CENG 115 - Discrete Structures
Homework 5

Exercise 1 (25 pts.)

a. (10 pts.) Guess a formula for the sum of the first n even positive integers. Prove this using mathematical induction.
b. (5 pts.) Dominos and ladders are useful analogies for teaching weak induction. Please write a good analogy for strong induction. If you cannot find any, please explain it using simple words and/or a drawing.
c. (5 pts.) Mathematical induction can be used to prove predicates for natural numbers. Can you extend this technique to prove predicates for all integers (including the negative ones)? Hint: As usual, you have to prove both $P(0)$ and $P(n) \Rightarrow P(n+1)$ for all natural numbers n. What else do you have to prove?

Exercise 2 (25 pts.)

<u>Examples</u>

1. Assume that the following propositions are all true:
   I. $P(0)$
   II. $P(n) \Rightarrow P(2n+1)$ for all natural numbers n

   Are these sufficient to conclude that $P(n)$ is true for all natural numbers n? If so, prove it. If not, what is the simplest proposition that is still need to be proven to draw such conclusion?

   Answer: No.
   $P(n) \Rightarrow P(2n)$ still needs to be proven.

2. Assume that the following propositions are all true:
   I. $P(\{1, 2, 3\})$
   II. $P(n) \Rightarrow P(m)$ for all sets n and m such that $m \subseteq n$ and $|m|=|n|-1$

   Are these sufficient to conclude that $P(\{1\})$ is true? If so, prove it. If not, what is the simplest proposition that is still need to be proven to draw such conclusion?

   Answer: Yes.
   Proof:
   III. $P(\{1, 2\})$ is true. We obtain this using I. and II.
   IV. $P(\{1\})$ is true. We obtain this using III. and II.

   (Note: Your proofs can be verbal explanations or drawings as long as they are convincing!)

Exercises

a. (10 pts.) Assume that the following propositions are all true:
   I. $P(0)$
   II. $P(n)$ and $P(n+1) \Rightarrow P(n+2)$ for all natural numbers n.

   Are these sufficient to conclude that P(n) is true for all natural numbers n? If so, prove it. If not, what is the simplest proposition that is still need to be proven to draw such conclusion?

b. (10 pts.) Assume that the following propositions are all true:
   I. $P(0)$ and $(P(100)$ or $P(2))$
   II. $P(n)$ and $P(m) \Rightarrow P(k)$ for all natural numbers n, m, k such that $n < k < m$

   Are these sufficient to conclude that P(n) is true for all integers n in the range [1, 30]? If so, prove it. If not, what is the simplest proposition that is still need to be proven to draw such conclusion?

c. (10 pts.) Assume that the following propositions are all true:
   I. $P((1,1))$
   II. $P((n,m)) \Rightarrow P((n+1, m+k))$ for all natural numbers n, m, k
   III. $P((n,m)) \Rightarrow P((n-1, m))$ for all integers n, m
   IV. $P((n,m)) \Rightarrow P((n, m-1))$ for all integers n, m

   Are these sufficient to conclude that P(n) is true for all elements n of ZxZ? If so, prove it. If not, what is the simplest proposition that is still need to be proven to draw such conclusion?

Exercise 3 (50 pts.)

a. (7.5 pts.) How many times the digit 2 is used when we write down all the numbers between [1-5000]?

b. (12.5 pts.) Assume that a password cracker software checks 10^6 passwords in a second. At most how many seconds does it need to crack the password of a user whose password
   1. (2.5 pts) is the length of 6 and only contains digits (0-9)
   2. (2.5 pts) is the length of 10 and can contain digits, uppercase and lowercase English letters, and symbols from the set **#!$()**
   3. (7.5 pts) is the length of 6 and contains only 1 lowercase English letter, at least 1 uppercase English letter, and any number of digits

c. (15 pts.) You are supposed to select 4 letters from the English alphabet consisting of 26 letters. How many 4-letters strings possibly can be formed?
   1. (2.5 pts) when the letters' order matters and the letters can be the same
   2. (2.5 pts) when the letters' order matters and each letter is distinct
   3. (7.5 pts.) when the letters' order doesn't matter and the letters can be the same
   4. (2.5 pts.) when the letters' order doesn't matter and each letter is distinct

d. (15 pts.) You have a group of 10 people containing a sibling of 3 and a sibling of 2 and the each of remaining 5 people is not relative to any group members, and you want to form a committee of

5 people from this group. In how many ways can you do this if no two people on the committee can be siblings, i.e. there mustn't be any siblings in the selection?