CENG 115 - Discrete Structures

HOMEWORK - 5

Ahmet Kurt - 290201034

Exercise 1

a.

The sum of the first n even positive integers is equal to 2 times the sum of the first n positive integers. This can be expressed as the following formula:

$$Sum = 2 * (1 + 2 + 3 + 4 + 5 + ... + n)$$

We can write like:

$$\forall n \geq \sum_{i=0}^{n} (2i) = n(n+1)$$

Now we need to prove this formula by mathematical induction. For that we need to define P(n).,

Definiton:

$$P(n) : 2 + 4 + ... + (2n-2) + 2n = n(n+1)$$

Base Case:

$$P(0) = 0(0+1) = 0$$

Inductive case:

Assume that the formula holds for some value of n. We need to prove that it also holds for n+1.

$$P(k) : \forall k, k \in \mathbb{N}, P(k) \implies P(k+1)$$

Since we assumed that the formula holds for some value of n, Sum(n+1) = Sum(n) + 2 * (n+1). Therefore, the formula holds for n+1.

Since the formula holds for the base case and the inductive case, **it holds for all positive integers.**

b.

An analogy for strong induction **could be to build a house.** Let's imagine we are trying to build a house and we have a plan that tells us in what order to build the different parts of the house.

In this scenario, the plan represents the strong inductive hypothesis, a statement that must be true for all steps in the process of building the house. For example, for the strong inductive hypothesis, we might say: "For every step in my process of building this house, the construction will be done right and it will go to plan."

c.

We also need to prove $P(n) \implies P(n-1)$ for negative integers.

Exercise 2

a.

It is not sufficient. $P(1)$ is still need to be proven.

b.

We can conclude $P(0)$ is true but to conclude that $P(n)$ is true for all integers n in the range [1, 30], we need to show that $P(n)$ is true for every integer n in that range.

We can conclude that $P(2)$ is true by II proposition as:

$$(P(0) \land P(100)) \implies P(1)$$

and,

$$(P(0) \land P(2)) \implies P(1)$$

We can conclude that $P(1)$ is true.

But we cannot go further. For example we cannot prove $P(9)$ by the given information. For $P(9)$ we need to prove $P(10)$. For $P(10)$, we need to prove $P(11)$. We can go to the $P(30)$ like this.

At least $P(30)$ is still need to be proven.

c.

Yes. It is sufficient,

1 - $P((0,1))$ is true, we obtain it by using III.

2- $P((0,0))$ is true, we obtain it by using IV.

3- $P((1,0),(1,1),(1,2),...(1,Z))$ is true, we obtain it by using II.

4 - $P((0,0),(0,1),(0,2),...,(0,Z))$ is true, we obtain it by using III.

5 - $P((2,0),(2,1),(2,2),...,(2,Z)) \implies P((Z,0),(Z,1),(Z,2),...,(Z,Z)$ is true, we obtain it by using II.

Exercise 3

a.

1-9 = 1 time.

10-19 = 1 times.

20-29 = 11 times

30-99 = 7 times.

100-199 = 20 times.

200-299 = 120 times.

300-999 = 140 times.

1000-1999 = 200 times.

2000-2999 = 1200 times.

3000-5000 = 400 times.

$\sum = 1800 \text{ times.}$

b.1

$10^6/10^6 = 1$ second.

b.2

Every digit has 67 options, length of the password = 67^{10}

Password cracker = 10^6

$\implies 67^{10}/10^6$ seconds.

b.3

$(6!/4) * (6!/12) * (6!/12) * (6!/4!) * (6!/5!) = 9417123456$

Password cracker = 10^6

$\implies 9,417,123,456/10^6 = 9,417$ seconds.

c.1

$26^4 = 456976$

c.2

$26 * 25 * 24 * 23$

c.3

$(26, 4) = 26!/(22! * 4!)$

c.4

$P(26, 4)/P(4, 4) = 14950$

d.

$C(5, 5) + C(5, 4)C(3, 1)C(5, 4)C(2, 1) + C(5, 3)C(3, 1)C(2, 1) = 86$ different ways.