Cmpe-300

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1 Introduction

Q3

First, check $\log(n!)$.

$$\log(n!) = \log(1{\cdot}2{\cdot}3{\cdot}\ldots{\cdot}n) = \log 1 + \log 2 + \log 3 + \ldots + \log n \leq \underbrace{\log n + \log n + \ldots + \log n}_{nterms} \leq n{\cdot}\log n$$

Now, check $n^{\cos n}$.

$$1 \geq \cos n \quad so \quad n^1 \geq n^{\cos n}, for, n \geq 1$$

Next, compare n and $\log(n!)$.

$$\lim_{n\to\infty} \frac{n}{n \cdot \log n} = \lim_{n\to\infty} \frac{1}{\log n} = 0 \quad so, according to this, n^{\cos n} \leq n \leq \log(n!)$$

Compare $\log(n!)$ and $3^{\log n}$.

$$\lim_{n\to\infty} \frac{n \cdot \log n}{3^{\log n}} = \lim_{n\to\infty} \frac{n \cdot \log n}{n^{\log 3}} = \lim_{n\to\infty} \frac{\log n}{n^{\log 3 - 1}}$$

Apply L'Hôpital's Rule:

$$= \lim_{n\to\infty} \frac{(1/n)}{(\log 3 - 1) \cdot n^{\log 3 - 2}} = \lim_{n\to\infty} \frac{1}{(\log 3 - 1) \cdot n^{\log 3 - 1}} = 0 \quad since (\log_2 3 - 1) > 0.$$

Therefore, as n goes to infinity, $\log(n!)$ grows at a slower rate than $3^{\log n}$.
Now, compare $3^{\log n}$ and $n^{\log n}$.

$$\lim_{n\to\infty} \frac{3^{\log n}}{n^{\log n}} = \lim_{n\to\infty} \frac{n^{\log 3}}{n^{\log n}} = \lim_{n\to\infty} \frac{1}{n^{\log n - \log 3}} = 0$$

Therefore, as n goes to infinity, $3^{\log n}$ grows at a slower rate than $n^{\log n}$.
Compare $n^{\log n}$ and $(\log n)^n$.

$$\lim_{n\to\infty} \frac{n^{\log n}}{(\log n)^n} = \lim_{n\to\infty} \frac{\log(n^{\log n})}{\log((\log n)^n)} = \lim_{n\to\infty} \frac{\log n \cdot \log n}{n \cdot \log(\log n)} \leq \lim_{n\to\infty} \frac{(\log n)^2}{n} \quad (n \log(\log n) \geq n)$$

Apply L'Hôpital's Rule:

$$= \lim_{n\to\infty} \frac{2 \cdot \log n \cdot (1/n)}{1} = \lim_{n\to\infty} \frac{2 \cdot \log n}{n}$$

Apply L'Hôpital's Rule:

$$\lim_{n\to\infty} 2 \cdot \frac{1}{n} = 0$$

Therefore, as n goes to infinity, $n^{\log n}$ grows at a slower rate than $(\log n)^n$. Thus, the order from the lowest to highest grow rate is: The list of functions is as follows:

1. $n^{\cos n}$
2. n
3. $\log(n!)$
4. $3^{\log n}$
5. $n^{\log n}$
6. $(\log n)^n$