Q1. With the usual matrix matrix addition and scalar matrix multiplication is D a linear combination of A, B and C where

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 4 & -1 & 1 \\ 0 & 0 & -8 \\ 1 & 6 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 2 & 0 \\ 4 & 3 & 1 \\ 3 & 0 & -2 \\ 1 & 5 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 2 & 9 \\ 1 & 0 & 8 \\ 0 & -1 & 0 \\ 7 & 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} -8 & 4 & 1 \\ 11 & -11 & -5 \\ -6 & 1 & -36 \\ -4 & 20 & 8 \end{pmatrix}.$$

Q2. Recall the CVS vector space discussed in class with the following operations:

vector addition

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \oplus \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 - 4 \\ y_1 + y_2 - 3 \end{pmatrix}$$

scalar vector multiplication

$$\alpha \odot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \alpha x - 4\alpha + 4 \\ \alpha y - 3\alpha + 3 \end{pmatrix}$$

In CVS are the vectors $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ linearly dependent?

Q3. Are the continuous functions $e^{2\ln x}, x^2, \cos x$ linearly dependent or independent, under the standard operations on functions?

Q4. Suppose the following two equalities hold:

$$\vec{v} = \alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + \cdots + \alpha_k \vec{u}_k$$
$$\vec{v} = \beta_1 \vec{u}_1 + \beta_2 \vec{u}_2 + \cdots + \beta_k \vec{u}_k$$

such that for at least one index i, we have $\alpha_i \neq \beta_i$. Does it mean that $\vec{u}_i = \vec{0}$, where $\vec{0}$ is the zero vector?

Q5. Suppose that $\vec{u}_1, \vec{u}_2, \vec{u}_3$ are linearly dependent. What can you say about the linear dependence or independence of the vectors $\vec{v}_1 = \vec{u}_1 - 2\vec{u}_2$ and $\vec{v}_2 = 3\vec{u}_2 + 2\vec{u}_3$?