Q1. Consider the following system of linear equations S in $x_1, x_2 \ldots, x_7$:

$$\begin{aligned} x_7 &= 1 \\ -x_1 + x_3 + 2x_5 &= 2 \\ 5x_4 + 2x_5 + 2x_7 &= 9 \\ x_2 + 2x_5 &= 4 \end{aligned}$$

Give its matrix, its augmented matrix and write the system in vector form.

Q2. Let S be the system of linear equations from the previous question. Give the matrix that transforms S into a new system of linear equations T where the first equation of T is three times the forth equation of S, the second equation of T is the sum of the first and fourth equations of S and the third equation of T is the three times the second equation of S plus the last equation of S. Give your answer as a multiplication of matrices and explain how you derived the matrix.

Q3. Let $A = \begin{pmatrix} 3 & -1 & 5 & -1 & 9 & 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 8 \\ 4 \\ 5 \end{pmatrix}$. Compute AB, BA and BAB, whenever possible else explain why the computation cannot be performed.

Q4. Let $U = \begin{pmatrix} 7 & 0 \\ 1 & -1 \\ 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $V = \begin{pmatrix} 1 & 6 & 2 & 6 & 3 \\ 4 & 4 & 4 & 6 & -1 \end{pmatrix}$, then $W = UV = \begin{pmatrix} 7 & 42 & 14 & 42 & 21 \\ -3 & 2 & -2 & 0 & 4 \\ 9 & 14 & 10 & 18 & 1 \\ 19 & 34 & 22 & 42 & 5 \end{pmatrix}$. Write the third column of W as a linear combination of the columns of U. Write the second and forth row of W as linear combinations of the rows of V.

Q5. Using the definition of a zero divisor, show that $\begin{pmatrix} 4 & 1 & 7 \\ 5 & 2 & 8 \\ 6 & 3 & 9 \end{pmatrix}$ is a zero divisor. If the matrix is not a zero divisor justify your claim.