EE 210 Chapter 6 Solutions

6.2 a) $0 < t \le 2ms$:

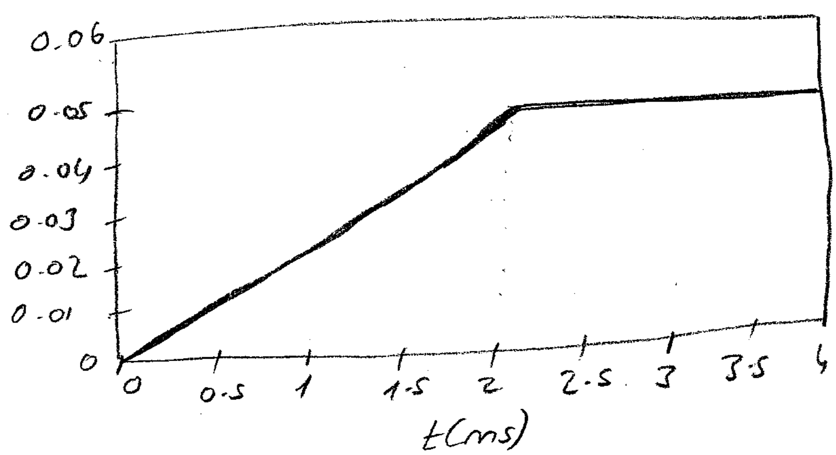
$$i_L = \frac{1}{L}\int_0^t v_s dx + i_L(0) = \frac{1}{200\times10^{-6}}\int_0^t 5\times10^{-3} dx + 0$$

$$= 25x \Big|_0^t = 25t\,A$$

$2ms \le t < \infty$:

$$i_L = \frac{1}{200\times10^{-6}}\int_{2\times10^{-3}}^t (0)dx + 25(2\times10^{-3}) = 50mA$$

b)



4 (a) $v = L\frac{di}{dt}$

$$\frac{di}{dt} = 18[t(-10e^{-10t}) + e^{-10t}] = 18e^{-10t}(1-10t)$$

$$v = (50\times10^{-6})(18)e^{-10t}(1-10t)$$

$$= 0.9e^{-10t}(1-10t)mV, \quad t>0$$

(b) $p = vi$

$$v(200\,ms) = 0.9e^{-2}(1-2) = -121.8\mu V$$

$$i(200ms) = 18(0.2)e^{-2} = 487.2mA$$

$$p(200ms) = (-121.8\times10^{-6})(487.2\times10^{-3}) = -59.34\mu W$$

(c) The inductor is delivering power, because the power is negative.

(d) $w = \frac{1}{2}Li^2 = \frac{1}{2}(50\times10^{-6})(487.2\times10^{-3})^2 = 5.93\mu J$

(e) The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 18[t(-10)e^{-10t} + e^{-10t}) = 18e^{-10t}(1-10t)]$$

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad t = 0.1s$$

$$i_{max} = 18(0.1)e^{-1} = 662.2\,mA$$

$$w_{max} = \frac{1}{2}(50\times10^{-6})(662.2\times10^{-3})^2 \cong 10.96\,\mu J$$

6.7 (a)
$$i = \frac{1}{15\times10^{-3}}\int_0^t 30\sin 500x\,dx - 4$$

$$= 2000\int_0^t \sin 500x\,dx - 4$$

$$= 2000\left[\frac{-\cos 500x}{500}\right]_0^t - 4$$

$$= 4(1-\cos 500t) - 4$$

$$i = -4\cos 500t\ A$$

(b)
$$p = vi = (30\sin 500t)(-4\cos 500t)$$

$$= -120\sin 500t\cos 500t \quad \Rightarrow (\sin 2\alpha = 2\sin\alpha\cos\alpha)$$

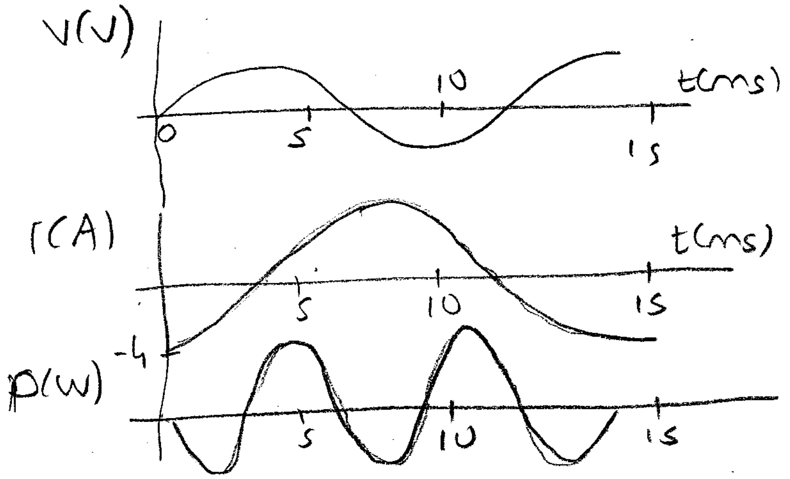
$$p = -60\sin 1000t\ W$$

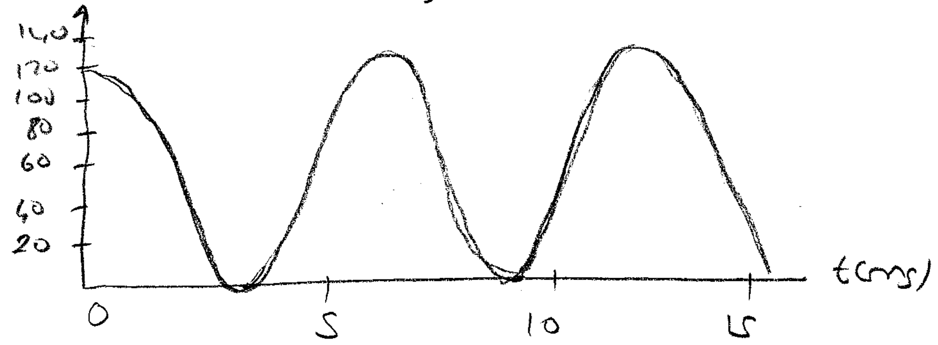
$$w = \frac{1}{2}Li^2$$

$$= \frac{1}{2}(15\times10^{-3})\,16\cos^2 500t$$

$$= 120\cos^2 500t\ mJ$$

$$w = [60 + 60\cos 1000t]\,mJ.$$





(c) Absorbing power: $\pi < t \leq 2\pi$ ms, $3\pi \leq t \leq 4\pi$ ms

Delivering power: $0 \leq t \leq \pi$ ms, $2\pi \leq t \leq 3\pi$ ms

6-11 For $0 \leq t \leq 1.6$ s

$$i_L = \frac{1}{5}\int_0^t 3\times10^{-3}dx + 0 = 0.6\times10^{-3}t$$

$$i_L(1.6s) = (0.6\times10^{-3})(1.6) = 0.96mA$$

$$R_m = (20)(1000) = 20k\Omega$$ (The resistance of the voltmeter when it is measuring 20 V.)

$$v_m(1.6s) = (0.96\times10^{-3})(20\times10^3) = 19.2V$$

6.12 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$$W = \int_0^\infty p\,dx = \int_0^\infty 40x\,[e^{-10x} - 10xe^{-20x} - e^{-20x}]dx = 0.2J$$

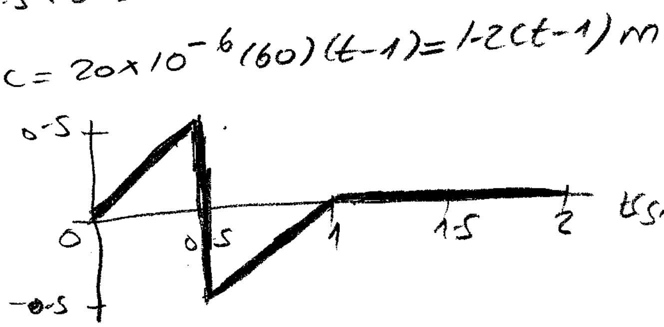
This is the energy stored in the inductor at $t=\infty$.

6.14 $i_C = C(dv/dt)$ $\quad 0 < t < 0.5$

$v_C = 30t^2 V$ $\quad i_C = 20\times10^{-6}(60)t = 1.2t\,mA$

$0.5 < t < 1$: $\quad v_C = 30(t-1)^2 V$

$i_C = 20\times10^{-6}(60)(t-1) = 1.2(t-1)\,mA$



6.15 (a) $0 \le t \le 5\mu s$ $C = 5\mu F$ $\frac{1}{C} = 2\times10^5$

$$v = 2\times10^5 \int_0^t 4dx + 12$$

$$v = 8\times10^5 t + 12V \qquad 0 \le t \le 5\mu s$$

$$v(5\mu s) = 4 + 12 = 16V$$

(b) $5\mu s \le t \le 20\mu s$

$$v = 2\times10^5 \int_{5\times10^{-6}}^t -2dx + 16 = -4\times10^5 t + 2 + 16$$

$$v = -4\times10^5 t + 18V \qquad 5 \le t \le 20\mu s$$

$$v(20\mu s) = -4\times10^5(20\times10^{-6}) + 18 = 10V$$

(c) $20\mu s \le t \le 25\mu s$

$$v = 2\times10^5 \int_{20\times10^{-6}}^t 6dx + 10 = 12\times10^5 t - 24 + 10$$

$$v = 12\times10^5 t - 14V, \qquad 20\mu s \le t \le 25\mu s$$

$$v(25\mu s) = 12\times10^5(25\times10^{-6}) - 14 = 16V$$

(d) $25\mu s \le t \le 35\mu s$

$$v = 2\times10^5 \int_{25\times10^{-6}}^t 4dx + 16 = 8\times10^5 t - 20 + 16$$

$$v = 8\times10^5 t - 4V, \qquad 25\mu s \le t \le 35\mu s$$

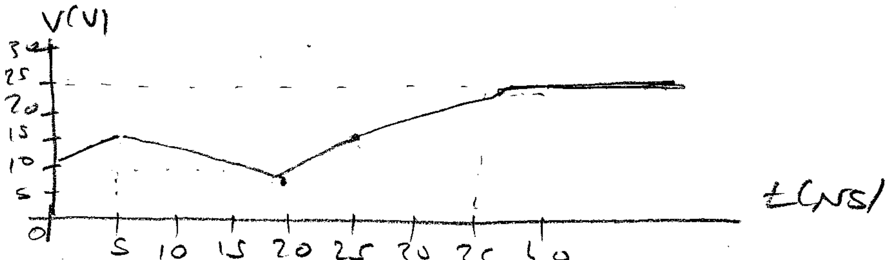
$$v(35\mu s) = 8\times10^5(35\times10^{-6}) - 4 = 24V$$

(e) $35\mu s \le t < \infty$

$$v = 2\times10^5 \int_{35\times10^{-6}}^t 0dx + 24 = 24$$

$$v = 24V, \qquad 35\mu s \le t < \infty$$

(f)



6.16 $v = -10V, \quad t \leq 0; \quad C = 0.8\mu F$

$v = 40 - e^{-1000t}(50\cos 500t + 20\sin 500t) V, \quad t \geq 0$

(a) $i = 0, \quad t < 0$

(b) $\frac{dv}{dt} = 1000e^{-1000t}(50\cos 500t + 20\sin 500t)$

$- e^{-1000t}(-25{,}000\sin 500t + 10{,}000\cos 500t)$

$= e^{-1000t}(50{,}000\cos 500t + 20{,}000\sin 500t$

$+ 25{,}000\sin 500t - 10{,}000\cos 500t$

$= (40{,}000\cos 500t + 45{,}000\sin 500t)e^{-1000t}$

$i = C\frac{dv}{dt} = (32\cos 500t + 36\sin 500t)e^{-1000t}$ mA

(c) NO

(d) Yes, at $t < 0$ $i = 0$ at $t > 0$ $i = 32$, thus there is a change from 0 to 32mA.

(e) $v(\infty) = 40V$

$W = \frac{1}{2}Cv^2 = \frac{1}{2}(0.8 \times 10^{-6})(40)^2 = 640\mu J$

6.17 (a) $i = \frac{400 \times 10^{-3}}{5 \times 10^{-6}} t = 8 \times 10^4 t \qquad 0 \leq t \leq 5\mu s$

$i = 400 \times 10^{-3} \qquad 5 \leq t \leq 20\mu s$

$q = \int_0^{5\times10^{-6}} 8 \times 10^4 t\,dt + \int_{5\times10^{-6}}^{15\times10^{-6}} 400 \times 10^{-3}\,dt$

$= 8 \times 10^4 \frac{t^2}{2}\Big|_0^{5\times10^{-6}} + 400 \times 10^{-3}(10 \times 10^{-6})$

$= 8 \times 10^4 \left(\frac{1}{2}\right)(25 \times 10^{-12}) + 4 \times 10^{-6}$

$= 5\mu C$

[b] $v = \frac{1}{0.25\times10^{-6}}\left[\int_0^{5ns} 8\times10^4 x\,dx + \int_{5ns}^{20ns} 0.4\,dx + \int_{20ns}^{30ns} (10^6 x - 0.5)\,dx\right]$

$= \frac{1}{0.25\times10^{-6}}\left[4\times10^4 t^2 \Big|_0^{5ns} + 0.4t \Big|_{5ns}^{20ns} + (5000t^2 - 0.5t)\Big|_{20ns}^{30ns}\right]$

$= \frac{1}{0.25\times10^{-6}}\left[1\times10^{-6} + 6\times10^{-6} - 10.5\times10^{-6} + 8\times10^{-6}\right] = 18V$

(c) $v(50ns) = 18 + \frac{1}{0.25\times10^{-6}}(5000t^2 - 0.5t)\Big|_{30ns}^{50ns}$

$= 18 + \frac{1}{0.25\times10^{-6}}(-12.5\times10^{-6} + 10.5\times10^{-6}) = 10V$

$W = \frac{1}{2}Cv^2 = \frac{1}{2}(0.25\times10^{-6})(10)^2 = 12.5\mu J$

6.18 (a) $v = \frac{1}{0.5\times10^{-6}}\int_0^{500\times10^{-6}} 50\times10^{-3} e^{-2000t}\,dt - 20$

$= 100\times10^3 \frac{e^{-2000t}}{-2000}\Big|_0^{500\times10^{-6}} - 20$

$= 50(1 - e^{-1}) - 20 = 11.61V$

$W = \frac{1}{2}CV^2 = \frac{1}{2}(0.5)(10^{-6})(11.61)^2 = 33.7\mu J$

(b) $v(\infty) = 50 - 20 = 30V$

$w(\infty) = \frac{1}{2}(0.5\times10^{-6})(30)^2 = 225\mu J$

6-19 (a) $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(0.25)\times10^{-6}(50)^2 = 312.5\,nJ$

(b) $v = (A_1t + A_2)e^{-4000t}$

$v(0) = A_2 = 50\,V$

$\frac{dv}{dt} = -4000e^{-4000t}(A_1t + A_2) + e^{-4000t}(A_1)$

$= (-4000A_1t - 4000A_2 + A_1)e^{-4000t}$

$\frac{dv}{dt}(0) = A_1 - 4000A_2$

$i = C\frac{dv}{dt}, \quad i(0) = C\frac{dv(0)}{dt}$

$\therefore \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{400\times10^{-3}}{0.25\times10^{-6}} = 16\times10^5$

$\therefore 16\times10^5 = A_1 - 4000(50)$

Thus, $A_1 = 16\times10^5 + 2\times10^5 = 18\times10^5\,\frac{V}{s}$

(c) $v = (18\times10^5t + 50)e^{-4000t}$

$i = C\frac{dv}{dt} = 0.25\times10^{-6}\frac{d}{dt}(18\times10^5t + 50)e^{-4000t}$

$i = \frac{d}{dt}\left[(0.45t + 12.5\times10^{-6})e^{-4000t}\right]$

$= (0.45t + 12.5\times10^{-6})(-4000)e^{-4000t} + e^{-4000t}(0.45)$

$= (-1800t - 0.05 + 0.45)e^{-4000t}$

$= (0.40 - 1800t)e^{-4000t}\,A, \quad t \geq 0$

6.21 $30 \| 20 = 12H$

$80 \| (8+12) = 16H$

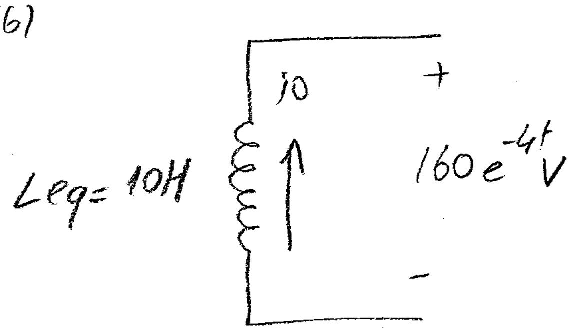
$60 \| (14+16) = 20H$

$15 \| (20+10) = 20H$

$L_{ab} = 5 + 10 = 15H$

6.23 (a) $i_o(0) = i_1(0) + i_2(0) = 4A$

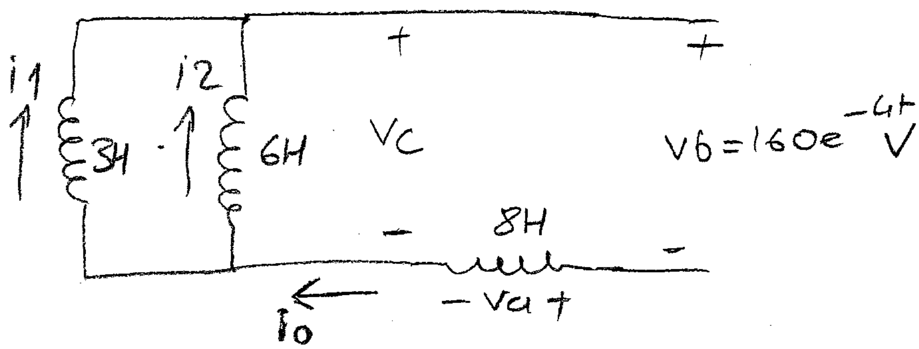
(b)



$$i_o = -\frac{1}{10}\int_0^t 160 e^{-4x}\,dx + 4 = -16\left[\frac{e^{-4x}}{-4}\right]_0^t + 4$$

$$= 4(e^{-4t} - 1) + 4 = 4e^{-4t} A, \quad t \geq 0$$

(c)



$$v_a = 8\frac{d}{dt}(4e^{-4t}) = -128e^{-4t} V$$

$$v_c = v_a + v_b = -128e^{-4t} + 160e^{-4t}$$

$$= 32e^{-4t}$$

$$i_1 = -\frac{1}{3}\int_0^t 32e^{-4x}\,dx + 1$$

$= 2.67e^{-4t} - 2.67 + 1$

$i_1 = 2.67e^{-4t} - 1.67A,$

$t \geq 0$

(d) $i_2 = -\frac{1}{6}\int_0^t 32e^{-4x}dx + 3$

$= 1.33e^{-4t} - 1.33 + 3$

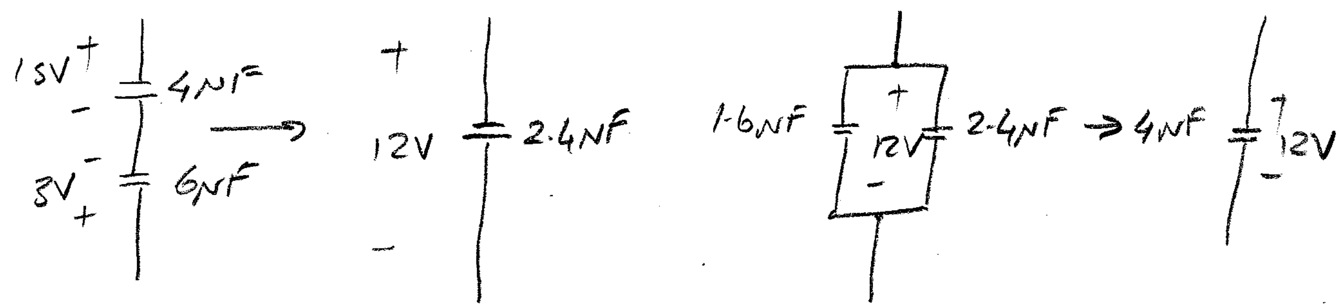
$i_2 = 1.33e^{-4t} + 1.67A, \quad t \geq 0$

(e) $w(0) = \frac{1}{2}(3)(1)^2 + \frac{1}{2}(6)(3)^2 + \frac{1}{2}(8)(4)^2 = 92.5J$

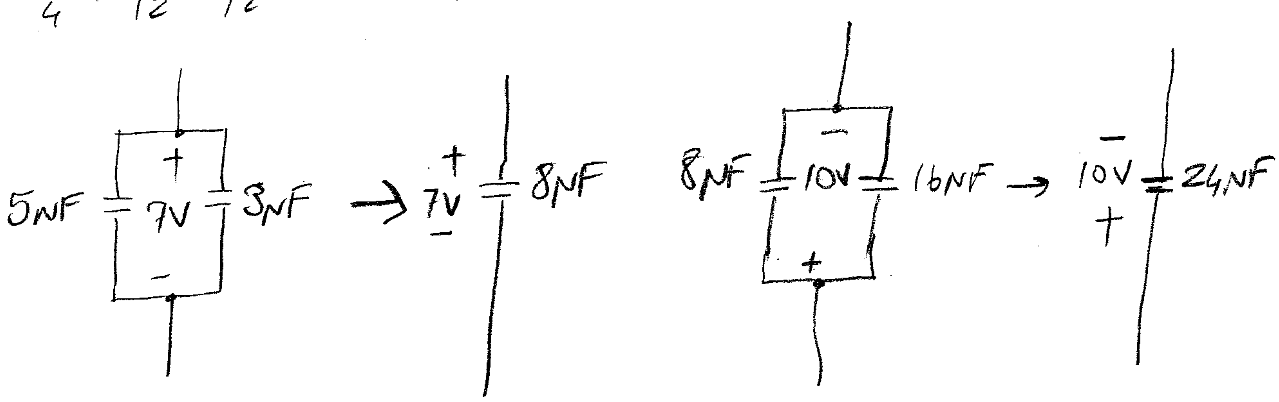
(f) $w_{del} = \frac{1}{2}(10)(4)^2 = 80J$

(g) $w_{trapped} = 92.5 - 80 = 12.5J$

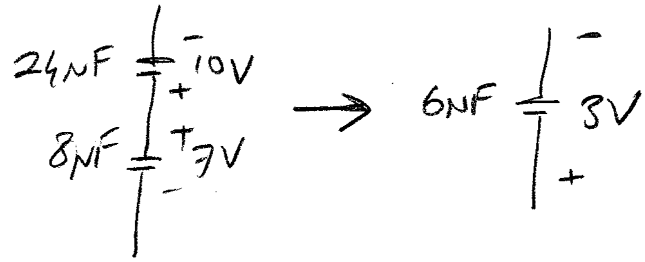
.25 $\frac{1}{4} + \frac{1}{6} = \frac{5}{12} \therefore C_{eq} = 2.4\mu F$



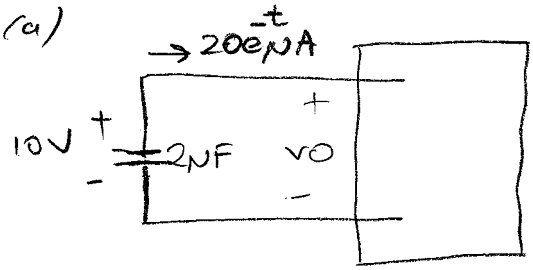
$\frac{1}{4} + \frac{1}{12} = \frac{4}{12} \therefore C_{eq} = 3\mu F$



$\frac{1}{24} + \frac{1}{8} = \frac{4}{24} \therefore C_{eq} = 6\mu F$



P 6.27 (a)



$$v_o = -\frac{1}{2\times10^{-6}}\int_0^t 20\times10^{-6}e^{-x}dx + 10$$

$$= 10e^{-x}\Big|_0^t + 10$$

$$= 10e^{-t}V, \quad t \geq 0$$

(b) $$v_1 = -\frac{1}{3\times10^{-6}}(20\times10^{-6})e^{-x}\Big|_0^t + 4$$

$$= 6.67e^{-t} - 2.67V, \quad t \geq 0$$

(c) $$v_2 = -\frac{1}{6\times10^{-6}}(20\times10^{-6})e^{-x}\Big|_0^t + 6$$

$$= 3.33e^{-t} + 2.67V, \quad t \geq 0$$

(d) $$p = vi = (10e^{-t})(20\times10^{-6})e^{-t}$$

$$= 200\times10^{-6}e^{-2t}$$

$$w = \int_0^\infty 200\times10^{-6}e^{-2t}dt$$

$$= 200\times10^{-6}\frac{e^{-2t}}{-2}\Big|_0^\infty$$

$$= -100\times10^{-6}(0-1) = 100\mu J$$

(e) $$w = \frac{1}{2}(3\times10^{-6})(4)^2 + \frac{1}{2}(6\times10^{-6})(6)^2$$

$$= 132\mu J$$

(f) $$w_{trapped} = \frac{1}{2}(3\times10^{-6})(8/3)^2 + \frac{1}{2}(6\times10^{-6})(8/3)^2 = 32\mu J$$

$(\frac{1}{2}C\cdot V^2)$ $\quad$ V at $t\to\infty = 2.67V$.

Check: $100+32=132\,\mu J$

(g) Yes, they agree.

6.32 $$\frac{di_o}{dt} = 5\{e^{-2000t}[-8000\sin 4000t + 4000\cos 4000t]$$

$$-2000e^{-2000t}[2\cos 4000t + \sin 4000t]\}$$

We need to find $\frac{di_o(0^+)}{dt}$ to find the inductor voltage.

$$\frac{di_o(0^+)}{dt} = 5[1(4000)+(-2000)(2)] = 0$$

$$v_2(0^+) = 10\times10^{-3}\frac{di_o}{dt}(0^+) = 0 \quad \left(v_2 = v_L = L\frac{di}{dt}\right)$$

$$v_1(0^+) = 40\,i_o(0^+) + v_2(0^+) = 40(10)+0 = 400V$$

6.33 $$v_C = -\frac{1}{0.625\times10^{-6}}\left(\int_0^t 1.5e^{-16{,}000x}\,dx - \int_0^t 0.5e^{-4000x}\,dx\right) - 50$$

$$= 150(e^{-16{,}000t}-1) - 200(e^{-4000t}-1) - 50$$

$$= 150e^{-16{,}000t} - 200e^{-4000t}\,V$$

$$v_L = 25\times10^{-3}\frac{di_o}{dt}$$

$$= 25\times10^{-3}(-24{,}000e^{-16{,}000t} + 2000e^{-4000t})$$

$$= -600e^{-16{,}000t} + 50e^{-4000t}\,V$$

$$v_o = v_C - v_L$$

$$= (150e^{-16{,}000t} - 200e^{-4000t}) - (-600e^{-16{,}000t} + 50e^{-4000t})$$

$$= 750e^{-16{,}000t} - 250e^{-4000t}\,V, \quad t>0.$$