BLG 202E - Assignment 1

Student Name: Ahmet Furkan Karaz Student Number : 150190024

Q3)

Given theorems: $fl(x_1+y_1) = (x_1+y_1)(1+\epsilon)$, $|\epsilon| \leq \eta$

$fl(\ln z) = (\ln z)(1+\epsilon)$, $|\epsilon| \leq \eta$

→ From first equation, let $x_1+y_1 = x^y$:

① $fl(x^y) = x^y(1+\epsilon)$, $|\epsilon| \leq \eta$

→ The question gives us an equation:

$$x^y = e^{y \ln(x)}$$

$$fl(x^y) = fl(e^{y \ln x})$$

→ The equation says us that the error of $fl(x^y)$ comes from $\ln(x)$ and the others are exact. So we replace $fl(e^{y \ln x})$ with $e^{y \cdot \ln(x)(1+\epsilon)}$, $|\epsilon| \leq \eta$

$$fl(x^y) = fl(e^{y \ln x}) = e^{y \cdot \ln(x)(1+\epsilon)}, \quad |\epsilon| \leq \eta$$

→ Just simplify the equation:

$fl(x^y) = e^{y \cdot \ln(x)(1+\epsilon)}$, $|\epsilon| \leq \eta$

Since $x^y = e^{y \cdot \ln(x)}$

$fl(x^y) = (x^y)^{(1+\epsilon)}$, $|\epsilon| \leq \eta$

② $fl(x^y) = x^y \cdot (x^y)^{\epsilon}$, $|\epsilon| \leq \eta$

In upper part, we marked an equation as ① and now, we have also ② equation.

① $fl(x^y) = x^y(1+\epsilon)$, $|\epsilon| \leq \eta$

We know that this equation is always true.

② $fl(x^y) = x^y \cdot (x^y)^{\epsilon}$, $|\epsilon| \leq \eta$

this equation is always true?

As a result, $|1+\epsilon| \geq |(x^y)^{\epsilon}|$ must satisfy, because we know ① is always true. Then, the error of second equation should be smaller than the error of first equation.

This property does not hold for large "x^y"s. Because $|(x^y)^{\epsilon}|$ can be greater than $|1+\epsilon|$ for big values of "x^y".