BLG 202E - Assignment 1

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Q2)

$$\underbrace{\begin{bmatrix} a & b \\ b & a \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} \qquad \det(A) = a^2 - b^2 \qquad x = \frac{\begin{vmatrix} 1 & b \\ 0 & a \end{vmatrix}}{a^2-b^2} = \frac{a}{a^2-b^2}$$

$$y = \frac{\begin{vmatrix} a & 1 \\ b & 0 \end{vmatrix}}{a^2-b^2} = \frac{-b}{a^2-b^2}$$

a) The numerical problem of this linear system is calculating det(A). Since $a \approx b$, det(A) is very close to zero. The "x" and "y" values depend on det(A). When det(A) changes small, "x" and "y" changes so much.

b) As we calculate x and y values using Cramer's Rule, we can use them $z = x + y = \frac{a}{a^2-b^2} + \frac{-b}{a^2-b^2} = \frac{a-b}{(a-b)(a+b)} = \frac{1}{a+b}$

c) As we said in part-a, since $a \approx b$ solving this system has numerical difficulty and the solutions are not stable. When we change "a" or "b" small, the "x" and "y" values change so much since their formulas depend det(A) and det(A) is very close to zero. Also, computing "x+y" is not ill-conditioned. Because, x+y is not depend det(A) or another thing that causes unstability. The formula of $x+y = \frac{1}{a+b}$ is just a normal formula.

As a result, statement is TRUE.

Example:

For $a = 100.01$, $b = 100$, $x = 50.0025$, $y = -49.9975$, $x+y = 0.005$

For $a = 100.02$, $b = 100$, $x = 25.0025$, $y = -24.9975$, $x+y = 0.005$

Ill conditioned: Small perturbation in data causes large difference in result.

Remains nearly same. not Ill conditioned.