

Name:_____

This exams has 5 sheets to a total of 10 pages!

Write your name on the first page and your student number at the top-right corner of each page.

The exam contains two parts: required (pages 2-6) and advanced (pages 7-8). You can obtain at most 20 points out of 24 from the required part. You can achieve a grade of up to 25 points by solving addition problems from the advanced part.

The total of points in all the questions exceed 25. Your grade, however, cannot exceed 25 points. Since points have equal value, you are better off solving one question completely instead of solving partially multiple questions.

If necessary use the page labeled **extra space** for presenting your answers. Indicate clearly which question(s) you are writing on that page.

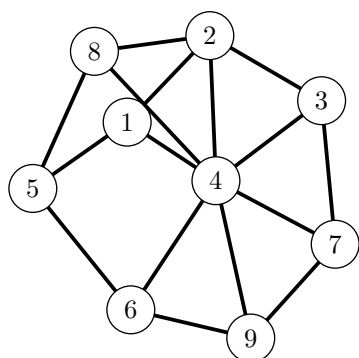
If necessary use the last page (page number 10) for draft work. Do *not* detach pages.

Provide justification for all your answers. Answers without justification will not be given any credit. Remember that obviously wrong answer may earn up to one *negative* point per problem.

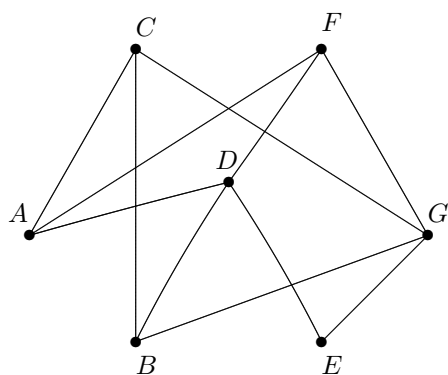
If you are using results from class state them (without proofs).

Don't panic and good luck!

/2pts Write the adjacency matrix of \mathcal{G}_1 pictured bellow. (label the rows and columns)



/2pts Write the incidence matrix of \mathcal{G}_{alp} , given bellow. Index both the rows and the columns of the matrix. To index say the edge between vertex B and vertex D use the label (BD) .



\mathcal{G}_{alp}

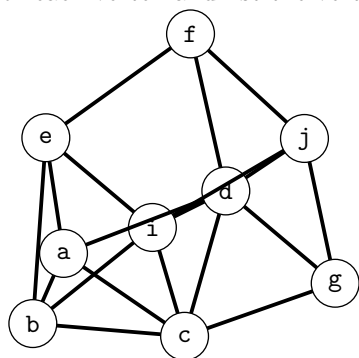
/2pts Are \mathcal{G}_1 and \mathcal{G}_{alp} isomorphic?

/2pts Is K_4 a subgraph of the graph \mathcal{G}_1 from the previous page

/2pts What is the girth of \mathcal{G}_1 ?

/2pts Is there a graph with a degree sequence 4, 4, 4, 4, 4, 4, 4, 4?

/3pts Draw a BFS tree for the graph pictured below with root g . Identify the parent child relation, the level for each vertex and list the vertices in order they entered your queue.



/3pts What is the total number of vertices in a tree that has four vertices of degree two, five vertices of degree three and six vertices of degree four and all remaining vertices of degree one?

/3pts Can a graph with no cut edges contain a cut vertex?

/3pts What is the number of edges in $K_{7,5}$?

/3pts Let K_5 be the complete graph on five vertices. Find the determinant of the adjacency matrix of K_5 .

/3pts Let A be a matrix such that $A^2 = A$. What are the possible eigenvalues of A ?

/3pts Show that the length of the shortest path from u to v in a connected graph G equals the level of v in *any* BFS tree of G rooted at u .

/3pts Let G be a graph on n vertices, where $n \geq 3$. Suppose that $\Delta(G) \geq \frac{n}{2}$. Can G have more than one component?

/3pts Is it true that any 20 regular graph has a path of length at least 10?

extra space:

draft work: