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Q1

a) Does the min-weight edge of G have to be on MST of G?

Solution a)

Yes, it have to be on MST. Contradiction;

① Let's say minimum weight edge e_{min} is not in the MST.

② And the T doesn't include the e_{min}.

③ Add e_{min} to T, since T is a spanning tree, adding e_{min} creates a cycle.

④ Remove e' which is a edge from the spanning tree. Cycle will break and T' is the new spanning tree.

⑤ Since e_{min} is the minimum-weight edge in G, $weight_{e_{min}} < weight_{e'}$.

⑥ Contradiction: The weight of T' is less than the weight of T, therefore e_{min} must be included in the MST of G.

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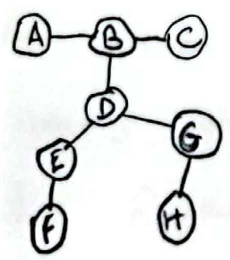
b) Can the max-weight edge of G belong to MST?

Solution b)

Let' assume a graph;



And think about $edge_{BC}$. If this edge is the maximum of spanning tree it can belong the MST.

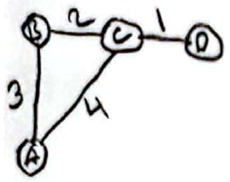
Because there is a just one path to reach. It is not form a cycle, and no other smaller weight edges can substitute it to keep the graph connected.

Yes it can belong to the MST.

c) Adding the same positive value to every edge of G can change MST or not?

Solution c)

No, Let' assume a graph:



Original edge weights;

$w(C,D) = 1$

$w(B,C) = 2$

$w(A,B) = 3$

$w(A,C) = 4$

MST is (A,B), (B,C), (C,D).

and the c constant = 2,

New edge weights;

$w'(C,D) = 1+2 = 3$

$w'(B,C) = 2+2 = 4$

$w'(A,B) = 3+2 = 5$

$w'(A,C) = 4+2 = 6$

New MST is (A,B), (B,C), (C,D) again.

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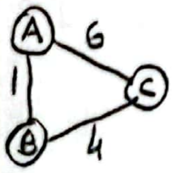
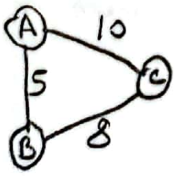
d) Adding the same positive value to every edge of G can change shortest path between two vertices or not?

Solution d)

Yes, Let's assume a graph:



For reaching A to C : $A \xrightarrow{6} C$ and $A \xrightarrow{1} B \xrightarrow{4} C$ is 5. If we add for example 4 to each edge, A-B-C path more increases



$A \xrightarrow{5} B \xrightarrow{8} C$ is 13, but now $A \xrightarrow{10} C$ direct is 10.

The shortest path is changed.

Q2

a) Define the density of a rod of length i to be p_i / i, that is, value per inch.
A greedy strategy for cutting a rod of length n cuts first the piece with the highest density.

Solution a)

length i	price p_i	density $\frac{p_i}{i}$
1	1	1
2	4	2
3	8	2.67
4	9	2.25
5	10	2
6	17	2.83
7	18	2.57
8	22	2.75
9	22	2.44
10	30	3

Let's sort highest to the lowest

$\Rightarrow L_{10} > L_6 > L_8 > L_3 > L_7 > L_9 > L_4 > L_2 > L_5 > L_1$ //

The greed strategy based on the highest density.
It can effective;
For example length of rod 5;
Greedy choices $L_3 + L_2 = 8 + 4 = 12$ (price)

b) Show by counterexample (at least for one n) that this greedy strategy does not always determine an optimal way to cut rods.

Solution b)

But, greedy may not always provide optimal solution.
For example the length of rod 8;
Greedy choices 6 an 2 $\rightarrow L_6 + L_2 \rightarrow 17 + 4 = 21$
But if you select 8 instead of cut, the price (L_8) will be 22.

$\Rightarrow$ This shows greedy strategy does not always select a optimal way to cut rods.

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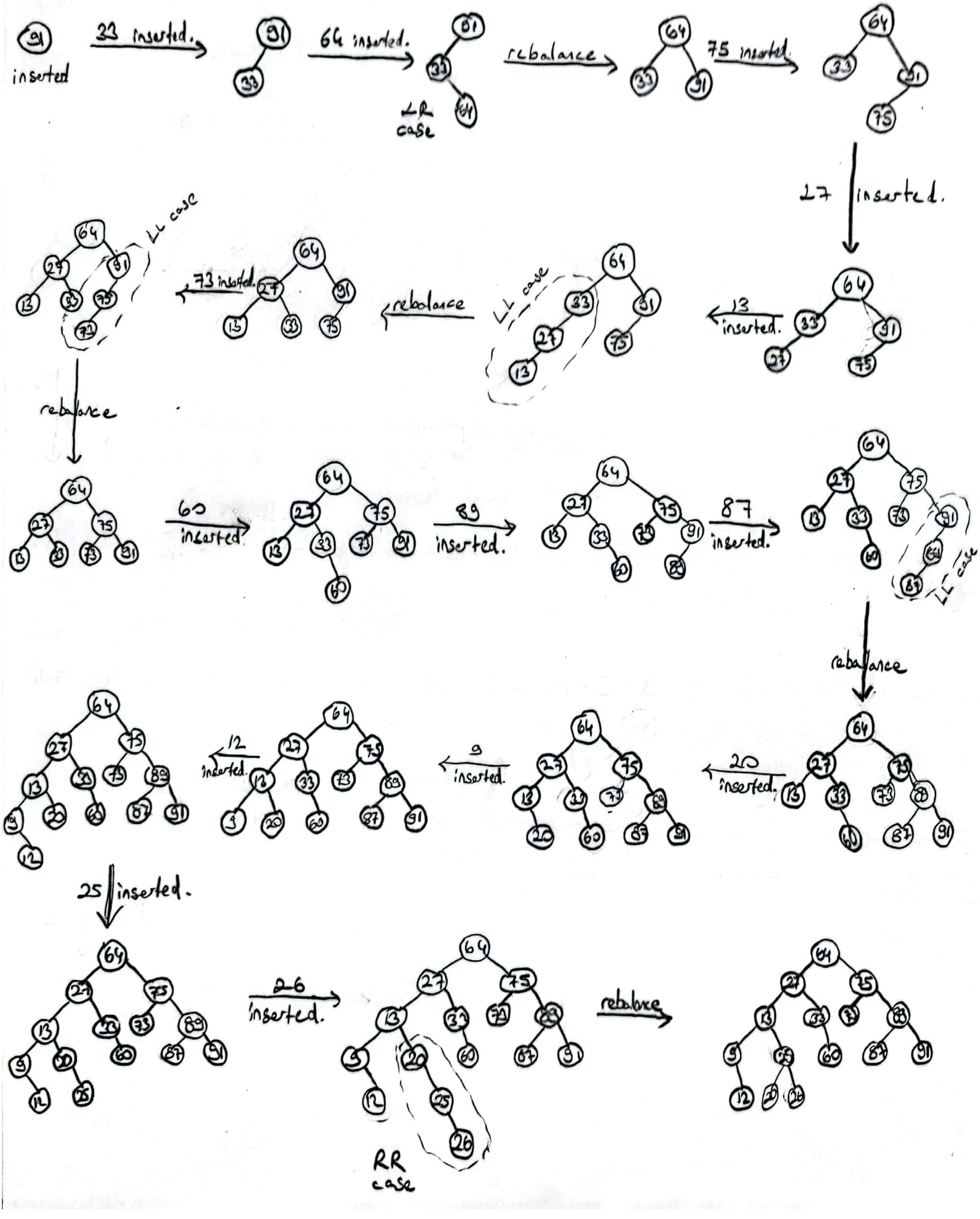
Q3

a) Build an AVL tree after successively inserting the keys: 91, 33, 64, 75, 27, 13, 73, 60, 89, 87, 20, 9, 12, 25, 26, 79, 74, 5. Show and explain every iteration.

Solution a)



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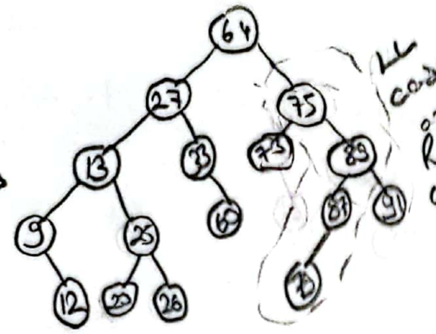
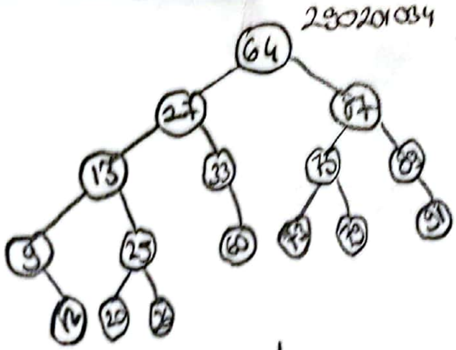
79 inserted.

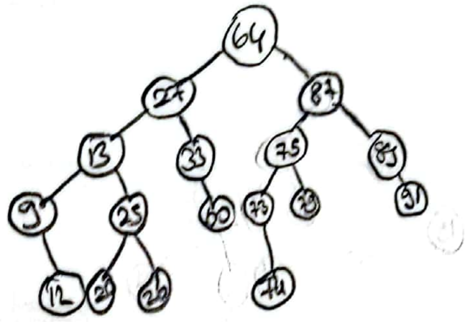


LL case and RL case

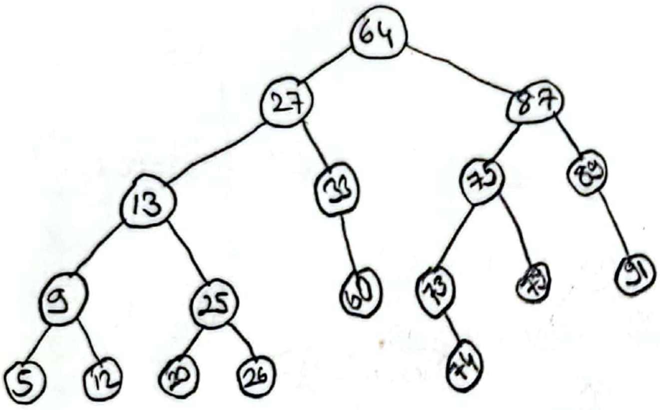
rebalance



74 inserted.



5 inserted.



b) Traverse the tree you build in-order, pre-order and post-order.

Solution b)

INORDER → 5-9-12-13-20-25-26-27-33-60-64-73-74-75-79-87-89-91
(when we saw bottom.)

PREORDER → 64-27-13-9-5-12-25-20-26-33-60-87-75-73-74-79-89-91
(when we saw left.)

POSTORDER → 5-12-9-20-26-25-13-60-33-27-74-73-79-75-91-89-87-64
(when we saw right.)