CmpE300 Analysis of Algorithms

Assignment

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Submission Rules

You must submit your work through Moodle. For each question, create a separate file and ensure that it complies with `<Your Student Number>_Q<Question Number>.pdf` rule. Example: `202300000000_Q5.pdf`. Please solve this assignment on computer. Scanned hand-written submissions will not be accepted. Your work must be done by yourself and yourself only. If you cheat, your work will not be graded and further actions will be taken if necessary. Late submissions will not be accepted.

Questions

Q1

Given functions f_1 and f_2, $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$. Prove or disprove that $f_1(n) + f_2(n) \in O(max\{g_1(n), g_2(n)\})$

Q2

You are given pseudocode of an algorithm. Write the formula for the output of this algorithm in terms of the input n(write the output a in terms of input n). Find the worst-case time complexity of the algorithm by showing all the steps.

```
procedure NESTEDLOOPS(n)
    a ← 0
    for i ← 1 to n do
        for j ← 1 to i^2 do
            for k ← 1 to i do
                a ← a + 1
            end for
        end for
    end for
    return a
end procedure
```

Q3

List the following functions from the lowest to highest order. Explain your answers in detail, and show the comparisons between the functions clearly. Otherwise, you will not receive any points.

$$n \quad \log(n!) \quad n^{\cos(n)} \quad n^{\log(n)} \quad log(n)^n \quad 3^{log(n)}$$

Q4

Let $f(n) = f(\sqrt{n}) + 1$ and $f(1) = 0$. Prove or disprove that $f(n) \in \Theta(\log(n))$.

Q5

Let $f(x) : \mathbb{N} \to \mathbb{R}^+$ be a non-decreasing function. Show that the following statement is true (you may use diagrams):

$$\int_0^n f(x)\,dx \leq \sum_{k=1}^{n} f(k) \leq \int_1^{n+1} f(x)\,dx$$

Provide 3 examples for f, show that the inequality is satisfied for your examples.

Q6

Please write a modified insertion sort algorithm to reduce the total number of comparisons (the total number of other operations may increase). Explain how your approach reduces the total number of comparisons and discuss its drawbacks compared to the original insertion sort.