

CENG 115 - Discrete Structures

Homework 4

December 09, 2022

Due Date: December 15, 2022

Exercise 1 O-notation Simplify

Simplify the following O-notation expressions with input variable n as much as possible. Please show the steps.

(a) $O(n + 2 \cdot n + 3)$

(b) $O(n \cdot 2^{n+1})$

(c) $O(\sqrt{n^2 \cdot (2n^2 + n)})$

(d) $O(\log \frac{n^n}{2} + \log \frac{2}{n})$

(a)	(b)	(c)	(d)

Exercise 2 Complexity Analysis

Please calculate the time complexities of given functions and represent it in the simplest form of their order of growth with input variable n and execution time t . Please show the steps.

(a) $f(n) = \sum_{i=1}^n t_1 + \sum_{j=1}^n t_2 + t_3$

(b) $f(n) = \sum_{i=1}^5 t_1 + (\sum_{j=1}^n t_2) + t_3$

(c) $f(n) = \sum_{i=1}^n (\sum_{j=i}^n 2t)$

(a)	(b)	(c)

Exercise 2 Algorithms

Please complete the algorithms to fulfill the specified tasks and **calculate their time and space complexities**.

(a) Algorithm that takes a list of real numbers as input and sorts the list in ascending order.

procedure sort($a_1, a_2, a_3 \dots a_n$: real numbers with $n \geq 2$)

```

for i:= to 
  if  then
    interchange  $a_i$  and  $a_{i+1}$ 
    
  end if
end for

```

(b) Algorithm that takes a grayscale image as a multidimensional list (matrix) and creates a new image that is half the size of the original image with the average of the pixel values corresponding to each 2×2 block.

procedure resize(Integer Image($m \times n$ pixels) with m, n : real even numbers and $m, n \geq 2$)

Initialize temp:=0; newImage[$m/2$][$n/2$];

```

for i:= to 
  for j:= to 
    newImage[i][j]:=
  end for
end for

```

(c) Algorithm that takes a grayscale image as a multidimensional list (matrix) and creates a blurred new version of the original image by traversing a $k \times k$ mask over it by writing the average of the corresponding pixel values to the center pixel value. (The pixel in the center of the mask will not be included in the average calculation. The outermost pixels will be ignored according to the mask size.)

procedure blur(Integer Image($m \times n$ pixels), Integer mask($k \times k$ pixels) with m, n : real numbers and $m, n \geq 3$ with k : real odd number and $k \geq 3$, $k \leq \min(m, n)$)

Initialize temp:=0; newImage:=Image

```

for i:= to 
  for j:= to 
    for a:= to 
      for b:= to 
        if  then
          
        end if
      end for
    end for
  end for
  
  
end for

```

Exercise 4 Encryption

Please find the decryption function and the corresponding message according to the encryption function and the encrypted message.

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26/0

Encryption Function: $|7x - 4| \pmod{26}$

Encrypted Message: AE BWTEX AGFZ C BWTE FZCF ACY IWRE FZCP BWTE

Decryption Function:	Decrypted Message: