IE 469 INDUSTRIAL APPLICATIONS OF OPERATIONS RESEARCH

SPRING 2024 TERM PROJECT

GROUP 15

EBRU AYAN 22003741

AHMET SAYAN 21903426

DAMLA UÇAR 22103276

1.The Mathematical Model

This part is the starting point of our project. We started with developing IP so that we can determine the number of bikes to be transferred among the stations during a day while minimizing the total cost including shortage, surplus, and transfer costs.

Parameters:

C_i=the capacity of jth station $i \in \{1, \dots 48\}$

I_{ij} =expected number of bikes in to ith station at the jth hour $i \in \{1, \dots 48\}$ and $j \in \{5, \dots, 23\}$

O_{ij} =expected number of bikes out from the ith station at the jth hour $i \in \{1, \dots 48\}$ and $j \in \{5, \dots, 23\}$

t= transfer cost per each bike s_{ij} =shortage cost of ith station at the jth hour $i \in \{1, \dots 48\}$ and $j \in \{5, \dots, 23\}$

p_{ij} =surplus cost of ith station at the jth hour $i \in \{1, \dots 48\}$ and $j \in \{5, \dots, 23\}$

M = binary constraint (1000)

Decision Variables:

T_{ij}^{-}= number of bikes transferred out from ith station at the jth hour $i \in \{1, \dots 48\}$ and $j \in \{5, \dots, 23\}$

T_{ij}^{+}= number of bikes transferred into ith station at the jth hour $i \in \{1, \dots 48\}$ and $j \in \{5, \dots, 23\}$

N_{ij}= number of available bikes in the ith station at the end of jth hour $i \in \{1, \dots 48\}$ and $j \in \{5, \dots, 23\}$

$$z_{ij} = \begin{cases} 1 & \text{if there is a surplus,} \\ 0 & \text{otherwise.} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if there is a shortage,} \\ 0 & \text{otherwise.} \end{cases}$$

Model:

$$\min \sum_{i=1}^{48} \sum_{j=5}^{23} (z_{ij} p_{ij} + x_{ij} s_{ij} + T_{ij}^{+} t)$$

$$\begin{aligned}
\text{s.t. } & N_{ij} = N_{ij-1} + T_{ij-2}^{+} - T_{ij}^{-} + I_{ij} - O_{ij} && i = 1, \dots, 48 \quad j = 5, \dots, 23 \\
& \sum_{i=1}^{48} N_{i4} = 400 && \\
& \sum_{i=1}^{48} T_{ij}^{+} = \sum_{i=1}^{48} T_{ij}^{-} && j = 5, \dots, 23 \\
& \sum_{i=1}^{48} T_{ij}^{+} = 0 && j = 5, 6 \\
& \sum_{i=1}^{48} T_{ij}^{-} = 0 && j = 22, 23 \\
& 1 - N_{ij} \leq M x_{ij} && i = 1, \dots, 48 \quad j = 5, \dots, 23 \\
& N_{ij} + 1 - C_i \leq M z_{ij} && i = 1, \dots, 48 \quad j = 5, \dots, 23 \\
& N_{ij}, T_{ij}^{+}, T_{ij}^{-} \geq 0, \qquad N_{ij}, T_{ij}^{+}, T_{ij}^{-} \in \mathbb{Z}, && i = 1, \dots, 48 \quad j = 5, \dots, 23 \\
& z_{ij}, x_{ij} \in \{0, 1\} && i = 1, \dots, 48 \quad j = 5, \dots, 23
\end{aligned}$$

Explanation of The Constraints and The Objective Function:

1) The objective is to minimize the total cost related to shortage, surplus, and transfers. The shortage and surplus costs are incurred per each station whereas transfer costs are incurred per each bike. The objective is set according to that.
2) In the first constraint, we simply write a balance equation. The number of bikes at the end of the hour should be equal to the sum of the number of bikes at the end of the previous hour and transfer ins minus transfer outs and expected ins minus expected outs.
3) In the second constraint, for each hour the number of bikes transferred out should be equal to the number of bikes transferred in since we cannot get rid of bikes.
4) In the fourth and fifth constraint, we simply write the equation of "no transfer-outs are performed after 10 pm. No transfer-ins are performed before 7 am."
5) In the sixth and seventh constraint, if then statements related to shortage and surplus are linearized.
6) The last two constraints define the set in which decision variables are defined.

2.The Logic of The Model

In this part, we briefly want to explain the logic behind the model and give some assumptions in detail.

First of all, we found the arrival and departure dates for each station and each hour by utilizing the given past data. Our rates are “bikes/ hour”. However, since we are taking the average number of bikes the rates are not integer as expected. We cannot use these rates because in the balance equation above the number of transferred bikes should be an integer. In that case, the problem becomes infeasible.

Hence, we applied the rounding procedure and we rounded up incoming and outgoing rates. Then, we are able to obtain a feasible solution. However, there is a disadvantage coming with this procedure. The number of bikes at the end of the 23rd hour exceeded 400 because the expected numbers are rounded up, meaning that there might be incoming bikes even though we do not have that many bikes. Since this occurred at the end of the 23rd hour and we are dealing with a stochastic model, this constraint can be relaxed in a way. Also, the costs are updated according to the rounded rates.

Moreover, in the shortage constraint, since the number of bikes at the end of the hour cannot be negative, if it is zero, we accepted it as a shortage and linearized our if-then constraints accordingly.

The initial distribution of the bikes is also determined by the model such that the total number of bikes is 400. The total number of bikes at the end of each hour during a day can be less than 400, meaning that some customers leave bikes somewhere.

3. Cost Changes With Different Scenarios

In the second stage, after obtaining the optimal result of total cost and the transfers in the stations, we proceeded to examine the outcomes. In order to identify the problematic stations, we looked for the stations that have significantly large transfer out or in values. We observed that stations with notably high transfer in values may indicate high demand of bikes for that station that cannot be met sufficiently. This often occurs at stations located near houses during mornings. On the contrary, stations with high transfer out values may indicate a surplus of bikes being returned, possibly due to stations near subway stations during mornings.

Based on the observations of the transfer values, we identified potential scenarios to reduce the costs associated. Specifically, we noticed high transfer out values at stations Grove St PATH and Exchange Place during the early hours of the day. Therefore we decided to enhance the capacities of these stations in order to decrease the transfer numbers, thereby reducing the overall cost. Ten possible scenarios related to problematic stations are observed and they are as follows :

Case 1:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Transfer Payment After Case 1	Future worth in 2 years after case 1
640	-₺584.442,90

CASE 1: INCREASING GROOVE ST CAPACITY 12

Case 2:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Transfer Payment After Case 2	Future worth in 2 years after case 2
480	-₺547.426,91

CASE 2: INCREASING GROOVE ST CAPACITY 12, EXCHANGE PLACE + 8 CAPACITY

Case 3:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Transfer Payment after case 3	Future worth in 2 years after case 3
410	-₺536.191,26

CASE 3: INCREASING GROOVE ST CAPACITY 12, EXCHANGE PLACE + 12 CAPACITY

Case 4:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Transfer Payment After Case 4	Future worth in 2 years after case 4
480	-₺549.458,48

CASE 4: INCREASING GROOVE ST CAPACITY 12, EXCHANGE PLACE + 8 CAPACITY, ADDED 10 MORE BIKE

Case 5:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Transfer Payment After case 5	Future worth in 2 years after case 5
400	-₺528.918,91

CASE 5 : INCREASING GROOVE ST CAPACITY 16, EXCHANGE PLACE + 8 CAPACITY

Case 6:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Transfer Payment After case 6	**Future worth in 2 years after case 6**
330	-₺517.683,26

CASE 6: INCREASING GROOVE ST CAPACITY 16, EXCHANGE PLACE + 12

Case 7:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Trasfer Payment After Case 7	**Future worth in 2 years after case 7**
220	-₺517.029,03

CASE 7: INCREASING GROOVE ST CAPACITY 20, EXCHANGE PLACE + 16

Case 8:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Transfer Payment After	**Future worth in 2 years**
210	-₺628.769,11

CASE 8: INCREASING GROOVE ST CAPACITY 20, EXCHANGE PLACE + 16, 5 CORNERS LIBRARY +12

Case 9:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Transfer Payment After Case 9	**Future worth in 2 years after case 9**
580	-₺977.187,72

CASE 9: INCREASING GROOVE ST CAPACITY 20, EXCHANGE PLACE + 16, 5 CORNERS LIBRARY +12, BALDWIN +8

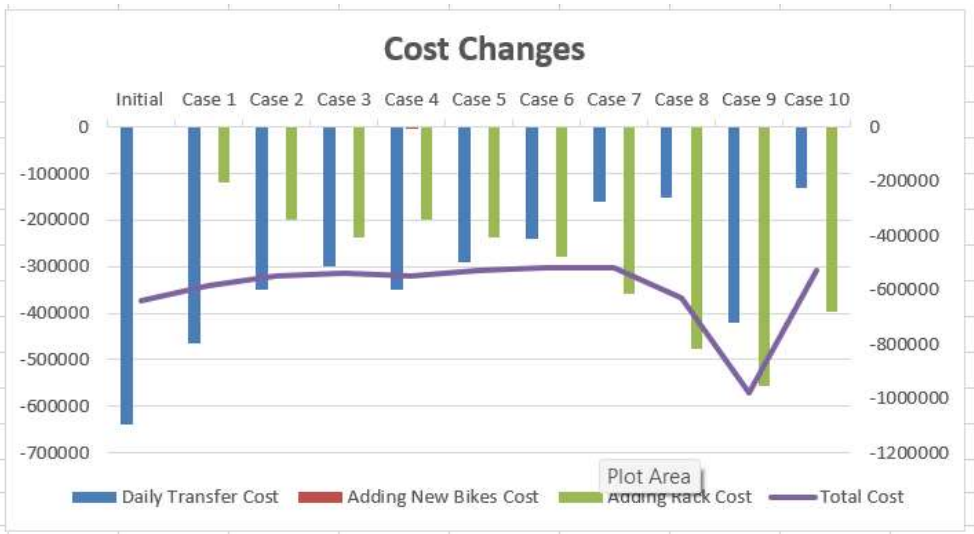
Case 10:

Daily Transfer Payment Initial	Future worth in 2 years initial
880	-₺639.966,89
Daily Transfer Payment after case 10	Future worth in 2 years after case 10
180	-₺527.610,43

CASE 10: INCREASING GROOVE ST CAPACITY 24, EXCHANGE PLACE + 16

We started with the increase in the capacity of Groove St and observed that it reduces the total cost. So, we increased it more and also increased the capacity of Exchange Place. The capacity increase in both stations resulted in reduction of the total cost with the reduction of transfer costs.Then, we tried to increase the number of bikes. However, this increase did not result in neither the total cost nor transfer costs. So, we decided that increasing the number of bikes is useless. We again increased the capacity of different stations but they did not give a decrease in total cost either. Hence, the minimum cost is obtained with case 7. In the last case, we increased the capacity of Groove St four more than case 7 but it did not reduce the cost. Hence, it is concluded that after some point, increasing the capacity is not useful. The table related to different types of costs for each case and the chart is given above.

	Daily Transfer Cost	Adding New Bikes Cost	Adding Rack Cost	Total Cost
Initial	-$639.966,89	$0,00	$0,00	-$639.966,89
Case 1	-$465.430,47	$0,00	-$119.012,43	-$584.442,90
Case 2	-$349.072,85	$0,00	-$198.354,06	-$547.426,91
Case 3	-$298.166,39	$0,00	-$238.024,87	-$536.191,26
Case 4	-$349.072,85	-$2.031,58	-$198.354,06	-$549.458,48
Case 5	-$290.894,04	$0,00	-$238.024,87	-$528.918,91
Case 6	-$239.987,58	$0,00	-$277.695,68	-$517.683,26
Case 7	-$159.991,72	$0,00	-$357.037,30	-$517.029,03
Case 8	-$152.719,37	$0,00	-$476.049,74	-$628.769,11
Case 9	-$421.796,36	$0,00	-$555.391,36	-$977.187,72
Case 10	-$130.902,32	$0,00	-$396.708,11	-$527.610,43



In our analysis, we used the future worth of the costs. The one with the higher future worth is a better solution. The present worth can also be used but in that case the one with the lower present worth would be a better solution since the values would be positive. The other important point here is to calculate the effective interest rate because the MARR is given as yearly but we need a daily interest rate. The following formula is utilized to calculate the effective interest rate :

Effective Interest Rate per Payment Period (*i*)

$$i = \left(1 + \frac{r}{CK}\right)^{C} - 1$$

- ❑ *C* = number of interest (compounding) periods per payment period
- ❑ *K* = number of payment periods per year
- ❑ *CK* = total number of interest periods per year, or *M*

4. User Form Explanation

In order to manage the input changes wanted in the system, we designed a user form using VBA. Clicking the “Input Changes” button, the interface can be accessed. This user form provides an interface for selecting stations, specifying the number of racks to add, and the number of bikes to add to the system.Since the increase of racks are done of 4 slots each, 1 rack is equivalent to 4 slots in the number selection of user form. Since no reduction in the number of bikes and number of racks in the system is allowed at any time, the values entered only work for numbers greater or equal than 0. We assumed that if there is a need to decrease or undo the numbers, such adjustments can be done directly from the worksheet .

After the selected items are added to the system, by pressing the solve button, the system uses opensolver to update the decision variables and objective function, reflecting the input changes and the financial implications of them. The updated information of present worth of all costs is then displayed in a message box, providing the insights.