IE-468 Case 4 Report

NETWORK MANAGEMENT FOR THAI LION AIR

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Before the analysis, an initial breakdown of the flight routings were done to serve as the basis for the questions. The three flights were segmented as shown in the following table, and the questions were answered with reference to these routings.

ODF Routing Breakdown by Flights

Flight	Route	ODFs Using This Flight
SL1001	Chiang Mai → Suvarnabhumi	101, 102, 103, 104
SL2002	Suvarnabhumi → Koh Samui	103, 104, 105, 106, 109, 110
SL3003	Koh Samui → Phuket	107, 108, 109, 110

Question 1

a)

Deterministic Linear Model

For this part, we used the linear programming approach below.

n products (origin-destination-fares for airlines)

m resources (flight legs for airlines)

d_j (mean) demand for product j

p_j price for product j

C_i capacity for resource i

$a_{ij} = 1$ if product j uses resource i, 0 otherwise

x_j capacity allocated for product j (decision variable)

$$
\begin{aligned}
\max \; & \sum_{j=1}^{n} p_j x_j \\
\text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} x_j \leq C_i, \; i = 1, \ldots, m \\
& x_j \leq d_j, \; j = 1, \ldots, n \\
& x_j \geq 0, \; j = 1, \ldots, n
\end{aligned}
$$

Model with Numerical Parameters

$max\ 750x_1 + 500x_2 + 1000x_3 + 600x_4 + 650x_5 + 450x_6 + 700x_7 + 350x_8 + 600x_9 + 400x_{10}$

s.t.

$x_1 + x_2 + x_3 + x_4 \leq 30$
$x_3 + x_4 + x_5 + x_6 + x_9 + x_{10} \leq 30$
$x_7 + x_8 + x_9 + x_{10} \leq 30$
$x_1 \leq 11,\ x_2 \leq 30,\ x_3 \leq 9,\ x_4 \leq 16,\ x_5 \leq 8,\ x_6 \leq 23,\ x_7 \leq 9,\ x_8 \leq 21,\ x_9 \leq 10,\ x_{10} \leq 13$
$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0,\ x_7 \geq 0,\ x_8 \geq 0,\ x_9 \geq 0,\ x_{10} \geq 0$

b)

By solving the model above, we obtained the values of x_j as follows:

	x
101	11
102	10
103	9
104	0
105	8
106	13
107	9
108	21
109	0
110	0

and the objective function is **46950**. From the solution, we observed that the capacity allocated as above is for each product. There is no capacity allocated for Suvarnabhumi - Phuket with a connection to Koh Samui because of the lower fare than other flights with the same capacity.

c) This question refers to the shadow price of the capacity constraint for the Suvarnabhumi to Koh Samui route. In this case, the answer is 450, since the capacity constraint is binding and the demand for 106 is not binding.

d) This time we are looking for the Chiang Mai to Koh Samui route. Since this route is the combination of two direct flights we will simply add the shadow prices of the flight capacity constraints. These being the CS and the SK flights.

	Constraints			Shadow Price
CS	30	<=	30	500
SK	30	<=	30	450
KP	30	<=	30	350

So the minimum amount the customer will have to pay will be 950.

Question 2

a)

The allocations from the DLP might not be the best solution as the deterministic values for the expected demand do not account for demand uncertainty. This could result in non-optimal allocations as actual demand can deviate from the mean. Furthermore since we're utilizing allocations rather than nesting there is a possibility of denying full-fare customers when we have enough capacity. Without protection levels we might have lower profits due to demand uncertainty.

b)

To determine protection levels for all types of tickets in their respective flights we first used indexing by removing the other legs from connecting tickets and then bucketing them with similarly index valued tickets. The results can be seen below.

Index CS	Bucket CS	Index SK	Bucket SK	Index KP	Bucket KP
750	B1				
500	B2				
500	B2	550	B1		
100	B3	150	B2		
		650	B1		
		450	B1		
				700	B1
				350	B2
		250	B2	150	B2
		50	B2	-50	B3

To determine the protection levels after forming these buckets, we would need to find the combined distribution of the buckets since they are the combination of multiple demand distributions. Then using their weighted average prices we can use Littlewood's rule to find the optimal booking limits directly if there are only 2 buckets (virtual fare classes) like the Suvarnabhumi to Koh Samui route.

However if there are more than 2 buckets we will need to use a heuristic like ESMRa or ESMRb first to find protection levels.

Question 3

a)

The problem was solved using the following model.

$$H^t(d^t) = max \sum_{j=1}^{n} p_j x_j$$

s.t.

$$\sum_{j=1}^{n} a_{ij} x_j \leq C_i \qquad i = 1, \ldots, m$$

$$x_j \leq d_j^t \qquad j = 1, \ldots, n$$

$$x_j \geq 0 \qquad j = 1, \ldots, n$$

The number of samples T was set to 10. For each of the 10 samples, demands were generated from the given distributions of each Origin-Destination-Fare (ODF) pair. These determined values can be seen in the table below.

n	1	2	3	4	5	6	7	8	9	10
101	11	9	14	12	12	8	9	11	9	12
102	32	30	30	32	33	28	28	28	28	29
103	10	9	11	9	11	9	6	6	9	11
104	14	15	14	18	18	15	14	17	15	18
105	6	10	10	9	8	9	7	8	6	10
106	23	26	23	25	21	26	20	25	25	23
107	7	10	6	6	9	12	7	7	7	10
108	16	21	15	24	21	26	17	28	23	16
109	11	10	11	9	9	9	11	8	11	11
110	12	12	10	17	11	14	9	15	16	9

Each of these scenarios was solved using the deterministic linear program defined in Question 1. The allocations (x) found for the samples can be seen in the table below. The average for the x values from x101 to x110 was also computed to be used in the model.

Sample	1	2	3	4	5	6	7	8	9	10	Average
x101	11	9	14	12	12	8	9	11	9	12	10.7
x102	9	12	5	9	7	13	15	13	12	7	10.2
x103	10	9	11	9	11	9	6	6	9	11	9.1
x104	0	0	0	0	0	0	0	0	0	0	0
x105	6	10	10	9	8	9	7	8	6	10	8.3
x106	7	11	0	12	11	12	11	16	15	5	10
x107	7	10	6	6	9	12	7	7	7	10	8.1
x108	16	20	15	24	21	18	17	23	23	16	19.3
x109	7	0	9	0	0	0	6	0	0	4	2.6
x110	0	0	0	0	0	0	0	0	0	0	0

b)

To calculate the shadow price for a new customer from Suvarnabhumi to Koh Samui, we used the following formula:

$$\pi_j^{RLP} = \frac{1}{T}\sum_{t=1}^{T} \pi_j^t(d_t)$$

$\pi_j^t(d_t)$ was found as 450 for each sample, therefore the minimum price that we would like to charge for this new request would be 450.

c)

Since we do not have the direct flight from Chiang Mai to Koh Samui, the shadow price is calculated as the summation of the shadow prices of the flights SL1001 and SL2002. The same calculations were done as in part b. For the flight SL2002, it was calculated as 500, and SL1001 was previously determined as 450. Therefore, the minimum price that we would like to charge for this new request would be 950.

Question 4

a)

The following model was used to solve the problem using the Probabilistic Non-Linear Program.

$$max \sum_{j=1}^{n} p_j \sum_{d=1}^{M_j} z_{jd} P(D_j \geq d)$$

s.t.

$$x_j = \sum_{d=1}^{M_j} z_{jd} \qquad j = 1, \ldots, n$$

$$\sum_{j=1}^{n} a_{ij} x_j \le C_i \qquad i = 1, \ldots, m$$

$$x_j \ge 0 \qquad j = 1, \ldots, n$$

$$0 \le z_{jd} \le 1 \quad j = 1, \ldots, n, \ d = 1, \ldots, M_j$$

b)

After solving the probabilistic non-linear program, we obtained the following results for decision variables

j	1	2	3	4	5	6	7	8	9	10
x	10	14	6	0	7	17	10	20	0	0
z_jd*P(D_j >=d)	9.57143	14	6	0	6.8	17	8.57143	18.6	0	0
p	750	500	1000	600	650	450	700	350	600	400

The objective value from the solution is:

Obj=	44758.6

c)

In previous cases, we could assign the shadow price by simply checking whether the price or demand constraints were tight. However, in this case, since the acceptance of bookings depended on probability, we did not have a binding capacity constraint. Therefore, we manually re-solved the model by adding one more booking and observed which additional ticket was accepted. The resulting increase in expected revenue was 450, which came from ODF 106. Thus, the minimum price we would like to charge for the new request is 450.

d)

Similarly, the same process was applied for each flight. For the request from Chiang Mai to Koh Samui, two connecting flights are required: Chiang Mai to Suvarnabhumi and Suvarnabhumi to Koh Samui. We manually re-solved the model for each leg to observe the additional accepted ticket and determine the corresponding increase in expected revenue. The shadow prices were found to be 450 for the Chiang Mai to Suvarnabhumi leg and 500 for the Suvarnabhumi to Koh Samui leg. Therefore, the minimum price we would like to charge for this new request is 950.

The solutions obtained from re-solving the model are presented in the table below. The table indicates which ODF would be used for the additional booking in each case, which we used to determine the shadow price.

	x	CS	SK	KP
101	10	10	10	10
102	14	15	14	14
103	6	6	6	6
104	0	0	0	0
105	7	7	7	7
106	17	17	18	17
107	10	10	10	11
108	20	20	20	20
109	0	0	0	0
110	0	0	0	0