What is the Characteristic Equation?

Given a $d \times d$ matrix V

a very important class of linear Equations is of the form

$$V\mathbf{x} = \lambda\mathbf{x}$$

$d \times d$ $d \times 1$	$d \times 1$

which can be rewritten as $(V - \lambda I)x = 0$

If V is real and symmetric

there are d possible solution vectors,
called Eigen Vectors, $e_1, \ e_d$
and associated Eigen values $\lambda_1, \ldots, \lambda_d$

Principal Component is obtained from the Covariance Matrix

If the matrix V is the Covariance matrix

Then its Characteristic Equation is

$$(V - \lambda I)\mathrm{a} = 0$$

Roots are Eigen Values
Corresponding Eigen Vectors are principal components

First principal component is the Eigen Vector associated with the largest Eigen value of V.

Other Principal Components

- Second Principal component is in direction orthogonal to first
- Has second largest Eigen value, etc