Q4

Let f(n) = f(√n) + 1 and f (1) = 0. Prove or disprove that f(n) ∈ Θ(log(n)). (f(2) = 1)

Disprove by counterexample

f(n) ∈ Θ(log(n)) if and only if there exist positive constants c_1,c_2, and n_0 such that c_1 * log(n) < f(n) < c_2 * log(n), whenever $n \geq n_0$

Take $n = 2^k$ such that $k = \log(n)$ and $\log(k) \in N$ and $2^k > n_0$

$$f(2^k) = f\left(2^{\frac{k}{2}}\right) + 1$$

$$f(2^k) = f\left(2^{\frac{k}{4}}\right) + 1 + 1$$

$$f(2^k) = f\left(2^{\frac{k}{8}}\right) + 1 + 1 + 1$$

$$*$$

$$*$$

$$*$$

We can divide k, log(k) times to get 1.

$$f(2^k) = f(2^1) + \log(k) * 1$$

Since f(2) = 1 also given

$$f(n) = 1 + \log(k)$$

Replace k with log(n)

$$f(n) = 1 + \log(\log(n))$$

Therefore $f(n) \in \mathrm{O}(\log(\log(n)))$ when $n = 2^k$

Since log(n) grows slower than n, log(log(n)) grows slower than log(n)

Therefore f(n) is bounded by a function grows slower than log(n)

Therefore there is no c_1 that can satisfy c_1 * log(n) < f(n)

Therefore we found a counterexample that doesn't satisfy f(n) ∈ Θ(log(n))

Therefore we disproved that f(n) ∈ Θ(log(n))