EE 210

Solutions of Problem Set 1

Instructor:
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1.9 $i = \frac{dq}{dt} = 20\cos 5000t$ $\quad dq = 20\cos 5000t\,dt$

$$\int_{q(0)}^{q(t)} dx = 20\int_0^t \cos 5000y\,dy = \frac{20}{5000}\sin 5000t$$

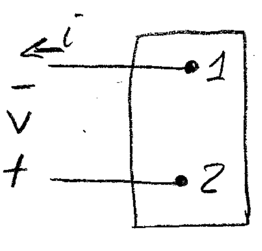
$q(0)=0 \Rightarrow$ The current reaches its maximum value at $t=0$

$q(t) = 4\times10^{-3}\sin 5000t\,C$
$= 4\times10^{-3}\sin 5000t\,mC.$

1-12 $p = v \cdot i$ due to passive sign convention. If $p>0$, B is absorbing power, thus the power must be flowing from A to B. If the power is negative, B is producing power which means that power must be flowing from B to A.

(a) $p = (20)(15) = 300W \Rightarrow 300W$ from A to B.
(b) $p = (100)(-5) = -500W \Rightarrow 500W$ from B to A.
(c) $p = (-50)(4) = -200W \Rightarrow 200W$ from B to A
(d) $p = (-25)(-16) = 400W \Rightarrow 400W$ from A to B.

1.13 (a)

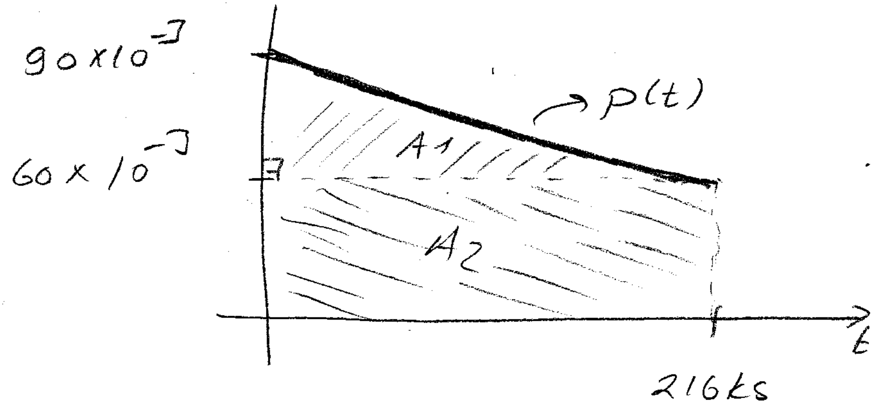


$p = vi = (-20)(5) = -100W$
Box is loosing power, so power is delivered by the box.

(b) Leaving

c) Gaining.

1-16 We sketch the power as function of time and recall that the energy is the integral of power = the area under power versus time curve.



$$60\text{ hours} = \frac{60\text{ seconds}}{1\text{ minute}} \times \frac{60\text{ minutes}}{1\text{ hour}} \times 60\text{ hours} = 216{,}000\text{ s}$$

$$p(0) = (6)(15\times10^{-3}) = 90\times10^{-3}\,W$$

$$p(216\text{ ks}) = (4)(15\times10^{-3}) = 60\times10^{-3}\,W$$

The area under the power line is the sum of the area of the right triangle A_1 plus the area of the rectangle A_2.

$$W = (60\times10^{-3})(216\times10^{3}) + \frac{1}{2}(90\times10^{-3} - 60\times10^{-3})(216\times10^{3}) = \underline{\underline{16.2\,kJ}}$$

1-18 a) Instantaneous power $p(t) = v(t)\,i(t)$

$$p(20\,ms) = v(20\,ms)\,i(20\,ms)$$

$$= (100e^{-1}\sin 3)(20e^{-1}\sin 3) = 5.39\,W$$

b) $$p = vi = 2000e^{-100t}\sin^2 150t$$

$$= 2000e^{-100t}\left[\frac{1}{2} - \frac{1}{2}\cos 300t\right]$$

$$= 1000e^{-100t} - 1000e^{-100t}\cos 300t$$

$$W = \int_0^\infty 1000e^{-100t}\,dt - \int_0^\infty 1000e^{-100t}\cos 300t\,dt = \underline{\underline{9J}}$$

1.19 (a) $0s \le t < 1s \Rightarrow v = 5V,\ i = 20A;\ p = 100t\,W$

$1s \le t < 2s \Rightarrow v = 0V;\ i = 20A;\ p = 0W$

$2s \le t < 3s \Rightarrow v = 0V;\ i = 20A;\ p = 0W$

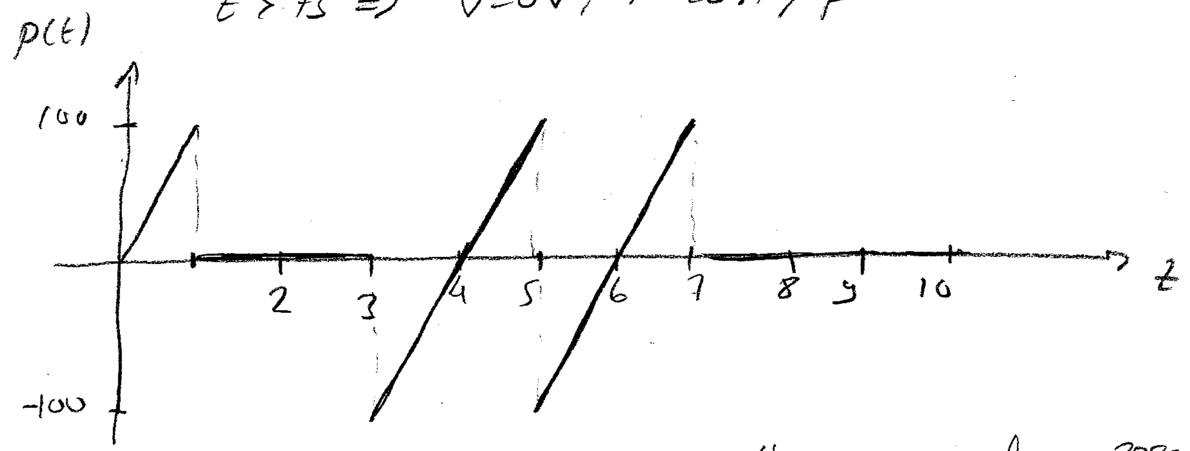
$3s \le t < 4s \Rightarrow v = 5V;\ i = 80 - 20t\ A;\ p = -400 + 100t\ W$

$4s \le t < 5s \Rightarrow V = -5V;\ i = 80 - 20t\,A;\ p = -600 + 100t\ W$

$5s \le t < 6s \Rightarrow V = 5V;\ i = -120 + 20t\ A;\ p = -600 + 100t\ W$

$6s \le t < 7s \Rightarrow V = 5V;\ i = -120 + 20t\ A;\ p = -600 + 100t\ W$

$t > 7s \Rightarrow V = 0V;\ i = 20A;\ p = 0W.$



(b) Calculate the area under the curve from zero up to the desired time.

$$W(1) = \frac{1}{2}(1)(100) = 50J$$

$$W(6) = \frac{1}{2}(1)(100) - \frac{1}{2}(1)(100) + \frac{1}{2}(1)(100) - \frac{1}{2}(1)(100) = 0J$$

$$W(10) = \frac{1}{2}(1)(100) - \frac{1}{2}(1)(100) + \frac{1}{2}(1)(100) - \frac{1}{2}(1)(100) + \frac{1}{2}(1)(100) = 50J.$$

1.24 To find the time at which the power is maximum, we write the expression for the instantaneous power, take its first derivative and set it equal to zero.

$p = 0 \quad t < 0, \quad p = 0 \quad t > 40s$

$$p = vi = (t - 0.025t^2)(4 - 0.2t)$$

$$= 4t - 0.3t^2 + 0.005t^3\ W \qquad 0 < t < 40s$$

$$\frac{dp}{dt} = 4 - 0.6t + 0.015t^2 = 0 \qquad t_1 = 8.453s \quad t_2 = 31.547s$$

$$p(t_1) = (8.453 - 0.025(8.453)^2)(4 - 0.2 \times 8.453) = 15.396W$$

$$p(t_2) = (31.547 - 0.025(31.547)^2)(4 - 0.2 \times 31.547) = -15.396W$$

Therefore maximum power is being delivered at $t = 8.453s$.

(b) $p_{max} = 15.396\,W$ (delivered)

(c) Maximum power being extracted equals to the minimum power at $t = 31.547s$.

(d) $P_{maximum\ extracted} = 15.396\,W$

(e) $$W = \int_0^t p\,dx = \int_0^t (4x - 0.3x^2 + 0.005x^3)\,dx$$
$$= 2t^2 - 0.1t^3 + 0.00125t^4$$

$W(0) = 0J \quad W(10) = 112.50J \quad W(20) = 200J$

$W(30) = 112.50J \quad W(40) = 0J$