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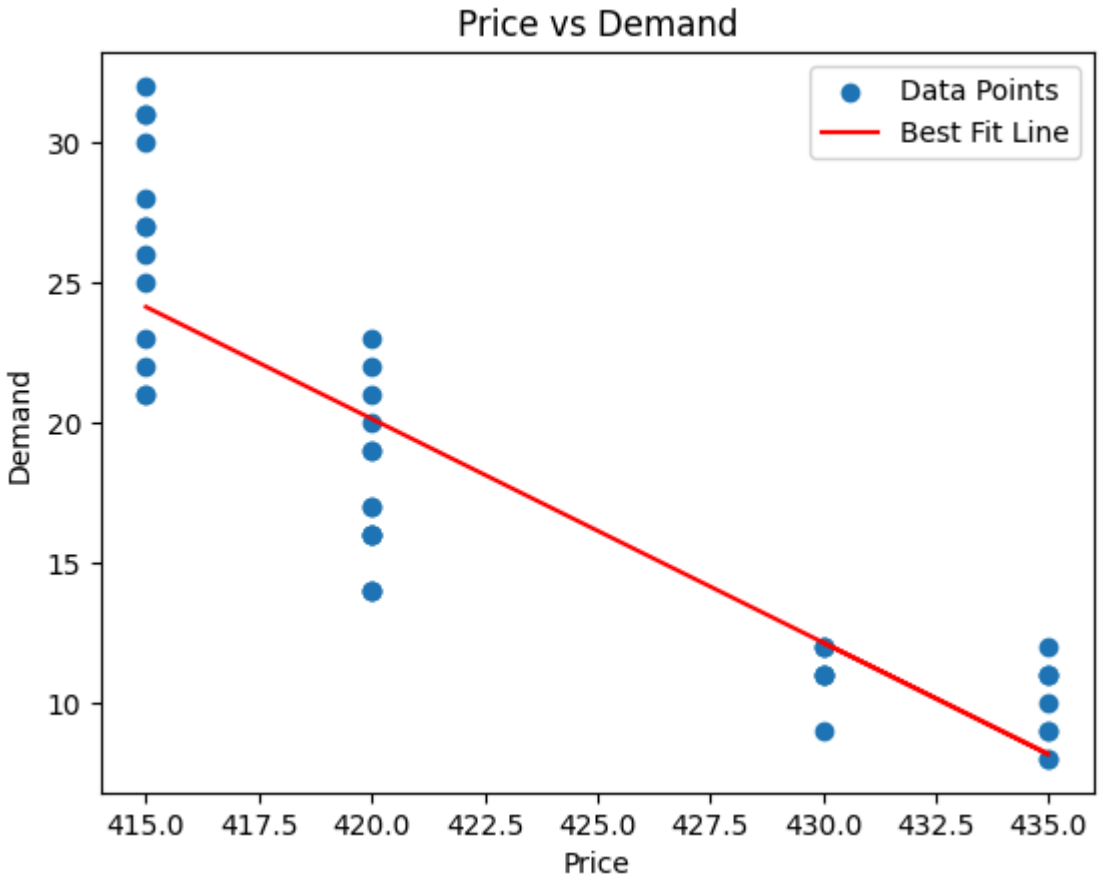
Q1)

Using the 45 days of price and demand data provided we created a linear price response function using regression. Dot plot below shows our data points and the best fitting line for the data. Price was our independent variable shown on the x axis and demand was our dependent variable shown on the y axis.



The results show that the best fitting price response function is:

$$d(p) = 355.3346 - x(0.7981)$$

We believe this function is a relatively good fit since our R^2 value is 0.7620, meaning we can explain 76.2% of the variation in our data. But there is certainly room for improvement through the use of other regression models.

Q2)

Based on the price response function we calculated in Q1 if the price of Komili was set to 450 TL the predicted demand would be:

$$d(p) = 355.3346 - 450(0.7981) = -3.8164$$

Since our predicted demand is negative we might assume at a price of 450 TL returns might be higher than actual sales or our linear model is having trouble predicting demand at relatively extreme prices. This could be because a linear model may not be able to accurately predict a price response function where there elasticity is not constant at every price point.

Assuming demand has normal distribution to not stock out with 95% probability we calculate our inventory level using:

Stock Amount= z * STDEV+ PREDICTED DEMAND

At 95% the z value is 1.96 and from our residuals we calculated our residual standard deviation as 3.46. With our predicted demand of -3.8164 our projected stock amount comes out to 2,97 (rounded to 3) to not stock out with 95% probability.

Q3)

The optimal price for the product would be found through taking the derivative of the total profit function with respect to the price and finding which values of p result in the function being zero.

$$m(p)= (\frac{4}{5}p - 275)(355.335 - 0.7981p)$$

Which results in:,

$$p^* = 394.484\,TL$$

And out expected demand at this price would be:

$$d(p) = 355.335 - 394.484\,(0.7981) = 40.49$$

So Komili's expected daily profit in this store is:

$$(\frac{4}{5}) * 394.484 - 275) * 40.49 = 1643.44\,TL$$

Q4)

For exponential and constant elasticity response functions, we first took derivative of the formulas to apply linear regression. Then calculate intercept and coefficient.

	Exponential Response	Constant Elasticity
Demand Function Form	d(p) = e^{a-bp}	d(p) = $Cp^{-\epsilon}$
Ln form	ln(d(p))= a - bp	ln(d(p))= ln(C) -ϵ ln(p)
Intercept	23,3335	127,688
Coefficient	-0,0486	-20,653
Predicted Demand for Price 450	4,412	4,545
R^2	0,8453	0,8472

We used R^2 values to compare models, and Constant Elasticity looks slightly better than Exponential response. However, both are significantly better than linear models.

Question 5

To find the average utility values of each product, we initially separated the components of the products. These were determined as Base (Migros Olive Oil), Komili, Kristal, Kırlangıç, Natural, Sızma and Display. Before running our model, we assigned random values for these components to have a starting point.

For each product, the average utility was formulated by considering these components. For example, for Kırlangıç Natural, the utility was calculated as the sum of Base + Kırlangıç + Natural.

μ was taken as 20 for all calculations. After calculating the utilities for these components, the preferences for each product were determined using the following formula, excluding those that were out of stock or in-store display.

$$e^{((average\ utility\ of\ product\ i - price\ of\ product\ i\ on\ day\ j)\ /\ \mu)}$$

For any stockout situation, we assign a preference value of 0 to the corresponding product's preference for the affected dates. And for any In-store Display, we calculated the corresponding products' preference using the following formula:

$$e^{((average\ utility\ of\ product\ i - price\ of\ product\ i\ on\ day\ j + price\ of\ component\ "Display") / \mu)}$$

After calculating the preference for each product, we determined the probability of each product being chosen on a given day using the formula:

$$Probability\ =\ Preference\ of\ product\ i\ on\ day\ j\ /\ Total\ \ preference\ value\ of\ products\ on\ day\ j$$

Further, the likelihoods were calculated using the following formula:

$$\Pi_i Demand\ of\ product\ i^{\ Preference\ of\ product\ i}$$

The demand data used in this calculation was obtained from the DetailedSalesData. Next, we took the logarithms of the likelihoods to calculate the log-likelihood for each date. Our goal was to maximize the sum of these log-likelihoods. To achieve this, we used Excel Solver, where the objective was the summation of the log-likelihoods, and the decision variables were the costs for each component previously determined (Base, Komili, Kristal, Kırlangıç, Natural, Sızma, and Display). The model was then solved to maximize this objective.

From the outcome of our model, we obtained the designated cost for each component as follows:

Base	Komili	Kristal	Kırlangıç	Naturel	Sızma	Display
300.052	130.088	110.109	89.8298	30.3127	49.7741	19.8857

Using these costs, we obtained the avg utility for each product as shown in the table below:

	Komili	Kristal	Kırlangıç	Kırlangıç + Natural	Kristal + Sızma	Migros	No purch
AVG UTILITY	430.1397068	410.1614	389.8819	420.1946286	459.9354486	300.0521	₺0.00

For the following questions, these attributions will be used.

Question 6

The expected demand for Komili was calculated using the probability of purchase of Komili for each day. Firstly, for linear price response, the predicted demand was calculated using the following formula:

$$Expected\ demand\ on\ day\ j\ =\ -\ 0.7981\ \times\ Price\ on\ day\ j\ +\ 355$$

For multinomial logit, it was found by multiplying the number of consumers arriving each day (100 consumers) with the probability of Komili.

$$Expected\ demand\ on\ day\ j\ =\ Probability\ of\ Komili\ on\ day\ j\ \times\ 100$$

These expected values obtained for these price-response models was then compared with the real demand. To assess which model performs better, Sum of Squared Errors (SSE) method was used.

$$Error_i\ =\ (\ Observed\ Demand\ on\ day\ j\ -\ Predicted\ Demand\ on\ day\ j\ )^2$$

From this comparison, we obtained the following results:

	Squared Error MNL	Squared Error Linear
SSE Error	4.097055922	541.9601777

Based on the SSE (Sum of Squared Errors) values, the Multinomial Logit model appears to fit the data better.

Question 7

What is the market share of each product predicted by the model that you found in Question 5. Assume the following prices. Assume that no product is out of stock and no product is selected as an "in-store display". Comment on the market shares that you found. To obtain the market share, first the preference for each product is calculated using the following formula:

$$Preference\ of\ Product\ i\ =\ e^{(\ Average\ utility\ found\ in\ Question\ 5\ -\ Price\ of\ product\ i\ )\ /\ \mu}$$

Furthermore, the probabilities were calculated by dividing the preference of each product by the sum of all preferences. The market shares were found as:

	Komili	Kristal	Kırlangıç	Kırlangıç Natural	Kristal Sızma	Migros Olive Oil	No purchase
Market Share	16.03%	5.91%	20.32%	12.52%	20.38%	12.43%	12.40%

The highest market shares are for Kırlangıç (20.32%) and Kristal Sızma (20.38%), while the lowest market share is for Kristal (5.91%).

Question 8

The same calculations from Question 7 were repeated with updated values of prices , and the values found are shown in the table below.

	Komili	Kristal	Kırlangıç	Kırlangıç Natural	Kristal Sızma	Migros Olive Oil	No purchase
Market Share	7.81%	2.88%	12.40%	6.25%	8.76%	11.32%	50.58%

It is observed that a 10% increase in the price of each product led to a significant increase in the "no purchase" probability. Additionally, the market shares for each product dropped considerably. This can be attributed to the fact that the new prices exceeded the average utility for each product, resulting in a higher likelihood of customers choosing not to purchase.

Question 9

For this model, our objective was to maximize Komili's profit by adjusting the prices of Komili and Kırlangıç products while keeping the prices of other products constant. The profit for the Komili products are calculated as:

$$Unit\ Profit\ of\ Product\ i\ =\ (\ Price\ of\ Product\ i\ \times\ 0.8\ )\ -\ Product\ cost\ of\ i$$

To find the total profit for each product, the unit profit was multiplied by its market share. Using the total profit values for Komili, Kırlangıç, and Kırlangıç Natural, we ran the model to maximize the sum of these three profits. As a result, the MNL model achieved a maximum profit of 68.45863147 and the optimal pricing for these 3 products as:

	Komili	Kırlangıç	Kırlangıç Natural
Price	449.3226494	355.5729219	380.5729203

Question 10

The comparison was made using the outcomes from Question 7. The same calculations were done after removing Kristal Sızma. Overall, the removal of Kristal Sızma led to a redistribution of its market share. This primarily benefited Kırlangıç and Komili, while also increasing the likelihood of no purchase. Even though all products had an increase in market share, Kristal had only a slight increase from 6% to 7.42%. Which led the total Kristal market shares from 26% to 7.42%. This suggests that Kristal Sızma had a significant impact on Kristal's overall sales, indicating that it was a key product for the brand.

	Komili	Kristal	Kırlangıç	Kırlangıç Natural	Migros Olive Oil	No purchase	
Market Share	20.14%	7.42%	25.53%	15.73%	15.62%	15.57%	
	Komili	Kristal	Kırlangıç	Kırlangıç Natural	Kristal Sızma	Migros Olive Oil	No purchase
Market Share (with Sızma)	16%	6%	20%	13%	20%	12%	12%

Question 11

With the introduction of Komili Sızma Olive Oil, the market share distribution changed significantly. This new product captured the largest portion of the market with 29.65%. Meanwhile, all other market shares dropped, showing that a significant portion of customers

switched to Komili Sızma. Further, it is observed that the Sızma component clearly adds significant value and utility to products, strongly influencing market dynamics. We first observed this when Kristal Sızma was removed, leading to a drop in Kristal's overall market share. Now, with Komili introducing its own Sızma variant, we see the opposite effect. As a result, Komili's total market share across all its products rose to 43.82%, proving that Sızma is a key differentiator that shapes consumer preferences and shifts the competitive landscape. The new market shares are seen in the table below.

	Komili	Kristal	Kırlangıç	Kırlangıç Natural	Komili Sızma	Migros Olive Oil	No purchase
Market Share	14.17%	5.22%	17.96%	11.06%	29.65%	10.98%	10.96%

Question 12

For this part, we changed the utility calculation for Kristal in display by adding the "Display" component's added value that we found in Question 5. As a result, Kristal's average utility increased from 410.16 to 430.05. This increase in utility directly impacted Kristal's market share, increasing it to 12.95%. This confirms that in-store displays play a big role in influencing customer preferences. Additionally, the no-purchase probability slightly decreased, suggesting that the in-store display encouraged more customers to make a purchase rather than opting out.

	Komili	Kristal - in display	Kırlangıç	Kırlangıç Natural	Komili Sızma	Migros Olive Oil	No purchase
Market Share	13.01%	12.95%	16.49%	10.16%	27.23%	10.09%	10.06%

Question 13

With the prices given in the question, the market share was calculated as it was explained in previous steps. From these market shares, the expected demand for each product was calculated by multiplying 100 (the number of customers arriving) by the market share for each product. The resulting values were then rounded to the nearest integer. The highest expected demand is for Kırlangıç, with 26 customers, followed by Komili with 22 customers. The Migros Olive Oil has the lowest expected demand, with only 6 customers.

	Komili	Kristal	Kırlangıç	Kırlangıç Natural	Kristal Sızma	Migros Olive Oil	No purchase
Market Share	22.48%	10.63%	25.78%	10.65%	13.50%	6.41%	10.55%
Expected Demand	22.48101959	10.6308051	25.78467665	10.64848332	13.49687898	6.412775438	10.54536092
Rounded	22	11	26	11	13	6	11