Cmpe-300

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1 Introduction

Q1

Given functions f_1 and f_2, $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$. Prove or disprove that $f_1(n) + f_2(n) \in O(\max\{g_1(n), g_2(n)\})$.

Answer: To prove;
Proof:
Since $f_1(n) \in O(g_1(n))$, there exist positive constants C_1 and n_1 such that for $n \geq n_1$,

$$f_1(n) \leq C_1 \cdot g_1(n).$$

Similarly, since $f_2(n) \in O(g_2(n))$, there exist positive constants C_2 and n_2 such that for $n \geq n_2$,

$$f_2(n) \leq C_2 \cdot g_2(n).$$

Now, consider the sum $f_1(n) + f_2(n)$. We want to show that there exist positive constants C and n_0 such that for $n \geq n_0$,

$$f_1(n) + f_2(n) \leq C \cdot \max\{g_1(n), g_2(n)\}.$$

Let $C_3 = C_1 + C_2$,$C = 2 \cdot C_3$,h(n) = $\max\{g_1(n), g_2(n)\}$, and $n_3 = \max\{n_1, n_2\}$ such that for $n_3 \geq n_0$,
f$_1(n) + f_2(n) \leq C_1 \cdot g_1(n) + C_2 \cdot g_2(n) \leq C_3 \cdot h(n) + C_3 \cdot h(n)$
$f_1(n) + f_2(n) \leq 2 \cdot C_3 \cdot h(n)$
$f_1(n) + f_2(n) \leq 2 \cdot C_3 \cdot \max\{g_1(n), g_2(n)\})$
$f_1(n) + f_2(n) \leq C \cdot \max\{g_1(n), g_2(n)\})$

Therefore, we have shown that $f_1(n) + f_2(n) \in O(\max\{g_1(n), g_2(n)\})$.