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CENG 115 - Discrete Structures
Homework 4

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Due Date: December 15, 2022

Exercise 1 Θ-notation Simplify

(a) $O(n+2 \cdot n+3)$

(b) $O(n \cdot 2^{n+1})$

(c) $O(\sqrt{n^2 \cdot (2n^2+n)})$

(d) $O(\log \frac{n^n}{2} + \log \frac{2}{n})$

(a)	(b)	(c)	(d)
$O(3n+3)$	$O(n \cdot 2 \cdot 2^n)$	$O(\sqrt{2n^4 + n^3})$	$O((n-1)\log n)$
$= O(3n) + O(3)$	$= O(2n \cdot 2^n)$	$= O(n^2)$	$= O(n\log n) + O(\log n)$
$= \Theta(n)$	$= O(n \cdot 2^n)$		$= O(n\log n)$

Exercise 2   Complexity Analysis

(a) $f(n) = \sum_{i=1}^{n} t_1 + \sum_{j=1}^{n} t_2 + t_3$

(b) $f(n) = \sum_{i=1}^{5} t_1 + (\sum_{j=1}^{n} t_2) + t_3$

(c) $f(n) = \sum_{i=1}^{n} (\sum_{j>i}^{n} 2+)$

(a)	(b)	(c)
$O(t_1 \cdot n) + O(t_2 \cdot n + t_3 n) = O_{(n)} + O_{(n)}$	$O_{(5t_1)} + O_{(n t_2)} + O_{(t_3)}$	$O_{(n \cdot 2t_1) \cdot n} = O_{(2t_1 n)} = O_{(n^2)}$
$= O_{(n)} + O_{(2n)}$	$= O_{(5)} + O_{(n)} + O_{(1)}$	
$= \boxed{O_{(n)}}$	$= \boxed{O_{(n)}}$	$= \boxed{O_{(n^2)}}$

Exercise 3   Algorithms

(a) Space   Complexity : $O(1)$
    Time   Complexity : $O(n^2)$
    procedure   sort $(a_1, a_2, a_3, \ldots, a_n :$ real num with $n \geqslant 2)$
        for $i := \boxed{1}$ to $\boxed{n}$
            if $\boxed{a_i > a_{i+1}}$ then
                interchange a_i and a_{i+1}
                sort $(a_1, a_2, \ldots, a_i)$
            end if
        end for

(b) Space   Complexity : $O(m \times n)$
    Time   Complexity : $O(m \times n)$
    procedure   resize $($ Integer Image $(m \times n$ pixels$)$ with $m, n :$ real eve
    numbers and $m, n \geqslant 2)$
        for $i := 0$ to $m/2$
            for $j := 0$ to $n/2$
                new Image $[i][j] := \dfrac{\text{Image } [2i+1]^2 + \text{Image } [2i+1][2j] + \text{Image} [2i]}{[2j+1] + \text{Image } [2i][2j]}$
                $\dfrac{}{4}$
            end for
        end for

(c) Space   Complexity : $O(m \times n)$
     Time   Complexity : $O(m \times n)$
     procedure blur $($ Integer Image $(m \times n$ pixels$)$, Integer mask $(k \times k$
     with $m, n :$ real numbers and $m, n \geqslant 3$ with $k :$ real
     odd   number   and $k \Rightarrow 3, k < \min (m, n))$

```
for i=0 to m-1 do
    for j=0 to n-1 do
        for a:= -k/2 to k/2 do
            for b:= -k/2 to k/2 do
                if i+a >= 0 and i+a < m and j+b >= 0 and
                    j+b < n then
                        temp + = Image [i+a] [j+b]
                end if
            end for
        end for
        new Image [i][j] := temp/((k x k) -1)
        temp := 0
    end for
end for
```

Exercise 4   Encryption

Encryption Function: $\implies$ Decryption Function:

$|7x-4|$ mod 26 $|15x+8|$ mod 26

Decrypted Message:

WE LOVED WITH A LOVE THAT

WAS MORE THAN

LOVE