For this homework $\mathbf{P}_3$ is the vector space of polynomials of degree at most three and $\mathbf{P}_2$ is the vector space of polynomials of degree at most two. You may assume without proof that $\langle A\rangle$ and $\langle B\rangle$ are basis for $\mathbf{P}_3$. Likewise $\langle E\rangle$, $\langle R\rangle$ and $\langle C\rangle$ are basis for $\mathbf{P}_2$.

$$\begin{aligned}\mathbf{P}_3 &= \langle B\rangle = \left\langle \vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4 \right\rangle = \left\langle x^3, x^2, x, 1 \right\rangle \\ &= \langle A\rangle = \langle \vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4 \rangle = \left\langle x^3 - 2x^2 + 2x + 1, 2x^3 + 3x + 1,\ 2x^3 + x^2 + 3x + 1, -x^3 + x^2 + x + 1 \right\rangle\end{aligned}$$

$$\begin{aligned}\mathbf{P}_2 &= \langle E\rangle = \langle \vec{e}_1, \vec{e}_2, \vec{e}_3 \rangle = \left\langle x^2, x, 1 \right\rangle \\ &= \langle R\rangle = \langle \vec{r}_1, \vec{r}_2, \vec{r}_3 \rangle = \left\langle x^2, x^2 + x, x^2 + x + 1 \right\rangle \\ &= \langle C\rangle = \langle \vec{c}_1, \vec{c}_2, \vec{c}_3 \rangle = \left\langle 4x^2 + 3x + 1, 2x^2 + 2x + 1, 3x^2 + x \right\rangle\end{aligned}$$

Q1. Give the matrix representation of the derivative $d : \mathbf{P}_3 \to \mathbf{P}_2$ for

1. $A \to E$;
2. $A \to R$;
3. $A \to C$;

Q2. Give the matrix representation of the identity map $id : \mathbf{P}_2 \to \mathbf{P}_2$ for

1. $C \to R$;
2. $R \to C$;
3. $C \to E$;

Q3. How are the matrices in Q1.2, Q1.3 and Q2.2 related?

Q4. How are the matrices in Q2.1, Q2.2 related?

Q5. Let $\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ -1 \end{pmatrix}_A$. Compute representation of the derivative of $\vec{v}$ in basis R, C and E.