BLG 202E

Ahmet Furkan Kavraz 150190024

Q3)

Q3a) $f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0) + \frac{h^3}{6}f'''(\xi_1)$

Multiply with -1 / And Sum the equations

$f(x_0-h) = f(x_0) - hf'(x_0) + \frac{h^2}{2}f''(x_0) - \frac{h^3}{6}f'''(\xi_2)$

$\hookrightarrow f(x_0+h) - f(x_0-h) = 2hf'(x_0) + \frac{h^3}{3}f'''(\xi_3)$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} - \frac{h^2}{6}f'''(\xi_3),$$

$x_0-h \leq \xi_3 \leq x_0+h$

Q3b)

The step size is h. The error formula for three point derivation is $e(h) = \frac{h^2}{6}f'''(\xi)$, which means if we reduced h value to half, the error will reduce quarter of previous error.

Q3c) ... the sum of Randoff error and Data error

Q3d) When we have unequally spaced data, we can not find numerical differentiation with using Taylor series. Alternative method is Deriving the Lagrange Polynomial of the function.

BLG 202E

Ahmet Furkan Kerroz 150190024

Q4a)

$$\int_1^5 f(x)\,dx = \int_1^5 (x^2+4x)\,dx$$

$$\frac{x^3}{3} + 2x^2 \Big|_1^5 = \frac{125}{3} + 50 - \left(\frac{1}{3}+2\right) =$$

$$= \frac{124}{3} + 48 \approx 89.333$$

Q4b) In Simpson Method, we need three point: $f(x)=x^2+4x$

$f(1) = 5$, $f(3) = 9+12 = 21$, $f(5) = 25+20 = 45$

Simpson formula: $\frac{b-a}{6}\left[f(a) + 4f\left(\frac{b+a}{2}\right) + f(b)\right]$

$$= \frac{5-1}{6}\left[5 + \underbrace{4.21}_{84} + 45\right] = \underbrace{\frac{2}{3}.134}_{89} = 89.333$$

The result of exact integral and result of Simpson method are same. Error is %0.

The error formula is $\Rightarrow \left| \frac{\text{Reel value} - \text{Prediction}}{\text{Reel value}} \right| .100$

Numerator of error will be 0. The error will be 0.

BLG 202E

Ahmet Furkan Kavraz 150190024

Q4c)

In Trapezoidal Method, we need two point: $f(x)=x^2+4x$

$f(1)=5$, $f(5)=45$

Trapezoidal Formula:

$$= \frac{b-a}{2}\left[f(a)+f(b)\right]$$

$$= \frac{5-1}{2}\left[5+45\right] = 2.50 = 100$$

The result of Trapezoidal Method = 100

The result of exact integral = 89.333

The error of the Trapezoidal Rule $= 100\left|\frac{100-89.333}{89.333}\right| = 100.\left|\frac{10.667}{89.333}\right| = 0.1194 \cdot 100$

$\Rightarrow \%\ 11.94$