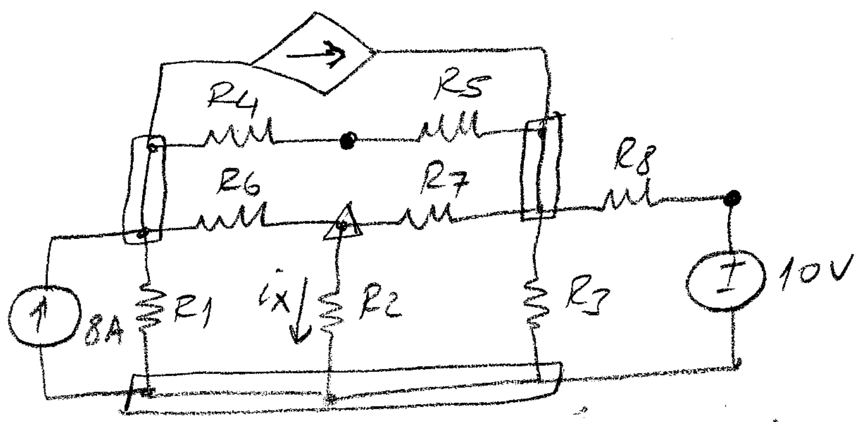
# SOLUTIONS OF PROBLEM SET 4-1

4-1




a) There are 8 branches with resistors, 2 branches with independent sources, 1 branch with a dependent source.

b) Except the branch containing the 8A current source, the current is unknown in all branches. Thus there are 10 branches with unknown current.

c) 9 essential branches. R4-R5 forms an essential branch as does R8-10V. The remaining seven branches are essential branches that contain a single element.

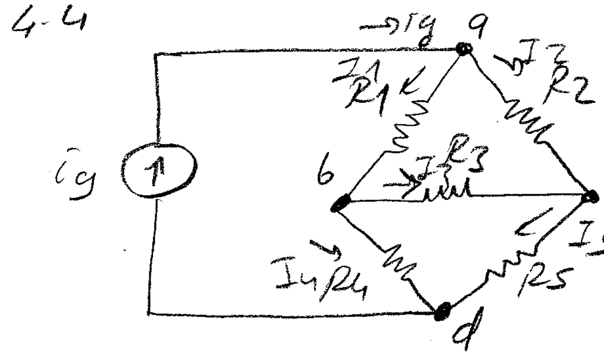
d) The current is known only in the essential branch containing the current source, and is unknown in the remaining 8 branches.

e) From the figure there are 6 nodes — three identified by rectangular boxes, two identified with single dots, and one identified by a triangle.

f) There are 4 essential nodes, three identified with rectangular boxes and one identified with a triangle.

g) A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.

4.4




a) There are 5 unknown currents.

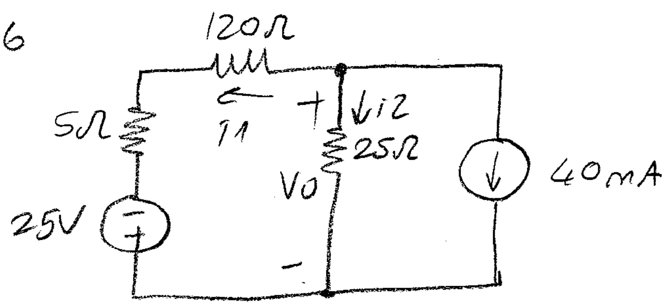
b) There are 4 nodes, 3 independent KCL equations can be written.

c) Node ⓐ $-i_g + I_1 + I_2 = 0$

" ⓑ $-I_1 + I_3 + I_4 = 0$

" ⓒ $-I_2 - I_3 + I_5 = 0$

6




$$\frac{V_o - (-25)}{125} + \frac{V_o}{25} + 0.04 = 0$$

$$V_o + 25 + 5V_o + 5 = 0$$

$$6V_o = -30$$

$$V_o = -5V$$

7 a) $P_{40mA} = (-5V)(40mA) = -200mW = -0.2W$

The power developed by the 40mA source is 0.2W.

b) $i_1 = \frac{V_o + 25}{125} = \frac{-5+25}{125} = \frac{20}{125} = \frac{4}{25} = 0.16\ A$

$P_{25V} = (-25V)(0.16A) = -4W$

The power developed by the 25V source is 4W.

c) $P_{5\Omega} = (0.16A)^2(5\Omega) = 0.128W$

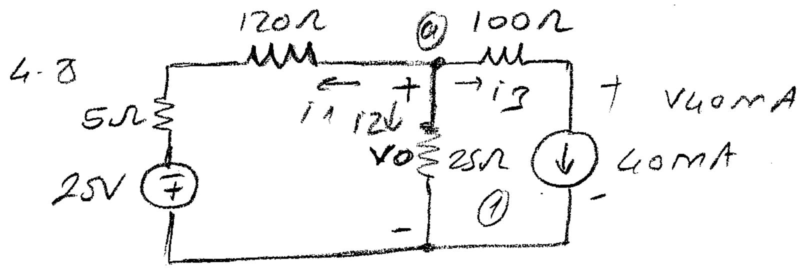
$P_{120\Omega} = (0.16A)^2(120\Omega) = 3.072W$

$P_{25\Omega} = \frac{V_o^2}{25} = \frac{(-5V)^2}{25} = 1W$

$\Sigma P_{dissipated} = 0.128 + 3.072 + 1 = 4.2W$

$\Sigma P_{delivered} = 0.2 + 4 = 4.2W$, the powers dissipated and delivered are equal.

4.8




K-C-L a) at node (a)

$$\frac{V_0-(-25)}{125} + \frac{V_0}{25} + 0.04 = 0$$

$$V_0 + 25 + 5V_0 + 5 = 0 \Rightarrow 6V_0 = -30 \quad V_0 = -5V.$$

b) $V_{40mA} = V_0 - 100 \times i_3 = -5V - 100(0.04A) = -9V$

$P_{40mA} = (-9V)(0.04\ A) = -0.36\ W = -360mW$

The power developed by the 40mA source is 360mW.

c) $i_1 = \frac{V_0-(-25)}{125} = \frac{-5+25}{125} = \frac{20}{125} = \frac{4}{25} = 0.16A$

$P_{25V} = (-25V)(0.16A) = -4W$

The power developed by the 40mA source is 4W.

d) $P_{5\Omega} = (0.16A)^2(5\Omega) = 0.128W$

$P_{120\Omega} = (0.16A)^2(120\Omega) = 3.072W$

$P_{25\Omega} = \frac{V_0^2}{25\Omega} = \frac{(-5V)^2}{25\Omega} = 1W$

$P_{100\Omega} = (0.04)^2 \cdot (100\Omega) = 0.16W$

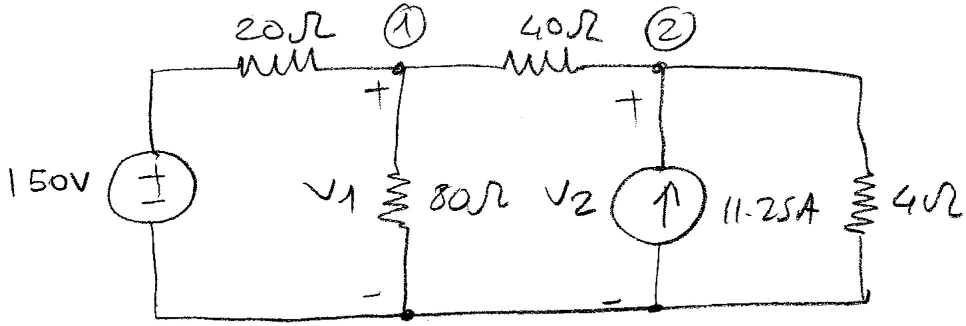
$\Sigma P_{dissipated} = 0.128 + 3.072 + 1 + 0.16 = 4.36W$

$\Sigma P_{developed} = 4 + 0.36 = 4.36W$

The total power dissipated equals the total power delivered.

e) It will have no effect as Vo is independent on the resistance connected in series, it only depends on the independent voltage source and on the independent current source.

4-12




At Node 1 $\quad \dfrac{V_1-150}{20}+\dfrac{V_1}{80}+\dfrac{V_1-V_2}{40}=0$

At Node 2 $\quad \dfrac{V_2-V_1}{40}-11.25+\dfrac{V_2}{4}=0$

Multiplying equation 1 by 80, we get:

$$4(V_1-150)+V_1+2(V_1-V_2)=0$$

$$\text{I} \quad 7V_1-2V_2=600$$

Multiplying equation 2 by 40 we get

$$V_2-V_1-450+10V_2=0$$

$$\text{II} \quad -V_1+11V_2=450$$

Solving for V1 from II $\quad V_1=11V_2-450$, and substituting this relationship into equation I

$$7(11V_2-450)-2V_2=600$$

$$75V_2-3150=600$$

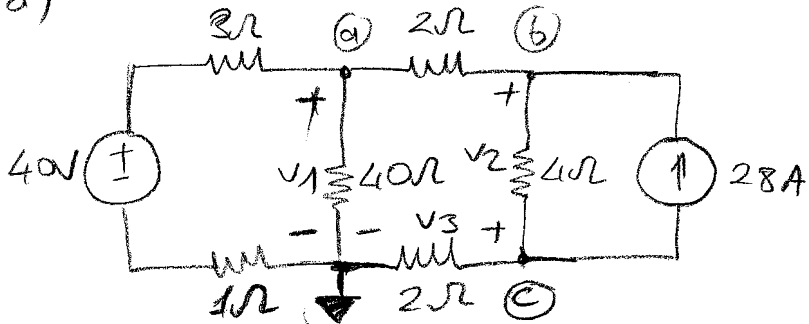
$$75V_2=3750$$

$$V_2=50V$$

substituting $V_2=50V$ into $V_1=11V_2-450$, we get

$$V_1=11\times 50-450=100V$$

4-14 a)




Reference Node.

K-C-L at Node ⓐ $\dfrac{V1-40}{4} + \dfrac{V1}{40} + \dfrac{V1-V2}{2} = 0$

" " " ⓑ $\dfrac{V2-V1}{2} + \dfrac{V2-V3}{4} - 28 = 0$

" " " ⓒ $\dfrac{V3}{2} + \dfrac{V3-V2}{4} + 28 = 0$

I $V1\left(\frac{1}{4}+\frac{1}{40}+\frac{1}{2}\right) - \frac{V2}{2} = 10$

II $V1\left(-\frac{1}{2}+\frac{1}{4}\right) + \frac{V2}{2} - \frac{V3}{4} = 28$

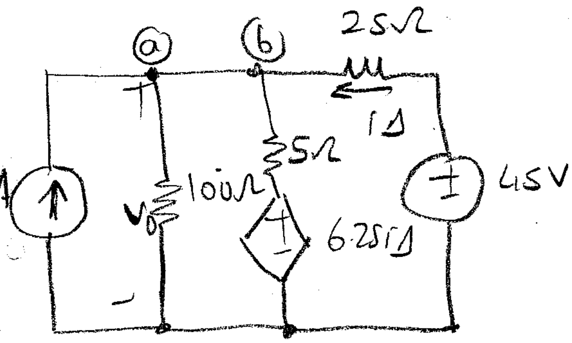
III $V2\left(-\frac{1}{4}\right) + V3\left(\frac{1}{2}+\frac{1}{4}\right) = -28$

solving : $v1 = 60V$ ; $v2 = 73V$, $V3 = -13V$

$P_{28A} = -Va(28A) = -(V_2-V_3)(28A) = -(73+13)(28) = -2408\,W$

The 28 A source delivers 2408W

4-17




a) at Node ⓐ K-C-L

$-0.45 + \dfrac{Vo}{100} + \dfrac{Vo-6.25i\Delta}{5} - i\Delta = 0$

$i\Delta = \dfrac{45-Vo}{25}$ ⇒ substituting into K-C-L at node a.

$-0.45 + Vo\left(\frac{1}{100}+\frac{1}{5}\right) + i\Delta\left(-\frac{6.25}{5} - 1\right) = 0$

I $Vo\left(\frac{1}{100}+\frac{1}{5}+\frac{1}{25}\right) + i\Delta\left(-\frac{6.25}{5}\right) = \frac{45}{25} + 0.45$

II $Vo\left(\frac{1}{25}\right) + i\Delta(1) = \frac{45}{25}$ solving $Vo = 15V$ ; $i\Delta = 1.2A$

[b] $p_{ds} = \frac{v_0 - 6.25 i_\Delta}{5} = \frac{15 - 7.5}{5} = 1.5A$

$p_{ds} = (6.25)(1.2)(1.5) = 11.25W$

Thus dependent source absorbs 11.25W.

(c) $p_{450mA} = -(0.45)(15) = -6.75W$

$p_{45V} = -(1.2)(45) = -54W$

$\sum P_{dev} = 6.75 + 54 = 60.75W$

Thus the independent source develops 60.75W

Also

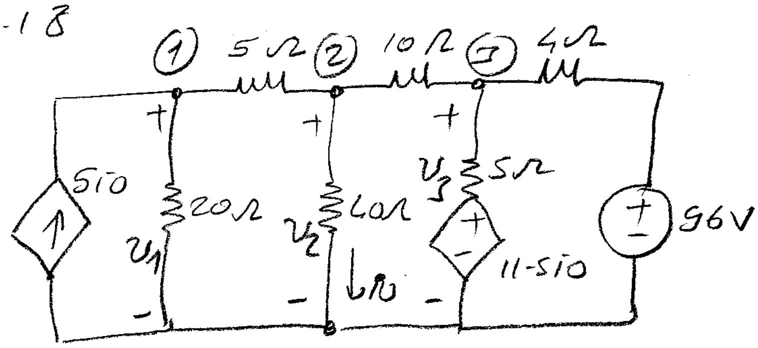
$\sum p_{dis} = p_{ds} + p_{100\Omega} + p_{5\Omega} + p_{25\Omega}$

$= 11.25 + (1.5)^2/100 + (1.5)^2(5) + (1.2)^2(25)$

$= 11.25 + 2.25 + 11.25 + 36 = 60.75W$

Total Power dissipated equals total power developed.

4.18




a) At node ①  $-5i_o + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$

At node ②  $\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} = 0$

At node ③  $\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5 i_o}{5} + \frac{v_3 - 96}{4} = 0$

The dependent source constraint equations:

$i_o = v_2/40$

We place these equations in standard form:

$$v_1\left(\frac{1}{20} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) + v_3(0) + i_o(-5) = 0$$

$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{5} + \frac{1}{40} + \frac{1}{10}\right) + v_3\left(-\frac{1}{10}\right) + i_o(0) = 0$$

$$v_1(0) + v_2\left(-\frac{1}{10}\right) + v_3\left(\frac{1}{10} + \frac{1}{5} + \frac{1}{4}\right) + i_o\left(-\frac{11.5}{5}\right) = \frac{96}{4}$$

$$v_1(0) + v_2\left(-\frac{1}{40}\right) + v_3(0) + i_o(1) = 0$$

Solving we get  $v_1 = 156V$; $v_2 = 120V$; $v_3 = 78V$; $i_o = 3A$

(b) Calculate the power:

$P_{currentcontrolledcurrentsource} = -[5(3)](156) = -2340W$

$P_{20\Omega} = (156)^2/20 = 1216.8W$

$P_{5\Omega} = (156-120)^2/5 = 259.2W$

$P_{40\Omega} = (120)^2/40 = 360W$

$P_{10\Omega} = (120-78)^2/10 = 176.4W$

$P_{5\Omega} = (78 - 11.5 \cdot 3)^2/5 = 378.45W$

$P_{4\Omega} = (78-96)^2/4 = 81W$

$P_{96V} = [(78-96)/4](96) = -432W$

$P_{current controlled voltage source} = [(78-3(11.5))/5](11.5-3) = 300.15W$

$\Sigma P_{delivered} = 2340+432 = 2772W$

$\Sigma P_{dissipated} = 1216.8+259.2+360+176.4+378.45+81+300.15 = 2772W$

Thus, the circuit dissipates 2772W.

4-24

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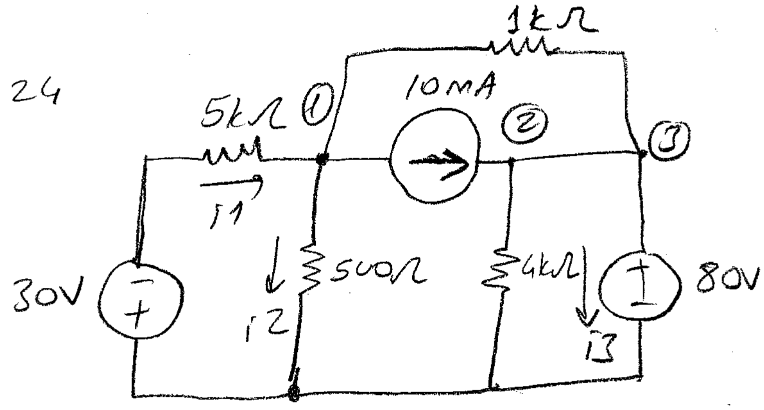



$$① \quad \frac{V1+30}{5} + \frac{V1}{0.5} + 10 + \frac{V1-80}{1} = 0$$

solving

$Va+30+10Va+5Va-400+50=0$, so $16Va=320$

$Va = 20V$.

$i_1 = -30-20/500 = -10mA$

$i2 = 20/500 = 40mA$

$i4 = 80/400 = 20mA$

$i3 = (80-20)/1000 = 60mA$

$i3+i4+i3-10mA=0$ so $i3 = 0.01-0.02-0.06 = -0.07 = -70mA$

(b) $P_{30V} = (30)(-0.01) = -0.3W$

$P_{10mA} = (20-80)(0.01) = -0.6W$

$P_{80V} = (80)(-0.07) = -5.6W$

$P_{5k} = (-0.01)^2(5000) = 0.5W$

$P_{500\Omega} = (0.04)^2(500) = 0.8W$

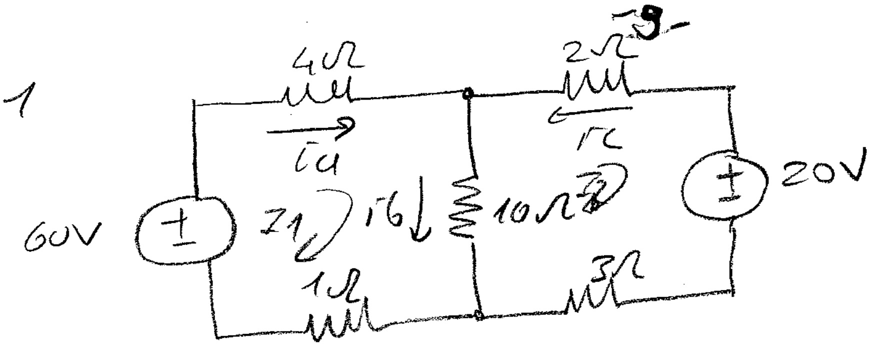
$P_{1k} = (80-20)^2/(1000) = 3.6W$

$P_{4k} = (80)^2/(4000) = 1.6W$

$\Sigma P_{TOTAL\ ABORBED} =$
$0.5+0.8+3.6+1.6 = 6.5W$

$\Sigma P_{TOTAL\ DELIVERED} =$
$0.3+0.6+5.6 = 6.5W$

4-31




(1) $-60+4I_1+10(I_1-I_2)+I_1=0$

(2) $2I_2+20+3I_2+10(I_2-I_1)=0$

Arrangy the terms

$15I_1-10I_2=60$

$-10I_1+15I_2=-20$

$\rightarrow I_1=\dfrac{\det\begin{vmatrix}60 & -10\\ -20 & 15\end{vmatrix}}{\det\begin{vmatrix}15 & -10\\ -10 & 15\end{vmatrix}}$

$I_2=\dfrac{\det\begin{vmatrix}15 & 60\\ -10 & -20\end{vmatrix}}{\det\begin{vmatrix}15 & -10\\ -10 & 15\end{vmatrix}}$

$I_1=\dfrac{700}{125}=5.6A$

$I_2=\dfrac{300}{125}=2.4A$, Now using these mesh currents

we solve for the requested currents:

$ia=i1=5.6A$; $ib=i1-i2=3.2A$; $ic=-i2=-2.4A$

(b) If the polarity of the 60V source is reversed, we have the following mesh current equations in standard form:

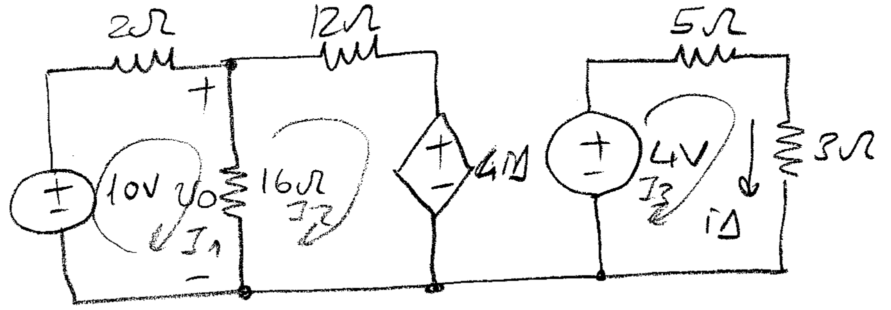
$i1(4+10+1)+i2(-10)=-60$

$i1(-10)+i2(3+10+2)=-20$

Solving $i1=-8.8A$; $i2=-7.2A$

$ia=i1=-8.8A$, $ib=i1-i2=-1.6A$, $ic=-i2=7.2A$

4.36




$-10 + 2I_1 + 16(I_1 - I_2) = 0$ I

$16(I_2 - I_1) + 12I_2 + 4i_\Delta = 0$ II

$-4 + 5I_3 + 3I_3 = 0$ III

solving for $I_3$ from III equation ⇒ $8I_3 = 4$ $I_3 = 0.5A$
thus $i_\Delta = 0.5A$, substituting this value into the second equation and arranging terms, we get:

$18I_1 - 16I_2 = 10$

$-16I_1 + 28I_2 = -2$

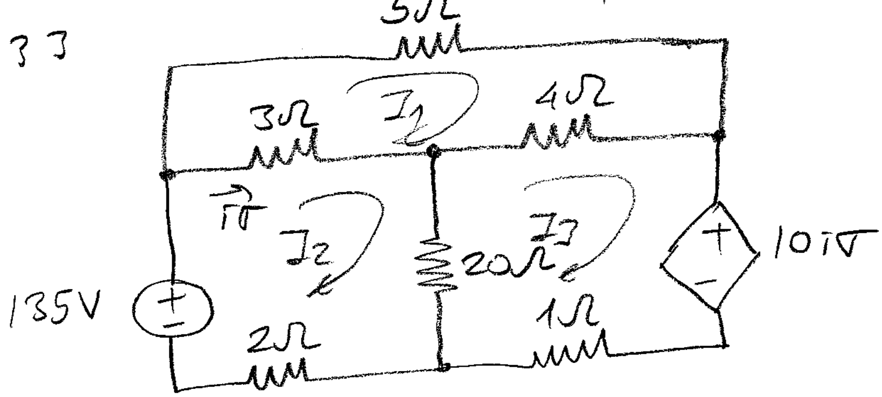
solving for $I_1 = \dfrac{\det\begin{vmatrix} 10 & -16 \\ -2 & 28 \end{vmatrix}}{\det\begin{vmatrix} 18 & -16 \\ -16 & 28 \end{vmatrix}} = \dfrac{248}{248} = 1A$

$I_2 = \dfrac{\begin{vmatrix} 18 & 10 \\ -16 & 2 \end{vmatrix}}{248} = \dfrac{124}{248} = 0.5A$

$v_o = 16(I_1 - I_2) = 16(1 - 0.5) = 8V$

(b) $P_{4i_\Delta} = 4i_\Delta i_2 = (4)(0.5)(0.5) = 1W$ absorbed
thus the power delivered by the $4i_\Delta$ source is
$P_{4i_\Delta} = -1W$.

4-33




Mesh ①: $5I_1 + 4(I_1 - I_3) + 3(I_1 - I_2) = 0$

Mesh ②: $3(I_2 - I_1) + 20(I_2 - I_3) + 2I_2 - 135 = 0$

Mesh ③: $4(I_3 - I_1) + 10 i_\sigma + I_3 + 20(I_3 - I_2) = 0$

We note that the branch current $i_\sigma = I_2 - I_1$

and put the equations into standard form

① $12I_1 - 3I_2 - 4I_3 = 0$

② $-3I_1 + 25I_2 - 20I_3 = 135$

③ $4(I_3 - I_1) + 10(I_2 - I_1) + I_3 + 20(I_3 - I_2) = 0$

$-14I_1 - 10I_2 + 25I_3 = 0$

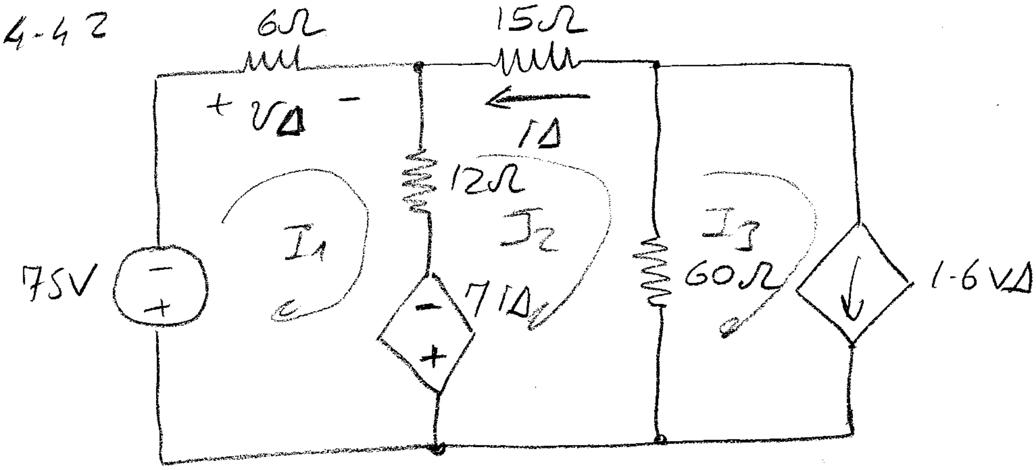
the coefficient matrix $A = \begin{vmatrix} 12 & -3 & -4 \\ -3 & 25 & -20 \\ -14 & -10 & 25 \end{vmatrix}$

$\det A = 12\begin{vmatrix} 25 & -20 \\ -10 & 25 \end{vmatrix} + 3\begin{vmatrix} -3 & -20 \\ -14 & 25 \end{vmatrix} - 4\begin{vmatrix} -3 & 25 \\ -14 & -10 \end{vmatrix} = 12 \times 425 + 3 \times (-355) - 4 * 32$

$= 5100 - 1065 - 1280 = 2755$

$I_1 = \dfrac{\det\begin{vmatrix} 0 & -3 & -4 \\ 135 & 25 & -20 \\ 0 & -10 & 25 \end{vmatrix}}{2755} = \dfrac{135\begin{vmatrix} -3 & -4 \\ -10 & 25 \end{vmatrix}}{2755} =$

4-42




1 $75 + 6I_1 + 12(I_1 - I_2) - 7i\Delta = 0$

2 $15I_2 + 60(I_2 - I_3) + 7i\Delta + 12(I_2 - I_1) = 0$

we see that the branch current $i\Delta = -I_2$ and

$I_3 = 1.6\,v\Delta$ as $v\Delta = 6I_1 \Rightarrow I_3 = 1.6 \times 6I_1 = 9.6I_1$

we can use $i\Delta = -I_2$ and $I_3 = 9.6I_1$ to eliminate two of the unknowns from the first two equations.

1 $75 + 6I_1 + 12(I_1 - I_2) + 7I_2 = 0$

2 $15I_2 + 60(I_2 - 9.6I_1) - 7I_2 + 12(I_2 - I_1) = 0$

Arranging terms

1 $18I_1 - 5I_2 = -75$ → $5I_2 = 18I_1 + 75$ substituting into the second equation.

2 $-588I_1 + 80I_2 = 0$

$-588I_1 + 16 \times 5I_2 \Rightarrow -588I_1 + 16 \times (18I_1 + 75) = 0$

$-588I_1 + 288I_1 + 1200 = 0$

$300I_1 = 1200 \quad I_1 = 4A$

$I_2 = \frac{18I_1 + 75}{5} = \frac{18 \times 4 + 75}{5} = 29.4A$ , $i\Delta = -I_2 = -29.4A$

$I_3 = 1.6\,v\Delta = 9.6 \times I_1 = 9.6 \times 4 = \underline{38.4A}$

Calculate the powe associated with the three sources:

$v = 60(i_2 - i_3) = -540V$

$v_\Delta = 6i_1 = 6(4) = 24V$

$P_{75V} = (75)(4) = 300W$

$P_{current controlled voltage source} = -7(-29.4)(4 - 29.4) = -5227.32W$

$P_{voltage controlled current source} = (-540)[1.6(24)] = -20736W$

The two dependent sources are generating a total of

$5227.32 + 20736 = 25,963.32W$.

[6] find the power dissipated. Remember that the 75V source is generating 300W, as calculated in part (a):

$P_{6\Omega} = 6(4)^2 = 96W$

$P_{12\Omega} = (12)(4 - 29.4)^2 = 7741.92W$

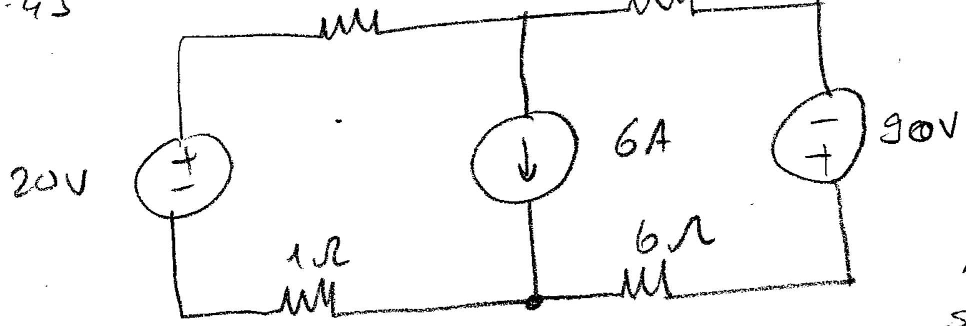
$P_{15\Omega} = (15)(29.4)^2 = 12,965.4W$

$P_{60\Omega} = (60)(29.4 - 38.4)^2 = 4860W$

$\Sigma P_{dissipated} = 300 + 96 + 7741.92 + 12,965.4 + 4860 =$

$25,963.32W$ (checks).

4.43




The supermesh equation is

$-20 + 4i_1 + 9i_2 - 90 + 6i_2 + i_1 = 0$

$i_1 - i_2 = 6$

Placing these equations in standard form

$i_1(4 + 1) + i_2(9 + 6) = 20 + 90$

$i_1(1) + i_2(-1) = 6$ solving $i_1 = 10A$, $i_2 = 4A$

Now we find the power: $P_{4\Omega} = 10^2(4) = 400W$ $P_{1\Omega} = 10^2(1) = 100W$

$P_{9\Omega} = 4^2(9) = 144W$ $P_{6\Omega} = 4^2(6) = 96W$ $P_{20V} = -(20)(10) = -200W$

$v_{6A} = 9i_2 - 90 + 6i_2 = (9)(4) - 90 + (6)(4) = -30V$ $P_{90V} = -(90)(4) = -360W$

$\Sigma P_{del} = 200 + 180 + 360 = 740W$ $\Sigma P_{diss} = 400 + 100 + 144 + 96 = 740W$.