Cmpe-300

Ahmet Hacıoğlu

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1 Introduction

Q4

I disprove $f(n) \in \Theta(\log(n))$ via contradiction.
Assume $f(n) \in \Theta(\log(n))$.
Let $n = 2^{2^k}$ where $k \geq 0$.

Therefore,

$$f(2^{2^k}) = f(2^{2^{k-1}}) + 1$$

$$f(2^{2^{k-1}}) = f(2^{2^{k-2}}) + 1$$

$\vdots$

$$f(2^{2^1}) = f(2^{2^{k-k}}) + 1$$

Using backward substitution:

$$f(2^{2^k}) = f(2) + k$$

$$f(2^{2^k}) = 1 + k$$

Now, since we assume $f(n) \in \Theta(\log(n))$, so there exist positive constants c_1, c_2, and n_0 such that $c_1 \log n \leq f(n) \leq c_2 \log n$ for $n \geq n_0$.

Consider the left part:

$$c_1 \log n \leq f(n)$$

$$c_1 \log(2^{2^k}) \leq f(2^{2^k})$$

$$c_1 \cdot 2^k \log 2 \leq k + 1$$

If we take the limit :

$$\lim_{k\to\infty} \frac{k+1}{c_1 \cdot 2^k \log 2} = \lim_{k\to\infty} \frac{1}{c_1 \cdot 2^k (\log 2)^2} = 0$$

So, the denominator grows faster than the numerator. Thus, we cannot find positive constants c_1 and n_0 such that this inequality(left part) is satisfied. Therefore, $f(n) \notin \Theta(\log(n))$.