COR}\,T} + R_{\mathrm{OIL}\,T}/f_T)]} \tag{1}$$

$$\theta_{\mathrm{ES}} = \left(\frac{a\left[\frac{f_I R_{\mathrm{OIL}\,I}}{(f_I R_{\mathrm{COR}\,I} + R_{\mathrm{OIL}\,I})^2}\right] + b\left[\frac{f_T R_{\mathrm{OIL}\,T}}{(f_T R_{\mathrm{COR}\,T} + R_{\mathrm{OIL}\,T})^2}\right]}{a\left(\frac{f_I}{f_I R_{\mathrm{COR}\,I} + R_{\mathrm{OIL}\,I}}\right) + b\left(\frac{f_T}{f_T R_{\mathrm{COR}\,T} + R_{\mathrm{OIL}\,T}}\right)}\right)\theta. \tag{4}$$

$$\theta' = \frac{(\rho_C q' s)(ab) + \theta_C(2a/R_{\mathrm{COR}\,I})}{2a/(R_{\mathrm{COR}\,I} + R'_{\mathrm{OIL}\,I}/f_I) + 2a/R_{\mathrm{COR}\,I} + 2b/(R_{\mathrm{COR}\,T} + R'_{\mathrm{OIL}\,T}/f_T)} \tag{6}$$

$$\theta'' = \frac{(\rho_C q'' s)(ab) + \theta_C(3a/R_{\mathrm{COR}\,I})}{a/(R_{\mathrm{COR}\,I} + R''_{\mathrm{OIL}\,I}/f_I) + 3a/R_{\mathrm{COR}\,I} + 2b/(R_{\mathrm{COR}\,T} + R''_{\mathrm{OIL}\,T}/f_T)} \tag{9}$$

expression for the effective thermal resistance of the block can be derived:

$$R = (1/4)(t/k). \tag{12}$$

Note that the above thermal resistance is based on the maximum temperature of the block and not the average. [If the average temperature of the block were used, then the denominator of the constant on the right-hand side of (12) would be six].

The above can now be used to derive expression for the thermal resistance of the core. This results in

$$R_{\mathrm{COR}\,I} = b/4k_I \tag{13}$$
$$R_{\mathrm{COR}\,T} = a/4k_T \tag{14}$$

IV. Boundary Layer

The author has already investigated the thermal resistances of cooling liquid boundary layers in the course of work on winding temperature gradients [14]. For the case in question (natural flow), he found the following expression:

$$R_{\mathrm{OIL}} = \frac{1}{0.1029}\left(\frac{\beta\rho_L^2\theta_{\mathrm{BL}}c_P g k_L^2}{\mu}\right)^{-1/3}. \tag{15}$$

The above equation can now be used to derive expression for the thermal resistance of the two cooling liquid boundary layers. This results in

$$R_{\mathrm{OIL}\,I} = \frac{1}{0.1029}\left(\frac{\beta\rho_L^2\theta_S c_P g k_L^2}{\mu}\right)^{-1/3} \tag{16}$$
$$R_{\mathrm{OILL}} = \frac{1}{0.1029}\left(\frac{\beta\rho_L^2\theta_E c_P g k_L^2}{\mu}\right)^{-1/3}. \tag{17}$$

Naturally, the above can also be used to calculate the thermal resistance of the boundary layers in the cooling liquid at the L- and T-joints. It is necessary to substitute the appropriate values for the plate edge or plate surface temperature gradient.

The thermal resistance of the boundary layer can be seen to depend on the temperature drop across it. Thus, one cannot be calculated without the other. For computer applications, it is easy to set up an iteration to perform the necessary calculations. For "pencil and paper" calculations, the iteration is laborious and it may be preferable to guess a value for the temperature drop across the cooling liquid boundary layer and afterwards verify that it was not too much in error.

V. Comparison

To assist in the development of the method described in this paper, the author's company made detailed core temperature rise measurements on two medium-power transformers—one rated at 30 MVA and the other at 60 MVA. A sample calculation based on the 60-MVA transformer is presented in Appendix B.

A comparison of the test results with the results of calculations using the method proposed in this paper may be found in Table I. (A blank column indicates that no reliable measurements of the quantity concerned were made). There is generally good agreement between the calculations and the measurements, especially for the core interior temperature gradients. The errors are largest for the plate surface temperature gradients. This suggests that the translaminar conductivity of the core may be underestimated.

TABLE I
COMPARISON OF CALCULATED AND MEASURED CORE TEMPERATURE GRADIENTS

Part of Core	Temperature Gradient (K)			
	30MVA Transformer		60MVA Transformer	
	Calc.	Meas.	Calc.	Meas.
Limb and Yokes-				
Interior	17.2	18.6	17.8	17.5
Plate Edges	14.0	13.1	13.4	12.7
Plate Surfaces	7.3	10.7	8.8	
L-Joints-				
Interior	17.6	16.2	19.2	
Plate Edges	14.3	15.7	14.4	12.2
Plate Surfaces	7.5	11.9	9.4	
T-Joints-				
Interior	20.6	18.6	24.1	27.1
Plate Edges	16.6	14.4	17.7	
Plate Surfaces	8.5	12.8	11.3	

TABLE II
THERMAL CONDUCTIVITY OF CORE MATERIAL

Direction	Thermal Conductivity (W/m/K)
Interlaminar	21.0
Translaminar	3.3

VI. Conclusion

The method described in this paper allows the core temperature gradient in power transformers to be calculated with reasonable accuracy and little effort. It is difficult to say whether the methods could successfully be applied by other transformer manufacturers. The formulation and the method for calculating the thermal resistance of the core material should be of universal validity. The method for calculating the thermal resistance of the boundary layer may not be applicable to all types of construction and all sizes of transformer. Other manufacturers might be able to use the same approach but with a different version of (15) and different values for the material properties.

VII. Recommendations

The author has obtained good results using simple methods. The results could be improved if a more accurate value for the translaminar thermal conductivity were available.

Appendix A
Thermodynamic Properties of Dielectric Materials

Recommended values for the thermal conductivity of the core material may be found in Table II. These were originally taken from [4].

Recommended values for the thermodynamic properties of oil may be found in Table III. These were originally taken from a table in [4], and are for a reference temperature of 75 °C.

TABLE III
THERMODYNAMIC PROPERTIES OF OIL

Property	Value (units)
Cubic expansivity	0.000795 (/K)
Density	849 (kg/m^3)
Dynamic viscosity	0.00399 (kg/m/s)
Specific heat capacity	2080 (J/kg/K)
Thermal conductivity	0.1272 (W/m/K)

APPENDIX B
WORKED EXAMPLE

The example chosen is the core of the 60-MVA transformer. The core cross-section is 0.153 435 m^2. It is divided by a single duct. The stacked depth on either side of the duct is 295 mm.

The equivalent plate width is

$$b = 0.520.$$

The specific losses in the core are 0.740 W/kg (supplier's data). The density of the core material is 7650 kg/m^3 and the ratio of nominal to effective cross-sectional area is 0.97. The ratio of exposed to total surface area at the plate edges and surfaces is assumed to be 0.6.

For the core material, this gives

$$R_{\mathrm{COR}\,I} = 0.006\,19$$
$$R_{\mathrm{COR}\,T} = 0.0223.$$

The temperature drops in the two boundary layers are equal to the temperature gradients of the plate edges and surfaces. By iteration, these are

$$\theta_E = 13.4$$
$$\theta_S = 8.79.$$

These give the following thermal resistances:

$$R_{\mathrm{OIL}\,I} = 0.0113$$
$$R_{\mathrm{OIL}\,T} = 0.0130.$$

These, in turn, give the following value for the core interior temperature gradient:

$$\theta = 17.8.$$

Finally, the weighted average temperature gradient at the edges and surfaces of the laminations and the average temperature gradient across the cross-section of the core are as follows:

$$\theta_{\mathrm{ES}} = 11.1$$
$$\theta_C = 15.6.$$

Assuming that the specific losses in the L-joints are 1.5 times the specific losses in the linear parts of the core [7], the temperature gradients of the plate edges and surfaces as obtained by iteration are

$$\theta'_E = 14.4$$
$$\theta'_S = 9.36.$$

These give the following thermal resistances:

$$R'_{\mathrm{OIL}\,I} = 0.0110$$
$$R'_{\mathrm{OIL}\,T} = 0.0127.$$

These, in turn, give the following value for the core interior temperature gradient:

$$\theta' = 19.2.$$

Assuming that the specific losses in the T-joints are 2.5 times the specific losses in the linear parts of the core [7], the temperature gradients of the plate edges and surfaces as obtained by iteration are

$$\theta''_E = 17.7$$
$$\theta''_S = 13.3.$$

These give the following thermal resistances:

$$R''_{\mathrm{OIL}\,I} = 0.0103$$
$$R''_{\mathrm{OIL}\,T} = 0.0120.$$

These, in turn, give the following value for the core interior temperature gradient:

$$\theta'' = 24.1.$$

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