Q5

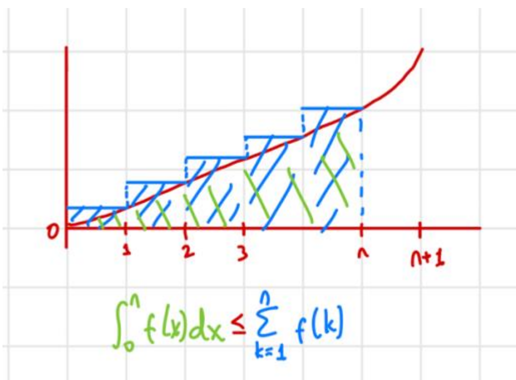
Let f(x) : N → R+ be a non-decreasing function. Show that the following statement is true (you may use diagrams):

$$\int_0^n f(x)\,dx \le \sum_{k=1}^{n} f(k) \le \int_1^{n+1} f(x)\,dx$$

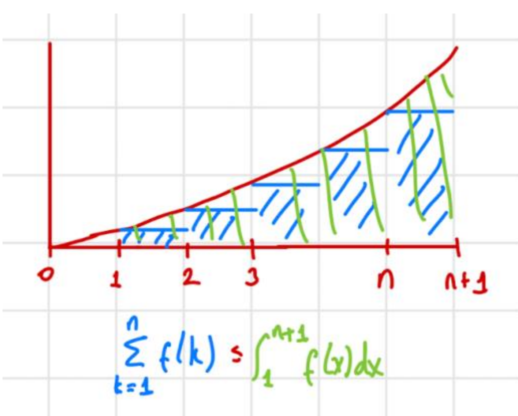
Provide 3 examples for f, show that the inequality is satisfied for your examples.

<u>Show</u>

First, consider the left inequality. Integral term is the area under f(x) from 0 to n. We can consider summation as calculating area under the curve from 0 to n, using rectangle method with right endpoint. So we consider the area between 0-1 as [f(1) * 1], between 1-2 as [f(2) * 1], … and (n-1)-n as [f(n) * 1]. This method overestimates the area when f(n) is non-decreasing (inspect the following graph), so we proved the inequality.



Second, consider the right inequality. Integral term is the area under f(x) from 1 to n+1. We can consider summation as calculating area under the curve from 1 to n+1, using rectangle method with left endpoint. So we consider the area between 1-2 as [f(1) * 1], between 2-3 as [f(2) * 1], … and n-(n+1) as [f(n) * 1]. This method underestimates the area when f(n) is non-decreasing (inspect the following graph), so we proved the inequality.



Therefore, we showed the inequality is true.