CENG 115 - Discrete Structures Homework 4

December 09, 2022

Due Date: December 15, 2022

Exercise 1 O-notation Simplify

Simplify the following O-notation expressions with input variable n as much as possible. Please show the steps.

- (a) $O(n+2\cdot n+3)$
- **(b)** $O(n \cdot 2^{n+1})$
- (c) $O(\sqrt{n^2 \cdot (2n^2 + n)})$
- (d) $O(\log \frac{n^n}{2} + \log \frac{2}{n})$

(a)	(b)	(c)	(d)

Exercise 2 Complexity Analysis

Please calculate the time complexities of given functions and represent it in the simplest form of their order of growth with input variable n and execution time t. Please show the steps.

(a)
$$f(n) = \sum_{i=1}^{n} t_1 + \sum_{i=1}^{n} t_2 + t_3$$

(b)
$$f(n) = \sum_{i=1}^{5} t_1 + (\sum_{j=1}^{n} t_2) + t_3$$

(c)
$$f(n) = \sum_{i=1}^{n} (\sum_{j=i}^{n} 2t)$$

(a)	(b)	(c)

Exercise 2 Algorithms

Please complete the algorithms to fulfill the specified tasks and calculate their time and space complexities.

(a) Algorithm that takes a list of real numbers as input and sorts the list in ascending order.

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procedure \operatorname{sort}(a_1, a_2, a_3...a_n): real numbers with n \ge 2)

for i:=

to

then

interchange a_i and a_{i+1}

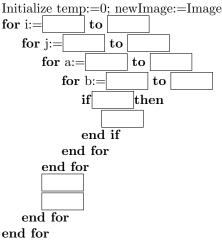
end if

end for
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(b) Algorithm that takes a grayscale image as a multidimensional list (matrix) and creates a new image that is half the size of the original image with the average of the pixel values corresponding to each 2×2 block.

(c) Algorithm that takes a grayscale image as a multidimensional list (matrix) and creates a blurred new version of the original image by traversing a $k \times k$ mask over it by writing the average of the corresponding pixel values to the center pixel value. (The pixel in the center of the mask will not be included in the average calculation. The outermost pixels will be ignored according to the mask size.)

procedure blur(Integer Image($m \times n$ pixels), Integer mask($k \times k$ pixels) with m,n: real numbers and m,n ≥ 3 with k: real odd number and $k=\geq 3$, $k\leq \min(m,n)$



Exercise 4 Encryption

Please find the decryption function and the corresponding message according to the encryption function and the encrypted message.

A	В	С	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	О	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26/0

Encryption Function: $|7x-4| \mod 26$

Encrypted Message: AE BWTEX AGFZ C BWTE FZCF ACY IWRE FZCP BWTE

Decryption Function:	Decrypted Message: