Name:_______________________________________

This exams has 5 sheets to a total of 10 pages!

Write your name on the first page and your student number at the top-right corner of each page.

The exam contains two parts: required (pages 2-6) and advanced (pages 7-8). You can obtain at most 20 points out of the required part. You can achieve a grade of up to 25 points by solving addition problems from the advanced part.

The total of points in all the questions exceed 25. Your grade, however, cannot exceed 25 points. Since points have equal value, you are better off solving one question completely instead of solving partially multiple questions.

If necessary use the page labeled **extra space** for presenting your answers. Indicate clearly which question(s) you are writing on that page.

If necessary use the last page for draft work. Do *not* detach pages.

Provide justification for all your answers. Answers without justification will not be given any credit. Remember that obviously wrong answer may earn up to one *negative* point per problem.

If you are using results from class state them (without proofs).

Don't panic **and good luck!**

/3pts Compute all eigenvalues and their eigenvectors for $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & -3 & 3 \\ 0 & 0 & 3 & -3 \end{pmatrix}$.

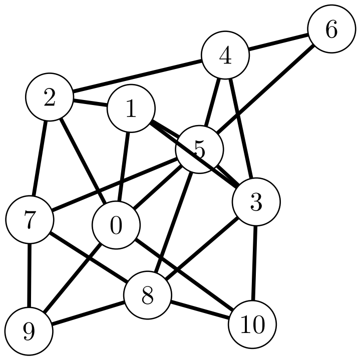
/3pts Is the set of 2×2 matrices with real entries and rank at most one a vector space under the usual matrix addition and scalar matrix multiplication operations?

/3pts Let $\vec{a} = \begin{pmatrix} \log_2(7) \\ 1 \\ 8.14 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} \log_2(14) \\ \pi \\ 16.14 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} \log_2(21) \\ 2\pi \\ 24.14 \end{pmatrix}$ and $\vec{d} = \begin{pmatrix} \log_2(7) \\ 0 \\ 8.14 \end{pmatrix}$. Are $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ linearly dependent?

/3pts Show that a tree on at least two vertices has at least two leaves.

/3pts Let the vertex set of the graph O be the set of all subsets of a five set. Two vertices in O are adjacent if and only if their symmetric difference contains exactly two elements. Draw a picture of H and of a spanning forest F of H. What is the rank of the incidence matrix of F that you drew?

/3pts Let G_m be the graph described bellow. Find a maximal matching M that is not maximum and an M-augmented path.



/3pts Can a graph on six vertices be isomorphic to its complement?

/3pts Let $\mathtt{U} = \{\vec{u}_1, \dots, \vec{u}_k\}$ and $\mathtt{V} = \{\vec{v}_1, \dots, \vec{v}_s\}$. Suppose every vector $\vec{v}_i$ is a linear combination of the vectors in $\mathtt{U}$ and suppose $k < s$. Is it true that every vector in $\mathtt{U}$ is a linear combination of the vectors in $\mathtt{V}$?

/3pts Let $M = \begin{pmatrix} 4, & -3, & 5, & \pi, & -4, & 7 \end{pmatrix}$ and $N = \begin{pmatrix} 2, & -7, & 3, & 5, & 1, & 8 \end{pmatrix}$. Find one eigenvalue and a corresponding eigenvector for $U = M^T N$.

/3pts A *diagonal* of a cycle in a graph is an edge that hoins vcertices that are not consecutive in the cycle. Prove that if a graph has a cycle then the cycle of shortest length has no diagonal.

/3pts Let G be a four regular graph G with cut vertex v. Can $G \setminus v$ have three componets?

extra space:

draft work: