BLG202E –Final Exam Part 2

Spring 2021, Duration: 35 minutes

Instructions:

- *Do NOT communicate with other people, including your friends, classmates, and family members!*
- *This is an open-book exam.*
- *Give your answers in English.*
- *Use an A4 paper for each question.*
- *Write the question number, your Name and İTÜ ID on the top of each page and **sign all pages**.*
- *Scan or take photo of your answers and upload them on Ninova within a pdf file **before the deadline**!*
- *There will be no extension for time without penalty. There will be a late submission option for 15 mins where you will lose 3 points for each late minute. Your latest submission will be considered only.*
- *Use last 10 minutes for submission.*

Q 3) **(25 pts**) Let *f(x)* be a given function that can be evaluated at points $x_0 \pm h$ and $x_0 \pm 2h$, for any fixed value of *h*, *0 < h*.

a) **(5 pts)** Derive the three points formula for $f'(x_0)$ using Taylor series! *(Hint: Taylor series* $f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2} f''(x_0) + \cdots + \frac{h^k}{k!} f^{(k)}(x_0) + \cdots$*)*

b) **(5 pts)** What is the relationship between step size, *h*, and truncation error?

c) **(5 pts)** If we apply the derived numerical derivative approximation algorithm in a computer system the total error is the sum of……error and ……error.

d) **(10 pts)** In some cases, Taylor series may become difficult to apply to derive numerical differentiation formulas, why? Which alternative methods can be used to compute the derivative of a function?

Q 4) **(25 pts**) The following table summarizes the basic quadrature rules and corresponding errors.

Method	Formula	Error
Midpoint	$(b-a)f(\frac{a+b}{2})$	$\frac{f''(\xi_1)}{24}(b-a)^3$
Trapezoidal	$\frac{b-a}{2}[f(a)+f(b)]$	$-\frac{f''(\xi_2)}{12}(b-a)^3$
Simpson	$\frac{b-a}{6}[f(a)+4f(\frac{b+a}{2})+f(b)]$	$-\frac{f''''(\xi_3)}{90}(\frac{b-a}{2})^5$

Use function $f(x) = (x^2 + 4x)$ for the following problems:

(a) **(5 pts)** Compute exact value of $\int_1^5 f(x)dx$.

(b) **(10 pts)** Approximate $\int_1^5 f(x)dx$ using Simpson quadrature rule. Find the absolute relative error.

(c) **(10 pts)** Approximate $\int_1^5 f(x)dx$ using Trapezoidal quadrature rule. Find the absolute relative error.