CENG222 HOMEWORK 1

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Experiment 1:

1)

Figure 1: ar_A which represents the outcomes of the dice A (6-faced). The histogram shows the results of ar_A. The die is fair, which means that every result between 1 and is equally likely. Histogram shows the uniform distribution.

Figure 2: ar_B, which represents the outcomes of the dice B (4-faced). Histogram shows the uniform distribution of the dice B.

Figure 3: ar_C, which represents the outcomes of the coin (heads or tails). Histogram shows the uniform distribution of the coin. 2 outcomes equals to 0.5.

Figure 4: ar_X, defined in the question (X = A+(B*C)). Histogram shows the distribution of the random variable X which depends on the outcomes of the dices and the coin. Values of the X between -3 to 10.

For -3; C = tails which assigned to -1. B = 4 on the dice and A = 1 on the dice => 1+(-1 * 4) = 3

For 10; C = heads which assigned to 1. B = 4 on the dice and A = 6 on the dice => 6+(1*4) = 10

In the histogram outcomes are mostly around 2 to 5.

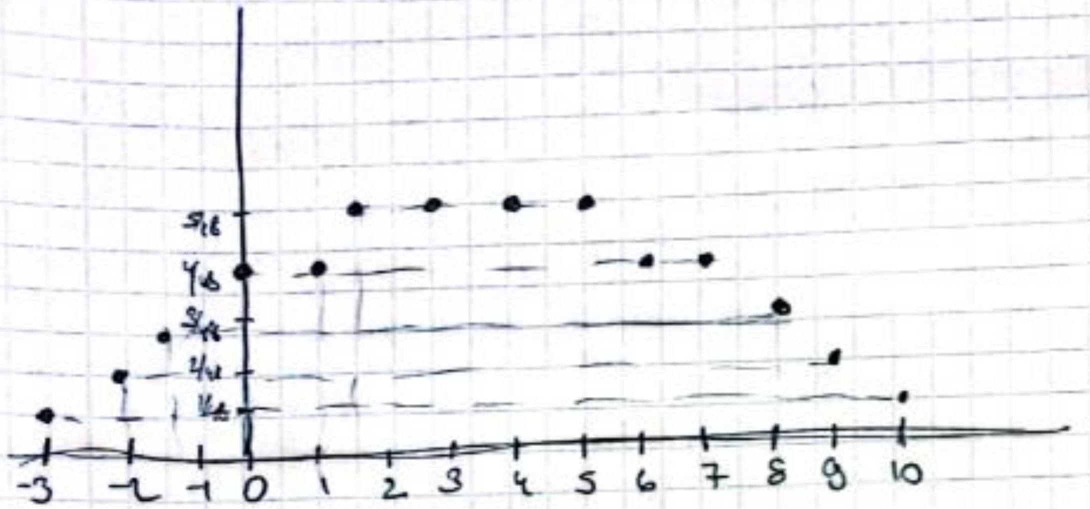
2)

A = {1,2,3,4,5,6}   B = {1,2,3,4}   C = {-1,1}   and   X = A + (B * C)

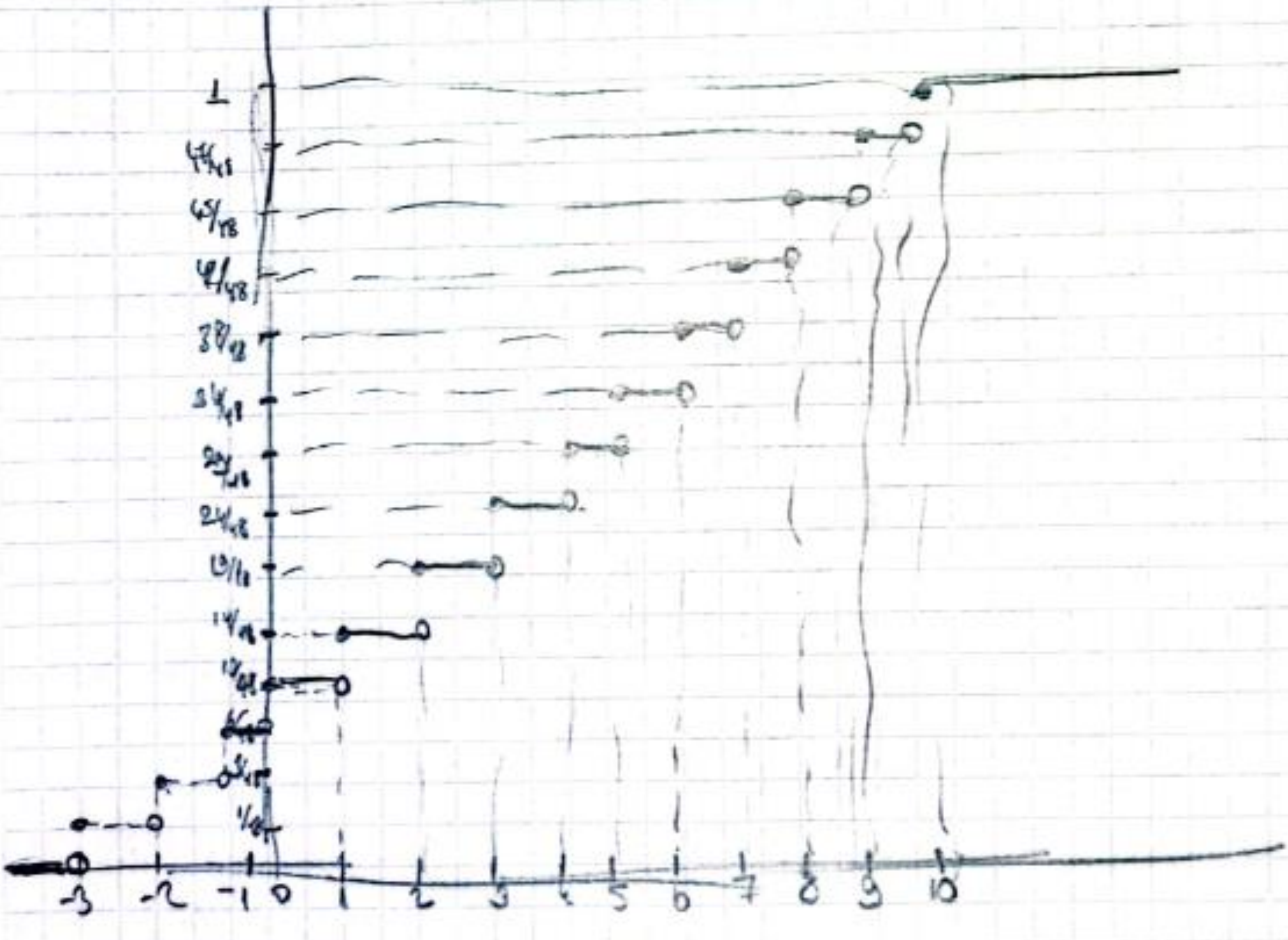
B*C **A**	**1**	**2**	**3**	**4**	**-1**	**-2**	**-3**	**-4**
1	2	3	4	5	0	-1	-2	-3
2	3	4	5	6	1	0	-1	-2
3	4	5	6	7	2	1	0	-1
4	5	6	7	8	3	2	1	0
5	6	7	8	9	4	3	2	1
6	7	8	9	10	5	4	3	2

X	**P(X)**
-3	1/48
-2	1/24
-1	1/16
0	1/12
1	1/12
2	5/48
3	5/48
4	5/48
5	5/48
6	1/12
7	1/12
8	1/16
9	1/24
10	1/48

PROBABILITY MASS FUNCTION (PMF OF X)



CUMULATIVE DISTRIBUTION FANCTION (CDF OF X)



3)

$$E(X) = \sum_{x}^{n} P(X) = X_1.P(x_1) + X_2.P(x_2) + \cdots + X_n + P(x_n)$$

$E(X) = \frac{-3-4-3+0+4+10+15+20+25+24+28+24+18+10}{48}$ = **3.5** and the Var (X) = E(X^2) – ((E(X)) ^2

$E(X^2) = \frac{9+8+3+0+4+20+45+80+125+144+196+192+162+100}{48}$ = **22.6**

Var(X) = 22.6 – 12.25= ***10.4***

4)

Our results in the python code which represents the outputs are very close to each other. In the histograms, we take the iteration number as 30000, but larger input the results are going to be closer to each other. Same way for low input like 100-1000 there is no more equality for the random variables. Also for me, theoretically for very large iteration histograms is equal.

<u>Experiment 2:</u>

a-1)

Figure 10: Shows the change between U to X. Every dot on X_a and U shows the lines passes. ((U(i), 1.2 to (X(i), 1)).

Figure 11: This figure shows the distribution histograms of X_a and U. U generated randomly which means random distribution shows. X_a is in the shape of its own distribution.

Figure 12: This figure is the cdf's (cumulative distribution function) of the values (X_a and U).

a-2)

$$\boldsymbol{(CDF)F(X) = x^2}$$
$$\boldsymbol{(PDF)\ f(x) = F'(x) = 2x}$$
$$\boldsymbol{E(X) = \int_{-30}^{\infty} x.f(x)dx = \int_{0}^{1} x.2xdx = \frac{2x^3}{3} = \frac{2}{3} = 0.666}$$
$$\boldsymbol{E(X^2) = \int_{-\infty}^{\infty} x^2.f(x)dx = \int_{0}^{1} x^2.2xdx = \frac{2x^4}{4} = \frac{1}{2} = 0.5}$$
$$\boldsymbol{Var(x) = \frac{1}{2} - \frac{4}{9} = \frac{1}{18} = 0.055}$$

a-3)

Calculations are matched with the python code data. 0.67 and 0.56, values which the figure 13 and figure 14 converges.

b-1)

Figure 15: Shows the distribution of random variable X_b.

Figure 16: Shows the cdf (cumulative distribution function) X_b. It is same as X_a's.

b-2)

The found and the output very similar. Also, they are converging to same E(X) and Var(X).