MARMARA UNIVERSITY

COMPUTER SCIENCE ENGINEERING

MATH2059 – NUMERICAL METHODS

REPORT OF HOMEWORK #2

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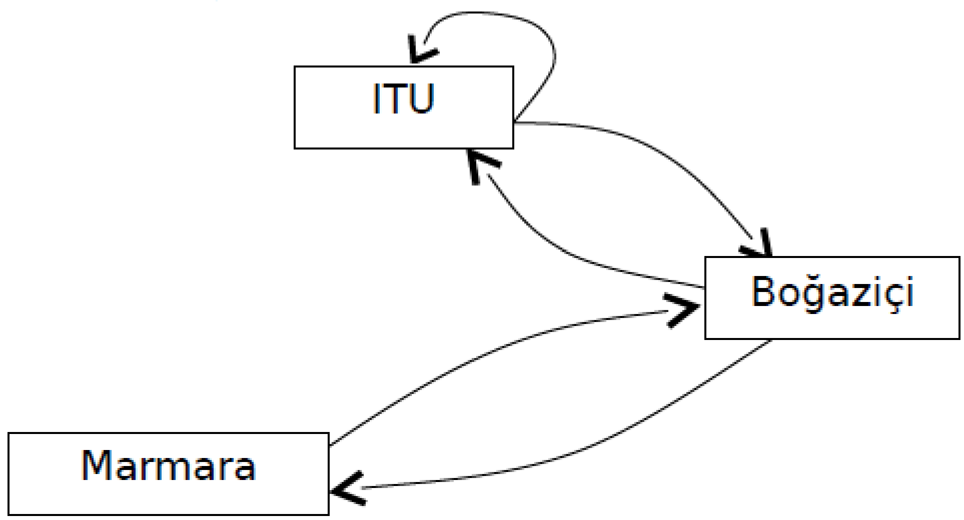
Problem #1:

Nowadays, almost all people search on the internet with using search engines when they need an information. How this search engines work?

Search engines search keyword(s) which people write into its search box and rank web pages which have information about the searching word(s). The rank run with an algorithm that scans connections between web pages. When a webpage is linked with many normal pages or some important pages, its rank is increased with computations of the algorithm.

The algorithm uses matrix multiplication to find eigenvector(s) of page(s) in order to put in order. In this problem, we have used this PageRank algorithm also.

Relation of the pages is:



Matrix relation:

$$\begin{bmatrix} I^{(t+1)} \\ M^{(t+1)} \\ B^{(t+1)} \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 0 & 1/2 \\ 1/2 & 1 & 0 \end{bmatrix} \begin{bmatrix} I^{(t)} \\ M^{(t)} \\ B^{(t)} \end{bmatrix}$$

where $I^0 = 1, M^0 = 1, B^0 = 1$

Matlab script of this problem works like this:

<u>Option a:</u>

How script works:

Step 1: Define the matrix of the recursive relation values between the pages (1/outdegree(s) of the site)

Step 2: Multiply the matrix with firstly initial values after obtained eigenvector until pagerank of the Boğaziçi will be equal to İTÜ.

Step 3: Calculate the magnitude of the values in eigenvector and divide them with this magnitude in order to obtain normalized vector of the calculated eigenvector.

After tracing the steps, we have obtained an eigenvector which is:

$$\begin{matrix} 0.666666666725002 \\ 0.333333333410473 \\ 0.666666666569761 \end{matrix}$$

Whereas, other eigenvector which is calculated by matlab with using eigs function is:

$$\begin{matrix} 0.309016994374948 \\ -0.809016994374948 \\ 1.000000000000000 \end{matrix}$$

If we think about the differences, we can understand that this matrix has different eigenvalues.

To find them, we must make some calculations:

Matrix eigenvectors

Let A be a square matrix, v a vector and λ a scalar that satisfy $Av = \lambda v$, then v is an eigenvector of A and λ is the eigenvalue associated with it

To find the eigenvectors v, solve $(A - \lambda I)v = 0$ for each eigenvalue λ

Find the eigenvalues for $\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 \end{pmatrix}$: $1, \frac{1}{4}(\sqrt{5} - 1), \frac{1}{4}(-1 - \sqrt{5})$

Eigenvectors for $\lambda = 1$: $\begin{pmatrix} 1 \\ \frac{1}{2} \\ 1 \end{pmatrix}$

Eigenvectors for $\lambda = \frac{1}{4}(\sqrt{5} - 1)$: $\begin{pmatrix} -\frac{3+\sqrt{5}}{2} \\ \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix}$

Eigenvectors for $\lambda = \frac{1}{4}(-1 - \sqrt{5})$: $\begin{pmatrix} -\frac{3-\sqrt{5}}{2} \\ -\frac{\sqrt{5}-1}{2} \\ 1 \end{pmatrix}$

The eigenvectors for $\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 1 \\ \frac{1}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} -\frac{3+\sqrt{5}}{2} \\ \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} -\frac{3-\sqrt{5}}{2} \\ -\frac{\sqrt{5}-1}{2} \\ 1 \end{pmatrix}$$

As we can see, this matrix has 3 different eigenvectors.

Option b:

Do the same things in option a part with using different initial values that the Marmara does not link the Boğaziçi.

When doing this, our test case loop never reaches the break point option which runs when page rank of İTÜ is equal to page rank of the Boğaziçi.

Thus, we obtained an eigenvector which values are infinite.

$$\begin{matrix} Inf \\ Inf \\ Inf \end{matrix}$$

Option c:

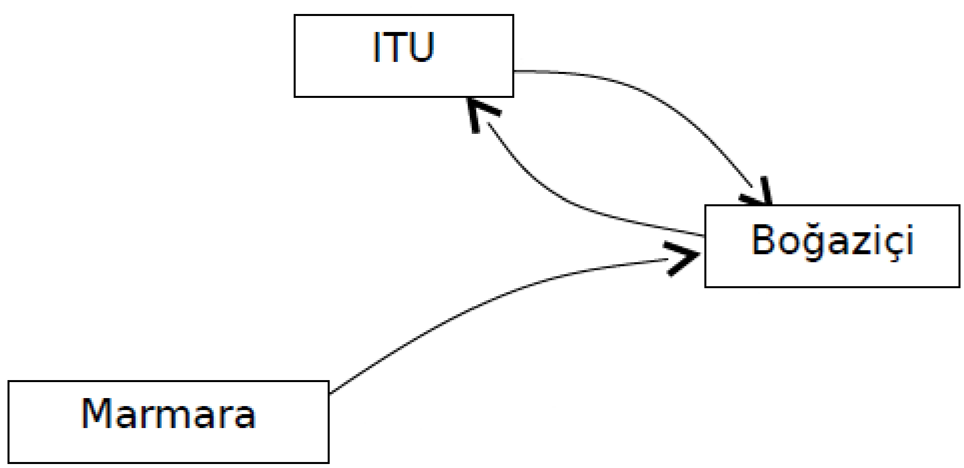
If we want to make the Marmara the most important web page, we must link it itself in order to increase its page rank. When we do this, we obtain the eigenvector (page ranks) below:

İTÜ's rank:	0.0506321539504784
Marmara's rank:	0.998226979620891
Boğaziçi's rank:	0.0312934840388374

With making this change, we obtained that the most popular web page is Marmara.

Options d:

If we delete the link from Boğaziçi to Marmara and İTÜ itself, we will obtain graph below:



Matrix of this graph is:

$$\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix}$$

Without using cyle, we have obtained a matrix which leads to iteration that will not converge.

Eigenvector of this matrix is:

$$\begin{matrix} 0.577350269189626 \\ 0.577350269189626 \\ 0.577350269189626 \end{matrix}$$

Problem #2:

In deriving the Gaussin Elimination with Backward Subsitition algorithm, we found that a row interchange was needed when one of the pivot elements a_{kk}^{k} is 0.

In deriving the Gaussian Elimination with Backward Substitution algorithm, we found that a row interchange was needed when one of the pivot elements a_{kk}^{k} is 0. This row interchange has the form (Ek) ↔ (Ep), where p is the smallest integer greater than k with $a_{pk}^{k} \neq 0$.

To reduce round-off error, it is often necessary to perform row interchanges even when the pivot elements are not zero.

If a_{kk}^{k} is small in magnitude compared to a_{jk}^{k} , then the magnitude of the multiplier

$$m_{jk}^{k} \frac{= \ a_{jk}^{k}}{a_{kk}^{k}}$$

will be much larger than 1.

Round-off error introduced in the computation of one of the terms a_{kl}^{k} is multiplied by m_{jk} when computing a_{kk}^{k+1} , which compounds the original error.[1]

In this problem, we have used Gaussian elimination with scaled column pivoting in order to find solution of the system Ax = b

Input 1: [singular1, x1] = [A1, b1]

A1 =

3	-13	9	3
-6	4	1	-18
6	-2	2	4
12	-8	6	10

b1 =

-19
-34
16
26

singular1 =

1

x1 =

3.0000
1.0000
-2.0000
1.0000

Input 2: [singular2, x2] = [A2, b2]

A2 =

25	5	1
64	8	1
144	12	1

b2 =

106.8000
177.2000
279.2000

singular2 =

1

x2 =

0.2905
19.6905
1.0857

Input 3: [singular3, x3] = [A3, b3]

A3 =

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

b3 =

1
3
5
7
7

singular3 =

1

x3 =

0.0035
0.0073
0.2708
0.0842
-0.0119

Problem #3:

In the natural cubic spline, we set the two end conditions:

σ1 = σn = 0.

Among different ways of picking these two conditions, we now choose the following. Let c1(x) and cn(x) be the unique cubics that interpolate the first four and last four data points, respectively. The two end conditions match the third derivatives of s(x) to the third derivatives of these cubics, namely,

$$s'''(x_1) = c_1''' \quad \text{and} \quad s'''(x_n) = c_n'''.$$

The constraints c ‴ 1 and c ‴ n can be determined directly from the data without actually finding c1(x) and c2(x). We have already introduced the quantities

$$\Delta_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i},$$

which are approximations to first derivatives. Let

$$\Delta_i^{(2)} = \frac{\Delta_{i+1} - \Delta_i}{x_{i+2} - x_i} \quad \text{and} \quad \Delta_i^{(3)} = \frac{\Delta_{i+1}^{(2)} - \Delta_i^{(2)}}{x_{i+3} - x_i}.$$

These quantities are known as divided differences; 2Δ(2) i and 6Δ(3) i are approximations to second and third derivatives. In particular,

$$c_1''' = 6\Delta_1^{(3)} \quad \text{and} \quad c_n''' = 6\Delta_{n-3}^{(3)}.$$

Since

$$s'''(x) = \frac{6(\sigma_{i+1} - \sigma_i)}{h_i} \quad \text{on} \quad [x_i, x_{i+1}],$$

we require that

$$\frac{\sigma_2 - \sigma_1}{h_1} = \Delta_1^{(3)} \quad \text{and} \quad \frac{\sigma_n - \sigma_{n-1}}{h_{n-1}} = \Delta_{n-3}^{(3)}.$$

To make the final system of equations symmetric, these last two equations should be multiplied by h 2 1 and –h^2(n−1) respectively to give

$$-h_1\sigma_1 + h_1\sigma_2 = h_1^2\Delta_1^{(3)} \quad \text{and} \quad h_{n-1}\sigma_{n-1} - h_{n-1}\sigma_n = -h_{n-1}^2\Delta_{n-3}^{(3)}.$$

The system is

$$\begin{bmatrix} -h_1 & h_1 & & & & \\ h_1 & 2(h_1+h_2) & h_2 & & 0 & \\ & h_2 & 2(h_2+h_3) & h_3 & & \\ & & \ddots & \ddots & \ddots & \\ & 0 & & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \\ & & & & h_{n-1} & -h_{n-1} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \vdots \\ \sigma_{n-1} \\ \sigma_n \end{bmatrix} = \begin{bmatrix} h_1^2\Delta_1^{(3)} \\ \Delta_2 - \Delta_1 \\ \Delta_3 - \Delta_2 \\ \vdots \\ \Delta_{n-1} - \Delta_{n-2} \\ -h_{n-1}^2\Delta_{n-3}^{(3)} \end{bmatrix}.$$

[2]

Resources:

[1]: https://www.math.ust.hk/~mamu/courses/231/Slides/CH06_2A.pdf

[2]: http://www.cas.mcmaster.ca/~qiao/courses/cs4xo3/assignments/a11f3s.pdf