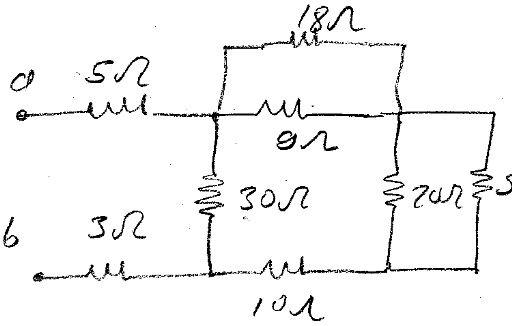
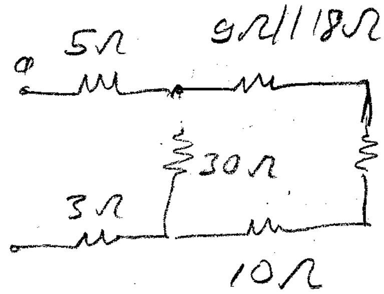
Solutions Problem set 3

3.8 a)



$\Rightarrow$

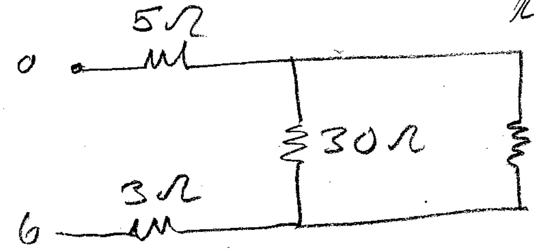


$9\Omega \| 18\Omega \Rightarrow 6\Omega$

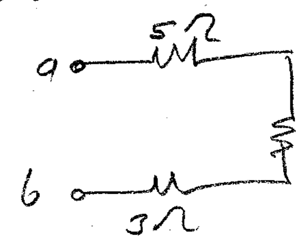
$20\Omega \| 5\Omega = 4\Omega$

$$9\Omega \| 18\Omega = \frac{9\times 18}{9+18} = 6\Omega \qquad 20\Omega \| 5\Omega = \frac{20\times 5}{20+5} = 4\Omega$$

$R_{series} = 6\Omega + 4\Omega + 10\Omega = 20\Omega$



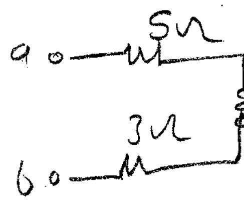
$\Rightarrow$



$20\Omega \| 30\Omega = 12\Omega$

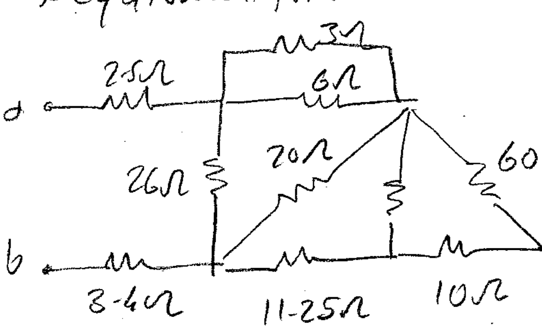
$$20\Omega \| 30\Omega = \frac{20\times 30}{20+30} = 12\Omega$$

$\Rightarrow$



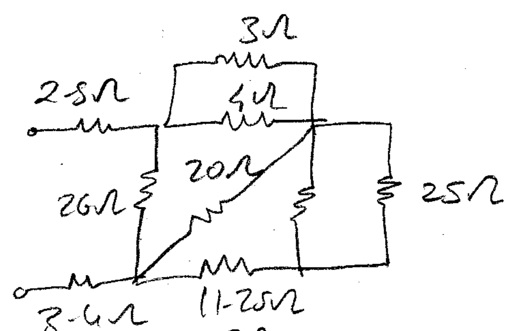
$R_{equivalent\,final} = 5\Omega + 3\Omega + 12\Omega = \underline{\underline{20\Omega}}$

b)

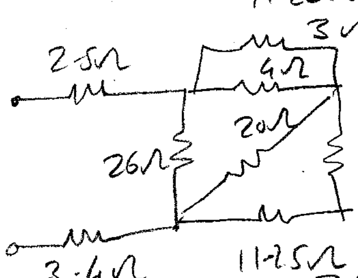


$60\Omega \| 20\Omega = 15\Omega$

$\Rightarrow$

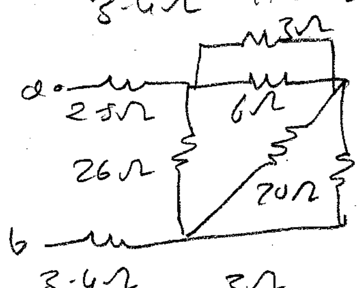


$\Rightarrow$



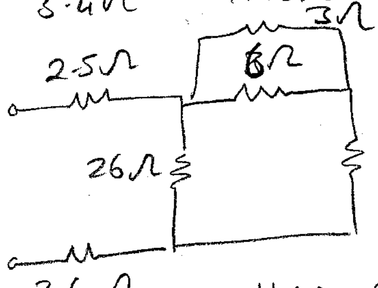
$75\Omega \| 25\Omega = 18.75\Omega$

$\Rightarrow$



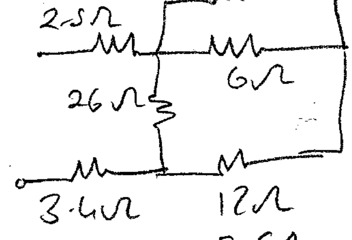
$18.75\Omega + 11.25\Omega = 30\Omega$

$\Rightarrow$



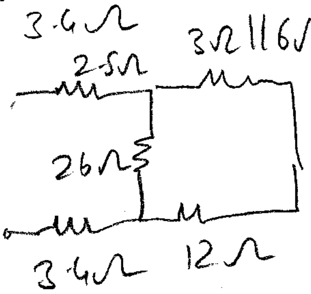
$20\Omega \| 30\Omega = 12\Omega$

$\Rightarrow$

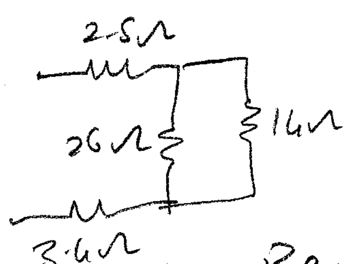


$3\Omega \| 6\Omega = 2\Omega$

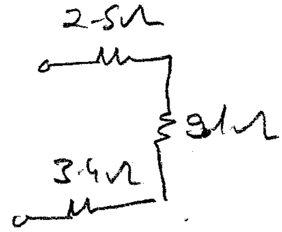
$\Rightarrow$



$\Rightarrow$

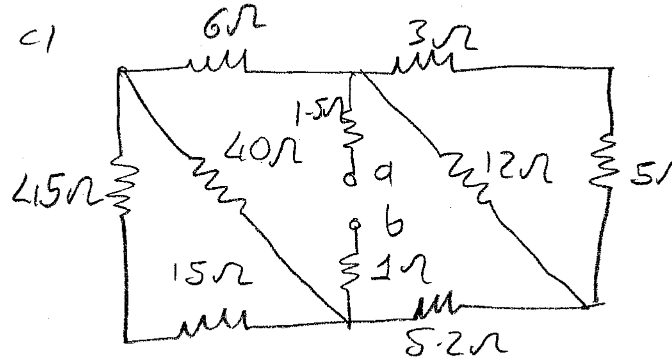


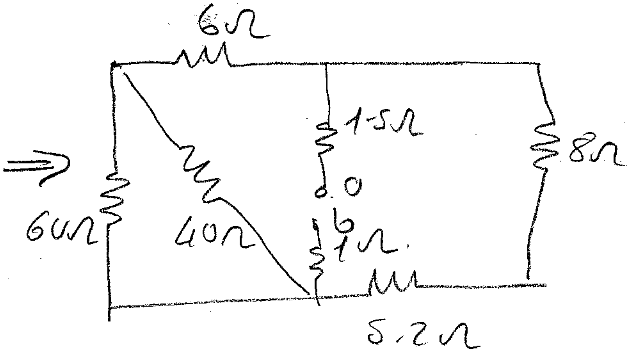
$\Rightarrow$



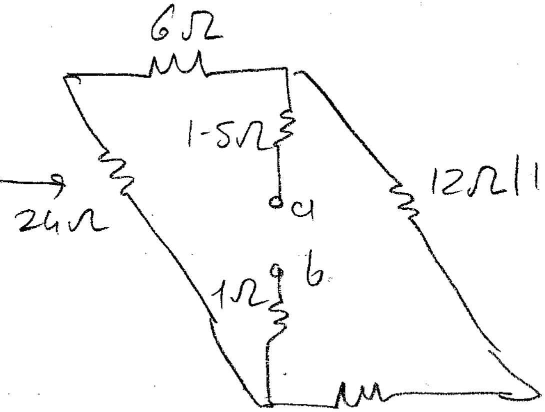
$R_{eq} = 2.5\Omega + 9.1\Omega + 3.4\Omega = \underline{15\Omega}$

3-8 c)

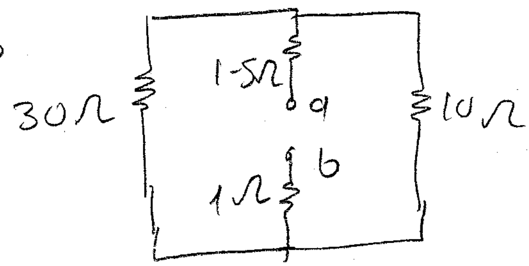




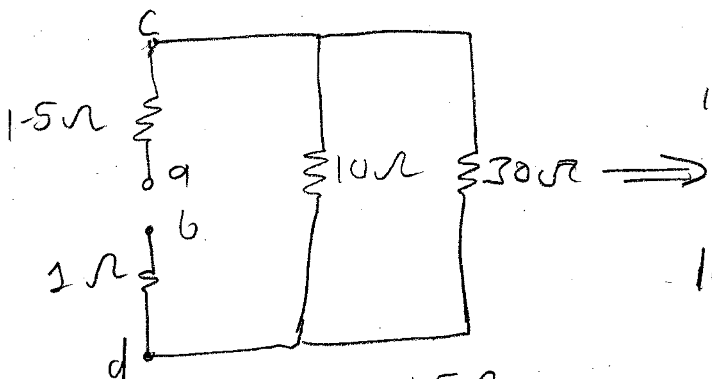
⇒ $60\Omega \| 40\Omega = 24\Omega$

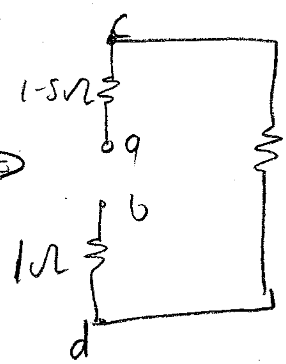


⇒



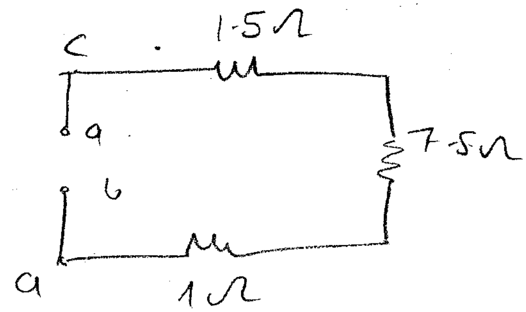
$V_{30\Omega} = V_{10\Omega} = V_{cd}$, thus the 30Ω and 10Ω resistors are in parallel, we can draw an equivalent circuit.





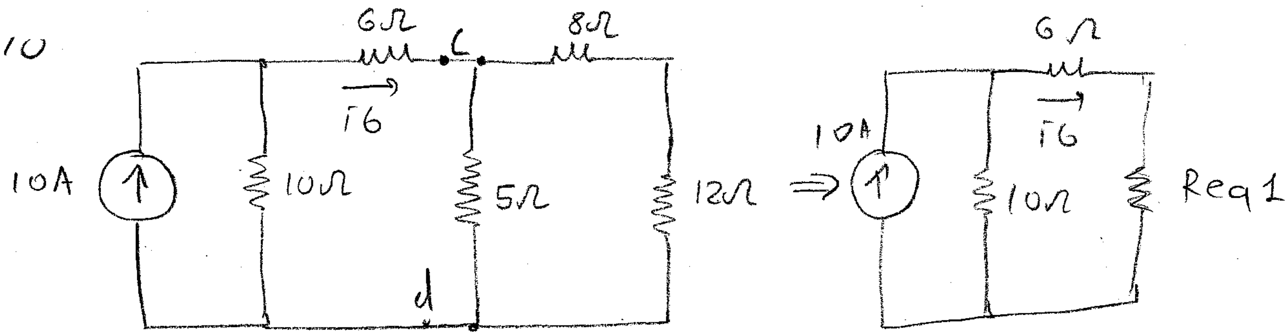
$10\Omega \| 30\Omega = 7.5\Omega$

⇒

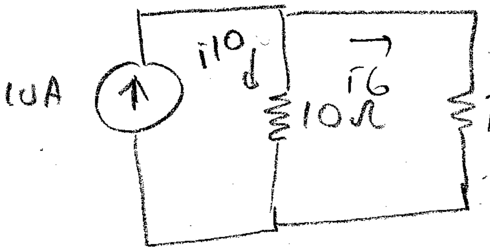


$R_{eq} = 1.5\Omega + 7.5\Omega + 1\Omega = \underline{10\Omega}$

3.10



$$Req1 = 5\Omega \,||\, (8\Omega + 12\Omega) = 5\Omega || 20\Omega = \frac{5\times 20}{5+20} = 4\Omega$$



$Req_2 = 6\Omega + 4\Omega = 10\Omega$

Using the current Division equation:

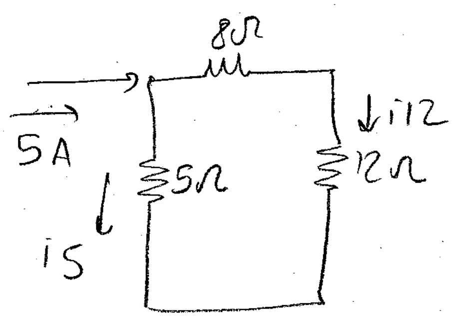
$$i_j = \frac{v}{R_j} = \frac{Req}{R_j}\, i$$

Applying to this case $i = 10A$

$$Req = 10\Omega || 10\Omega = 5\Omega$$

$$i6 = i_j = \frac{5\Omega}{10\Omega} \times 10A = \underline{\underline{5A}}$$

⇒ Now that we have determined i6, we go back to the circuit and compute the current left of terminals c and d, and apply current division equation again.



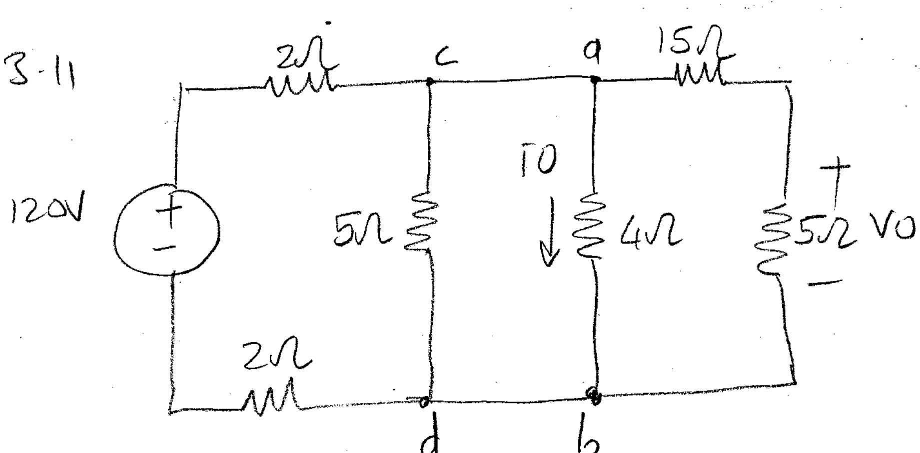
$$Req = 5\Omega || (8\Omega + 12\Omega) = 5\Omega || 20\Omega$$

$$= \frac{5\times 20}{5+20} = 4\Omega$$

$$I_5 = \frac{4\Omega}{5\Omega} \times 5A = \underline{\underline{4A}}$$

$$P5\Omega = (4A)^2 (5\Omega) = \underline{\underline{80\,W}}$$

3-11



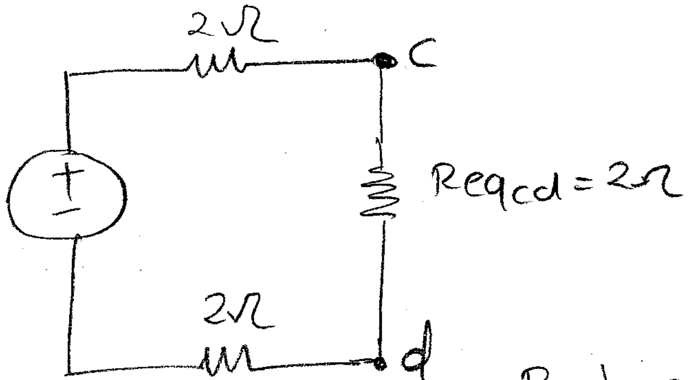
a) In order to determine vo, we need to determine vab first, once we determine vab, we can also determine vo.

But $v_{ab} = v_{cd}$, as the 5Ω resistor is connected in parallel to the 4Ω resistor and the series combination of 15Ω and 5Ω resistors. If we can compute the equivalent resistance of the circuit to the right of the terminals cd, we can use voltage division to determine V_{ab}.

$$R_{eq\,right\,to\,cd} = 5\Omega \parallel 4\Omega \parallel (15\Omega + 5\Omega)$$

$$\frac{1}{R_{eqcd}} = \frac{1}{5} + \frac{1}{4} + \frac{1}{20} = \frac{20 + 25 + 5}{100} = \frac{1}{2}$$

$R_{eqcd} = 2\Omega$, thus, the following equivalent circuit represents the circuit.



$$V_{cd} = \frac{2\Omega}{2\Omega + 2\Omega + 2\Omega} \times 120V$$

$$V_{cd} = \frac{1}{3} \times 120V = 40V$$

But $v_{ab} = v_{cd} = 40V$, thus

$$i_o = \frac{v_{ab}}{4\Omega} = \frac{40V}{4\Omega} = \underline{10A}$$. Now to determine the voltage v_o, we apply voltage division again. The 40V, between terminals ab is equal to the sum of the voltages across the 15Ω and 5Ω resistors. The voltage is simply distributed proportional to the resistors.

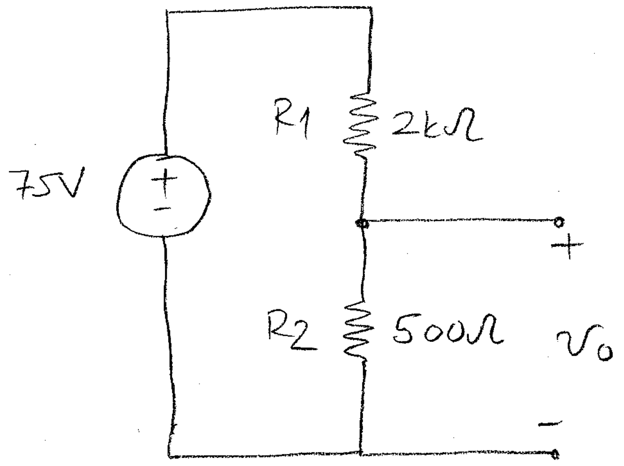
$$V_o = \frac{5\Omega}{5\Omega + 15\Omega} \times 40V = \frac{1}{4} \times 40V = \underline{10V}$$

b) $$i_{15} = \frac{V_{cd}}{15\Omega + 5\Omega} = \frac{40V}{20\Omega} = 2A \quad ; \quad P_{15\Omega} = (2A)^2 \cdot (15) = \underline{60W}$$

c) $$R_{TOTAL} = 2\Omega + 2\Omega + 2\Omega = 6\Omega \qquad I = \frac{120V}{6\Omega} = 20A$$

$P_{source} = (120V)(-20A) = -2400W$. The source is loosing 24,00W. The power delivered by the source to the circuit is 2400W = 2.4kW.

3.13



a) $V_{NOLOAD} = \dfrac{0.5k\Omega}{2k\Omega + 0.5k\Omega} \cdot 75V = \underline{\underline{15V}}$

b) $I = \dfrac{75V}{2k\Omega + 0.5k\Omega} = 30mA$

$P_{2k\Omega} = (30\times10^{-3}A)^2 (2\times10^{3}\Omega) = 900\times10^{-6}\times2\times10^{3} = 1800\times10^{-3}W = \underline{\underline{1800mW}}$

$P_{500\Omega} = (30\times10^{-3}A)^2 (500\Omega) = 900\times10^{-6}\times5\times10^{2} = 4500\times10^{-4}W = \underline{\underline{450mW}}$

c) We are given the power specification for the resistors, the power dissipated can not exceed 1W.

$p_{max} = U \cdot i \leq 1W$

Now if the no-load voltage has to be same as in part a) that is 15V, the voltages V_{R1} and V_{R2} across the resistors are fixed.

$V_{R2} = V_{NO-LOAD} = \underline{15V}$ $\qquad$ $V_{R1} = 75V - 15V = 60V$

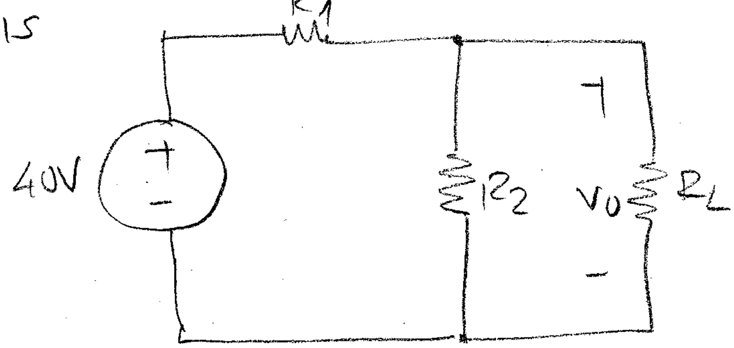
$i_{R1} \leq \dfrac{1W}{V_{R1}} = \dfrac{1W}{60V} = 16.67\times10^{-3}A = 16.67mA$

$i_{R2} \leq \dfrac{1W}{V_{R2}} = \dfrac{1W}{15V} = 66.67\times10^{-3} = 66.67mA$

So we get 2 Bounds for the currents, we select the lower one 16.67mA. Thus the current $I = I_{R1} = I_{R2}$ can be at most 16.67mA. $R_1 \geq \dfrac{V_{R1}}{I_{R1}} = \dfrac{60V}{16.67mA} = \underline{8.6k\Omega}$

$R_2 = \dfrac{V_{R2}}{I_{R2}} = \dfrac{15V}{16.67mA} = \underline{0.9k\Omega}$.

3.15



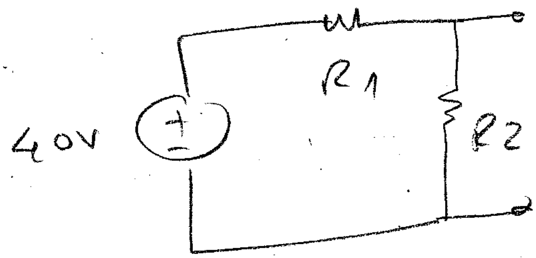
(a) $V_{NOLOAD} = 8V$

$R_{Lmin} = 3.6k\Omega$

$V_{omin} = 7.5V$

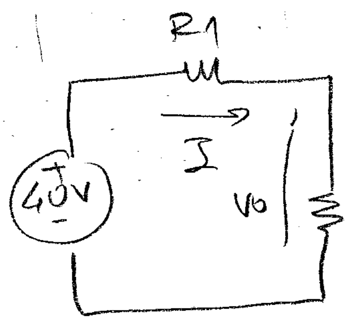
From the constraint on no-load voltage

$\frac{R_2}{R_1+R_2}(40) = 8$ so $R_1 = 4R_2$



→ NO LOAD CIRCUIT

→ LOADED CIRCUIT



$Req = R_2 \| R_L = \frac{R_2 \cdot R_L}{R_2 + R_L}$

$$\Rightarrow \frac{v_o}{40} = \frac{Req}{R_1 + Req} \rightarrow v_o = 40 \times \frac{Req}{R_1 + Req}$$

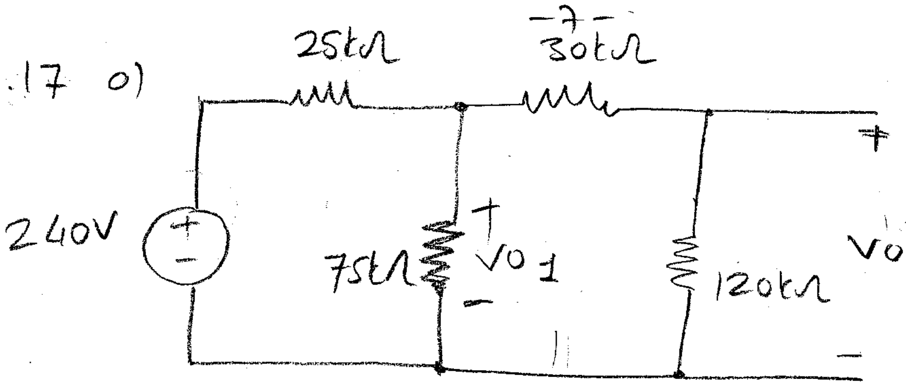
$$v_o = 40 \times \frac{\frac{R_2 R_L}{R_2 + R_L}}{R_1 + \frac{R_2 R_L}{R_2 + R_L}} = \frac{40 R_2 R_L}{R_1(R_2 + R_L) + R_2 R_L} = \frac{40 R_2}{\frac{R_1 R_2}{R_L} + R_1 + R_2}$$

Now from No load condition $R_1 = 4R_2$, also when R_L is minimum, v_o is also at its minimum 7.5V.

$$7.5V = \frac{R_2 3600}{4R_2(R_2 + 3600) + R_2 3600} = \frac{40 \times 3600 R_2}{4R_2^2 + 18000 R_2}$$

$$\frac{144000}{4R_2 + 18000} = 7.5 \quad R = 300\Omega \text{ and } R_1 = 4R_2 = 1200\Omega$$

3.17 a)



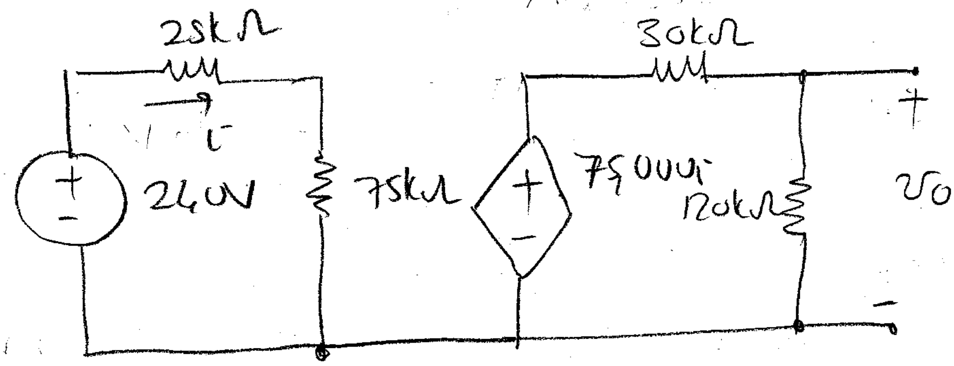
$120k\Omega + 30k\Omega = 150k\Omega$

$75k\Omega \,||\, 150k\Omega = \frac{75 \times 150}{75+150} = \frac{\cancel{75} \times 150}{\cancel{75}(1+2)} = 50k\Omega$

$V_{o1} = \frac{240 \times 50,000}{25,000 + 50,000} = 160V$

$V_o = \frac{120,000}{150,000}(V_{o1}) = \frac{4}{5} \times 160V = 128V$

b)



$i = \frac{240V}{25k\Omega + 75k\Omega} = \frac{240V}{100k\Omega} = \underline{2.4mA}$

$75,000\, i = 75 \times 2.4 = 180V$

$v_o = 180V \times \frac{120k\Omega}{120k\Omega + 30k\Omega} = \underline{144V}$

c) It removes loading effect of second voltage divider on the first voltage divider. The open circuit voltage of the first divider is

$V'_{o1} = \frac{75,000}{(100,000)}(240) = 180V$

Note this is the input voltage to the second voltage divider when the current controlled voltage source is used.

3.18 $I_s = \frac{P_s}{V_s} = \frac{36W}{24V} = 1.5A$

$(R_1+R_2)(1.5A) = 12V$ $\quad R_1+R_2 = 8\Omega$

$R_2(1.5A) = 6V$ $\quad R_2 = 4\Omega \Rightarrow R_1 = 8\Omega - 4\Omega = 4\Omega$

$R_3 I_3 = 12V$ $\quad R_3 \cdot (1.5A) = 12V$ $\quad R_3 = 8\Omega$

3.21 $i_g = i_1 + i_2 + i_3 + i_4 = 8\,mA \Rightarrow$ K-C-L

$i_1 = \frac{V_g}{R_1} = \frac{4}{R_1} \Rightarrow i_2 = \frac{i_1}{2} = \frac{2}{R_1}$ $\quad i_3 = \frac{i_2}{10} = \frac{i_1}{20} = \frac{1}{5R_1}$

since $i_3 = i_4$ $\quad i_4 = \frac{1}{5R_1}$

substituting the values of i_1, i_2, i_3 and i_4 into the K-C-L equation

$$\frac{4}{R_1} + \frac{2}{R_1} + \frac{1}{5R_1} + \frac{1}{5R_1} = 8\,mA$$

$40R_1 = 20 + 10 + 1 + 1 = 32 \Rightarrow R_1 = \frac{32}{40} = 0.8k\Omega$

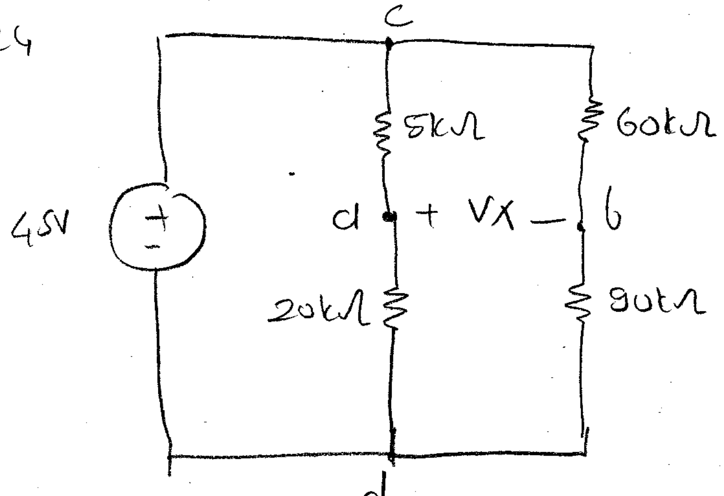
$i_2 = \frac{V_g}{R_2} \rightarrow R_2 = \frac{V_g}{i_2} \rightarrow i_1 = \frac{4V}{0.8k\Omega} = 5\,mA$ $\quad i_2 = \frac{i_1}{2} = 2.5\,mA$

thus $R_2 = \frac{4V}{2.5mA} = 1.6k\Omega$ $\quad R_3 = \frac{V_g}{i_3}$ but $i_3 = \frac{i_2}{10} = 0.25\,mA$

$R_3 = \frac{V_g}{i_3} = \frac{4V}{0.25mA} = 16k\Omega$ $\quad R_4 = \frac{V_g}{i_4}$ but $i_4 = i_3$

thus $R_4 = R_3 = 16k\Omega$

3.24



a) $V_x = V_{ad} - V_{bd}$

Using Voltage Division:

$V_{ad} = \frac{20k\Omega}{20k\Omega + 5k\Omega} \times 45V = 36V$

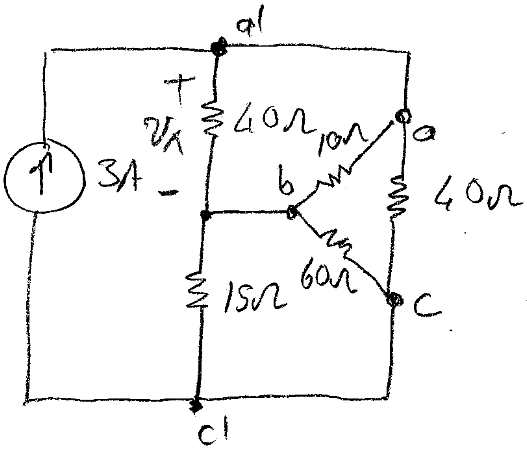
$V_{bd} = \frac{90k\Omega}{90k\Omega + 60k\Omega} \times 45V = 27V$

$V_x = V_{ad} - V_{bd} = 36V - 27V = 9V$

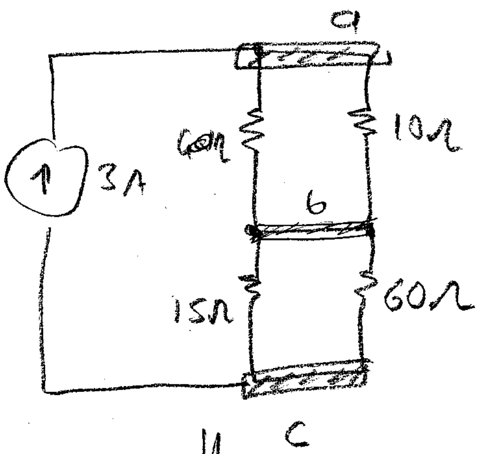
b) $V_{ad} = \frac{20}{25} \times V_s = 0.8V_s$ $\quad V_{bd} = \frac{90}{150} \times V_s = 0.6V_s$

$V_x = V_{ad} - V_{bd} = 0.2V_s$

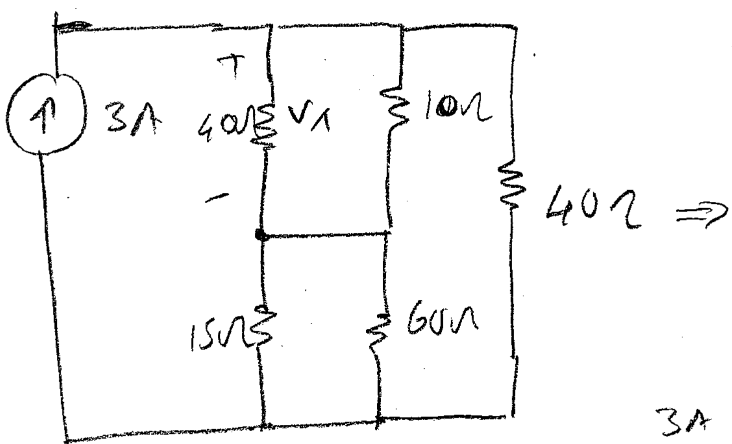
3-30 After connecting the device the circuit looks like this:



Note as a and a' and c and c' are at the same potential the 10Ω resistor is connected in parallel with the 40Ω resistors and the 60Ω resistor is connected in parallel with the 15Ω resistor. We can combine a and a' into a single node. Similarly we can combine c and c' into a single node.

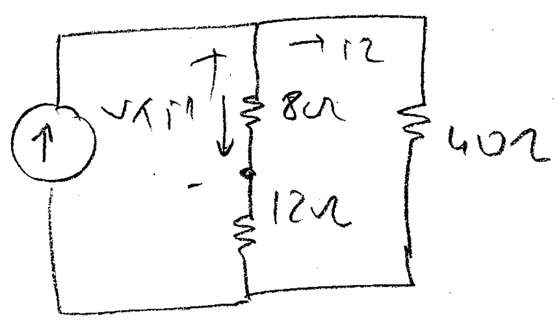


No we note that at the original circuit the 40Ω resistor on the right was connected between nodes a and c. So we connect it the same way between nodes a and c and obtain the final circuit



$40\Omega \| 10\Omega = 8\Omega$

$15\Omega \| 60\Omega = 12\Omega$



We can now apply current division to find i_1 and i_2.

$$R_{eq} = 20\Omega \| 60\Omega = \frac{800}{60} = \frac{40}{3}\Omega$$

$$i_2 = \frac{40}{3} \cdot \frac{1}{20} \times 3A = 2A$$

$$i_1 = \frac{40}{3} \cdot \frac{1}{40} \times 3A = 1A$$

$V_x = 8 i_1 = 16V$

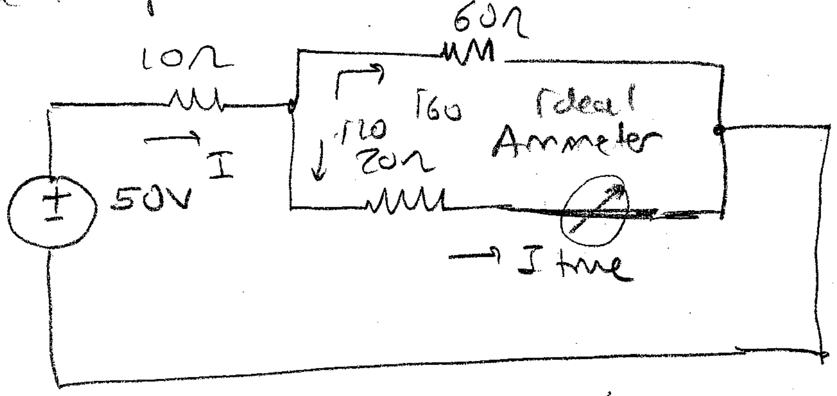
$V_g = 20 i_1 = 40V$

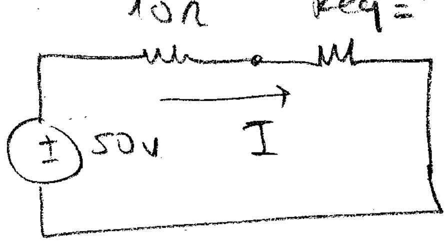
$V_{60} = V_g - V_x = 24V$

$$P_{device} = \frac{24^2}{60} + \frac{16^2}{10} + \frac{40^2}{40} = \underline{75.2W}$$

3.32 We have to determine both the true value of the current and the measured value of the current. Let's call them I_{true} and $I_{measured}$. The true value of the current I_{true} can be found by removing the ammeter or equivalently by replacing it with an ideal ammeter with 0 resistence that is a short circuit.

We the get the true or ideal equivalent circuit;





$$R_{eq} = 60\Omega \| 20\Omega = \frac{1200}{80} = 15\Omega$$

$$I = \frac{50V}{10\Omega + 15\Omega} = \frac{50V}{25\Omega} = 2A$$

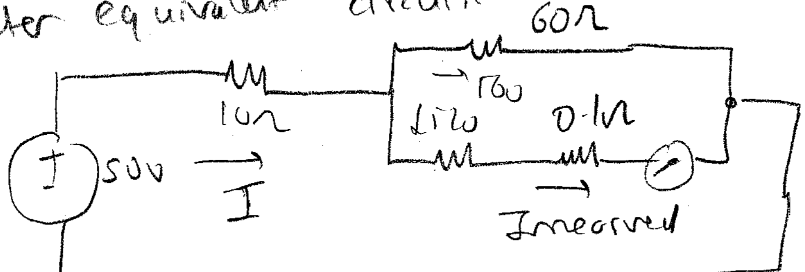
Now we will apply current division to determine the currents I_{60} and I_{20} on the 60Ω and 20Ω resistors.

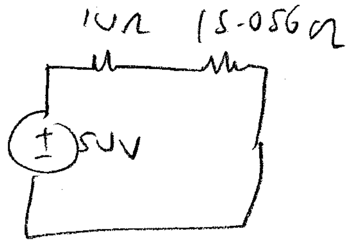
$$I_{20} = \frac{60\Omega}{60\Omega + 20\Omega} \times 2A = \frac{3}{4} \times 2A = 1.5A$$

$$I_{60} = \frac{20\Omega}{60\Omega + 20\Omega} \times 2A = \frac{1}{4} \times 2A = 0.5A$$

Thus $I_{true} = I_{20} = 1.5A$

Let's draw the equivalent circuit using the non-ideal ammeter equivalent circuit.





$$R_{eq} = 60\Omega \| 20.1\Omega = 15.056\Omega$$

$$I = \frac{50V}{10\Omega + 15.056\Omega} = 1.9955A$$

Using current division i_{60} and i_{20} become

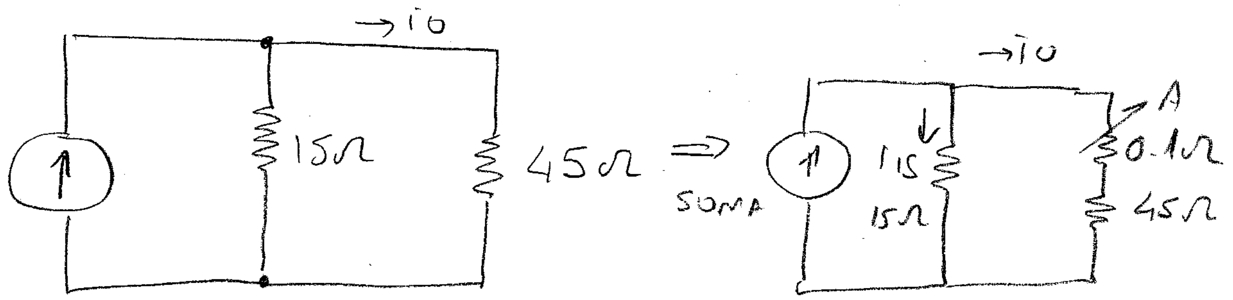
$$i_{20} = 0.75 \times 1.9955 = 1.495$$

$$i_{60} = 0.25 \times 1.9955 = 0.4988$$

$$I_{measured} = 1.69664$$

$$\% \text{ error} = \left(\frac{1.495}{1.5} - 1 \right) \times 100 = -0.3488\%$$

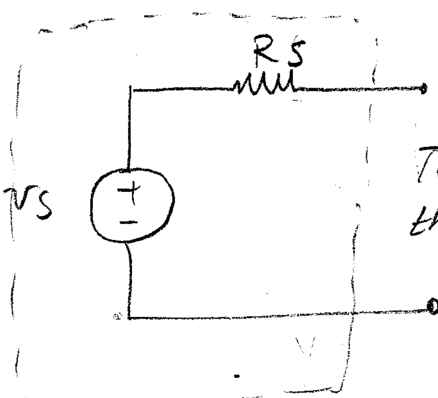
3.33



$$i_0 = \frac{15\Omega}{15\Omega + 45\Omega + 0.1\Omega} \times 50mA = 12.479\,mA$$

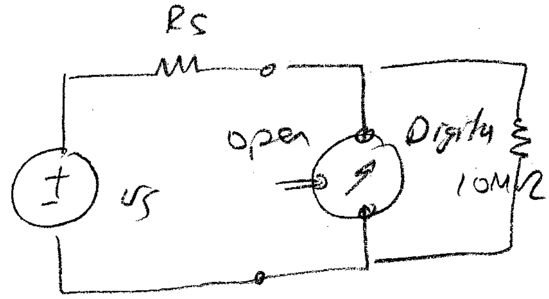
$$\% \text{ error} = \left[\frac{12.48}{12.5} - 1 \right] 100 = -0.1664\%$$

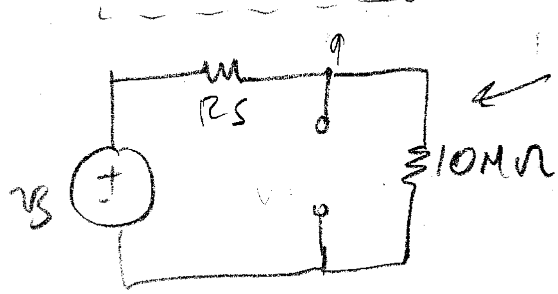
3.47



Terminals of the source

(1)





$$1) \quad V_S \times \frac{10M\Omega}{10M\Omega + R_S} = 50mV$$

$$2) \quad V_S \times \frac{6M\Omega}{6M\Omega + R_S} = 48.75mV$$

From Eq (1) $10 + R_S = 0.2V_S$, substituting into Eq (2)

yields $48.75 = \dfrac{6V_S}{0.2V_S - 6}$ or $V_S = 52mV$

(i) From Eq (1) $50 = \dfrac{520}{10 + R_S}$ or $50R_S = 20$ so $R_S = 400k\Omega$

-58 (a) Convert the upper delta to a wye.

$$R_1 = \frac{(50)(50)}{200} = 12.5\,\Omega$$

$$R_2 = \frac{(50)(100)}{200} = 25\,\Omega$$

$$R_3 = \frac{(50)(100)}{200} = 25\,\Omega$$

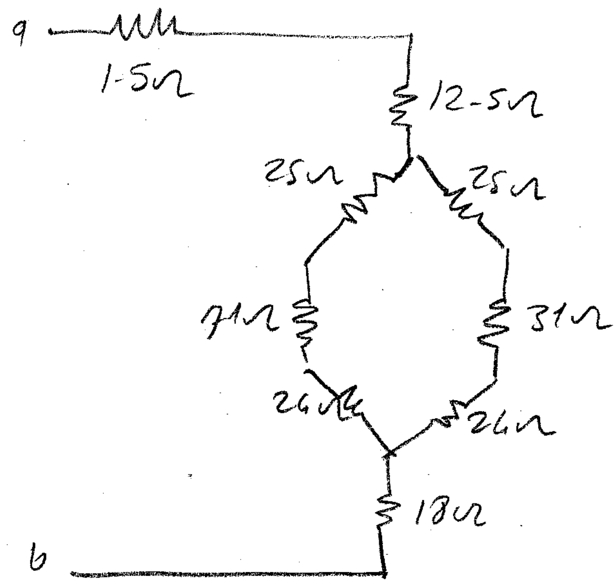
Convert the lower delta to a wye.

$$R_4 = \frac{(60)(80)}{200} = 24\,\Omega$$

$$R_5 = \frac{(60)(60)}{200} = 18\,\Omega$$

$$R_6 = \frac{(60)(80)}{200} = 24\,\Omega$$

Now redraw the circuit using the wye equivalents.



$R_{ab} = 1.5 + 12.5 +$
$(25+71+24) \| (25+31+24) + 18$
$= 1.5 + 12.5 + (120 \| 80) + 18$
$= 1.5 + 12.5 + 48 + 18 = 80\,\Omega$

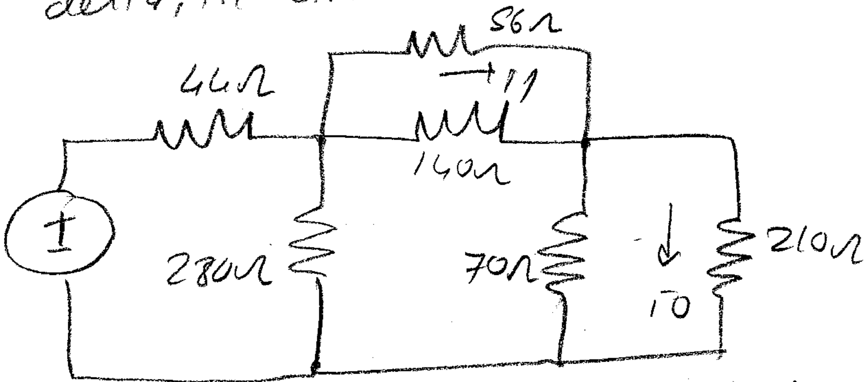
(b) $v_{ab} = 400\,V$

$$i_g = \frac{400}{80} = 5A$$

$$i_o = \frac{120}{120+80}(5) = 3A$$

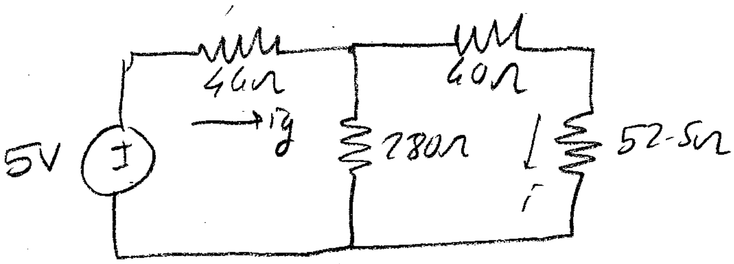
$$P_{31\Omega} = (31)(3)^2 = 279\,W$$

3-59 After the $20\Omega - 80\Omega - 40\Omega$ wye is replaced by its equivalent delta, the circuit reduces to



This circuit can be replaced to the following circuit:

$56\Omega \parallel 140\Omega = 40\Omega$ $\qquad 70\Omega \parallel 210\Omega = 52.5\Omega$



$R_{eq} = 44 + 280 \parallel 92.5 = 113.53\Omega$

$i_g = 5/113.53 = 44.04\,mA$

$i = (280/372.5)(44) = 33.11\,mA$

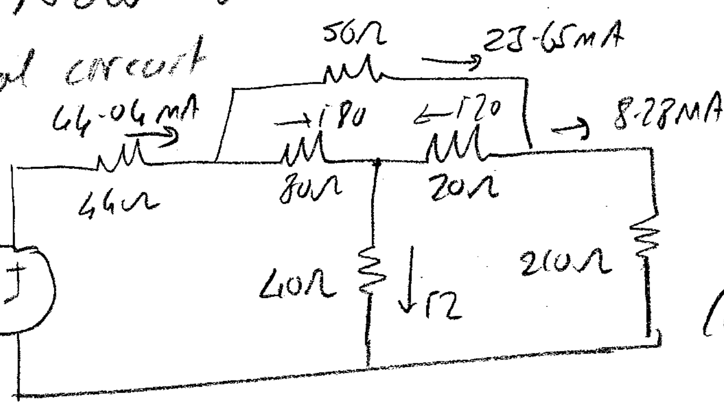
$V_{52.5\Omega} = (52.5)(33.11m) = 1.74V$

$i_o = 1.74/210 = 8.28\,mA$

(b) $V_{40\Omega} = (40)(33.11m) = 1.32V$

$i_1 = 1.32/56 = 23.65\,mA$

(c) Now that io and i1 are known we return to the original circuit



$i_{80\Omega} = 44.04m - 23.65m = 20.39mA$

$i_{20\Omega} = 23.65m - 8.28m = 15.37mA$

$i_2 = i_{80\Omega} + i_{20\Omega} = 35.76\,mA$

(d) $P_{delivered} =$

$(5)(44.04m) = \underline{220.2\,mW}$