Problem Session 5 - Questions

5.1

The impulse response $h[n]$ of an FIR filter is $h[n] = \delta[n-1] - 2\delta[n-4]$. Write the difference eq.

5.3-c

An LTI system is described by the difference eq.

$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$

Determine the response of this system to a unit impulse input; i.e., find the output $y[n] = h[n]$ when the input is $x[n] = \delta[n]$.

5.5

Consider a system defined by $y[n] = \sum_{k=0}^{M} b_k x[n-k]$

a) Suppose that the input $x[n]$ is nonzero only for $0 \le n \le N-1$. Show that $y[n]$ is nonzero at most over a finite interval of the form $0 \le n \le P-1$. Determine P and the support of $y[n]$ in terms of M and N.

b) Suppose that the input $x[n]$ is nonzero only for $N_1 \le n \le N_2$. What is the support of $x[n]$? Show that $y[n]$ is nonzero at most over a finite interval of the form $N_3 \le n \le N_4$. Determine N_3 and N_4 and the support of $y[n]$ in terms of N_1, N_2, and M.

a) Assume $b_0 \neq 0$ and $b_M \neq 0$ for the biggest interval.

Write the sum:

$y[n] = b_M x[n-M] + b_{M-1} x[n-M+1] + b_{M-2} x[n-M+2] + \dots + b_{M-M} x[n-M+M]$

We know that $x[n]$ is non-zero for $0 \le n \le N-1$.

So, $b_0 . x[n-M+M] = \boxed{b_0 . x[n]} \rightarrow$ This term is non-zero for the same interval.

$b_M . x[n-M]$ is non-zero if $0 \le n-M \le N-1$

$M \le n \le N+M-1$

The question asks the biggest interval s.t. $y[n] \neq 0$

$0 \le n \le N+M-1$ $\boxed{P = N+M}$

b) Similarly, if $x[n]$ is non-zero for $N_1 \le n \le N_2$

$b_0 . x[n-M+M] = b_0 . x[n]$ is non-zero for the same interval.

$b_M . x[n-M]$ is non-zero if $N_1 + M \le n \le N_2 + M$

So, $N_3 = N_1$ $N_4 = N_2 + M$

5.8 If the filter coefficients of an FIR system are

$b_k = \{13, -13, 13\}$ and the input signal is

$$x[n] = \begin{cases} 0, & \text{for } n \text{ even} \\ 1, & \text{for } n \text{ odd} \end{cases}$$

Determine output signal $y[n]$ for all n.

$h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$

$h[n] = 13\delta[n] - 13\delta[n-1] + 13\delta[n-2]$

$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$

$y[0] = h[0] . x[0] + h[1] \times [-1] + h[2] . x[-2]$

Convolution:

n:	...	-2	-1	0	1	2	3	4	...
$x[n]$:	...	0	1	0	1	0	1	0	
$h[n]$:	...			13	-13	13			

$$y[n] = \begin{cases} -13 & \text{for } n \text{ even} \\ 26 & \text{for } n \text{ odd} \end{cases}$$

5.9

For each following system, determine whether or not the system (1) linear, (2) time-invariant, (3) causal.

a) $y[n] = x[n] \cdot \cos(0.2\pi n)$ b) $y[n] = x[n] - x[n-1]$

c) $y[n] = |x[n]|$

5.10

Suppose that S is an LTI system. Suppose that the following input-output pair is the result of a test.

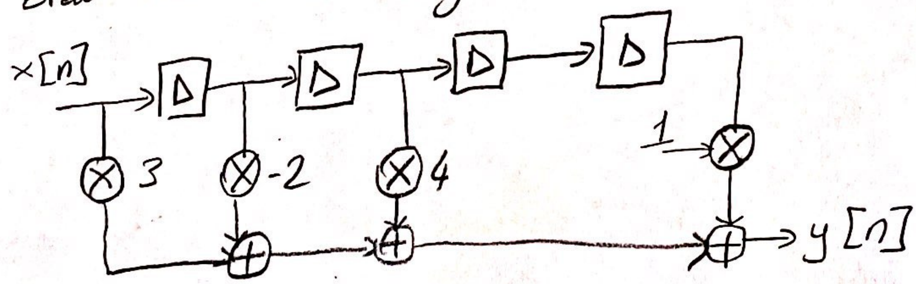
$x[n]$	$y[n]$
$\delta[n] - \delta[n-1]$	$\delta[n] - \delta[n-1] + 2\delta[n-3]$

b) Use linearity and time-invariance to find the output of the system $x[n] = 7\delta[n] - 7\delta[n-2]$

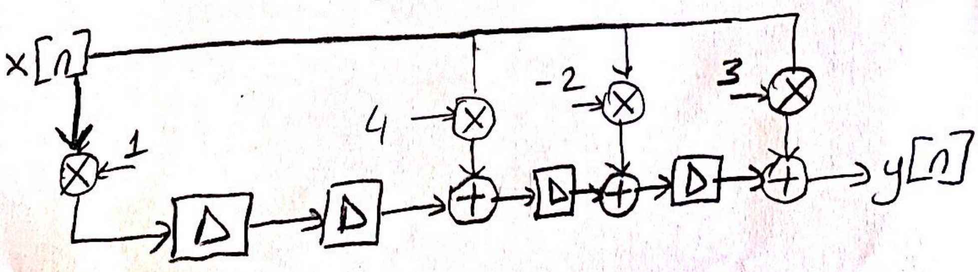
5.11

$$h[n] = 3\delta[n] - 2\delta[n-1] + 4\delta[n-2] + \delta[n-4]$$

a) Draw the block diagram in direct form



b) Draw the block diagram in transposed direct form.



5.15

Another form of deconvolution process starts with the output signal and the input signal, from which it should be possible to find the impulse response.

a) If the input signal is $x[n]=u[n]$, find FIR filter that will produce the output $y[n]=u[n-1]$

b) If the input signal is $x[n]=u[n]$, find FIR filter that will produce the output $y[n]=\delta[n]$

c) If the input signal is $x[n]=\left(\frac{1}{2}\right)^n u[n]$, find the FIR filter that will produce the output $y[n]=\delta[n-1]$.