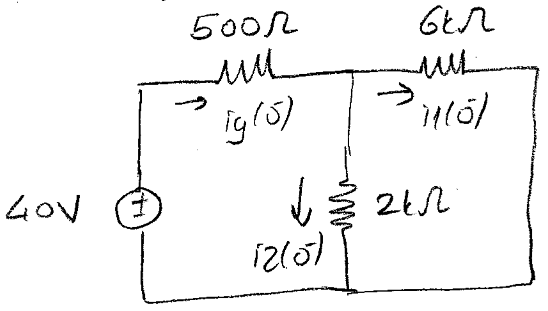
SOLUTIONS OF PROBLEM SET 7

1- (a) for $t<0$, the equivalent circuit is



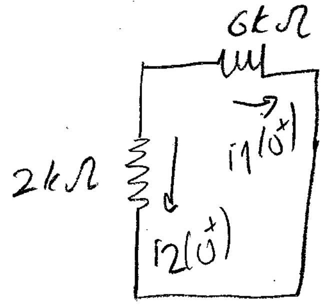
$$i_g(0^-) = \frac{40}{\frac{2\times 6}{2+6} + 0.5} = \frac{40}{2} = \underline{20\,mA}$$

The branch currents can be found using current division

$$i_1(0^-) = \frac{2}{8} \times 20 = 5\,mA \qquad i_2(0^-) = \frac{6}{8} \times 20 = 15\,mA$$

(b) Due to the continuity condition of the current in an inductor $i_1(0^+) = i_1(0^-) = 5\,mA$

The equivalent circuit for $t \geq 0$, just at the instant the switch opens at $t=0^+$.



$$i_1(0^+) = -i_2(0^+) \rightarrow i_2(0^+) = -i_1(0^+)$$
$$i_2(0^+) = -5\,mA$$

(c) $\tau = \frac{L}{R} = \frac{0.4\times 10^{-3}\,H}{8\times 10^{3}\,\Omega} = 5\times 10^{-5}\,s$, $\frac{1}{\tau} = 20{,}000$

$$i_1(t) = I_1(0^+)e^{-t/\tau} = 5e^{-20{,}000t}\ mA \qquad t \geq 0.$$

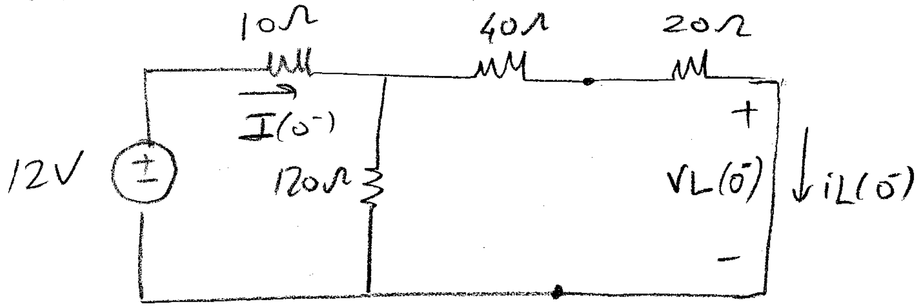
(d) when $t \geq 0^+$ $\quad i_2(t) = -i_1(t) \therefore i_2(t) = -5e^{-20{,}000t}\ mA,\ t \geq 0^+$

(e) Once the switch opens the circuit changes and the current in the resistor changes suddenly from 15mA to -5mA. The current in a resistor can be discontinous.

$$\therefore i_2(0^-) = 15\,mA \text{ and } i_2(0^+) = -5\,mA$$

7-3 (a) $i_0(0^-) = 0$, since the switch is open for $t<0$.

(b) For $t = 0^-$ the equivalent circuit is



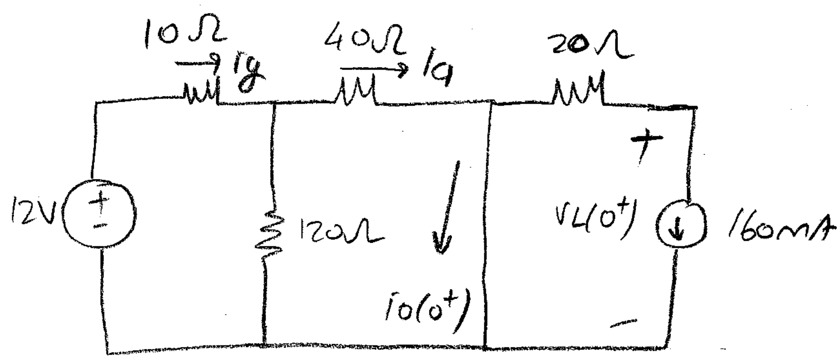
$$R_{eq} = 10 + 120 \,//\, 60 = 10 + \frac{120 \times 60}{120+60} = 50\Omega$$

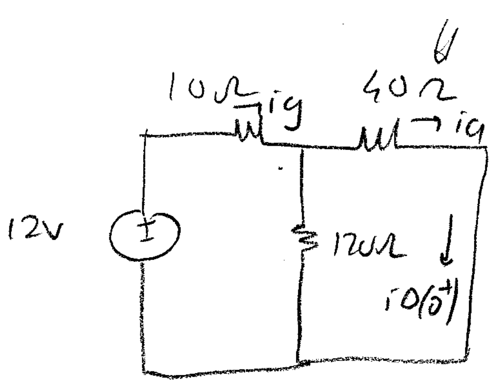
$$I(0^-) = \frac{12}{50} = 0.24A = 240mA$$

$i_L(0^-)$ can be found using current division rule.

$$i_L(0^-) = I(0^-) \times \frac{120}{120+60} = (240mA) \times \frac{2}{3} = 160\ mA$$

(c) For $t = 0^+$, the equivalent circuit is





We can ignore this part of the circuit as no current flows there due to the short circuit at the inductor branch.

$$R_{eq} = 40\Omega \,\|\, 120\Omega = 30\Omega$$

$$\therefore i_g = \frac{12}{10+30} = 0.30A = 300mA$$

$$i_a = \left(\frac{120}{160}\right) \times 300 = 225mA$$

$\therefore i_0(0^+) = 225 - 160 = 65mA$ (Kirchoff's current law.)

(d) $i_L(0^+) = i_L(0^-) = 160\,mA$

(e) $i_o(\infty) = i_a = 225\,mA$, becaus at $t \to \infty$ the inductor loads like a short circuit.

(f) $i_L(\infty) = 0$, since the switch short circuits the branch containing the 20Ω resistor and the $100mH$ inductor.

(g) $\tau = \frac{L}{R} = \frac{100 \times 10^{-3}}{20} = 5ms$; $\frac{1}{\tau} = 200$

$\therefore \quad i_L = 0 + (160 - 0)e^{-200t} = 160e^{-200t}\,mA, \quad t \geq 0$

(h) $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

(i) Refer to the circuit at $t = 0^+$ and note:

$20(0.16) + v_L(0^+) = 0 \quad \therefore \quad v_L(0^+) = -3.2V$

(j) $v_L(\infty) = 0$, since the current in the inductor is a constant at $t = \infty$.

(k) $v_L(t) = 0 + (-3.2 - 0)e^{-200t} = -3.2e^{-200t}\,V, \quad t \geq 0^+$

(l) $i_o = i_a - i_L = 225 - 160e^{-200t}\,mA, \quad t \geq 0^+$

7.4 (a) $i(0^+) = 10 \qquad v_o(t) = 400 \qquad R = \frac{v(0^+)}{i(0^+)} = 40\Omega$

b) $\tau = \frac{1}{5} = 0.2$ $\qquad$ c) $L = \tau R = (0.2 \times 40) = 8H$

d) $W = \frac{1}{2} L i(0^+)^2 = \frac{1}{2}(8) \times 10^2 = 400J$

e) $p(t) = v(t)\,i(t) = 4000\,e^{-10t}$

$$W = \int_0^{t'} 4000 e^{-10t}\, dt = -\frac{1}{10} \times 4000 \times \left[e^{-10t}\right]_0^{t'}$$

$$= -400 \times \left[e^{-10t'} - 1\right] = 400\left[1 - e^{-10t'}\right]$$

This expression gives the energy as a function of time at an arbitrary time t'. We want this to be equal to 80% of the initial energy, which is $w(0) = 400J$.

$$400\left[1 - e^{-10t'}\right] = 320$$

$$1 - e^{-10t'} = 0.8$$

$$e^{-10t'} = 0.2$$

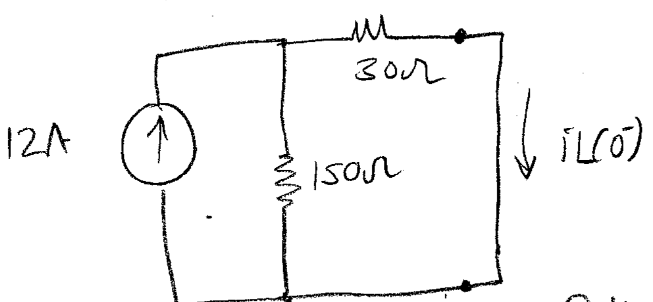
$$-10t' = \ln 0.2 \Rightarrow t' = -\frac{\ln 0.2}{10} = 0.1609s$$

$\underline{160.9ms}$

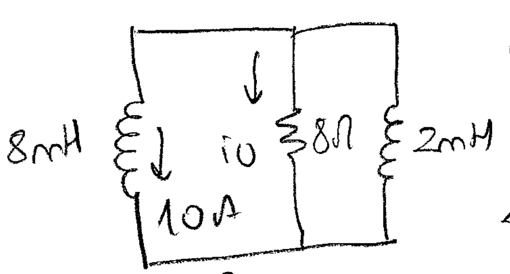
7-8 (a) For $t<0$, we obtain the following equivalent circuit as the inductor appears as a short circuit.

Using Current Division:

$$i_L(0^-) = \frac{150}{180}(12) = 10A$$

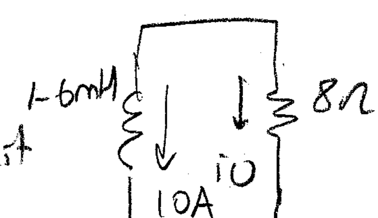


For $t \geq 0$, we get the following equivalent circuit



which can be represented with the following circuit

Using $L_{eq} = \frac{L_1 * L_2}{L_1 + L_2}$



$\tau = \frac{L}{R}$ $\quad \tau = \frac{1.6 \times 10^{-3}}{8} = 200 \times 10^{-6}$; $\quad 1/\tau = 5000$ $\quad i_0 = -10e^{-5000t} A \quad t \geq 0.$

(b) $W_{del} = \frac{1}{2}(1.6\times10^{-3})(10)^2 = 80\,mJ$ $\quad (w = \frac{1}{2} L_{eq} I_L(0^+)^2)$

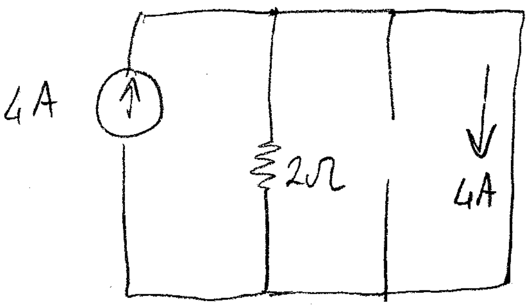
(c) $0.95\, W_{del} = 76\,mJ$

$\therefore 76\times10^{-3} = \int_0^{t_0} (100 \times e^{-10{,}000t})\,dt$ $\quad \left(\begin{array}{l} p(t) = v(t)\,i(t) \\ v(t) = L\,di/dt \end{array}\right)$

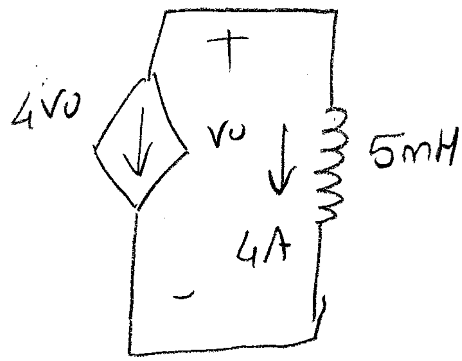
$\therefore 76\times10^{-3} = -80\times10^{-3} e^{-10{,}000t}\Big|_0^{t_0} = 80\times10^{-3}(1 - e^{-10{,}000t_0})$

$\therefore e^{-10{,}000t_0} = 4\times10^{-3}$ so $t_0 = 552.1\,\mu s$

-13 for $t<0$ at ($t=0^-$) the equivalent circuit is

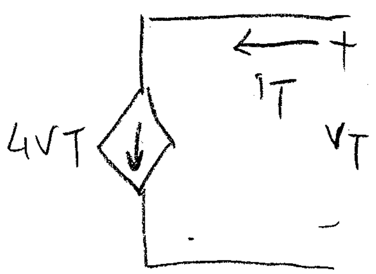


for $t>0$



$i_L(0^-) = i_L(0^+) = 4A$

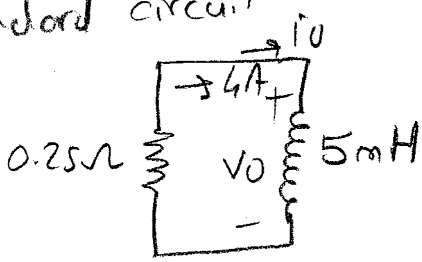
we find the Thevenin resistance seen by the inductor



$I_T = 4V_T$; $\quad \frac{V_T}{i_T} = R_{Th} = \frac{1}{4} = 0.25\,\Omega$

$\tau = \frac{L}{R} = \frac{5\times10^{-3}}{0.25} = 20\,ms$, $\quad \frac{1}{\tau} = 50$

We can now replace the above circuit with the following standard circuit.



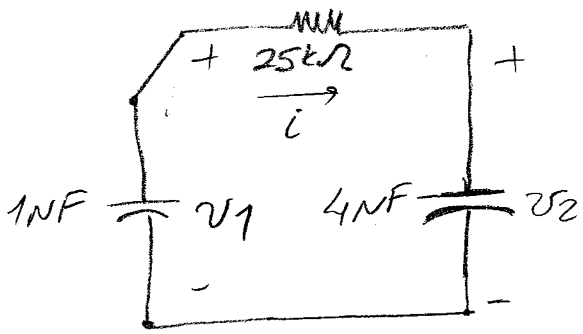
$i_o = 4e^{-50t}\,A, \quad t \geq 0$

$v_o = L\frac{di_o}{dt} = (5\times10^{-3})(-2\times10^{2}e^{-50t}) = -e^{-50t}\,V, \quad t \geq 0^+$

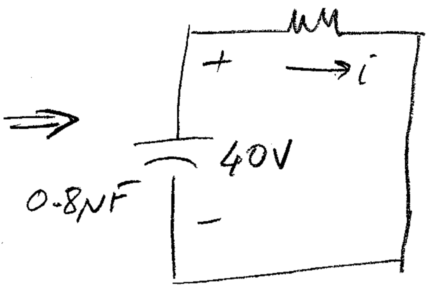
7.21 a) $v_1(0^-) = v_1(0^+) = 40V$, when steady state is reached the capacitor voltage is at 40V, the capacitor looks like an open circuit, no current flows through it.

When the switch moves to position b, initially the 4NF capacitor is uncharged, therefore the voltage across the capacitor is zero.

The circuit for $t \geq 0^+$



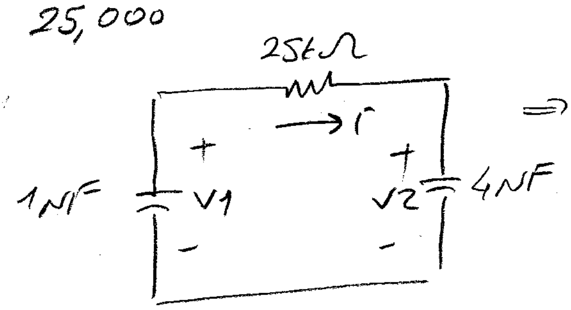
$\Rightarrow$



$$C_{eq} = (1)(4)/5 = 0.8NF \qquad \tau = RC = (25\times10^3)(0.8\times10^{-6}) = 20ms$$

$$\frac{1}{\tau} = 50$$

$$i = \frac{40}{25{,}000} e^{-50t} = 1.6e^{-50t} \, mA \qquad t \geq 0^+$$



$\Rightarrow$ original circuit

$$v_1 = \frac{-1}{10^{-6}} \int_0^t 1.6\times10^{-3} e^{-50x} dx + 40 = 32e^{-50t} + 8V, \quad t \geq 0$$

$$v_2 = \frac{1}{4\times10^{-6}} \int_0^t 1.6\times10^{-3} e^{-50x} dx + 0 = -8e^{-50t} + 8V, \quad t \geq 0$$

(b) The energy stored in the 1NF capacitor can be found using the voltage value $v_1(0^-) = 40V$ and $W = \frac{1}{2}CV^2$;

$$W = \frac{1}{2}(10^{-6}F)(40V)^2 = 800NJ$$

(c) $W_{trapped} = \frac{1}{2}(10^{-6})(8)^2 + \frac{1}{2}(4\times10^{-6})(8)^2 = 160NJ.$

The energy dissipated by the 25kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors (final voltage on the equivalent capacitor is zero):

$$w_{dis} = \frac{1}{2}(0.8\times10^{-6})(40)^2 = 640\,\mu J$$

Check: $w_{trapped} + w_{diss} = 160 + 640 = 800\,\mu J$;

$w(0) = 800\,\mu J$.

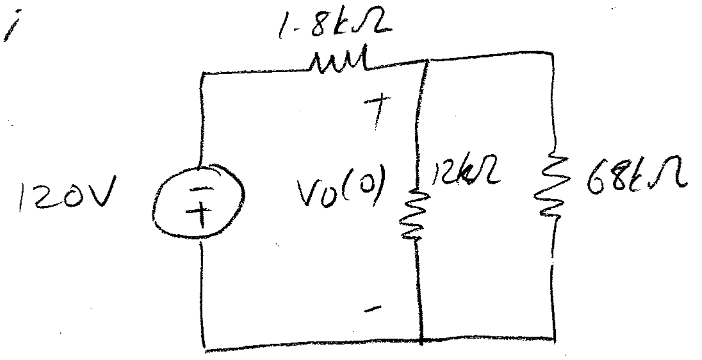
7.23 (a) $R = \frac{v}{i} = 4k\Omega$

(b) $\frac{1}{\tau} = \frac{1}{RC} = 25$; $\quad C = \frac{1}{(25)(4\times10^3)} = 10\,\mu F$

(c) $\tau = \frac{1}{25} = 40ms$

(d) $w(0) = \frac{1}{2}(10\times10^{-6})(48)^2 = 11.52mJ$

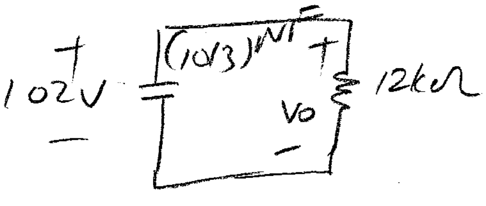
7.25 (a) $t<0$;



$R_e = 12k \| 68k = 10.2k\Omega$

$$v_o(0) = \frac{10{,}200}{10{,}200+1800}(-120) = -102V$$

$t>0$



$\tau = [(10/3)\times10^{-6})(12{,}000)] = 40ms$; $\quad \frac{1}{\tau} = 25$

$v_o = -102e^{-25t}V$, $\quad t \geq 0$

$$p = \frac{v_o^2}{12{,}000} = 867 \times 10^{-3} e^{-50t} \, W$$

$$w_{diss} = \int_0^{12 \times 10^{-3}} 867 \times 10^{-3} e^{-50t} \, dt = 17.34 \times 10^{-3} (1 - e^{-50(12 \times 10^{-3})}) = 7.82 \, mJ$$

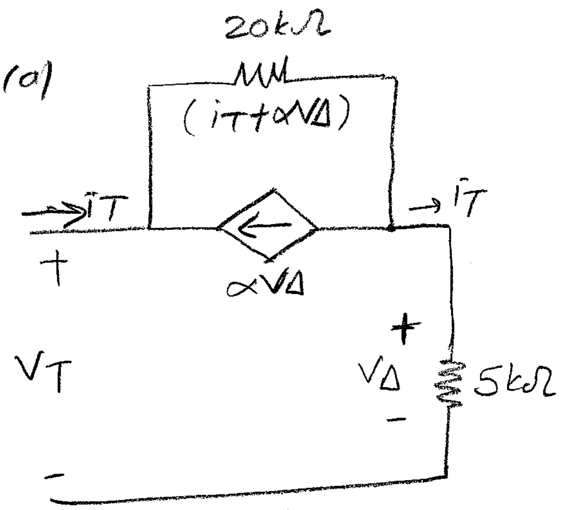
(b) $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34 \, mJ$

$0.75\,w(0) = 13 \, mJ$

$$\int_0^{t_o} 867 \times 10^{-3} e^{-50x} \, dx = 13 \times 10^{-3}$$

$\therefore \; 1 - e^{-50t_o} = 0.75; \quad e^{50t_o} = 4; \quad$ so $t_o = 27.73 \, ms$.

7.26 (a)



$v_T = 20 \times 10^3 (i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$

$v_\Delta = 5 \times 10^3 i_T$

$v_T = 25 \times 10^3 i_T + 20 \times 10^3 \alpha (5 \times 10^3 i_T)$

$R_{Th} = \frac{v_T}{i_T} = 25{,}000 + 100 \times 10^6 \alpha$

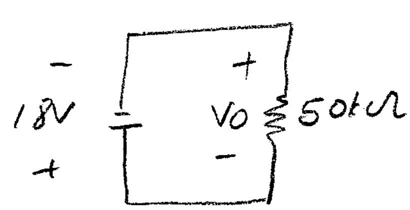
$\tau = R_{Th} C = 40 \times 10^{-3} = R_{Th} (0.8 \times 10^{-6})$

$R_{Th} = 50 \, k\Omega = 25{,}000 + 100 \times 10^6 \alpha$

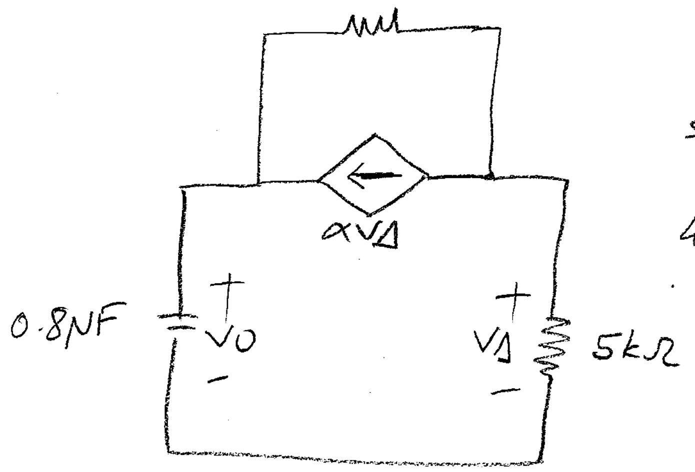
$$\alpha = \frac{25{,}000}{100 \times 10^6} = 2.5 \times 10^{-4} \, A/V$$

(b) $v_o(0) = (-5 \times 10^3)(3600) = -18 \, V \qquad t < 0$

$t > 0$:



$v_o = -18 e^{-25t} \, V, \; t \geq 0$

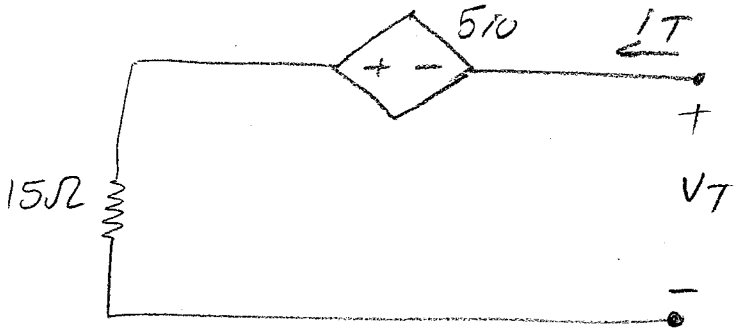


$$\frac{v_\Delta}{5000} + \frac{v_\Delta - v_o}{20{,}000} + 2.5\times10^{-6} v_\Delta = 0$$

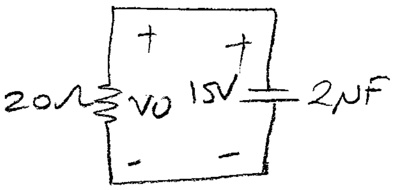
$$4v_\Delta + v_\Delta - v_o + 5v_\Delta = 0$$

$$\therefore v_\Delta = \frac{v_o}{10} = -1.8e^{-25t}\ \text{V},\quad t \geq 0^+$$

7.28



$$v_T = -5i_0 + 15i_0 = -20i_0 = 20i_T \quad \therefore \quad R_{Th} = \frac{v_T}{i_T} = 20\,\Omega$$



$$\tau = RC = 40\,\mu s; \quad \frac{1}{\tau} = 25{,}000$$

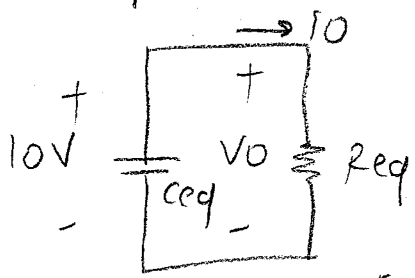
$$v_o = 15e^{-25{,}000t}\ \text{V}, \quad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\ \text{A}, \quad t \geq 0^+$$

7.29 (a) C_{eq} parallel combination $= 0.2\,\mu F + 0.8\,\mu F = 1\,\mu F$

$$C_{eq} = \frac{1 \times 0.25}{1 + 0.25} = \frac{0.25}{1.25} = 0.2\,\mu F$$

$$R_{eq} = 0.4 + \frac{24 \times 16}{24 + 16} = 0.4 + \frac{384}{40} = 0.4 + 9.6 = 10\,k\Omega$$



$$\tau = 2\,ms \quad \frac{1}{\tau} = 500$$

$$v_o = 10e^{-500t}\ \text{V}, \quad t \geq 0$$

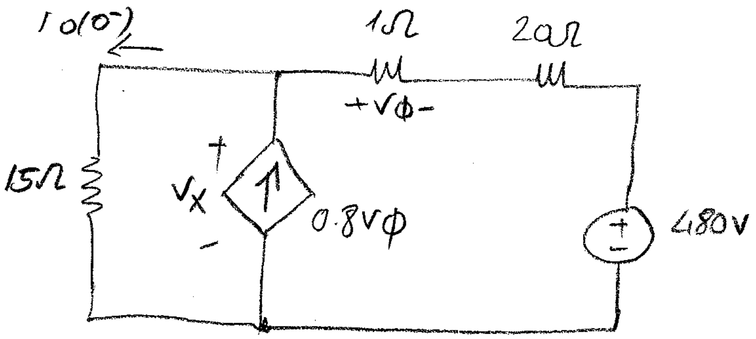
$$i_o = e^{-500t}\ \text{mA} \quad t \geq 0^+$$

$$i_{24k\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.4e^{-500t}\ \text{mA}, \quad t \geq 0^+$$

$$p_{24k\Omega} = (0.16\times10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\ \text{mW}$$

$$w_{24k\Omega} = \int_0^\infty 3.84\times10^{-3}e^{-1000t}\,dt = -3.84\times10^{-6}(0-1) = 3.84\,\mu J$$

7.39 For $t<0$



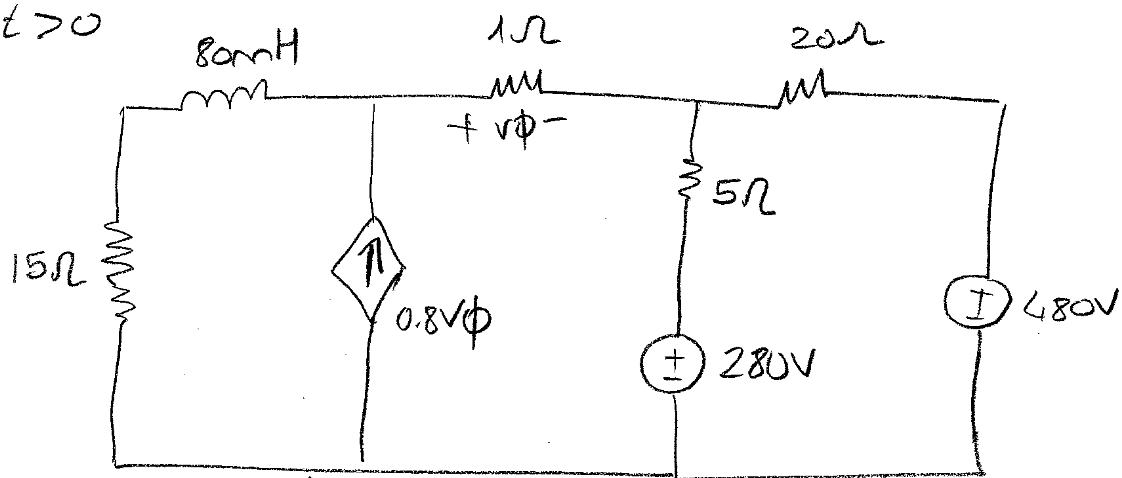
$$\frac{v_x}{15} - 0.8 v_\phi + \frac{v_x-480}{21} = 0 \Rightarrow v_\phi = \frac{v_x-480}{21}$$

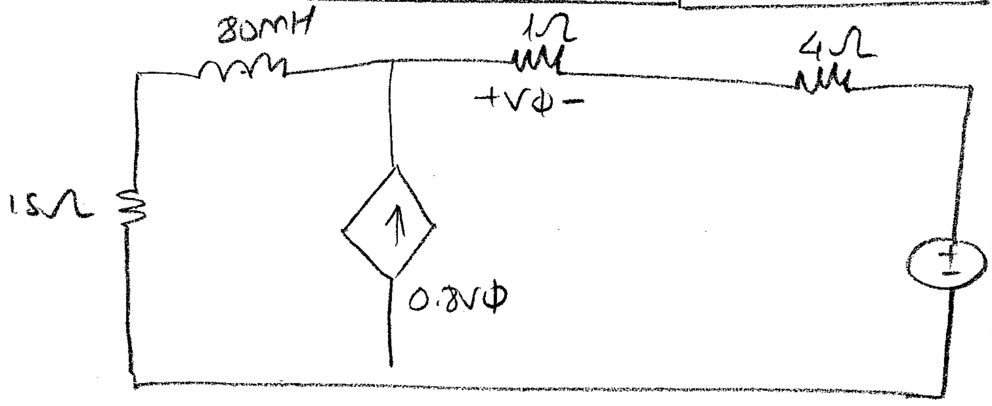
$$\frac{v_x}{15} - 0.8\left(\frac{v_x-480}{21}\right) + \left(\frac{v_x-480}{21}\right)$$

$$= \frac{v_x}{15} + 0.2\left(\frac{v_x-480}{21}\right) = 21 v_x + 3(v_x-480) = 0$$

$\therefore$ $24 v_x = 1440$ so $v_x = 60V$ $\quad i_o(0^-) = \frac{v_x}{15} = 4A.$

$t>0$





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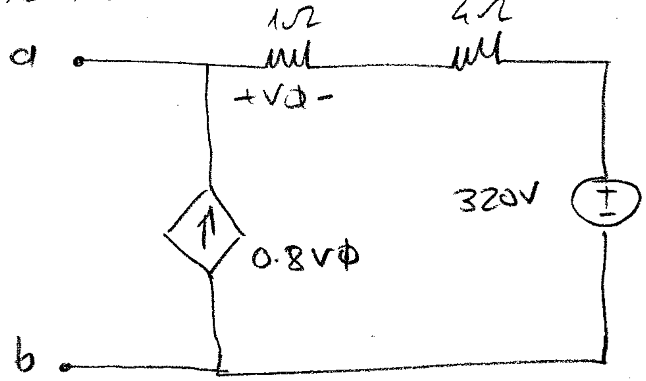
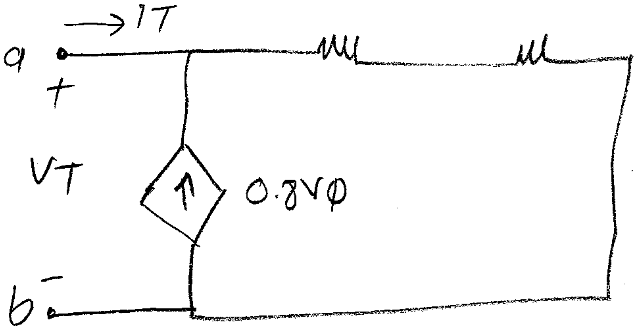
We Find the Thevenin equivalent with respect to a,b



$$\frac{v_{Th} - 320}{5} - 0.8\left(\frac{v_{Th} - 320}{5}\right) = 0 \qquad v_{Th} = 320V$$

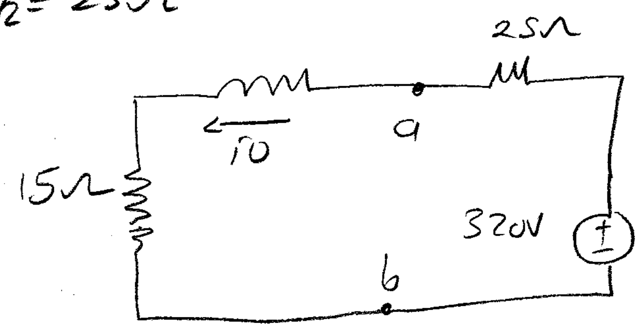


$$v_T = (i_T + 0.8 v_\phi)(5)$$

$$= \left(i_T + 0.8\frac{v_T}{5}\right)(5)$$

$$v_T = 5i_T + 0.8 v_T \therefore 0.2 v_T = 5 i_T$$

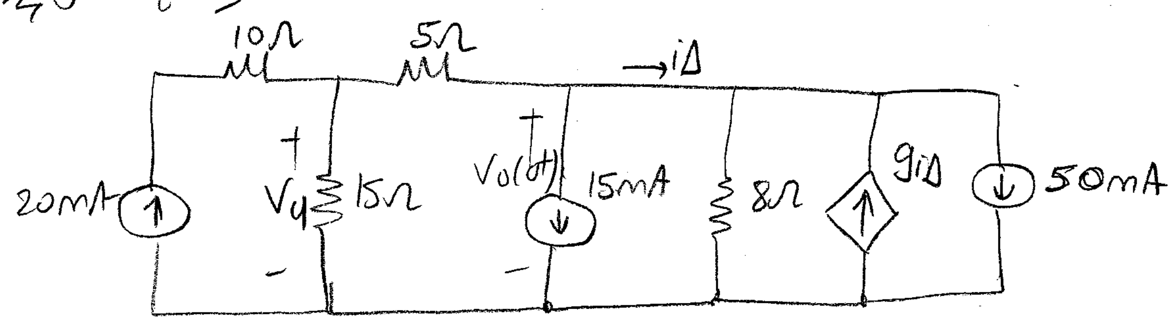
$$\frac{v_T}{i_T} = R_{Th} = 25\Omega$$



$$i_o(\infty) = 320/40 = 8A \qquad \tau = \frac{80 \times 10^{-3}}{40} = 2ms; \quad 1/\tau = 500$$

$$i_o = 8 + (4-8)e^{-500t} = 8 - 4e^{-500t} A, \quad t \geq 0.$$

7.40 $t >$



$$\frac{v_a}{15} + \frac{v_a - v_o(0^+)}{5} = 20\times10^{-3}$$

$$\therefore v_a = 0.75\, v_o(0^+) + 75\times10^{-3}$$

$$15\times10^{-3} + \frac{v_o(0^+) - v_a}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50\times10^{-3} = 0$$

$$13\, v_o(0^+) - 8v_a - 360\, i_\Delta = -2600\times10^{-3}$$

$$i_\Delta = \frac{v_o(0^+)}{8} - 9i_\Delta + 50\times10^{-3} \quad \therefore\ i_\Delta = \frac{v_o(0^+)}{80} + 5\times10^{-3}$$

$$\therefore\ 360\, i_\Delta = 4.5\, v_o(0^+) + 1800\times10^{-3}$$

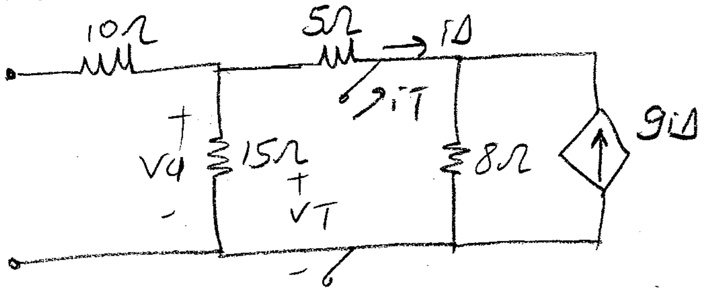
$$8v_a = 6\, v_o(0^+) + 600\times10^{-3}$$

$$\therefore\ 13\, v_o(0^+) - 6\, v_o(0^+) - 600\times10^{-3} - 4.5\, v_o(0^+) - 1800\times10^{-3} = -2600\times10^{-3}$$

$$2.5\, v_o(0^+) = -200\times10^{-3}; \quad v_o(0^+) = -80\text{ mV}$$

$$v_o(\infty) = 0$$

Find the Thevenin resistance seen by the 4 mH inductor.



$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_\Delta$$

$$i_\Delta = \frac{v_T}{8} - 9i_\Delta \quad \therefore\ 10\, i_\Delta = \frac{v_T}{8}; \quad i_\Delta = \frac{v_T}{80}$$

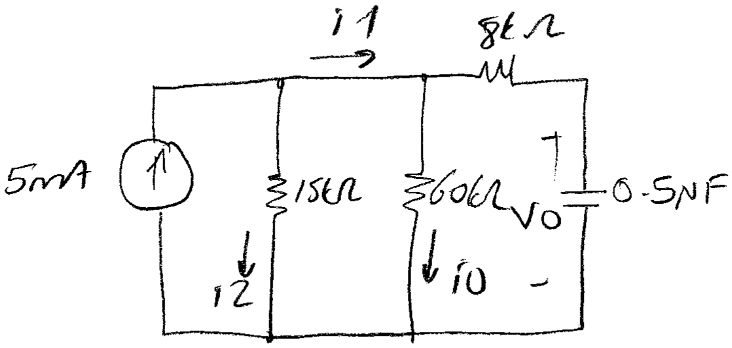
$$i_T = \frac{v_T}{20} + \frac{10v_T}{80} - \frac{9v_T}{80}$$

$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16}\text{ S} \qquad \therefore R_{Th} = 16\,\Omega$$

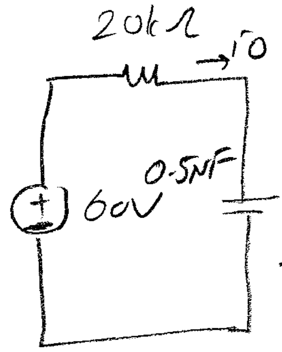
$$\tau = \frac{4\times10^{-3}}{16} = 0.25\text{ ms}; \qquad 1/\tau = 4000$$

$$\therefore\ v_o = 0 + (-80 - 0)e^{-4000t} = -80e^{-4000t}\text{ mV} \quad t \ge 0^+$$

7.48 (a) Simplify the circuit for $t>0$ using source transformation:



$\Rightarrow$



$$R_{eq} = \frac{15\times 60}{15+60} + 8 = 20k\Omega$$

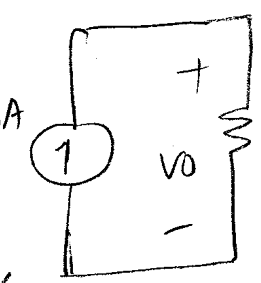
Since there is no source connected to the capacitor for $t<0$.

$$v_o(0^-) = v_o(0^+) = 0V$$

From the simplified circuit,

$$v_o(\infty) = 60V$$

$\Rightarrow$



$15 \| 60 = 12k\Omega$

$V_o = 5 \times R = 60V$

$$\tau = RC = (20\times 10^3)(0.5\times 10^{-6}) = 10ms$$

$$\frac{1}{\tau} = 100$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = (60 - 60e^{-100t})V, \; t \geq 0$$

(b) $$i_C = C\frac{dv_o}{dt} = 0.5\times 10^{-6}(-100)(-60e^{-100t}) = 3e^{-100t} mA$$

$$v_1 = 8000 i_C + v_o = (8000)(3\times 10^{-3})e^{-100t} + (60 - 60e^{-100t}) = 60 - 36e^{-100t} V$$

$$i_o = \frac{v_1}{60\times 10^3} = 1 - 0.6e^{-100t} mA, \quad t \geq 0^+$$

(c) $$i_1(t) = i_o + i_C = 1 + 2.4e^{-100t} mA \quad t \geq 0^+$$

(d) $$i_2(t) = \frac{v_1}{15\times 10^3} = 4 - 2.4e^{-100t} mA \quad t \geq 0^+$$

(e) $i1(0^+) = 1 + 2.4 = 3.4\,mA$ At $t = 0^+$:

$$Re = 15k \,||\, 60k \,||\, 8k = 4800\Omega$$

$$v1(0^+) = (5\times10^{-3})(4800) = 24V$$

$$i1(0^+) = \frac{v1(0^+)}{60{,}000} + \frac{v1(0^+)}{8000} = 0.4m + 3m = 3.4mA \text{ (check)}$$

7.64 (a) $v = I_sR + (V_0 - I_sR)e^{-t/RC}$ $\quad i = \left(I_s - \frac{V_0}{R}\right)e^{-t/RC}$

$\therefore I_sR = 40, \quad V_0 - I_sR = -24 \quad \therefore V_0 = 16V$

$$I_s - \frac{V_0}{R} = 3\times10^{-3}; \quad I_s - \frac{16}{R} = 3\times10^{-3}; \quad R = \frac{40}{I_s}$$

$$\therefore I_s - 0.4I_s = 3\times10^{-3}; \quad I_s = 5mA$$

$$R = \frac{40}{5}\times10^3 = 8k\Omega$$

$$\frac{1}{RC} = 2500; \quad C = \frac{1}{2500R} = \frac{10^{-3}}{20\times10^3} = 50nF;$$

$$\tau = RC = \frac{1}{2500} = 400\mu s$$

(b) $v(\infty) = 40V$ $\quad w(\infty) = \frac{1}{2}(50\times10^{-9})(1600) = 40\mu J$

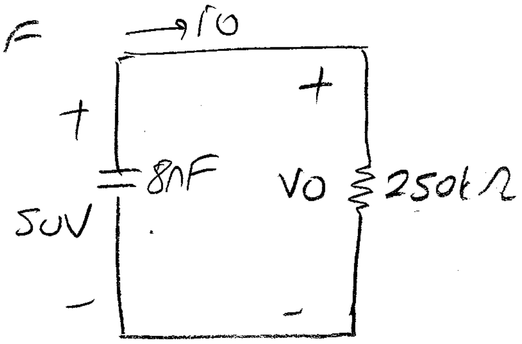
$$0.81\,w(\infty) = 32.4\,\mu J$$

$$v^2(t_0) = \frac{32.4\times10^{-6}}{25\times10^{-9}} = 1296; \quad v(t_0) = 36V$$

$$40 - 24e^{-2500t_0} = 36; \quad e^{2500t_0} = 6; \quad \therefore t_0 = 716.70\mu s$$

7-50 (a) For $t>0$: The equivalent circuit

$$C_{eq} = \frac{40\times10}{40+10} = 8\,nF$$



$$\tau = RC = 250\times10^{3} \times 8\times10^{-9} = 2ms; \qquad \frac{1}{\tau} = 500$$

$$v_o = 50\times e^{-500t}\,V, \quad t \geq 0^{+}$$

(b)
$$i_o = \frac{v_o}{250{,}000} = \frac{50e^{-500t}}{250{,}000} = 200\,e^{-500t}\,\mu A$$

$$v_1 = \frac{1}{C_1}\int_0^t i_o(\tau)\,d\tau + v_1(0^{+})$$

$$= \frac{-1}{40\times10^{-9}} \times 200\times10^{-6}\int_0^t e^{-500x}\,dx + 50 =$$

$$10e^{-500t} + 40V, \quad t \geq 0$$