Problem Session 5

5.1

$$h[n] = \delta[n-1] - 2\delta[n-4]$$

$$b_k = [0, 1, 0, 0, -2]$$

General formula of FIR filter.

$$y[n] = \sum_{k=0}^{M} b_k \cdot x[n-k]$$

$$y[n] = x[n-1] - 2x[n-4]$$

5.3

$$y[n] = \underbrace{2}_{b_0} x[n] - \underbrace{3}_{b_1} x[n-1] + \underbrace{2}_{b_2} x[n-2]$$

$$h[n] = \sum_{k=0}^{M} b_k \, \delta[n-k]$$

$$h[n] = 2\delta[n] - 3\delta[n-1] + 2\delta[n-2]$$

5.9

a) $y[n] = x[n] \cdot \cos(0.2\pi n)$

c) $y[n] = |x[n]|$

- Linearity:

Where $x[n] \rightarrow y[n]$, $\underbrace{\alpha x_1[n] + \beta x_2[n]}_{x[n]} \rightarrow \underbrace{\alpha y_1[n] + \beta y_2[n]}_{y[n]}$

a)

$$y[n] = (\alpha_1 x_1[n] + \alpha_2 x_2[n]) \cos(0.2\pi n)$$

$$y[n] = \alpha_1 x_1[n] \cdot \cos(0.2\pi n) \rightarrow y_1[n]$$
$$+ \alpha_2 \cdot x_2[n] \cdot \cos(0.2\pi n) \rightarrow y_2[n]$$

Yes, linear

c)

$$y[n] = |\alpha_1 x_1[n] + \alpha_2 x_2[n]|$$
$$\neq |\alpha_1 x_1[n]| + |\alpha_2 x_2[n]|$$

if $x_1[n]$ and $x_2[n]$ have different signs. No, not linear

Time Invariance:

where $x[n] \rightarrow y[n]$, $x[n-n_0] \rightarrow y[n-n_0]$

a)

$y[n-n_0] \neq x[n-n_0] . \cos(0.2\pi n)$

$y[n-n_0] = x[n-n_0] . \cos(0.2\pi(n-n_0))$

No, not time inveriant

b) $y[n-n_0] = |x[n-n_0]|$

Yes, time mverient.

Causolity :
If the filter uses only past and present values of input, then it is casual.

a) $y[n] = x[n] . \cos(0.2\pi . n]$ and $y[n] = |x[n]|$

depend on only presest values, they are casual.

b) $y[n] = x[n] - x[n-1]$, present and past still casud.

$y[n] = x[n] - x[n+1]$, not casual.

5.10

b) $x_1[n] = \delta[n] - \delta[n-1]$, we know $y_1[n] = \delta$

$X_2[n] = 7\delta[n] - 7\delta[n-2]$, we should write $X_2[n]$ in terms of $X_1[n]$ to use linearity and time-invariance.

$X_2[n] = 7X_1[n] + 7X_1[n-1] = 7\delta[n] - 7\delta[n-1] + 7\delta[n-1] - 7\delta[n-2]$

$Y_2[n] = 7Y_1[n] + 7Y_1[n-1]$ by LTI.

$7Y_1[n] = 7\delta[n] - 7\delta[n-1] + 14\delta[n-3]$

$7Y_1[n-1] = 7\delta[n-1] - 7\delta[n-2] + 14\delta[n-4]$

$+$

$Y_2[n] = 7\delta[n] - 7\delta[n-2] + 14\delta[n-3] + 14\delta[n-4]$

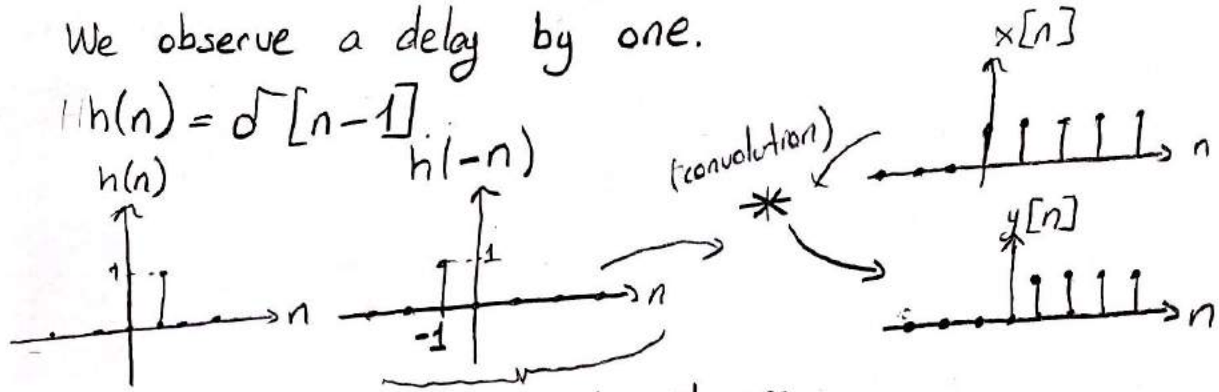
5.13

Deconvolution:

a) $x[n] = u[n]$ and $y[n] = u[n-1]$

We observe a delay by one.

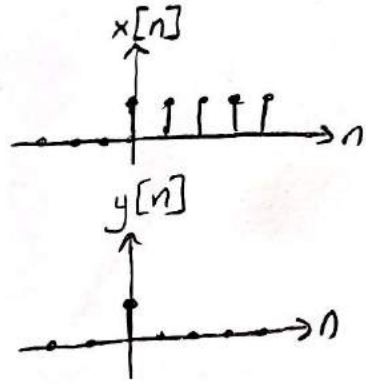
$h(n) = \delta[n-1]$



shift this from $-\infty$ to $+\infty$

on the graph of $x[n]$

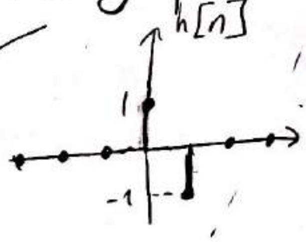
b) $x[n] = u[n]$ and $y[n] = \delta[n]$



So our filter should detect the change from 0 to 1 in $x[n]$ graph.

First difference filter ✓

$h[n] = \delta[n] - \delta[n-1]$



c) $x[n] = \left(\frac{1}{2}\right)^n . u[n]$ and $y[n] = \delta[n-1]$

Graphically you can find $h[n] = \delta[n-1] - \frac{1}{2}\delta[n-2]$

like we did above, let's try mathematical approach.

Convolution sum: $y[n] = \sum_{k=0}^{M} h[k] . x[n-k]$

$y[0] = 0 = \boxed{h[0]} . \underbrace{x[0]}_{1} + h[1] . \cancel{x[-1]} + h[2] . \cancel{x[-2]} \ldots$ $x[n]=0$, when $n<0$

$\Downarrow$ 0

$y[1] = 1 = \underbrace{h[0]}_{0} . \underbrace{x[1]}_{1/2} + \boxed{h[1]} . \underbrace{x[0]}_{1} + \cancel{h[2] . x[-1]} \to 0$

$\Downarrow$ 1

$y[2] = 0 = \underbrace{h[0]}_{0} . \underbrace{x[2]}_{1} + \underbrace{h[1]}_{1} . \underbrace{x[1]}_{1/2} + \boxed{h[2]} . \underbrace{x[0]}_{1} + h[3] . \cancel{x[-1]} \to 0$

$\Downarrow$ $-1/2$

$$y[3] = 0 = \underbrace{h[0]}_{0} \cdot \underbrace{x[3]}_{1/8} + \underbrace{h[1]}_{1} \cdot \underbrace{x[2]}_{1/4} + \underbrace{h[2]}_{-1/2} \cdot \underbrace{x[1]}_{1/2} + \underbrace{h[3]}_{0} \cdot \underbrace{x[0]}_{1}$$

$$h[4],\ h[5],\ \ldots = 0$$

So; $h[n] = \delta[n-1] - \frac{1}{2}\delta[n-2]$