Content

- Complexity Theory

Introduction

Complexity theory defines a methodology to analyse the computational complexity of crypto algorithms.

Definitions

Information Theory, (Except One-Time Pad) say that all cryptosystems are breakable.

Complexity Theory, explains the cryptosystems which are breakable or unbreakable without any dependency to exist computational powers.

Algorithmic Complexity :

The power of crypto; define by the required computational power to break it.

Argument #1 : Time - T

(required time to break it),

Argument #2 : Space - S

(required memory to break it)

Algorithmic Complexity :

Power of Cryptosystem is defined by big O .

Big O notation: Computational Complexity degree (as an upper bound).

Complexity Function

If the time complexity of an algorithm is defined by

$$3n^3+5n+23$$

Then ; its computational complexity is

$$O(n^3)$$

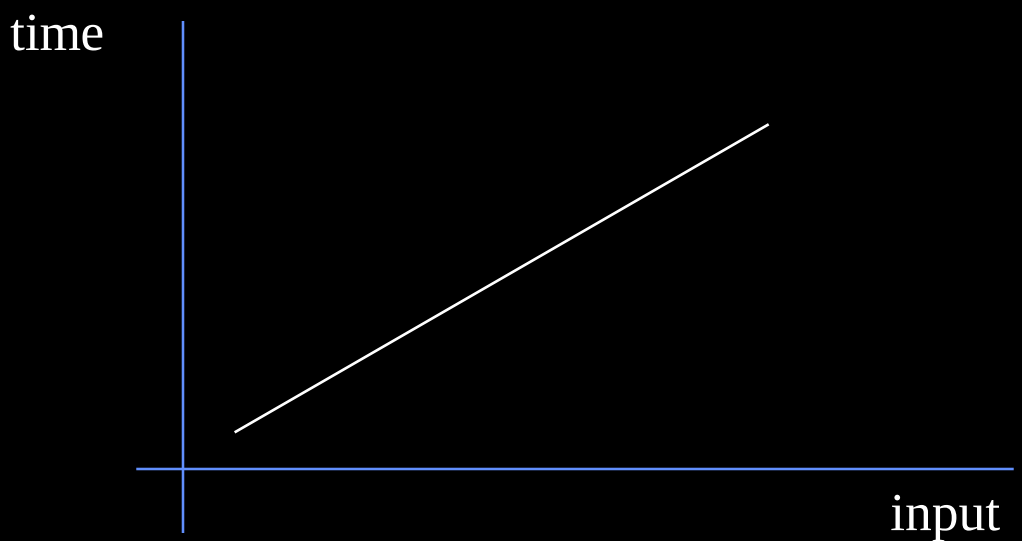
This notation is independent from any computer architecture.

Complexity Function:

This notation defines the impact of input size on Time and Space parameters of algorithm.

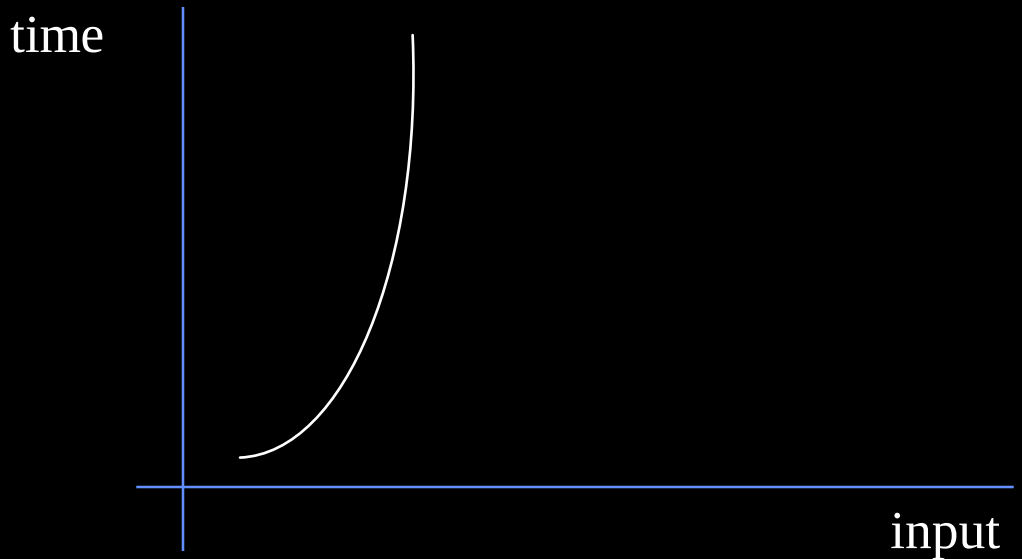
Complexity Function :

$O(n^t)$ ⟹ polynomial



Complexity Function :

$O(t^{f(n)})$ ⇒ exponential



Complexity Function :

Class	Complexity	Operation Count	Time
Constant	O(1)	1	1 micro sec.
Linear	O(n)	10^6	1 sec.
Quadratic	$O(n^2)$	10^{12}	11.6 day
Cubic	$O(n^3)$	10^{18}	32.000 year
Exp	$O(2^n)$	$10^{301.030}$	$10^{301.006}$

>> age of universe..

Big Numbers :

Age of Earth	10^{9} year
Age of Universe	10^{10} year
Total life time of Universe	10^{11} year
Total count of atoms in planet	10^{51}
Total count of atoms in Sun	10^{57}
Total count of atoms in Galaxy	10^{67}
Total count of atoms in Universe	10^{77}

Complexity of a problem:

Problem Classes

Solvable: these problems are solvable by a polynomial time algorithm.

Unsolvable: they are not solvable by polynomial time algorithm. HARD !

Complexity Classes :

Categorical classifications of known problems according to their difficulty degree...

Complexity Classes :

Class P (Polynomial) :

If a problem can be solved in polynomial time by a deterministic computer, this problem is belong to

P

Class.

Complexity Classes :

Class NP (Non deterministic but Polynomial) :

If a problem can be solved in polynomial time by a non-deterministic computer (such as unlimited parallel computing capability), this problem is belong to

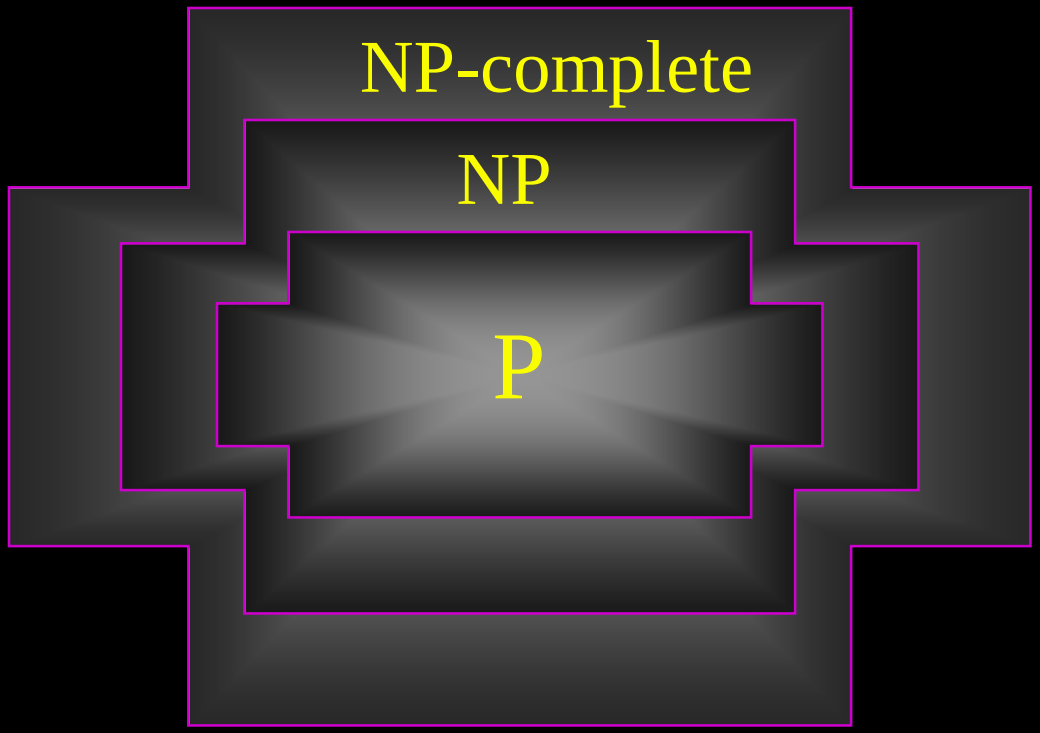
NP

class.

Complexity Classes :

Class NP – Complete: finding a solution for an instance of the NP-complete problem, which can solve other all similar problems of this NP class.

Complexity Classes :



Complexity Classes : Examples

Travelling Salesman : A salesman have to visit n different cities.

What is the shortest path which is every city visited only one time?

Complexity Classes : Examples

Travelling Salesman : n=2, 3, 4 and 5 is easy problem but if n=25 or more !!??...

Complexity Classes : Examples

Factorization : Finding of multiplicative prime factors of a number.

Example: 60 = 2 x 2 x 3 x 5

Complexity Classes : Examples-Factorization

Prime	Prime	Multiplication	Time
p	q	n=pxq	
223	293	65339	30 sec.

Class : P

Complexity Classes : Examples-Factorization

Multiplication	Prime	Prime	Time
n=pxq	p	q	
65339	223	293	1 hour

Class : NP

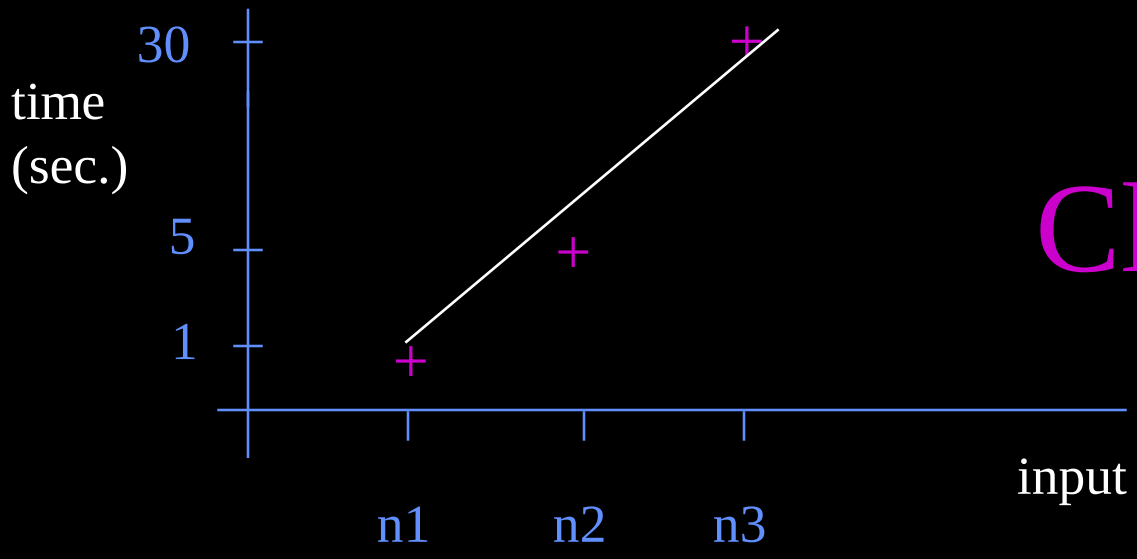
Complexity Classes : Examples-Factorization

Example : n=pxq

input	operation	time
n1=p1xq1	3x 5 = 15	1 sec.
n2=p2xq2	11x 17 = 187	5 sec.
n3=p3xq3	223x293 = 65339	30 sec.

Complexity Classes : Examples-Factorization

Example: n=pxq



Class : P

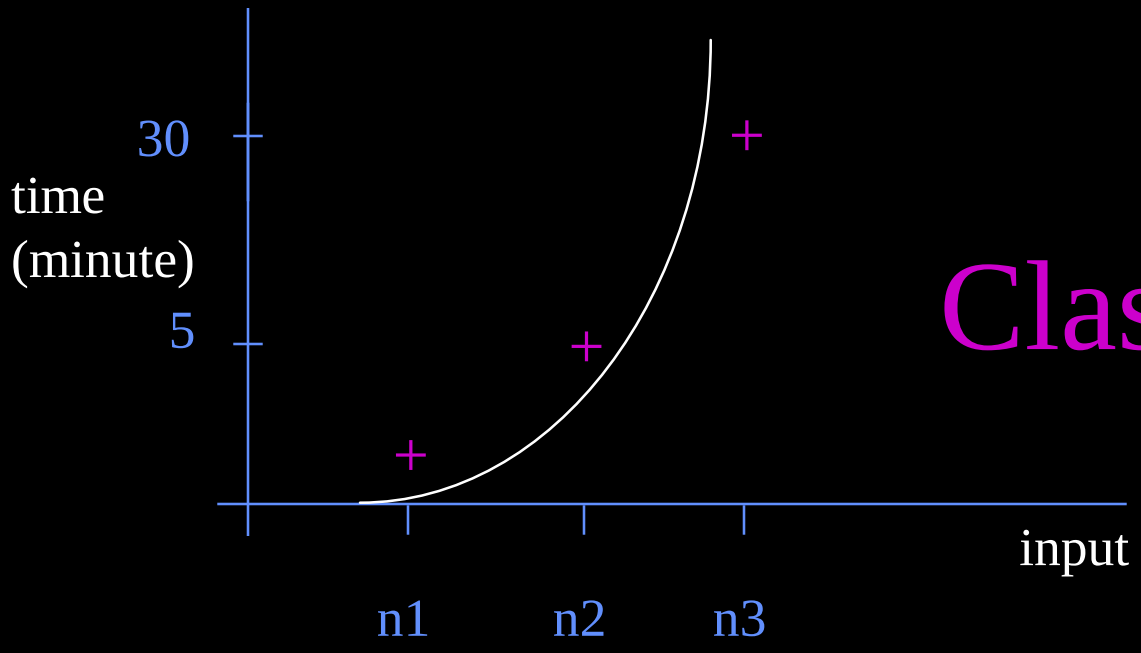
Complexity Classes : Examples-Factorization

Example : n is the multiplication of two prime numbers.
Try to find these prime numbers.

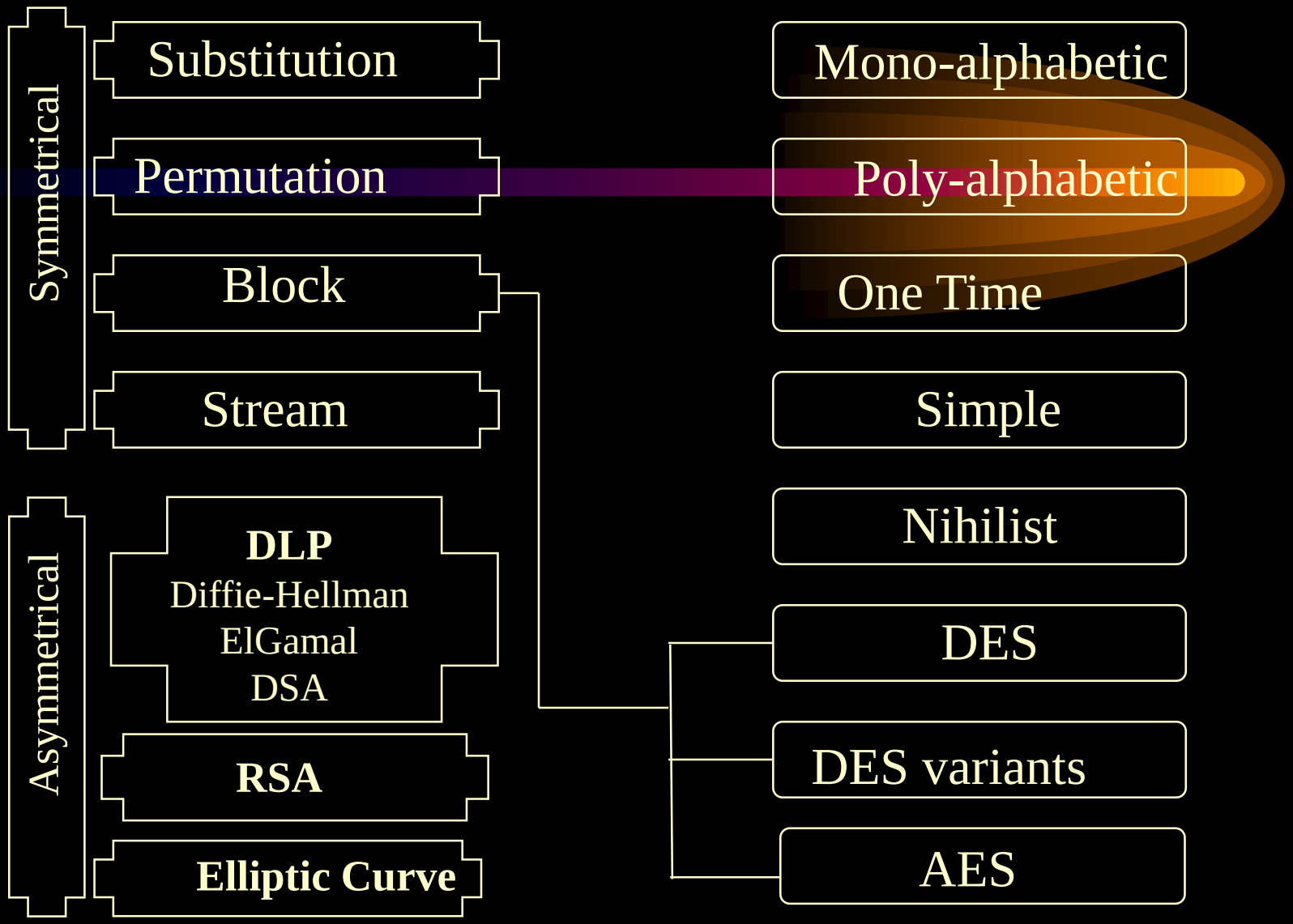
input	operation	time
n1	15 = 3x5	1 sec.
n2	187 = 11x17	5 minute
n3	65339 = 223x293	60 minute

Complexity Classes : Examples-Factorization

example: n; p,q



Class: NP


Symmetrical
Substitution
Permutation
Block
Stream
Asymmetrical
DLP
Diffie-Hellman
ElGamal
DSA
RSA
Elliptic Curve
Mono-alphabetic
Poly-alphabetic
One Time
Simple
Nihilist
DES
DES variants
AES