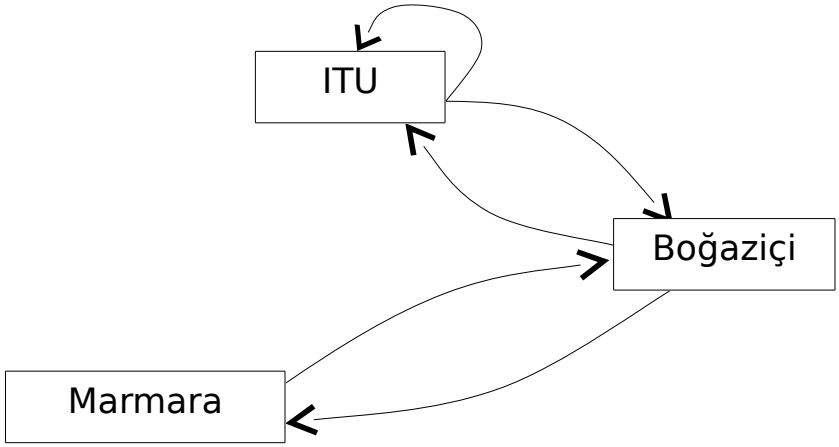
MATH 2059 – Assignment 2
Due: 21.5.2017 at 23:59

The assignment can be done in group of 2 students.

Problem 1 (30 pts)

The aim of this problem is to illustrate how linear algebra can be made use of in Internet search engines. When several pages match a search query, the pages should be displayed in order of their importance. So how is this page importance determined? Consider the patent pending PageRank scheme used by Google to determine the importance of each web page. A page is important if important pages link to it. This is a recursive definition of importance which needs to be solved. If we imagine that each page has one unit importance initially, then we can iterate a process of each page sharing whatever importance it has among its successors and receiving new importance from its predecessors. This can be represented as a matrix vector product as illustrated in the following example. Consider the following link structure:



Let I, M and B denote the importance of ITU, Marmara and Boğaziçi respectively. Then, in the following iteration, a column means that the corresponding site is giving 1/*k*th of its importance (with *k* being the outdegree of the site) to the sites it points to. The iteration for this example is given as:

$$\begin{bmatrix} I^{(t+1)} \\ M^{(t+1)} \\ B^{(t+1)} \end{bmatrix} = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 0 & 1/2 \\ 1/2 & 1 & 0 \end{bmatrix} \begin{bmatrix} I^{(t)} \\ M^{(t)} \\ B^{(t)} \end{bmatrix}$$

where $I^{(0)}=M^{(0)}=B^{(0)}=1$.

Eventually, the above iteration will reach a limit which happens to be its component in the principal eigenvector of this example matrix. In the limit, the solution will be I = 6/5, B = 6/5 and M = 3/5, i.e. ITU and Boğaziçi will have the same importance. Marmara, on the other hand, will have half the importance.

Answer the following questions:

a. Implement the above procedure by using sparse matrix representation in MATLAB. Normalize the final vector. Compare the result returned with that of MATLAB function which returns the principal eigenvector (do not forget to normalize vectors when comparing).

b. What will happen if Marmara removes the link from Marmara to Boğaziçi?

c. If after the action in (b), Marmara wants to become the most important site, what should it do?

d. (Bonus 5 pts) Give an example link graph whose link matrix will lead to an iteration which will not converge, but rather cycle through some values.

Problem 2 (30 pts)

Gaussian Elimination with Scaled Column Pivoting

Implement a function in MATLAB called mygauss which inputs an n-by-n matrix A and a vector b of size n, and outputs the solution x of the system Ax=b using Gaussian Elimination Method with Scaled Column Pivoting.

The definition of the function is as follows:

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function [singular, x] = mygauss (A,b)
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where singular is a scalar indicating whether the matrix A is non-singular (0) or singular (1) (i.e. whether the Gaussian elimination process is successfully finished (0) or not (1)), and x is the solution of the system Ax=b.

For this part of the assignment, you are expected to submit mygauss.m file (should be fully commented) and outputs of 3 runs (input values and output values) with different parameters (examples for both non-singular and singular cases). Also, include the outputs for x = A\b (built-in solver in MATLAB) for each input pair for checking the correctness of the results.

Problem 3 (30 pts)

In this problem, you are expected to implement a general curve drawing program (script curvedraw.m) using MATLAB. Your program should have the following features:

- The user should be able to enter a sequence of any number of points graphically (i.e. you display a square drawing area with a grid and the user enters the coordinates graphically by clicking on the drawing area). You can use MATLAB ginput function to input coordinates of the

clicked point. The number of data points is not restricted (use ginput function without input parameter).

- You should determine two natural cubic splines for the parametric curves x(t) and y(t) for the data points obtained using ginput function. The algorithm for determining the coefficients of the natural cubic spline was introduced in the class. You must implement this algorithm also as a separate function (called cubicspline).
- After finding x(t) and y(t), you should draw the whole curve afterwards including the data points entered using ginput.
- You are not allowed to use the built-in spline function.

For this part of the assignment, you are expected to submit curvedraw.m and cubicspline.m files (should be fully commented) and outputs of 3 different runs (graphs).

SUBMISSION POLICY OF THE ASSIGNMENT

1. **The assignment can be done in group of 2 students.**
2. For the first question, you should give details about how you implemented the procedure in your report. Please, give adequate and explanatory details for each option of the question.
3. Write a test script for the second question in which there exists creation of your inputs and call of your scripts/functions for each run.
4. For the third question, indicate which inputs are entered by you and which output is created for these inputs for each run in your report.
5. Write a detailed report which includes explanation about for each question. Write how your scripts and functions work, i.e., which parts of your scripts/functions accomplish which task and how it is accomplished. Put also your functions and scripts' outputs into your report.
6. Put your report, Matlab codes, plots etc. into a zip file. Name your zip file as your name_surname_studentnumber_hw2.zip. For example, a student whose name is Kaan Demir and student number is 150119099 will name the file as : kaan_demir_150119099_hw2.zip. Also, write your name, surname and student number as comments at the beginning of your codes.
7. To submit your homework, send your zip file to serap.korkmaz@marmara.edu.tr. Write the name of the zip file to subject part of your e-mail.
8. Write explanatory and sufficient comments on each line of your codes and indicate your inputs and outputs if exist.
9. Show your own work and keep away from plagiarism.