Q1. Let $V = \{(x_1, x_2) \mid x_1, x_2 \in \mathbb{R}\}$ and let

$\oplus$: for $\vec{u}, \vec{v} \in V$ define $\vec{u} \oplus \vec{v} = (u_1, u_2) \oplus (v_1, v_2) = (u_1 + v_1 - 2, u_2 + v_2 + 1)$

$\odot$: for $\alpha \in \mathbb{R}, \vec{v} \in V$ define $\alpha \odot \vec{v} = \alpha \odot (v_1, v_2) = (\alpha v_2 + \alpha - 1, \alpha v_1 - 2\alpha + 2)$

Is V a vector space over $\mathbb{R}$ with the above operations?

Q2. Let $V = \{(x_1, x_2) \mid x_1, x_2 \in \mathbb{R}, x_1 x_2 \neq 0\}$ and let

$\oplus$: for $\vec{u}, \vec{v} \in V$ define $\vec{u} \oplus \vec{v} = (u_1, u_2) \oplus (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$

$\odot$: for $\alpha \in \mathbb{R}, \vec{v} \in V$ define $\alpha \odot \vec{v} = \alpha \odot (v_1, v_2) = (\alpha v_1, \alpha v_2)$

Is V a vector space over $\mathbb{R}$ with the above operations?

Q3. Is the set of all invertible two by two matrices a vector space?

Recall the vector space discussed in class: $\mathbf{V} = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \mid x, y \in \mathbb{C} \right\}$ with:

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \oplus \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 - 4 \\ y_1 + y_2 - 3 \end{pmatrix} \qquad \text{and} \qquad \alpha \odot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha x - 4\alpha + 4 \\ \alpha y - 3\alpha + 3 \end{pmatrix}$$

Q4. Compute

1. $\left\{(-1) \odot \left[\begin{pmatrix} 8 \\ 1 \end{pmatrix} \oplus \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]\right\} \oplus \left\{3 \odot \left[\begin{pmatrix} 2 \\ 0 \end{pmatrix} \oplus \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right]\right\}$
2. $\left\{\left[0 \odot \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right] \oplus \begin{pmatrix} 4 \\ -1 \end{pmatrix}\right\} \oplus \left\{\left[3 \odot \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right] \oplus \left[2 \odot \begin{pmatrix} 9 \\ 1 \end{pmatrix}\right]\right\}$
3. $\left\{0 \odot \left[\begin{pmatrix} 2 \\ 3 \end{pmatrix} \oplus \begin{pmatrix} 4 \\ -1 \end{pmatrix}\right]\right\} \oplus \left\{7 \odot \left[\begin{pmatrix} 1 \\ 2 \end{pmatrix} \oplus \begin{pmatrix} 9 \\ 1 \end{pmatrix}\right]\right\}$

Q5. Find the additive inverses of

1. $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
2. $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$
3. $(-4) \odot \left[\begin{pmatrix} 4 \\ 3 \end{pmatrix}\right]$