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CEUG 115 - HW 2

Exercise : 1

(a) "Each card dealt is black or red."

$R(x) \Leftrightarrow x$ is red, $B(x) \Leftrightarrow x$ is black x consists of all cards dealt.

$\forall x\,(R(x) \vee B(x))$

(b) "Anyone taller than 140 cm can ride roller coaster."

$C(x) \Leftrightarrow x$ can ride roller coaster, $T(x) \Leftrightarrow x$ is taller than 140 cm.

x consists of all people.

$\forall x\,(T(x) \rightarrow C(x))$

(c) "A bright future is possible if even one person keeps trying."

$K(x) \Leftrightarrow$ keeps trying, $P(y) \Leftrightarrow y$ is possible, x consists of all people, y consist of all bright futures.

$\exists x\, \exists y\, (K(x) \rightarrow P(y))$

Exercise : 2

(a) $\underbrace{\neg A \rightarrow (B \wedge C),\quad A \rightarrow D,\quad \neg D}_{\text{Premises}}$ $\quad \underbrace{B}_{\text{Prove}}$ $\xrightarrow{\text{SOLUTION}}$

1) $\dfrac{\neg D \quad A \rightarrow D}{\therefore \neg A}$ Modus Tollens

2) $\dfrac{\neg A \quad \neg A \rightarrow (B \wedge C)}{\therefore B \wedge C}$ Ponens

3) $\dfrac{B \wedge C}{\therefore B}$ Simplification

(b) $\underbrace{A \wedge B,\quad A \rightarrow \neg(B \wedge C),\quad D \rightarrow C}_{\text{Premises}}$ $\quad \underbrace{\neg D}_{\text{Prove}}$ $\xrightarrow{\text{SOLUTION}}$

1) $\dfrac{A \wedge B}{\therefore A}$ $\quad \dfrac{A \wedge B}{\therefore B}$ Simplification

2) $\dfrac{A \rightarrow \neg(B \wedge C)}{\therefore \neg(B \wedge C)}$ Modus P.

3) $\neg(B \wedge C) \Leftrightarrow \neg B \vee \neg C$ De Morgan

4) $\dfrac{\neg B \vee \neg C \quad B}{\therefore \neg C}$ D. Syllogism

5) $\dfrac{\neg C \quad D \rightarrow C}{\therefore \neg D}$ M.T

(c) $A \to B$

$\neg B \vee C$

$C \to (E \vee D)$

$\underbrace{\neg D \wedge A}_{\text{Premises}}$ $\qquad \underbrace{E}_{\text{Prove}}$

SOLUTION

1) $\neg B \vee C \Leftrightarrow B \to C$

2) $A \to B$
$B \to C$
$\therefore A \to C$

3) $A \to C$
$C \to (E \vee D)$
$A \to (E \vee D)$

Hypothetical

A

4) $\therefore (E \vee D)$

Modus P.

5) $E \vee D$
$\neg D$
$\therefore E$

Disjunctive S.

(d) $(\neg A \wedge B) \to (C \vee D)$

$\neg A \to (C \to F)$

$(D \to E) \vee A$

$\underbrace{\neg A \wedge B}_{\text{Premises}}$ $\qquad \underbrace{E \vee F}_{\text{Prove}}$

SOLUTION

1) $\neg A \wedge B$
$\therefore \neg A$

Simplification

2) $(D \to E) \vee A$
$\neg A$
$\therefore D \to E$

D.S. Sy.

3) $\neg A$
$\neg A \to (C \to F)$
$\therefore (C \to F)$

Modus P.

4) $\neg A \wedge B$
$(\neg A \wedge B) \to (C \vee D)$
$\therefore C \vee D$

Modus P.

5) $\neg C \to D$
$D \to E$
$\therefore \neg C \to E$

Hyphothetical S.

6) $C \to F$
$\neg C \to E$
$\therefore E \vee F$

Exercise : 3

a) Suspect 1

$A(x) \equiv x$ leads to freedom

$B(x) \equiv x$ leads to prison

x consist of door 1 and 2

door 1 → 1 door 2 → 2

Promises:

Door 1 : $\Rightarrow A(1) \wedge B(2)$

Door 2 : $\Rightarrow (A(1) \wedge B(2)) \vee (A(2) \wedge B(1))$

↓

~~Exactly both clues~~

Door 1 $\oplus$ Door 2 $\Leftrightarrow$ True

$\neg A(x) \Leftrightarrow B(x)$

$\neg B(x) \Leftrightarrow A(x)$

Solution

If we assume that Door 1 is true then, Door 2 is true that means Door 1 is false, Door 2 is true

Door 2 leads to freedom.

$F \vee (A(2) \wedge B(1)) \Leftrightarrow A(2) \wedge B(1)$

$\Leftrightarrow$ TRUE

b) Suspect 2

$A(x) \equiv x$ leads to freedom.

$B(x) \equiv x$ leads to jail.

x consist of door 1 and 2

door 1 → 1 door 2 → 2

Promises:

Door 1 ⇔ $B(1) \vee A(2)$

Door 2 ⇔ $A(2)$

↓

either both clues are true or both are false.

Door 1 ⇔ Door 2 True

Solution:

If we assume that Door 2 is false, then Door 1 would be $B(1) \vee A$, we don't know that $B(1)$ is T or F, so Door 1 could be either. Another hand Door 2 is True, then Door 1 is true this means $A(2)$ is true (Freedom)

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c) Suspect 3

$A(x) \equiv x$ leads to freedom

$B(x) \equiv x$ leads to jail.

x consist of door 1 and 2

door 1 → 1 door 2 → 2

Promises:

Door 1 $\equiv \exists x\ A(x)$

Door 2 $\equiv A(1)$

⇓

$A(1) \rightarrow$ door 1

$B(1) \rightarrow \neg$ door 1

$A(2) \rightarrow \neg$ door 2

$B(2) \rightarrow$ door 2

$\neg A(x) \Leftrightarrow B(x)$

$\neg B(x) \Leftrightarrow A(x)$

SOLUTION

Assume door 2 is true then we can say $A(1) \rightarrow$ door 1 if door 1 is true

$$\frac{\exists x\ A(x)}{\therefore A(1)} \longrightarrow \text{Which means } A(1) \text{ is true}$$

$\boxed{\text{Door 1 leads to freedom.}}$