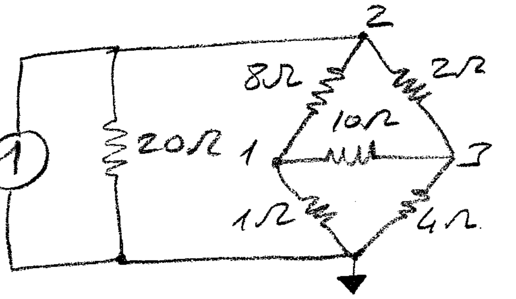
Solutions of Problem Set 4-2

4.53 [a] There are three unknown voltages and three unknown mesh currents, so the number of simultaneous equations required is the same for both methods. If either the top or bottom node is selected as the reference node, the node-voltage method has the advantage of having to solve three simultaneous equations. Therefore, the node-voltage method is preferred.

[b]



The node voltage equations are:

$$\frac{v_1}{1} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{10} = 0$$

$$-4 + \frac{v_2}{20} + \frac{v_2 - v_1}{8} + \frac{v_2 - v_3}{2} = 0$$

$$\frac{v_3 - v_1}{10} + \frac{v_3 - v_2}{2} + \frac{v_3}{4} = 0$$

Putting the equations in standard form:

$$v_1\left(1 + \frac{1}{8} + \frac{1}{10}\right) + v_2\left(-\frac{1}{8}\right) + v_3\left(-\frac{1}{10}\right) = 0$$

$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{20} + \frac{1}{8} + \frac{1}{2}\right) + v_3\left(-\frac{1}{2}\right) = 4$$

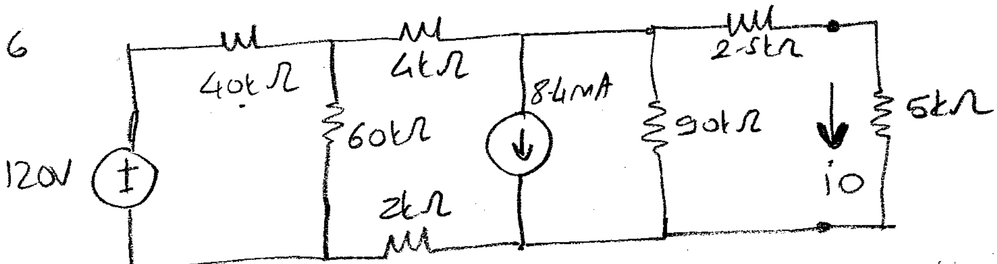
$$v_1\left(-\frac{1}{10}\right) + v_2\left(-\frac{1}{2}\right) + v_3\left(\frac{1}{2} + \frac{1}{10} + \frac{1}{4}\right) = 0$$

Solving, $v_1 = 1.72V$; $v_2 = 11.33V$; $v_3 = 6.87V$

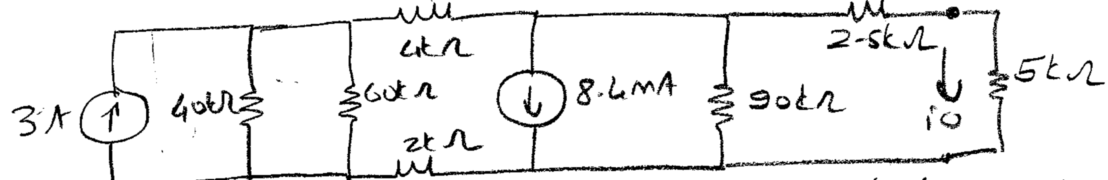
$$P_{4A} = -(11.33)(4) = -45.32W$$

Therefore, the 4A source is developing 45.32W

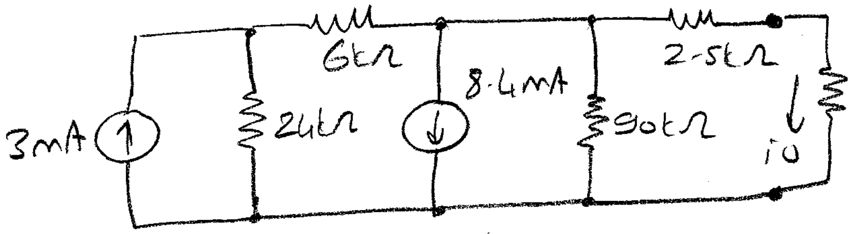
4-56



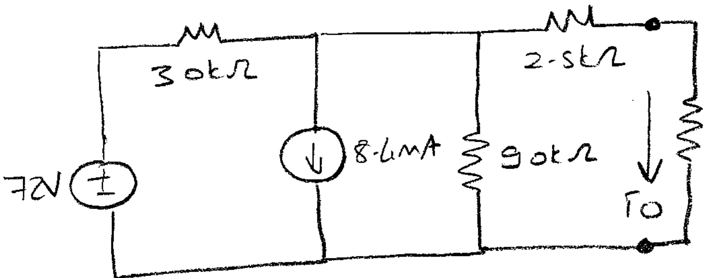
We convert the 120V source in series with the 40kΩ resistor, into a current source of 3A in parallel with the 40kΩ resistor first.



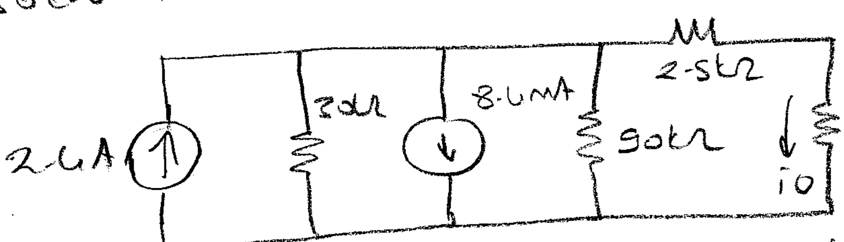
We next combine the 40kΩ and 60kΩ parallel resistors into a single parallel equivalent resistor and the 4kΩ and 6kΩ resistors into a single series equivalent resistance, and obtain the following circuit $R_{eq}\ 40k\Omega \| 60k\Omega = \frac{40 \times 60}{40+60} = 24k\Omega$ $R_{eq\,series} = 4k\Omega + 2k\Omega = 6k\Omega$



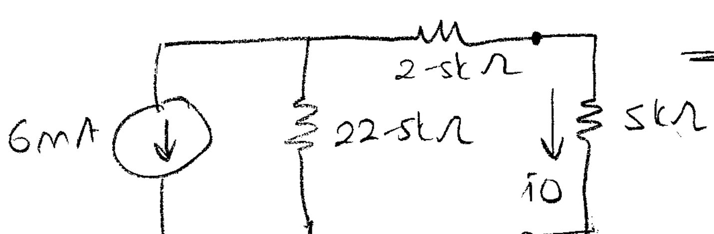
Now we perform another source conversion and convert the 3mA current source to a voltage source in series with the 24kΩ resistor $V_s = (3mA)(24k\Omega) = 72V$, and we combine the 24kΩ which now becomes a series connected resistor with the 6kΩ resistor $R_{series} = 24k\Omega + 6k\Omega = 30k\Omega$. The resulting circuit is shown below.



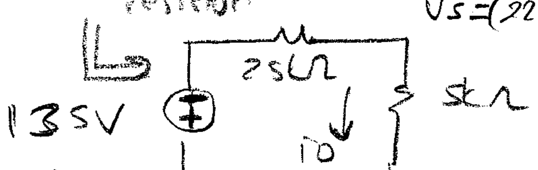
Now, after combining the 24kΩ and 6kΩ resistors, we have a different resistance value of 30kΩ and with this resistor, we perform a source conversion again to a current source in parallel with the 30kΩ resistor



No we can simply combine the two resistances 30kΩ and 90kΩ into their parallel equivalent and the two current sources into a single current source thanks to KCL to obtain the following circuit. $R_{eq} = 30 \times 90 / 30 + 90 = \frac{90}{4} = 22.5k\Omega$

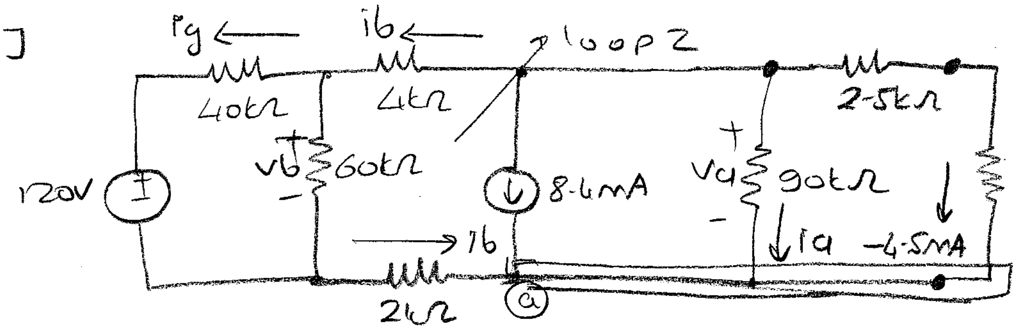


Finally we convert the 22.5kΩ in series with the 6mA current source into a voltage source in series with the 22.5kΩ resistor $V_s = (22.5k\Omega)(-6mA) = -135V$



$I_o = -135/30{,}000 = -4.5mA$

[6]



$v_a = (7500)(-0.0045) = -33.75\,V$

$i_a = \dfrac{v_a}{90{,}000} = \dfrac{-33.75}{90{,}000} = -0.375\,mA$

K.C.L at node a

$i_b = -8.4 + 0.375\times10^{-3} + 4.5\times10^{-3} = -3.525\,mA$

K.V.L for loop 2

$v_b = (6000)(3.525\times10^{-3}) - 33.75 = -12.6\,V$

$i_g = \dfrac{-12.6-120}{40{,}000} = -3.315\,mA$

$P_{120V} = (120)(-3.315\times10^{-3}) = -397.8\,mW$

check:

$P_{8.4\,mA} = (-33.75)(8.4\times10^{-3}) = -283.5\,mW$

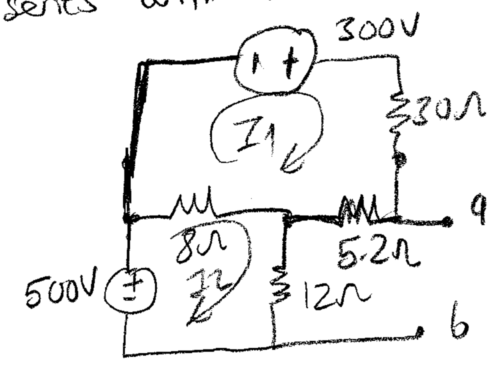
$\Sigma P_{delivered} = 397.8 + 283.5 = 681.3\,mW$

$\Sigma P_{dissipated} = (40{,}000)(-3.315\times10^{-3})^2 + \dfrac{(-12.6)^2}{60{,}000}$

$+ \dfrac{(-33.75)^2}{90{,}000} + (6000)(-3.525\times10^{-3})^2 + (7500)(-4.5\times10^{-3})^2$

$= 681.3\,mW$

4-61 We transform the 10A current source in parallel with the 30Ω resistor into a voltage source of 300V in series with the 30Ω resistor, and obtain the following circuit.



Then we use the mesh-current method, to determine the mesh currents

Mesh I $\;-300 + 35.2I_1 + 8(I_1 - I_2) = 0$

Mesh II $\;-500 + 8(I_2 - I_1) + 12I_2 = 0$

Putting these into standard form, we get:

Mesh I Eqn. $43.2I_1 - 8I_2 = 300$

Mesh II Eqn. $-8I_1 + 20I_2 = 500$

From II $\quad I_1 = \dfrac{20I_2 - 500}{8} = 2.5I_2 - 62.5$

Substituting into Eqn 1.

$$43.2(2.5I_2 - 62.5) - 8I_2 = 300$$

$$108I_2 - 2700 - 8I_2 = 300$$

$$100I_2 = 3000$$

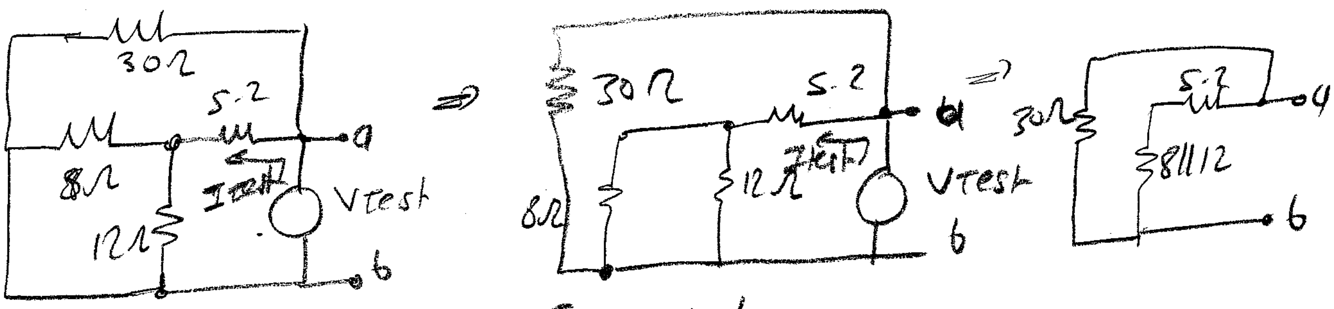
$$I_2 = 30A \;,\; I_1 = 2.5 \times 30 - 62.5 = 12.5A$$

$$V_{ab} = 5.2I_1 + 12I_2 = 5.2 \times 12.5 + 12 \times 30$$

$$= 65 + 360 = 425V$$

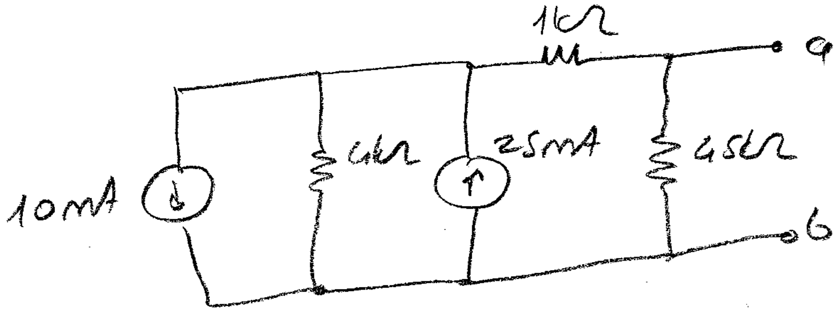
The open circuit voltage $V_{ab} = V_{Th} = 425V$.

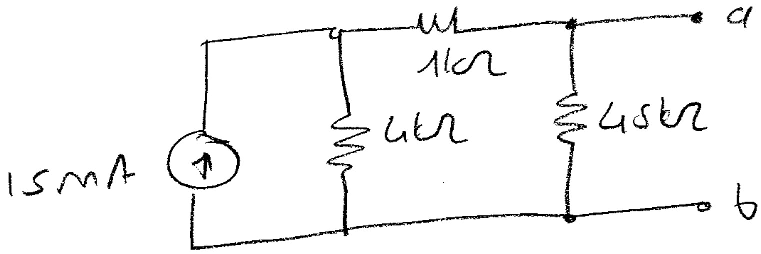
To find the thevenin resistance ① remove the voltage sources from the circuit, by replacing them with short-circuits ② Apply a test voltage between the terminals a, b ③ $R_{TH} = \dfrac{V_{test}}{I_{test}}$

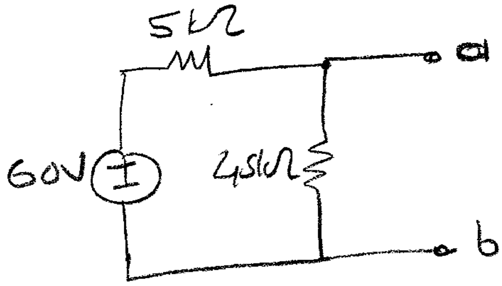


$$R_{TH} = (8\|12 + 5.2) \| 30 = 7.5\Omega$$

4-63 First, we find the Thevenin equivalent with respect to a,b using a succession of source transformations.

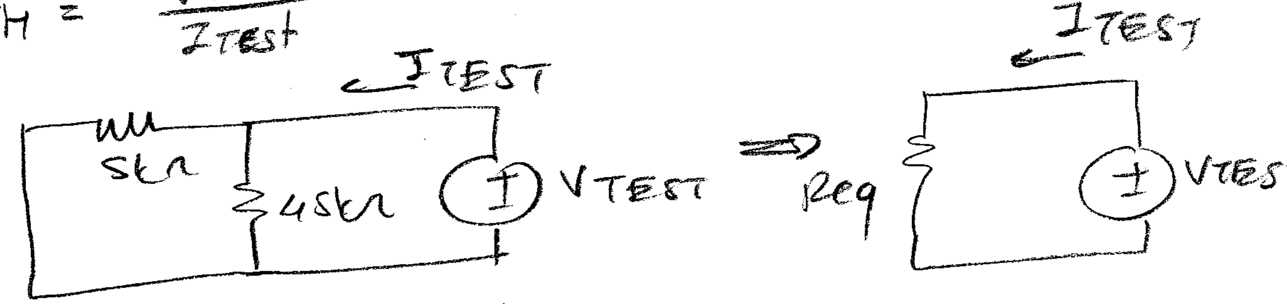


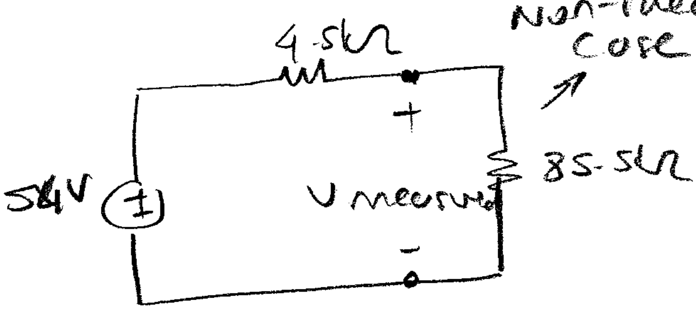




$$V_{Th} = \frac{45}{45+5} \times 60 = 54V$$

To find the thevenin resistance (1) we remove the independent voltage source by a short-circuit.
between terminals a,b
(2) Apply a test voltage
(3) $R_{TH} = \frac{V_{TEST}}{I_{TEST}}$





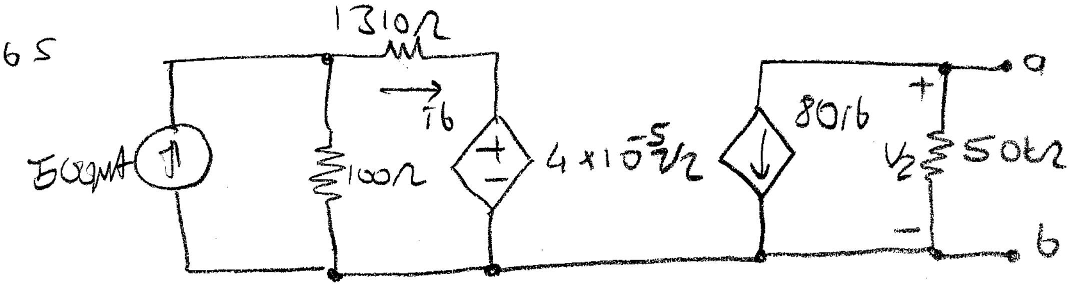
Non-ideal case

$$Req = \frac{5 \times 45}{5+45} = 4.5k\Omega$$

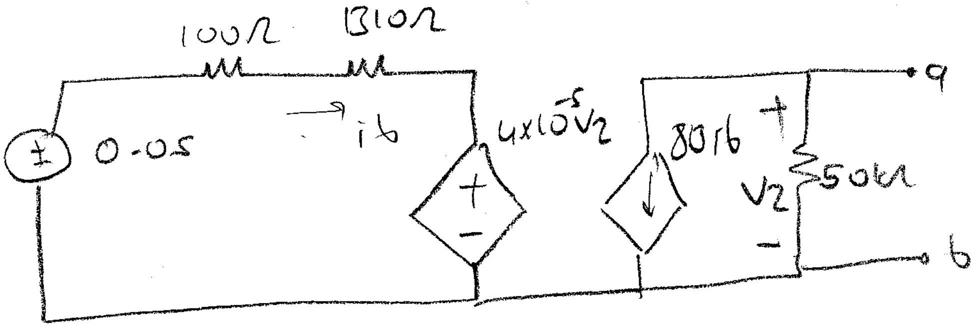
$$V_{measured} = \frac{85.5}{90}(54) = \underline{\underline{51.3V}}$$

(c) $\%error = \left(\frac{51.3 - 54}{54}\right) \times 100 = -5\%$

4-65



We convert the 500μA source into a voltage source in series with the 100 Ω resistor and obtain the following circuit



using ohm's Law:

$V_2 = -(80 i_b)(50k\Omega) = -40\times10^5 i_b$

$-0.05 + 1410 i_b + 4\times10^{-5} V_2 = 0$

$-0.05 + 1410 i_b + 4\times10^{-5}(-40\times10^5 i_b) = 0$

$-0.05 + 1410 i_b - 160 i_b = 0$

$1250 i_b = 0.05$

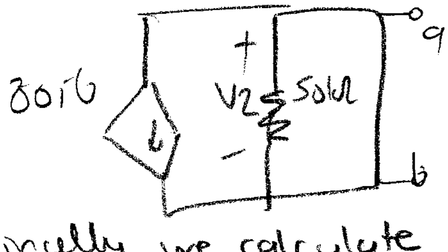
$i_b = \frac{0.05}{1250} = 40 nA$

Thus, $V_{Th} = V_2 = -40\times10^5 i_b = -40\times10^5(40\times10^{-6}) = -160V$

We calculate i_b using current division on the left hand side of the circuit:

$i_b = \frac{100}{100+1310} 500\times10^{-6} = 35.461 nA.$

Calculate the short circuit current from the right hand side of the circuit when a short circuit is placed between a to b



⇒ Note all current goes through the short circuit and the current through 50kΩ resistor is zero, thus

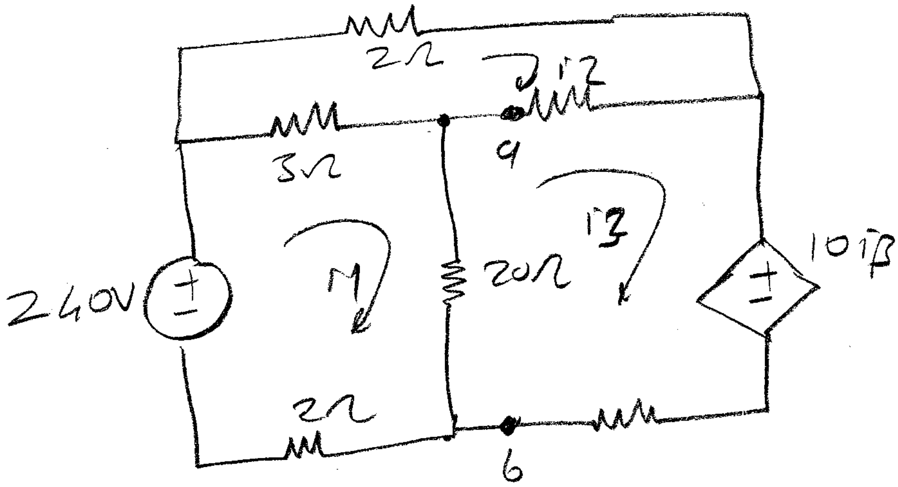
$V_2 = V_{ab} = 0$

$I_{sc} = -80(35.461\times10^{-6}) = -2.8369\times10^{-3} mA$

Finally, we calculate R_{Th} from the short circuit current & open circuit voltage $R_{TH} = -160/-2.8369\times10^{-3} = 56.4k\Omega$

4.76 We find the Thevenin equivalent with respect to the terminals of R_L.

Open circuit voltage:



The mesh current equations are:

$-240 + 3(i_1 - i_2) + 20(i_1 - i_3) + 2i_1 = 0$

$2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$

$10 i_B + i_3 + 20(i_3 - i_1) + 4(i_3 - i_2) = 0$

The dependent source constraint equation is

$i_B = i_2 - i_1$

Placing these equations in standard form:

$i_1(3 + 20 + 2) + i_2(-3) + i_3(-20) + i_B(0) = 240$

$i_1(-3) + i_2(2 + 4 + 3) + i_3(-4) + i_B(0) = 0$

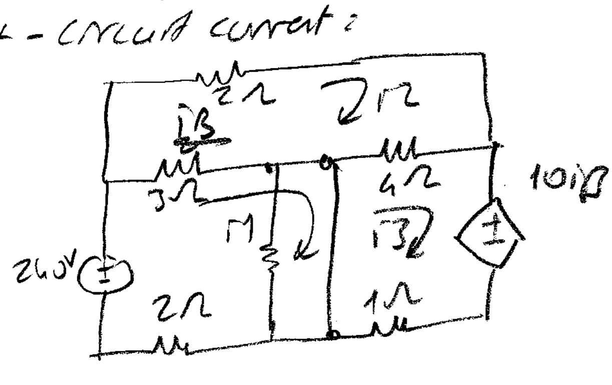
$i_1(-20) + i_2(-4) + i_3(4 + 1 + 20) + i_3(10) = 0$

$i_1(1) + i_2(-1) + i_3(0) + i_B(1) = 0$

Solving $i_1 = 99.6A$; $i_2 = 78A$; $i_3 = 100.8A$; $i_B = -21.6A$

$V_{Th} = 20(i_1 - i_3) = -24V$

Short-circuit current:



The mesh current equations are:

$$-240 + 3(i_1 - i_2) + 2i_1 = 0$$

$$2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$$

$$10i_\beta + 1i_3 + 4(i_3 - i_2) = 0$$

The dependent constraint equation is:

$$i_\beta = i_2 - i_1$$

Place these equations in standard form:

$$i_1(3+2) + i_2(-3) + i_3(0) + i_\beta(0) = 240$$

$$i_1(-3) + i_2(2+4+3) + i_3(-4) + i_\beta(0) = 0$$

$$i_1(0) + i_2(-4) + i_3(4+1) + i_\beta(10) = 0$$

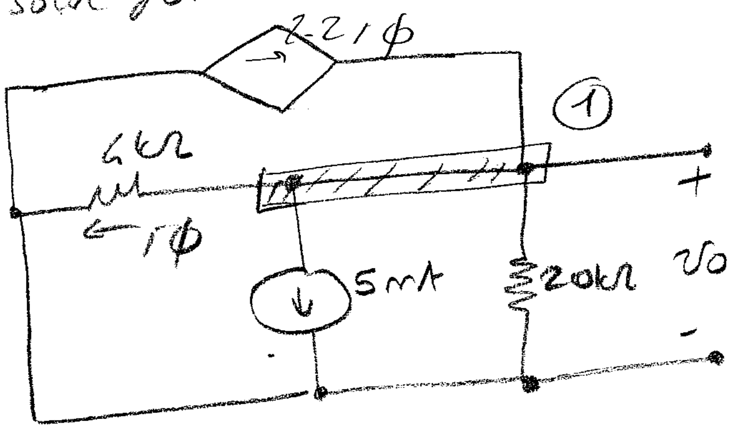
$$i_1(1) + i_2(-1) + i_3(0) + i_\beta(1) = 0$$

Solving, $i_1 = 92A$; $i_2 = 73.33A$; $i_3 = 96A$; $i_\beta = -18.67A$

$i_{sc} = i_1 - i_3 = -4A$; $R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-24}{-4} = 6\Omega$

[6] $P_{max} = \frac{12^2}{6} = 24W$

4-90 We first remove the 25V independent voltage source and solve for v_o in the resultant circuit



We write the node equation at Node 1.

I $i_\phi + 5 - 2.2i_\phi + \frac{V_1}{20} = 0$

but $i_\phi = \frac{V_1}{4}$, substituting into I.

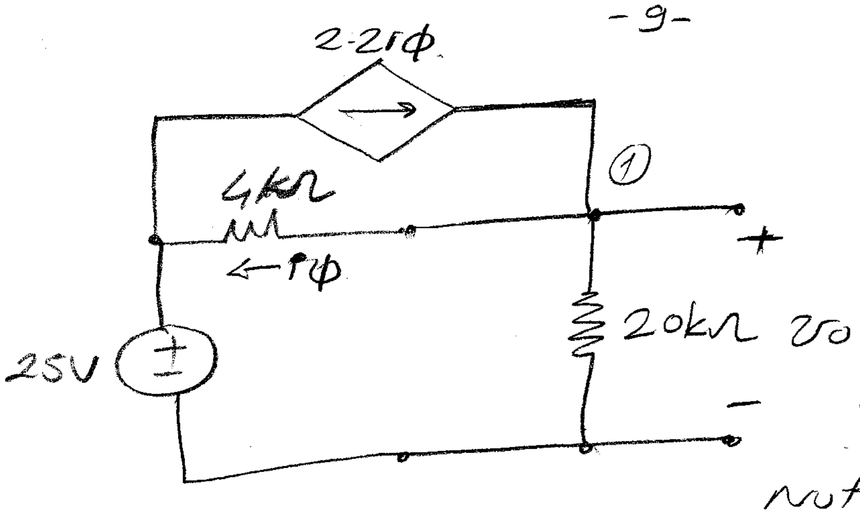
$$\frac{V_1}{4} - 2.2\left(\frac{V_1}{4}\right) + \frac{V_1}{20} = -5$$

$$\Rightarrow V_1(0.25 - 0.55 + 0.05) = -5$$

$$-0.25V_1 = -5$$

$V_1 = 20V$ $v_o = V_1 = 20V$, Let's call this

v_o due to 5mA $\Rightarrow$ v_{o5mA}, next we remove the current source from the circuit, and solve for v_o in the resultant circuit



We write the node-voltage equation at node 1

$$i_\phi - 2.2 i_\phi + \frac{v_o}{20} = 0$$

$$\text{I} \quad -1.2 i_\phi + \frac{v_o}{20} = 0$$

noting that $i_\phi = \frac{v_o - 25}{4}$

and substituting into I

$$-1.2\left[\frac{v_o - 25}{4}\right] + \frac{v_o}{20} = 0$$

$$v_o(-0.3 + 0.05) = -7.5$$

$$-0.25 v_o = -7.5$$

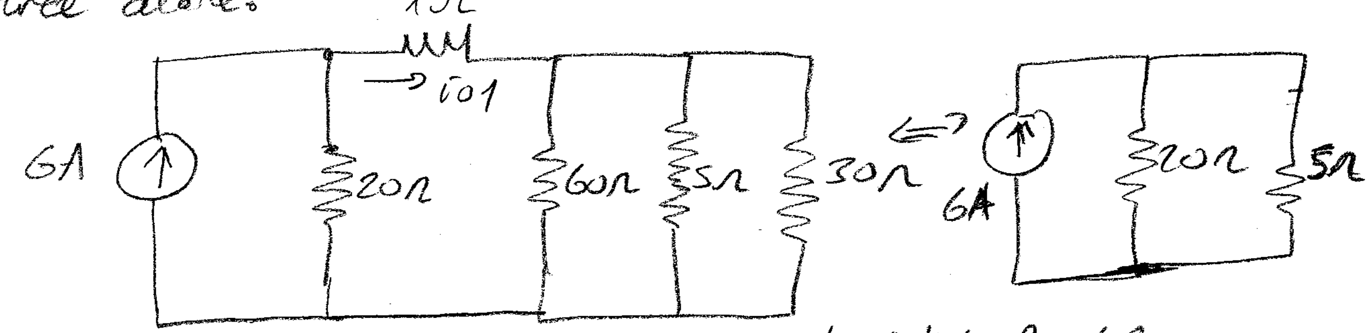
$$v_o = 30V$$

v_o due to 25V source $V_{o25V} = 30V$

The actual value of v_o is the sum of v_o due to 5mA current source and v_o due to 25V voltage source.

$$v_o = v_{o5mA} + v_{o25V} = 20V + 30V = 50V$$

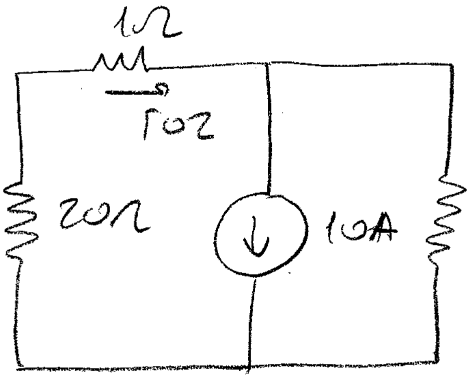
4-92 We first eliminate the 10A current source and the 75V voltage source from the circuit leaving the 6A source alone.



The equivalent resistance is $30\Omega \parallel 5\Omega \parallel 60\Omega = 4\Omega$

$$i_{o1} = \frac{20}{20+5} \times 6 = 4.8A$$

Next we remove the 6A current source and 75V voltage source and keep the 10A current source only

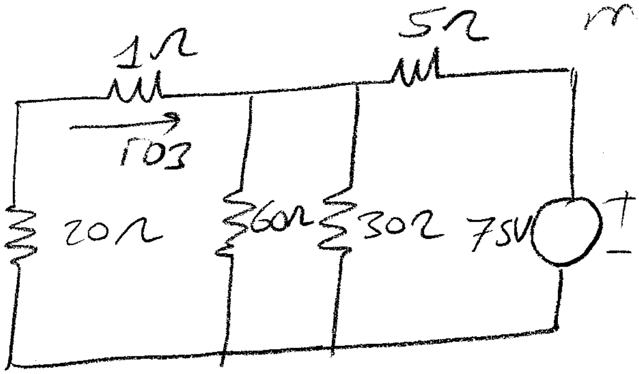


$= 60\Omega \| 5\Omega \| 30\Omega$

$$i_{02} = \frac{4}{25} \times (10) = 1.6A$$

Finally, we remove both current sources and leave the 75V voltage source in the circuit

converting the 5Ω resistance in series with 75V source to a parallel resistance with current source



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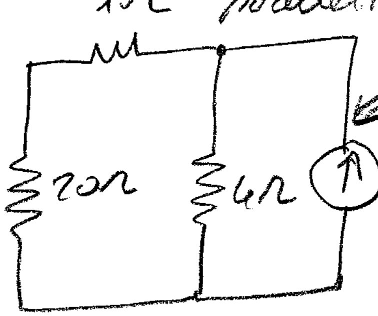

$$i_{03} = \frac{-4}{25}(15) = -2.4A$$

We find the output current i_0 by adding the output currents due to the three sources $i_0 = i_{01} + i_{02} + i_{03} =$

$4.8 + 1.6 - 2.4 = \underline{4A}$