EE 210

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Solutions of Problem Set 2

2-5 a) Yes the connection is valid, because independent sources can provide all the current required by the connection.

b) Considering the direction of 5mA current with respect to the voltage sources, we see that it enters both voltage sources at the positive terminal and leaves at the negative terminal, thus both independent voltage sources are absorbing power. The 5mA current source, however, is delivering power to the circuit.

(c) $P_{18V} = (5\times10^{-3})(18) = 90\,mW$ (absorbing)

$P_{5mA} = -(5\times10^{-3})(25) = -125\,mW$ (delivering)

$P_{7V} = (5\times10^{-3})(7) = 35\,mW$ (absorbing)

$\sum P_{absorbed} = \sum P_{delivered} = 125\,mW.$

(d) Yes, in that case 18V is delivering power, the 5mA current source and 7V voltage source are absorbing power.

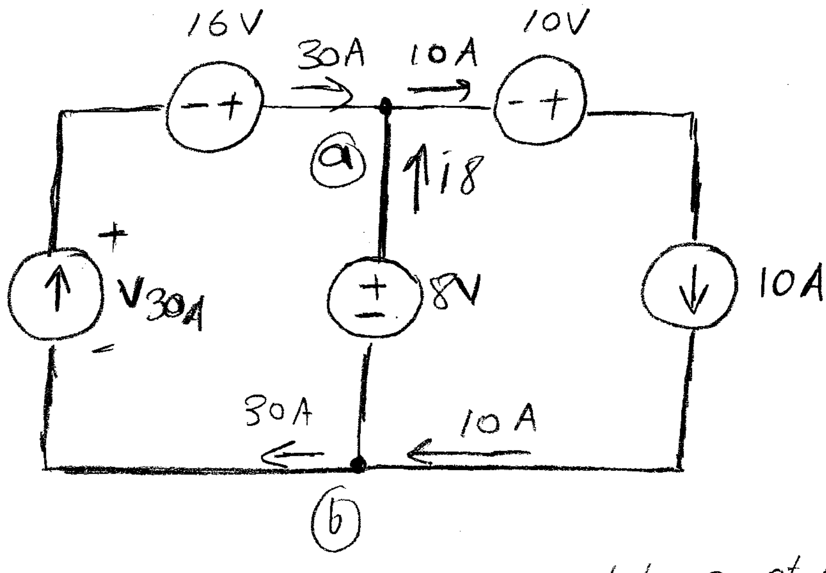
$P_{18V} = -(5\times10^{-3})(18) = -90\,mW$ (delivering)

$P_{5mA} = (5\times10^{-3})(11) = 55\,mW$ (absorbing)

$P_{7V} = (5\times10^{-3})(7) = 35\,mW$ (absorbing)

$\sum P_{absorbed} = \sum P_{delivered} = 90\,mW.$

2.7 We redraw the circuit assigning a current to the branch containing the 8V Voltage source.



We write Kirchoff's current law at node a

$-30 - i_8 + 10 = 0 \qquad i_8 = -20A$

Next we write Kirchoff's current law at node b

$-10 + i_8 + 30 = 0 \qquad i_8 = -20A$

As we get the same current value at both nodes, Kirchoff's current law is not violated. We next write 3 Kirchoff's voltage law equations, for the three possible loop (closed paths) of the circuit.

L_1: left loop: $v_{30} + 16V - 8V = 0$ I

L_2: right loop: $8V + 10V - V_{10} = 0$ II

L_3 = Main outer loop consisting of L_1 & L_2: $V_{30} + 16V + 10V - V_{10} = 0$ III

From equation I $\quad v_{30} = -8V$

From equation II $\quad v_{10} = 18V$

Substituting these two voltage values into the third equation, we get $-8V + 16V + 10V - 18V = 0$, thus Kirchoff's voltage law is not violated either. Because both Kirchoff's current law and Kirchoff's voltage law are satisfied at all nodes and loops, the circuit is valid.

⇒ Next we use i_8, v_{30} and v_{10} to determine the power associated with each source.

$P_{30A} = -(30)(-8) = 240W \Rightarrow$ absorbed

$P_{16V} = -(30)(16) = -480W \Rightarrow$ delivered

$P_{8V} = (-20)(8) = 160W \Rightarrow$ absorbed

$P_{10V} = -(10)(10) = -100W \Rightarrow$ delivered

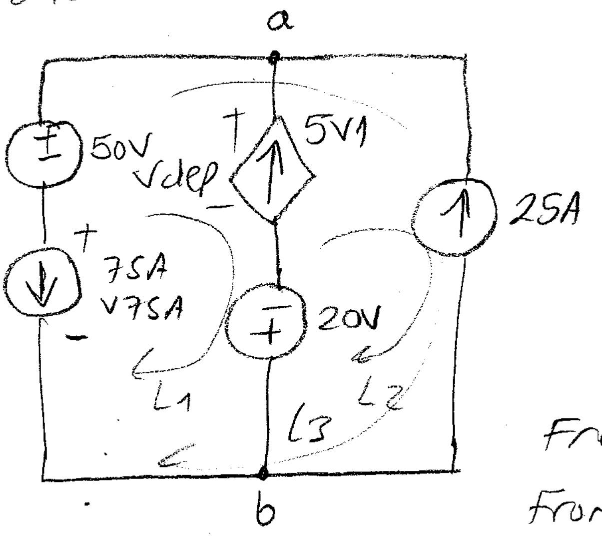
$P_{10A} = (10)(18) = 180W \Rightarrow$ absorbed

$\Sigma P_{absorbed} = \Sigma$ Delivered $= 580W$

$P_{I\text{-sources}} = P_{30A} + P_{10A} = 240 + 180 = 420W$

Since the power is positive, the sources are absorbing 420W power.

2.11 We redraw the circuit labeling the nodes, loops, voltages and current sources.



Kirchoff's Current law (K-C-L) at Node a ⇒

$75A - 5V_\Delta - 25A = 0$

(K-C-L) at Node b ⇒

$-75A + 5V_\Delta + 25A = 0$

From a $5V_\Delta = 50$ $V_\Delta = 10V$

From b, we get the same result thus, K-C-L is satisfied at both nodes.

Next, we write the three loop equations:

Loop 1: $V_{75A} + 50V - V_{dep} + 20V = 0$

Loop 2: $-20V + V_{dep} - V_\Delta = 0$

Loop 3: $V_{75A} + 50V - V_\Delta = 0$

Substituting $V_\Delta = 10V$ into the second loop equation $\Rightarrow V_{dep} = 20 + V_\Delta = 30V$

Substituting $V_\Delta = 10V$ into the third Loop equation $\Rightarrow V_{75A} = V_\Delta - 50V = 10V - 50V = -40V$

Finally, we substitute these values to check that they satisfy the first loop's Kirchoff's Voltage Law equation.

$-40V + 50V - 30V + 20V = 0V \Rightarrow$ the first equation is satisfied. Kirchoff's voltage law is not violated either. As neither Kirchoff's current law, nor Kirchoff's voltage law is violated, the circuit is valid.

$\Rightarrow$ V_Δ, V_{75} and v_{dep} will be used to determine the unknown power.

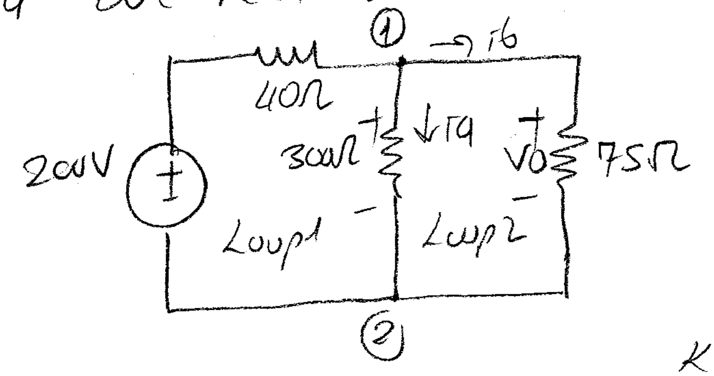
$P_{50V} = (75)(50) = 3750W$ $\quad P_{75A} = (75)(-40) = -3000W$

$P_{20V} = [5(10)](20) = 1000W$ $\quad P_{\text{dependent source}} = -(50)(30) = -1500W$

$P_{25A} = -(25)(10) = -250W$

$\Sigma P_{delivered} = 3750 + 1000 = 4750W = \Sigma P_{absorbed}.$

2.14 We redraw the circuit



Kirchoff's current law at Node ①

$i_b + i_a - I = 0$

Kirchoff's voltage law at Loop ① $-200 + 40I + 300 i_a = 0$

Kirchoff's voltage law at Loop ②

$75 i_b - 300 i_a = 0$

From II equation $i_b = 4 i_a$ from I. equation

$40I = 200 - 300 i_a \Rightarrow I = \frac{1}{40}(200 - 300 i_a) = 5 - 7.5 i_a$

Substituting I and i_b in terms of i_a into the first K-CL equation

$4 i_a + i_a - (5 - 7.5 i_a) = 0 \Rightarrow 5 i_a - 5 + 7.5 i_a = 0$

$12.5 i_a \Rightarrow 5$ $\quad i_a = \underline{0.4A}$

(b) $i_b = 4 \times i_a = 4 \times (0.4A) = \underline{1.6A}$

(c) $v_o = 75 \times i_b = 75 \times (1.6A) = \underline{120V}$

(d) $P_{40\Omega} = (i_{40})^2 \cdot 40\Omega$ $\qquad I = 5 - 7 - 5i_4 = 5 - 7 - 5\times 0.6A = \underline{\underline{2A}}$

$P_{40\Omega} = (2A)^2 \times (40\Omega) = 160W$

$P_{300\Omega} = (i_{300})^2 (300\Omega) = i_0^2 \cdot 300\Omega = (0.4A)^2 (300\Omega)$

$P_{300\Omega} = 48W$

$P_{75\Omega} = (i_{75})^2 \cdot (75\Omega) = (i_b)^2 \cdot 75\Omega = (1.6A)^2 (75\Omega) = 192W$

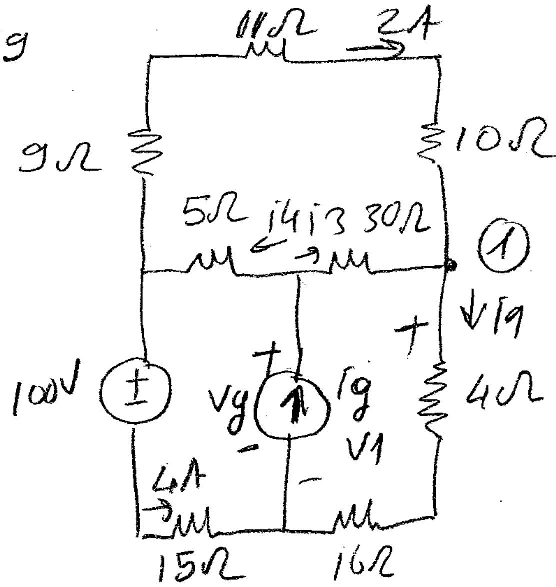
$P_{200V} = -(200V) I = -(200V)(2A) = -400W$

↳ The minus sign comes from passive sign convention.

(e) $\Sigma P_{dissipated} = 160 + 48 + 192 = 400W$

$\Sigma P_{\text{delivered by the voltage source}} = 400W.$

2-19



$v_2 = 100 + 4(15) = 160V$

$V_1 = 160 - 30(2) = 100V$

$i_1 = \frac{v_1}{20} = \frac{100}{20} = 5A,$

Applying Kirchhoff's Current Law at node ①

$i_1 - 2 - i_3 = 0 \qquad i_3 = i_1 - 2 = 5 - 2 = 3A$

$v_g = v_1 + 30 i_3 = 100 + 30(3) = 190V$

$v_g - 5i_4 = v_2$ thus $5i_4 = v_g - v_2 = 190 - 160 = 30V$

Thus $i_4 = \frac{30}{5} = 6A.$

We can now apply Kirchhoff's current law at node 2, to determine the unknown current i_g.

$i_g = i_3 + i_4 = 3 + 6 = 9A.$

(b) We calculate the powers dissipated by the resistors using the formula $P = RI^2$

$P_{9\Omega} = (9)(2)^2 = 36W$; $P_{11\Omega} = (11)(2)^2 = 44W$

$P_{10\Omega} = (10)(2)^2 = 40W$; $P_{30\Omega} = (30)(3)^2 = 270W$

$P_{5\Omega} = (5)(6)^2 = 180W$; $P_{4\Omega} = (4)(5)^2 = 100W$

$P_{16\Omega} = (16)(5)^2 = 400W$, $P_{15\Omega} = (15)(4)^2 = 240W$

(c) $v_g = 190V$

(d) $\sum P_{dissipated} = 36 + 44 + 40 + 270 + 180 + 100 + 400 + 240$
$= 1310W$

The power of voltage source: $P_V = (100V)(4A) = 400W$
" " current source: $P_C = -v_g i_g = -(190V)(9A)$
$= -1710W$.

Thus the total power dissipated is 1710W and the total power developed is 1710W, so the power balances.

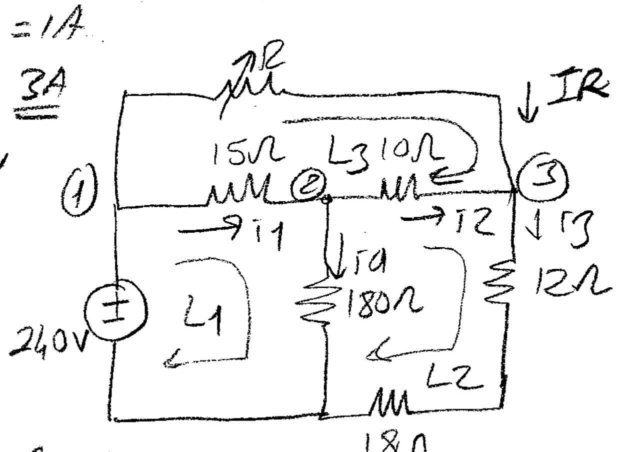
2.24. When $i_a = 1A$, the voltage drop across the 180Ω resistor must be $V_{180\Omega} = (1A)(180\Omega) = 180V$.

⇒ Applying Kirchoff's Voltage law around the Loop 4
$V_{12} = 240V - 180V = 60V$ and $i_1 = \frac{60V}{15\Omega} = 4A$.

⇒ Applying Kirchoff's current law at node 2: $-4A + i_a + i_2 = 0$ but $i_a = 1A$
$-4A + 1A + i_2 = 0 \Rightarrow i_2 = \underline{\underline{3A}}$

⇒ Now to determine the value of the variable resistor R, we have to use ohm's law: $R = \frac{V}{I}$, but we do not know the voltage and the current. Let's first determine V_R, the voltage across the variable resistor.

Circuit Redrawn



From Kirchoff's Voltage law for Loop 3 : $V_R = (15\Omega)(4A) + (10\Omega)(3A)$
$= 90V$

⇒ Now, we apply Kirchoff's current law at node 3, to determine I_R.

K-C-L III $\quad i_3 - i_2 - i_R = 0$ but $i_2 = 3A$ and $i_3 = \frac{V_3}{30\Omega}$

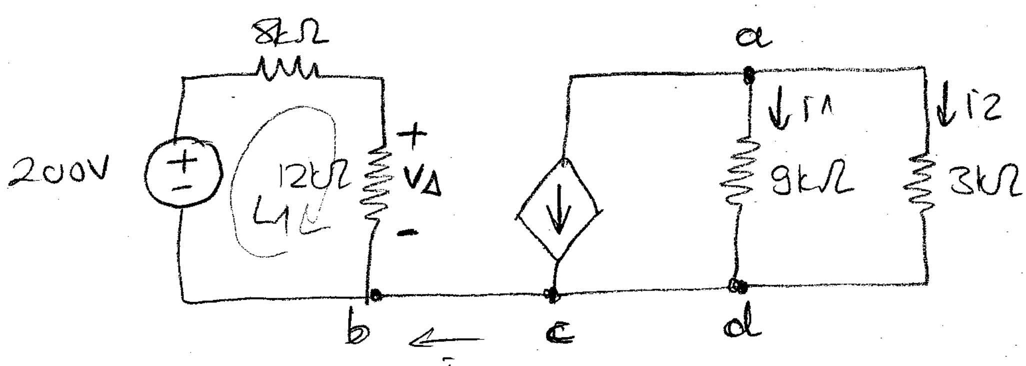
$$V_3 = 180V - (10\Omega) i_2 = 180V - (10\Omega)(3A) = 150V$$

$i_3 = \frac{150V}{30\Omega} = 5A$ ⇒ substituting the value of i_3 into the

K-C-L equation III $\quad 5A - 3A - i_R = 0 \Rightarrow i_R = 2A$

Finally $R = \frac{90V}{2A} = \underline{45\Omega}$

2-28 We redraw the circuit labeling the nodes:



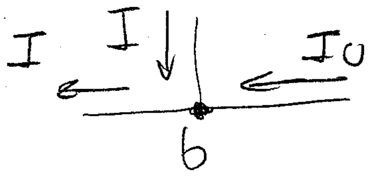
⇒ Solution Approach: If we had known the voltage at node a, we could have computed i1 & i2 and using Kirchoff's current law at node c, we could have found the unknown current Io. But we do not know the voltage at node a. We have to think of a different strategy. We first write the Kirchoff's voltage law equations for the loop L1 containing the 200V voltage source.

K-V-L L1 : $-200V + 8k\Omega I + 12k\Omega I = 0$

$$20k\Omega I = 200V$$

$$I = \frac{200V}{20k\Omega} = 10mA$$

Now we write, Kirchoff's current law at node b.



$I - I - I_o = 0 \quad I_o = 0$

b) In order to find I_1 and I_2, we first need to find the current supplied by the dependent source $5\times10^{-3}V_\Delta$, thus we determine V_Δ first.

$$V_\Delta = 12k\Omega I = 12k\Omega \times 10mA = 120V$$

$$5\times10^{-3}V_\Delta = 5\times10^{-3}\times120 = 0.6A$$

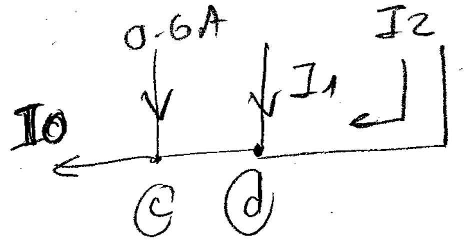
Now, we write Kirchoff's voltage law equation for the loop containing the $9k\Omega$ and $3k\Omega$ resistors.

$$V_{9k\Omega} = V_{3k\Omega} = V_a$$

From Ohm's law $\quad 9k\Omega I_1 = 3k\Omega I_2 = V_a$

$$\frac{I_1}{I_2} = \frac{1}{3} \quad \text{I}$$

We need one more equation to relate I_1 and I_2, it can be found by writing Kirchoff's current law at node d.



$$-0.6 - I_1 - I_2 = 0$$

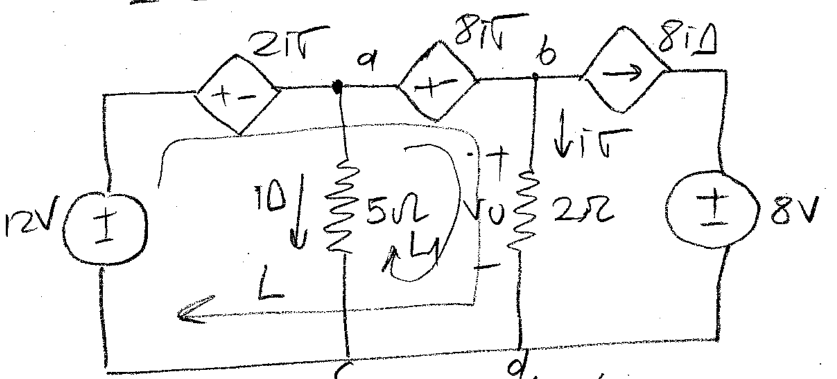
$$I_1 + I_2 = -0.6 \quad \text{II}$$

substituting $I_2 = 3I_1$ from equation I,

$$I_1 + 3I_1 = -0.6 \qquad 4I_1 = -0.6 \qquad I_1 = -0.15A$$

$$I_2 = -0.45A$$

2.30 We first redraw the circuit indicating the nodes:



Solution Analysis: To find i_Δ, we need to know v_a as $i_\Delta = \frac{v_a}{5\Omega}$, but we don't know it. If we write Kirchoff's voltage law for the loop containing the 12V source, we see that in addition to i_Δ, there will be a second unknown i_Δ due to the dependent voltage source $2i_\Delta$. Thus writing Kirchoff's voltage law equation for the leftmost loop, will not lead us any further. We should try another approach. Consider the loop L, consisting of the leftmost loop containing the voltage source and the loop bounded by nodes a, b, c, d.

If we write Kirchoff's voltage law equation for this loop, the only unknown will be i_σ, we should be able to determine i_σ, from this equation alone. Next we consider the loop surrounded by branches, ab, bd, dc, da, if i_σ is determined, the only unknown in this loop will be i_Δ, thus we will find the solution.

Kirchoff's Voltage Law for Loop L:

$$-12V + 2i_\sigma + 8i_\sigma + 2i_\sigma = 0$$

$$12 i_\sigma = 12V \Rightarrow i_\sigma = 1A$$

Kirchoff's Voltage Law for loop L1:

$$-12V + 2 \times 1A + 5 i_\Delta = 0$$

$$5 i_\Delta = 10V \qquad \underline{\underline{i_\Delta = 2A}}$$

$$v_o = (2\Omega)(1A) = \underline{\underline{2V}}$$

b) In order to show that the power developed equals to the power absorbed in the circuit, we have to determine all all powers delivered or absorbed by the sources and all powers dissipated (absorbed) by the resistances in the circuit. As we know the currents through the resistors, we can first find the power dissipated by them.

$$P = I^2 \cdot R \qquad P_{5\Omega} = (2A)^2 (5\Omega) = 20W$$

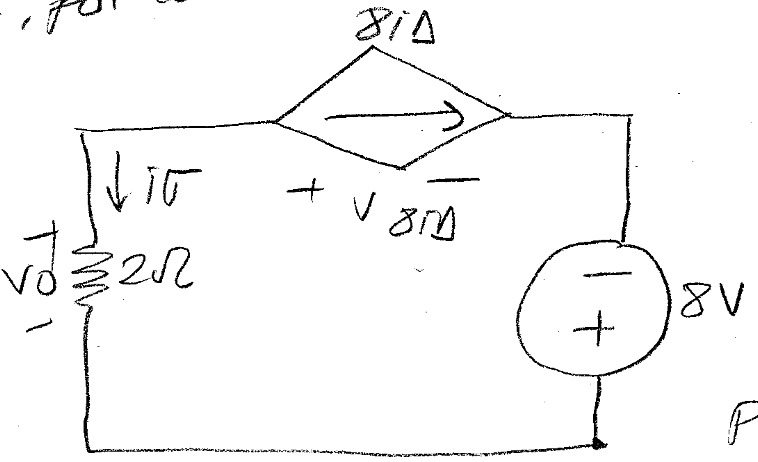
$$P_{2\Omega} = (1A)^2 (2\Omega) = 2W$$

To determine the power of sources, we have to know the voltage and current of all sources. We start with the rightmost section of the circuit and work our way backwards to determine the unknown voltages and currents.

The current through the 8V voltage source is $8i_\Delta = 8(2A) = 16A$

$$P_{8V} = (8V)(8 \times 2A) = 128W.$$

We will now compute the voltage across the $8i_\Delta$ dependent source. But we first have to determine the voltage across this dependent current source, in order to find the power absorbed or delivered by it. Note the sign of the voltage across this source will tell us whether it is absorbing or delivering power to the circuit. We redraw the loop containing the dependent source here, for convenience.



writing Kirchoff's voltage law equation for this loop:

$V_{8i\Delta} - 8V - V_0 = 0 \Rightarrow V_0$ had bee found to be 4V.

$V_{8i\Delta} = 8V + 4V = 12V$.

$P_{8i\Delta} = (12V)(8i\Delta) =$

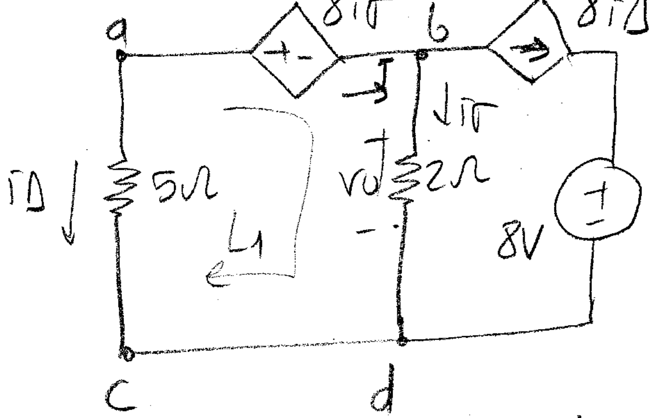
$-(10V)(8\times 2A) = -160W$

dissipating.

$\Rightarrow$ Next, we need to determine the current through the dependent voltage source $8i_\sigma$, to find the power supplied or delivered by it. The voltage of the source can be found easily as we already know $i_\sigma = 1A$.

$V = 8i_\sigma = 8(1A) = 8V$.

We redraw the circuit's 2nd loop containing the $8i_\sigma$ dependent voltage source, and third loop containing $8i\Delta$ dependent current source. Kirchoff's current law at node b, yields:



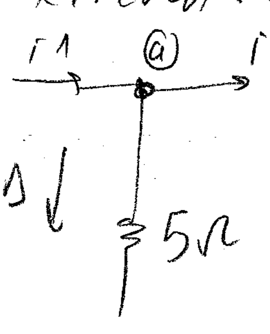
$8i\Delta + i_\sigma - i = 0$ where $i\Delta = 2A$, $i_\sigma = 1A$,

$i = 8\cdot(2A) + (1A) = 17A$,

$P_{8i_\sigma} = -(8\times 1)(17A) = -136W$, dissipating

Finally, we compute the current at node a, to determine the power supplied by the 12V independent voltage source and $2i_\sigma$ dependent source.

Kirchoff's Current law at node ⓐ



$i\Delta + i - i1 = 0 \quad i1 = i\Delta + i = 2A + 17A = 19A$

$P_{2i_\sigma} = -(2\times 1A)(19A) = -38W$ dissipating

$P_{12V} = (12V)(19A) = 228W$ delivering.

$\Sigma P_{dissipated} = 38W + 136W + 160W + 20W + 2W = 356W$

$\Sigma P_{delivered} = 228W + 128W = 356W$