

Q1. Let $A = \begin{pmatrix} \frac{1}{2}\sqrt{3} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2}\sqrt{3} \end{pmatrix}$. Compute A^{101} . Show all your steps.

Q2. Let $B = \begin{pmatrix} 1 & 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & -1 & 5 \\ 2 & 1 & 0 & 7 & 10 \\ -2 & -1 & 0 & 7 & 0 \\ 0 & 0 & 3 & 0 & -1 \end{pmatrix}$. Compute $\det B$.

Q3. Let $A = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{pmatrix}$. If $\det(A) = -3$, find $\det \begin{pmatrix} c_1 & -b_1 + 6c_1 & a_1 & d_1 \\ c_2 & -b_2 + 6c_2 & a_2 & d_2 \\ c_3 & -b_3 + 6c_3 & a_3 & d_3 \\ c_4 & -b_4 + 6c_4 & a_4 & d_4 \end{pmatrix}$.

Q4. Compute the determinant of the 10×10 matrix M whose entry in row i and column j is $i + j$.

Q5. Compute $\det \begin{pmatrix} -1 & 2 & -1 & 0 & -9 & -4 \\ 2 & -1 & 8 & 22 & 4 & 2 \\ 2 & 3 & 5 & -1 & 1 & 20 \\ 1 & 8 & 1 & 7 & 7 & 6 \\ 5 & -7 & 5 & 7 & 2 & 1 \\ 2 & 2 & 1 & 1 & 1 & 2 \end{pmatrix}$. You may use that

$$\det \begin{pmatrix} -1 & 2 & -1 & 0 & -9 & -4 \\ 2 & -1 & 8 & 22 & 4 & 2 \\ 2 & 3 & 5 & -1 & 1 & 20 \\ 1 & 8 & 1 & 7 & 7 & 6 \\ 5 & -7 & 5 & 7 & 2 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} = -8691, \det \begin{pmatrix} -1 & 2 & -1 & 0 & -9 & -4 \\ 2 & -1 & 8 & 22 & 4 & 2 \\ 2 & 3 & 5 & -1 & 1 & 20 \\ 1 & 8 & 1 & 7 & 7 & 6 \\ 5 & -7 & 5 & 7 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} = 105990,$$

$$\det \begin{pmatrix} -2 & 4 & -2 & 0 & -18 & -8 \\ 4 & -2 & 16 & 44 & 8 & 4 \\ 4 & 6 & 10 & -2 & 2 & 40 \\ 2 & 16 & 2 & 14 & 14 & 12 \\ 10 & -14 & 10 & 14 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix} = 6227136$$