# IE-468 Case 3 Report

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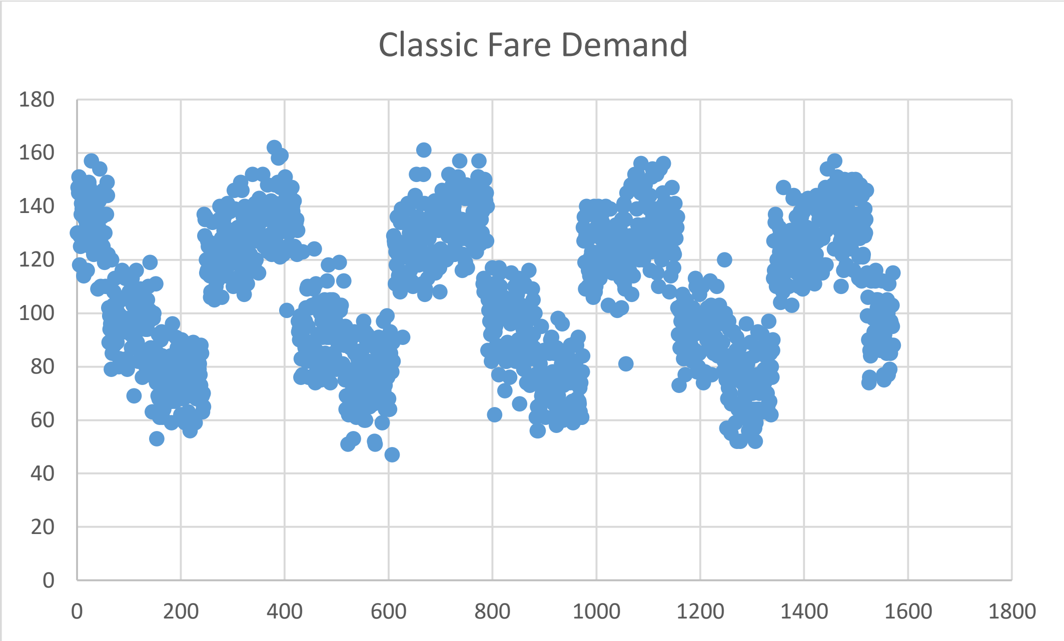
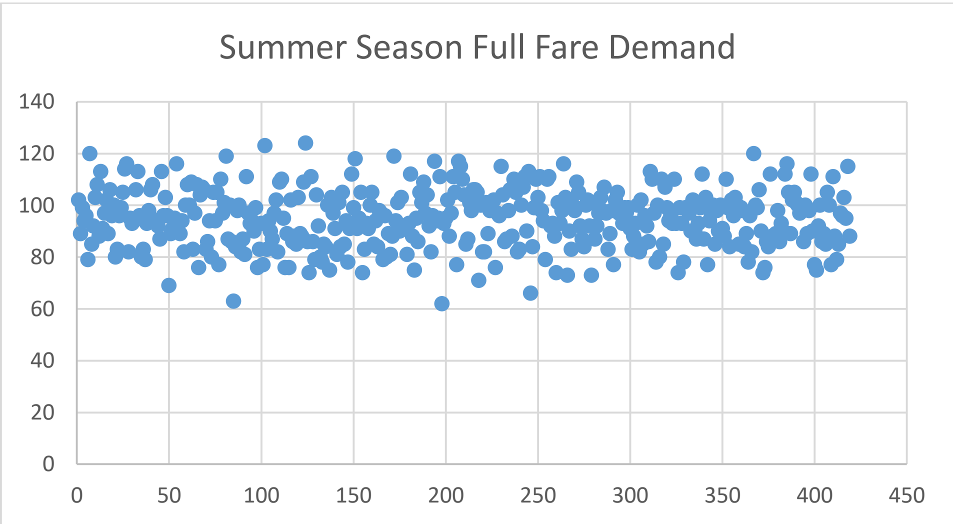
Q1)

To give us an initial idea on estimating a distribution for the full fare customer demand we graphed the demand. For this case we didn't estimate a distribution for discount customers as it is assumed that the demand for this class is greater than capacity.




From the graph we can see the effects of seasonality as there is a clear "on" and "off" season for this route. Investigating further there seems to be a partial "off" season in the summer; June, July and August, which is the period we'll focus on in this question.




For each day of the week June 16-22, 2025 we estimated the parameters of its distribution using regression. So we assumed the demand for full fare tickets was normal within a few categories. Using linear regression, we determined that the month and the day-of-week are significant attributes for estimating demand.

Using the coefficients from our model we estimated the means for each day. We then utilized our residual sum of squares to estimate the standard error of regression as our standard deviation for all days.

$$Standard\ Error = \frac{\sqrt{\text{Residual Sum of Squares}}}{\text{\# of Data Points} - \text{\# of Model Parameters}}$$

So we estimated the parameters for the normal distribution of demand for these days as:

| Date | Predicted_Mean | Standard Error |
|---|---|---|
| 6/16/2025 | 70.37712937 | 10.05733202 |
| 6/17/2025 | 71.34806194 | |
| 6/18/2025 | 71.01319372 | |
| 6/19/2025 | 77.47891231 | |
| 6/20/2025 | 78.87460995 | |
| 6/21/2025 | 78.87538835 | |
| 6/22/2025 | 70.61417632 | |

The full results of our regression model:

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                Classic   R-squared:                       0.853
Model:                            OLS   Adj. R-squared:                  0.852
Method:                 Least Squares   F-statistic:                     531.0
Date:                Thu, 24 Apr 2025   Prob (F-statistic):               0.00
Time:                        12:53:43   Log-Likelihood:                -5846.1
No. Observations:                1572   AIC:                         1.173e+04
Df Residuals:                    1554   BIC:                         1.182e+04
Df Model:                          17
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
const            87.4156      0.207    421.830      0.000      87.009      87.822
day_Friday       17.2795      0.620     27.870      0.000      16.063      18.496
day_Monday        8.7820      0.621     14.137      0.000       7.563      10.001
day_Saturday     17.2803      0.620     27.872      0.000      16.064      18.496
day_Sunday        9.0191      0.620     14.547      0.000       7.803      10.235
day_Thursday     15.8838      0.620     25.619      0.000      14.668      17.100
day_Tuesday       9.7529      0.621     15.700      0.000       8.534      10.971
day_Wednesday     9.4181      0.621     15.161      0.000       8.200      10.637
month_1          35.7131      0.778     45.933      0.000      34.188      37.238
month_2          33.7887      0.812     41.625      0.000      32.196      35.381
month_3          -4.5224      0.777     -5.817      0.000      -6.047      -2.997
month_4          -5.5990      0.814     -6.875      0.000      -7.196      -4.001
month_5          -5.6803      0.861     -6.596      0.000      -7.369      -3.991
month_6         -25.8205      0.874    -29.531      0.000     -27.535     -24.105
month_7         -24.9293      0.861    -28.948      0.000     -26.619     -23.240
month_8         -24.7515      0.861    -28.741      0.000     -26.441     -23.062
month_9          23.0638      0.874     26.378      0.000      21.349      24.779
month_10         26.0609      0.861     30.261      0.000      24.372      27.750
month_11         24.8493      0.874     28.420      0.000      23.134      26.564
month_12         35.2429      0.858     41.075      0.000      33.560      36.926
==============================================================================
Omnibus:                        3.983   Durbin-Watson:                   2.040
Prob(Omnibus):                  0.136   Jarque-Bera (JB):                3.935
Skew:                          -0.097   Prob(JB):                        0.140
Kurtosis:                       2.850   Cond. No.                     5.21e+15
==============================================================================
```

a)

To determine the protection and booking levels we utilized Littlewood's rule, since our goal is to maximise profit rather than revenue we first deducted the 4 € cost for food from prices.

$$p_f P(D_f \geq C - b^*) = p_d$$

$$P(D_f \leq 161 - b^*) = 0.34266$$

Meaning our z value will be -0.40522, so we calculated the booking limit with the cdf of the normal distribution using the parameters we estimated previously for each day:

$$C - b^* = \sigma_f z^* + \mu_f$$

Our rounded protection and booking limits are:

| | b | y |
|---|---|---|
| 6/16/2025 | 95 | 66 |
| 6/17/2025 | 94 | 67 |
| 6/18/2025 | 94 | 67 |
| 6/19/2025 | 88 | 73 |
| 6/20/2025 | 86 | 75 |
| 6/21/2025 | 86 | 75 |
| 6/22/2025 | 94 | 67 |

b)

To calculate the expected load factor, we first calculated the expected number of empty seats using our previous protection levels and booking limits. As this number can't be negative we used the expectation of a max function.

$$E(\max\{y^* - D_f, 0\}) = \sigma_f E(\max\{w - z^*, 0\}) - \mu_f + y^*$$

$$= \sigma_f L(z^*) - \mu_f + y^*$$

Where w is the standard normal an L(z) is the standardized loss function. Using the previously calculated z value for the value of the standardised loss function we get:

$$L(-0.40522) = 0.63387$$

Using the previously found formula we calculated the expected number of non-empty seats which is the same for all days since the probability of having empty seats is the same for all days with their respective protection levels.

$$158.7005 = 161 - 2.2995$$

And the expected load factor as:

98.572% = 158.7005/161

c)

We calculated the Revenue Per Available Seat Kilometer (RASK) by utilizing both the yields and changing load factor contributions of full fare and discount customers. Using revenue rather than profit, the yields are:

0.04047 €/km for full fare tickets
0.02703 €/km for discount tickets

And their contributions towards the load factor change day by day, so we found each day's values with their respective booking limits. Multiplying load factors and yields within their ticket classes finally gave us the daily RASK as can be seen below.

| Load-discount | Load-full | RASK |
|---|---|---|
| 0.588188405 | 0.397529 | 0.031984 |
| 0.582157768 | 0.403559 | 0.032065 |
| 0.584237695 | 0.401479 | 0.032037 |
| 0.544077952 | 0.441639 | 0.032577 |
| 0.535409023 | 0.450308 | 0.032693 |
| 0.535404188 | 0.450313 | 0.032693 |
| 0.586716064 | 0.399001 | 0.032004 |

d)

Using the previously found standardized loss function value again we calculated the expected number of full fare customers denied booking as: $\sigma_f L(z^*) = 6.375$

Q2)

a)

For Question Three, we used data only from Thursday, Friday, and Saturday demand data as requested, resulting in a sample size of 675 for the Sample Average Approximation method. Next, we created our mathematical model to find booking limits and protection levels.

**Parameters:**

$\boldsymbol{d_f^t}: full\ fare\ demand\ for\ sample\ t \qquad t = 1,2,\ldots 675$
$\boldsymbol{C}: capacity\ of\ the\ vueling$
$\boldsymbol{p_d}: discounted\ profit\ of\ a\ ticket$
$\boldsymbol{p_f}: full\ fare\ profit\ of\ a\ ticket$

**Decision Variables:**

$\boldsymbol{y}: protection\ level$
$\boldsymbol{x^t}: number\ of\ full\ fare\ tickets\ we\ sell\ in\ sample\ t$

**Objective Function**

$$\max\ \frac{1}{n}\left[\sum_{1}^{675}[p_d(C-y) + p_f x^t]\right]$$

**Constraints**

$x^t \leq y$

$x^t \leq d_f^t$

By solving mathematical model, we found **y = 98** (protection level) and **b = 63** (booking limit).

b)

Since we used only the Thursdays, Fridays, Saturdays data; We didn't include the seasonality factor in this approach resulting in higher protection level than off-season which we found in Question 1. Therefore, it is less profitable and explain worse the patterns.

c)

We used the same approach to calculate the load factor as Question 1. Then calculated the load factory with new protection level and booking limit. Except we calculated mean with simple average and standard deviation with the simple standard deviation formula as mentioned in the question.

| | |
|---|---|
| z= | -0,5482 |
| Loss | 0,731533 |
| | |
| Expected empty seats | 4,704172 |
| Expected non-empty | 156,2958 |
| Load Factor | 0,970782 |

d)

Similar to first question we calculated expected denied customers by multiplying the standard deviation with standard loss function.

| | |
|---|---|
| Expected Denied Customers | 18,77083916 |

Q3)

The parameters of these days' demand distributions were again calculated with the results from our regression model in Q1 and are assumed to be normal. We again deducted the costs from prices in our calculations so we used the profit margin $p_p = 205.49$ for premium tickets.

a)

Using the EMSRa heuristic, first we found the individual protection levels of premium (1) and full fare (2) tickets against discount (3) tickets. Using Littlewood's rule we calculated the z values for each days' full fare ticket distribution and found the protection levels with $y_{13} = \sigma_1 z + \mu_1$ and $y_{23} = \sigma_2 z + \mu_2$ which is calculated separately for each day as the mean changes from day to day.

Summing these values gave us $y_2$ which is the protection level for both class 1 and 2 against class 3. And finally we calculated $y_1 = \sigma_1 z + \mu_1$ to give us the protection level for class 1 against class 2. The final protection levels and booking limits that don't change from day to day are:

| | |
|---|---|
| y13 | 14 |
| y1 | 12 |
| b2 | 149 |
| b1 | 161 |

And the values that change day to day are:

| | y23 | y2 | b3 |
|---|---|---|---|
| 6/17/2025 | 67 | 81 | 80 |
| 6/18/2025 | 67 | 81 | 80 |
| 6/19/2025 | 73 | 87 | 74 |
| 6/20/2025 | 75 | 89 | 72 |
| 6/21/2025 | 75 | 89 | 72 |
| 6/22/2025 | 67 | 81 | 80 |
| 6/23/2025 | 66 | 80 | 81 |

b)

Using the EMSRb heuristic we artificially create a unified class by summing the demand for class 1 and 2 tickets, $D_{12}$.

This class has a normal distribution with parameters N($\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2}$) that change day to day due to the change in parameters for the full fare demand distribution.

| D12 | Mean | Std |
|---|---|---|
| 6/17/2025 | 84.34806 | 10.49523 |
| 6/18/2025 | 84.01319 | 10.49523 |
| 6/19/2025 | 90.47891 | 10.49523 |
| 6/20/2025 | 91.87461 | 10.49523 |
| 6/21/2025 | 91.87539 | 10.49523 |
| 6/22/2025 | 83.61418 | 10.49523 |
| 6/23/2025 | 83.37713 | 10.49523 |

Now to use Littlewood's rule to calculate the protection levels we first came up with the prices for this unified class for each day using the formula for weighted prices.

$$\frac{\mu_1 p_1 + \mu_2 p_2}{\mu_1 + \mu_2}$$

| | P12 |
|---|---|
| 6/17/2025 | 137.8199 |
| 6/18/2025 | 137.869 |
| 6/19/2025 | 136.9844 |
| 6/20/2025 | 136.8098 |
| 6/21/2025 | 136.8097 |
| 6/22/2025 | 137.9281 |
| 6/23/2025 | 137.9634 |

Then, using Littlewoods rule, we calculated the protection level for the artificial class $y_2$ as:

$$y_2 = \sigma_{12} z + \mu_{12}$$

And using the same method we calculated the protection level $y_1 = \sigma_1 z + \mu_1 \cong 12$, which is the same for every day. We calculated the booking limits by deducting the respective protection levels from capacity such as $b_2 = C - y_1 = 149$.

The results that don't change day to day are:

| | |
|---|---|
| y3 | 161 |
| y1 | 12 |
| b1 | 161 |
| b2 | 149 |

And the results that change day to day are:

| | y2 | b3 |
|---|---|---|
| 6/17/2025 | 82 | 79 |
| 6/18/2025 | 81 | 80 |
| 6/19/2025 | 88 | 73 |
| 6/20/2025 | 89 | 72 |
| 6/21/2025 | 89 | 72 |
| 6/22/2025 | 81 | 80 |
| 6/23/2025 | 81 | 80 |