CENG 115 - Discrete Structures
Homework 2

October 30, 2022

Due Date: November 8, 2022

Exercise 1 Predicates & Quantifiers

Express the following statements using quantifiers. Please specify the domains and propositional functions.

(a) "Each card dealt is black or red."

(b) "Anyone taller than 140cm can ride the roller coaster."

(c) "A bright future is possible if even one person keeps trying."

Exercise 2 Rules of Inference

Prove the given statements based on premises. Please specify the steps and rules you use.

	Premises	**Prove**
(a)	• $\neg A \rightarrow (B \wedge C)$ • $A \rightarrow D$ • $\neg D$	Prove: B
(b)	• $A \wedge B$ • $A \rightarrow \neg(B \wedge C)$ • $D \rightarrow C$	Prove: $\neg D$
(c)	• $A \rightarrow B$ • $\neg B \vee C$ • $C \rightarrow (E \vee D)$ • $\neg D \wedge A$	Prove: E
(d)	• $(\neg A \wedge B) \rightarrow (C \vee D)$ • $\neg A \rightarrow (C \rightarrow F)$ • $(D \rightarrow E) \vee A$ • $\neg A \wedge B$	Prove: $E \vee F$

Exercise 3 Proofs

In an unsolved case, it is decided to give the suspects a chance to determine their own destiny. Each suspect is presented with two doors. Every door leads to either freedom or jail (not both). The clues are written on doors and the suspect decides which door to open based on these clues and the given information about them. Prove the correct choice (freedom) for each situation and please explain your abbreviations and premises.

(a) Suspect 1 is told that exactly one of the following clues is true and exactly one is false.

- Door 1: This door leads to freedom, and the other door leads to jail.
- Door 2: One of the doors leads to freedom, and the other one leads to jail.

(b) Suspect 2 is told that either both clues are true or both are false.

- Door 1: Either this door leads to imprisonment or the other door leads to freedom. (Not both.)
- Door 2: This door leads to freedom.

(c) Suspect 3 is told that if door 1 leads to freedom then the clue on door 1 is true, but if door 1 leads to jail then the clue on that door is false. Door 2 follows the opposite rule: if door 2 leads to freedom the clue on door 2 is false, but if door 2 leads to jail the clue on that door is true.

- Door 1: At least one of these doors leads to freedom.
- Door 2: The other door leads to freedom.