BLG 202E – Assignment 2

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Q-2

a) Permutation matrix is a matrix when we multiply the A and permutation matrix, it provides a matrix that same rows with A but the order is different. With the help of permutation matrix, we can create row echelon form. We should change 3rd and 4th rows with each other. Permutation matrix is:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$PA = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{B}$$

→ LU Decomposition provides lower and upper triangular matrices, product of them equals to matrix B.

$$B = LU$$

→ Since B matrix is already upper triangular matrix; L equals to Identity matrix:

$$U = B \qquad L = I$$

$$U = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{U}$$

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b) We should solve $Ax = b$. However, finding inverse of A is difficult for us. We should use LU Decomposition since U matrix has row echelon form and L matrix is identity matrix for PA matrix. (from the part-a)

$$Ax = b$$

$$PAx = Pb$$

→ We know LU Decomposition of PA:

$$PA = LU$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{U}$$

$$LUx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 26 \\ 9 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 26 \\ 9 \\ -3 \\ 1 \end{bmatrix}$$

If $L = I$, we can eliminate it:

$$Ux = \begin{bmatrix} 26 \\ 9 \\ -3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 26 \\ 9 \\ -3 \\ 1 \end{bmatrix}$$

Backward Substitution:

$$x_4 = 1$$

$$-x_3 - 2x_4 = -3 \qquad x_3 = 3 - 2x_4 \quad x_3 = 1$$

$$4x_2 + 3x_3 + 2x_4 = 9 \qquad x_2 = \frac{9 - 2x_4 - 3x_3}{4} = 1$$

$$5x_1 + 6x_2 + 7x_3 + 8x_4 = 26 \qquad x_1 = \frac{26 - 6x_2 - 7x_3 + 8x_4}{5} = 1$$

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$