Q1. Compute the eigenvalues and corresponding eigenvectors for the matrix

$$A = \begin{pmatrix} 6 & -4 & 1 \\ 5 & -3 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Q2. Find all eigenvalues and their corresponding eigenvectors for the matrix B where

$$B = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}.$$

Q3. Compute the eigenvalue and corresponding eigenvectors for the matrix $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$

Q4. Let $U = \begin{pmatrix} -1 \\ 1 \\ 5 \\ -1 \\ 1 \\ 5 \\ 7 \end{pmatrix}$ and $V = \begin{pmatrix} 1 & -2 & 3 & -4 & 1 & 5 & 3 \end{pmatrix}$. Find two distinct eigenvalues and

corresponding eigenvectors of the matrix UV. Hint: you can do it without evaluating determinants!

Q5. Suppose a 10×10 matrix has eigenvalue zero of multiplicity ten (in other words the set of eigenvalues contains only one value which is zero). Is this matrix necessarily the 10×10 zero matrix? If yes prove it, if no give a counterexample.