BLG 202E – Midterm 1

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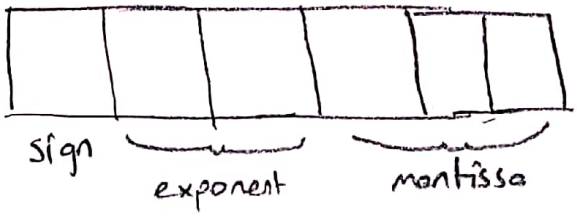
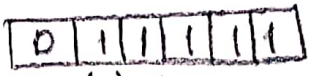
Q-3



a) smallest and largest positive numbers

for largest 

$(-1)^0 \cdot (1,111)_2 \cdot 2^{(11)_2 - 1} = 1 \cdot (1+0,5+0,25+0,125) \cdot 2^2$

$= 1,875 \cdot 4 = 7,5 \leftarrow$ largest positive number

for smallest 

$(-1)^0 \cdot (1,001)_2 \cdot 2^{0-1} = 1 \cdot (1+0,125) \cdot 2^{-1}$

$= 0.0625 \leftarrow$ smallest positive number

b) $(13.25)_{10} \leftarrow$ We can not represent it.

Because the number is too big from 7.5 which is the largest positive number. Or we can represent it as 7.5.

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Q-4)

$f(x) = x^2 - 2x - 3 = (x-3)(x+1)$ roots = 3, -1

a) Fastest ⟶ Slowest

Newton, Secant, Bisection

14,0625

9,765

25-10-3=12

625

b)

Iteration	x_l	x_r	x_u	$f(x_l)$	$f(x_r)$	$f(x_u)$	Sign $f(x_l) \cdot f(x_r)$
1	0	2.5	5	-3	-1	12	+
2	2.5	3.75	5	-1	3,5625	12	-
3	2.5	3.125	3.75	-1	0,5156	3.5625	-

↳ result of iteration 3.125

real root = 3

$$\epsilon = \left| \frac{3.125 - 3}{3} \right| = \left| \frac{0.125}{3} \right| = 0.0416$$

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Q-4

c) Newton's Method: $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

$f(x) = x^2 - 2x - 3$ $\quad x_0 = 7$

$f'(x) = 2x - 2$

$$x_{k+1} = x_k - \frac{x_k^2 - 2x_k - 3}{2x_k - 2} =$$

$$x_1 = 7 - \frac{49 - 14 - 3}{14 - 2} = 7 - \frac{32}{12} = \frac{84 - 32}{12} = \frac{52}{12} = \frac{13}{3}$$

$$x_2 = \frac{13}{3} - \frac{\frac{169}{9} - \frac{26}{3} - 3}{\frac{26}{3} - 2} = \frac{13}{3} - \frac{169 - 78 - 9}{3(26 - 6)} = \frac{13 - 4,1}{3} \approx 2,97$$

$$x_3 = 2,97 - \frac{(2,97)^2 - 5,94 - 3}{5,94 - 2} = 2,97 - \frac{8,82 - 5,94 - 3}{3,94}$$

$x_4 = 2,97 + 0,0304 = 3.0004$

$$\epsilon = \left| \frac{3 - 3.0004}{3} \right| = \left| \frac{0.0004}{3} \right| \approx 0,00013$$

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d)

$f(x) = x^2 - 2x - 3$

$x_0 = 7 \quad x_1 = 6$

$$x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$$

$36 - 12 - 3 = 21$ $\quad 36 - 15 = 21$ $\quad 49 - 17 = 32$

First

$$x_2 = 6 - \frac{f(6)(6-7)}{f(6) - f(7)} = 6 + \frac{21 \cdot (+1)}{21 - 32}$$

$$= 6 + \frac{-7}{+3} = \frac{-11}{3} = 3.666$$

Secant $\quad x_1 = 6 \quad x_2 = 3.66$ $\quad f(x_1) = 21 \quad f(x_2) = 3.07$

$$x_3 = 3.66 - \frac{f(3.66)(3.66-6)}{f(3.66) - f(6)} = 3.66 + \frac{4.32\ (2.34)}{4.32 - 21}$$

$$= = 3.66 + \frac{10.108}{-16.68} = 3.055$$

$x_2 = 3.66 \quad x_3 = 3.055 \quad f(x_2) = 4.32 \quad f(x_3) = 0.22$

$$x_4 = 3.055 - \frac{0.22 \cdot (3.055 - 3.66)}{0.22 - 4.32}$$

$$= 3.055 + \frac{(0,22) \cdot (+0,605)}{-4,1} = 3.055 - 0.032 = 3.025$$

$$\epsilon = \left| \frac{3 - 3.025}{3} \right| = \left| \frac{-0.025}{3} \right| = 0.0083$$

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Q-4/

e)

Relative Errors

$\epsilon_b = 0,0416$

$\epsilon_N = 0.00013$ $\epsilon_N < \epsilon_S < \epsilon_b$

$\epsilon_S = 0,0083$