PS-3

a) $x(t) = [12 + 7\sin(\pi t - \pi/3)]\cos 13\pi t$

$$x(t) = \left[12 + \frac{7}{2j} e^{j(\pi t - \pi/3)} - \frac{7}{2j} e^{-j(\pi t - \pi/3)}\right]\left(\frac{1}{2} e^{j13\pi t} + \frac{1}{2} e^{-j13\pi t}\right)$$

Multiply it:

$$x(t) = 6 e^{j13\pi t} + 6 e^{-j13\pi t} + \frac{7}{4} e^{-j\pi/2} \cdot e^{-j\pi/3} \cdot e^{j14\pi t}$$

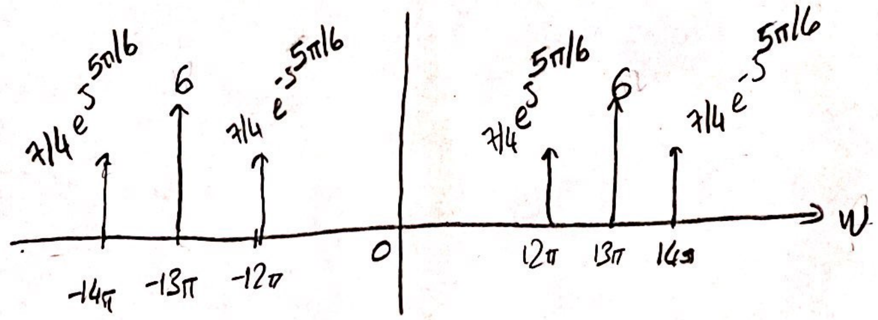
$$+ \frac{7}{4} \cdot e^{-j5\pi/6} \cdot e^{-j12\pi t} + \frac{7}{4} e^{j\pi/2} e^{j\pi/3} e^{j12\pi t} + \frac{7}{4} e^{j5\pi/6} \cdot e^{-j14\pi t}$$

$$x(t) = 12\cos(13\pi t) + \frac{7}{2}\cos(14\pi t - 5\pi/6) + \frac{7}{2}\cos(12\pi t + 5\pi/6)$$

$w_1 = 12\pi$ $\quad A_1 = 7/2 \quad \varphi_1 = 5\pi/6$

$w_2 = 13\pi$ rad/sec $\quad A_2 = 12 \quad \varphi_2 = 0$

$w_3 = 14\pi$ $\quad A_3 = 7/2 \quad \varphi_3 = -5\pi/6$



Q. 3.6

a) Use the phasors to show that $x(t)$ can be expressed in the form $x(t) = A_1 \cdot \cos(w_1 t + \varphi_1) + A_2 \cdot \cos(w_2 t + \varphi_2) + A_3 \cdot \cos(w_3 t + \varphi_3)$

where $w_1 < w_2 < w_3$

b) Sketch 2-sided spectrum of the signal on a freq. axis.

Fourier Analysis Eq.

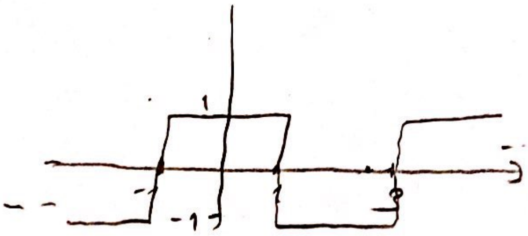
$$a_k = 1/T_0 \int_0^{T_0} x(t).e^{-j(2\pi/T_0).kt} .dt$$

Fourier Synthesis Equation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T_0)kt}$$

Last year Quiz:

$$x(t) = \begin{cases} 1 & -1 \le t \le 1 \\ -1 & 1 \le t \le 3 \end{cases}$$



$$a_0 = \frac{1}{4} \int_{-1}^{3} x(t).e^{-j(2\pi/4).k.t} = 0$$

why? There is no DC.

$$\boxed{w_0 = \pi/2}$$

$$a_k = \frac{1}{4}\left(\int_{-1}^{1} 1 e^{-jw_0kt} + \int_{1}^{3} (-1)\, e^{-jw_0kt}\right).dt$$

$$= \frac{1}{4}\left(\frac{-e^{-j\frac{\pi}{2}kt}}{+j\frac{\pi}{2}.k}\Bigg|_{-1}^{1} + \frac{e^{-j\frac{\pi}{2}.k.t}}{j\frac{\pi}{2}.k}\Bigg|_{1}^{3}\right)$$

$$= \frac{1}{4}\left(\frac{-e^{-j\frac{\pi}{2}k} + e^{j\frac{\pi}{2}k}}{j\frac{\pi}{2}k} + \frac{e^{-j\frac{3\pi}{2}k} - e^{-j\frac{\pi}{2}k}}{j\frac{\pi}{2}.k}\right)$$

$$e^{-j\frac{3\pi}{2}k} = \cos\left(-\frac{3\pi}{2}k\right) + j\sin\left(-\frac{3\pi}{2}k\right)$$

$$= \cos\left(\frac{\pi}{2}k\right) + j\sin\left(\frac{\pi}{2}k\right)$$

$$= e^{j\frac{\pi}{2}k}$$

$$= \frac{1}{j2\pi k}\left(\frac{2.e^{j\frac{\pi}{2}k} - 2e^{-j\frac{\pi}{2}k}}{1}\right) = \frac{1}{j\pi k}\left(e^{\frac{j\pi k}{2}} - e^{\frac{-j\pi k}{2}}\right)$$

$$a_1 = \frac{1}{j\pi}\left(e^{j\frac{\pi}{2}} - e^{-j\frac{\pi}{2}}\right) = \frac{2}{\pi}$$

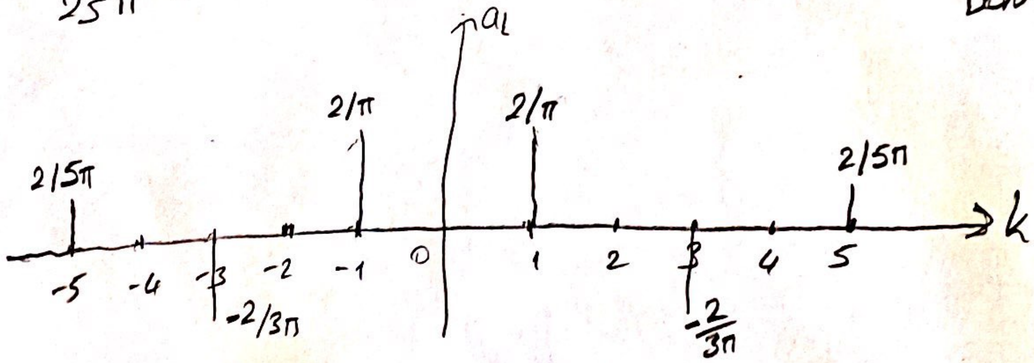
$$a_5 = \frac{2}{5\pi}$$

$$a_3 = \frac{1}{3j\pi}\left(e^{j\frac{3\pi}{2}} - e^{-j\frac{3\pi}{2}}\right) = -\frac{2}{3\pi}$$

$$a_k = \begin{cases} 0, & k \text{ is even} \\ \frac{2}{k\pi}, & k = \pm 1, \pm 5, \pm 9 \\ \frac{-2}{k\pi}, & k = \pm 3, \pm 7 \end{cases}$$

$$a_2 = \frac{1}{2j\pi}\left(e^{j\pi} - e^{-j\pi}\right) = 0$$

DON'T FORGET



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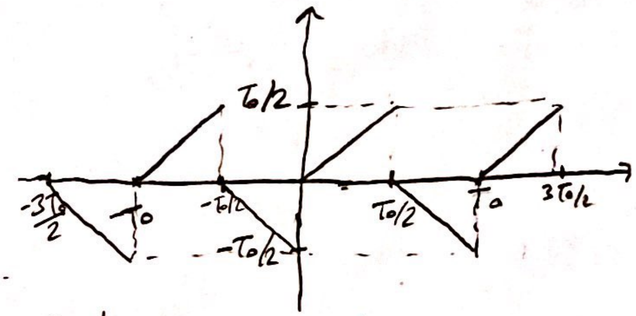
Q 3.10

A periodic waveform has the half-wave symmetry iff the last half of the period is the negative of the first half. More precisely, $x(t+T_0/2) = -x(t) \quad -\infty < t < \infty$

a) Suppose $x(t)$ is a periodic signal with half-wave symmetry and is defined over half a period by

$x(t) = t$ for $0 \le t < T_0/2$ where T_0 is period

Plot this signal.



b) Prove that DC coefficient a_0 is 0 for any half-wave symmetric signal.

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)\,dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} x(t)\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)\,dt + \frac{1}{T_0}\int_0^{T_0/2} \underbrace{x(u+T_0/2)}_{=-x(u)}\,du$$

Change of variables $u = t - T_0/2$

$\longrightarrow$ Half-wave symmetry

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)\,dt - \frac{1}{T_0}\int_0^{T_0/2} x(u)\,du = \underline{\underline{0}}$$

c) Prove that all even indexed Fourier series coeff. are 0 for a half-wave symmetric signal

If k is even, then $k=2l$ where $l \in \mathbb{Z}$

$$a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t).e^{-j(2\pi/T_0)2lt}.dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} x(t).e^{-j(2\pi/T_0)2lt}.dt$$

Change of variables $(u = t - T_0/2)$

$$a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t).e^{-j(2\pi/T_0)2lt}.dt + \frac{1}{T_0}\int_0^{T_0/2} \underbrace{x(u+T_0/2)}_{-x(u)}.\underbrace{e^{-j(2\pi/T_0)2l(u+T_0/2)}}_{A}.du$$

$$A = e^{-j(2\pi/T_0).2l.u}.e^{-j(2\pi/T_0).2.l(T_0/2)} = e^{-j(2\pi/T_0).2.l.u}.\underbrace{e^{-j2\pi l}}_{1}$$

$$\Rightarrow a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t).e^{-j(2\pi/T_0)2l.t}.dt - \frac{1}{T_0}\int_0^{T_0/2} x(u).e^{-j(2\pi/T_0).2lu}$$

$\Rightarrow a_k = 0$ for k even.

P. 3-11

A signal $x(t)$ is periodic with period $T_0 = 8$. Therefore, it can be represented as Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{j(2\pi/8)kt}$$

It is known that Fourier series coeff. for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{8}(4+t)e^{-j(2\pi/8)kt}.dt$$

a) In the integral expression for a_k above, the integrand and the limits define the signal $x(t)$. Determine an equation $x(t)$ that is valid for one period.

b) Using the result from part (a), draw a plot of $x(t)$ over the range $-8 \le t \le 8$ seconds.

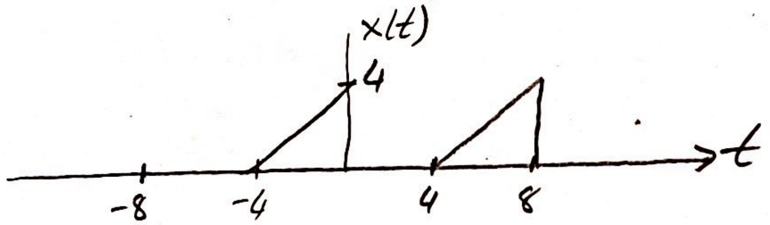
c) Determine the DC value of $x(t)$.

$$a_k = \frac{1}{8} \int_{-4}^{0} (4+t) \cdot e^{-j(2\pi/8)kt} \cdot dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} x(t) \cdot e^{-j(2\pi/T) \cdot kt} \cdot dt$$

a) $T = 8$ sec. $\quad x(t) = \begin{cases} (4+t) & \text{for } -4 \le t \le 0 \\ 0 & \text{for } 0 < t < 4 \end{cases}$

b)



Another possibility

$$x(t) = \begin{cases} (4+t) & \text{for } -4 \le t \le 0 \\ 0 & \text{for } -6 \le t \le -4 \\ 0 & \text{for } 0 \le t \le 2 \end{cases}$$

c) $a_0 = \frac{1}{8} \int_{-4}^{0} (t+4) \cdot dt = 1/8 \left(\frac{t^2}{2} + 4t\right) \Big|_{-4}^{0}$

$$= 0 - \frac{1}{8} \left(\frac{16}{2} - 16\right) = 1$$

or $\quad a_0 = \frac{1}{8} \cdot$ Area in one period $= 1/8 \cdot \left(\frac{1}{2} \cdot 4 \cdot 4\right) = 1$

3.14

We know that $x(t)$ can be represented by its Fourier series (3.19). You will learn that we can transform many time-domain operations to Fourier domain. E.g $y(t) = \frac{dx(t)}{dt}$

$$y(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[\sum_{k=-\infty}^{\infty} a_k \cdot e^{jk\omega_0 t}\right]$$

$$= \sum_{k=-\infty}^{\infty} a_k \cdot \frac{d}{dt}\left[e^{jk\omega_0 t}\right] = \sum_{k=-\infty}^{\infty} a_k \cdot \left[(jk\omega_0) e^{jk\omega_0 t}\right]$$

Thus, $y(t) = \sum_{k=-\infty}^{\infty} b_k \, e^{jk\omega_0 t}$, where $b_k = (jk\omega_0) \cdot a_k$

Taking derivative equals to multiplication with $jk\omega_0$ in Fourier domain

Similarly a) $y(t) = A \cdot x(t)$ b) $y(t) = x(t - t_d)$

a) $y(t) = A \cdot \sum_{k=-\infty}^{\infty} a_k \cdot e^{jk\omega_0 t} = \sum_k (A \cdot a_k) \cdot e^{jk\omega_0 t} \Rightarrow b_k = A a_k$.

b) $y(t) = x(t - t_d)$

$$= \sum_k a_k \, e^{j\omega_0 k(t-t_d)} = \sum_k \left(a_k e^{-j\omega_0 k t_d}\right) e^{j\omega_0 k t}$$

$$\Rightarrow b_k = a_k e^{-jk\omega_0 t_d}$$