M7
Predictive Analytics

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Seasonal ARIMA models

- We can add seasonal terms to original ARIMA:

$$\text{ARIMA}\ \underbrace{(p,d,q)}_{\uparrow \atop \left(\text{Non-seasonal part of the model}\right)}\ \underbrace{(P,D,Q)_m}_{\uparrow \atop \left(\text{Seasonal part of the model}\right)}$$

where m is the number of periods per season.

- Use backshift operator to write concisely.
- Example: ARIMA(1,1,1)(1,1,1)$_4$

$$\underbrace{(1-\phi_1 B)}_{\left(\text{Non-seasonal AR(1)}\right)}\ \underbrace{(1-\Phi_1 B^4)}_{\left(\text{Seasonal AR(1)}\right)}\ \underbrace{(1-B)}_{\left(\text{Non-seasonal difference}\right)}\ \underbrace{(1-B^4)}_{\left(\text{Seasonal difference}\right)}y_t = \underbrace{(1+\theta_1 B)}_{\left(\text{Non-seasonal MA(1)}\right)}\ \underbrace{(1+\Theta_1 B^4)}_{\left(\text{Seasonal MA(1)}\right)}e_t.$$

Seasonal ARIMA models

- Example: ARIMA(1,1,1)(1,1,1)$_4$

$$(1-\phi_1 B)(1-\Phi_1 B^4)(1-B)(1-B^4)y_t = (1+\theta_1 B)(1+\Theta_1 B^4)\varepsilon_t$$

All the factors can be multiplied out and the general model written as follows:

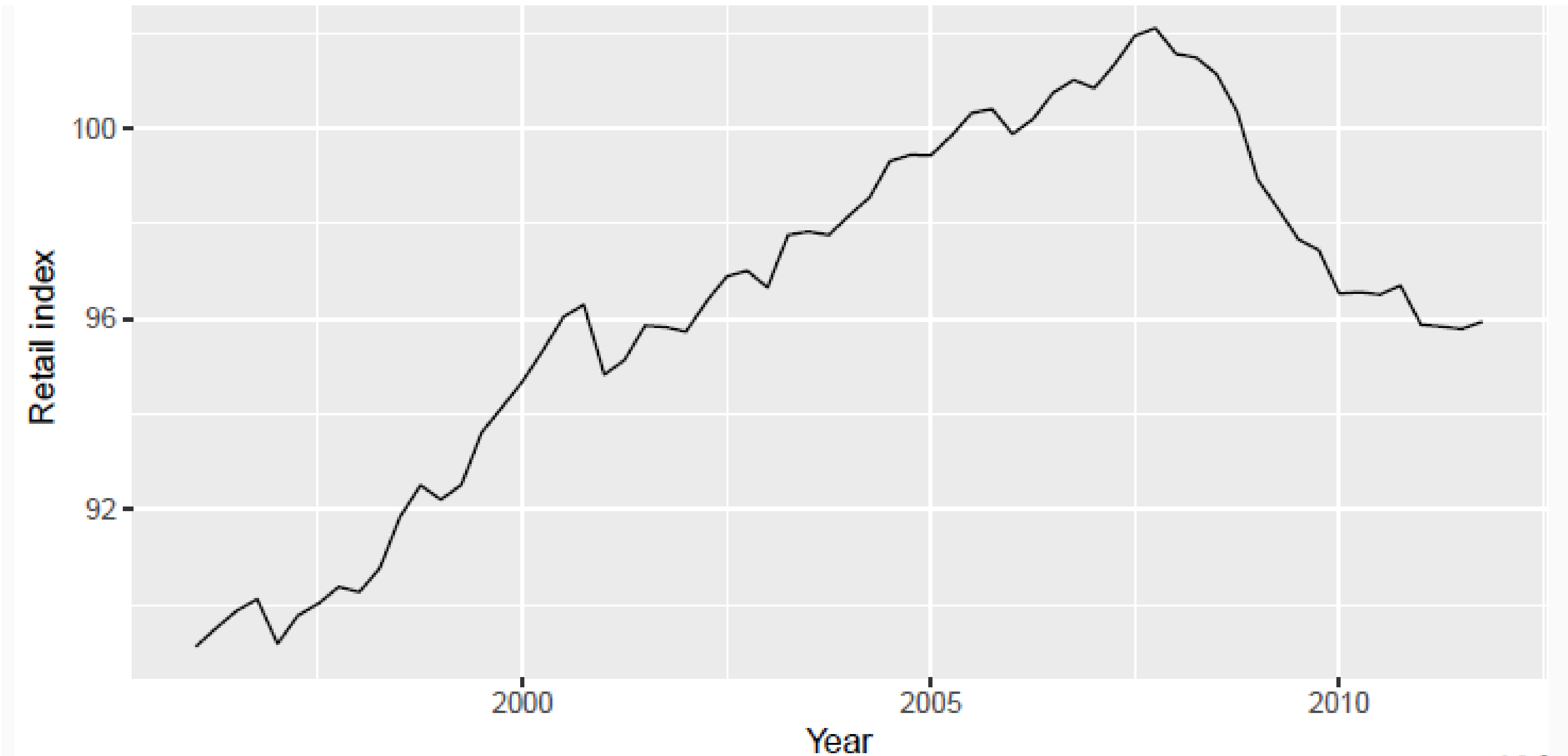
$$\begin{aligned} y_t = {} & (1+\phi_1)y_{t-1} - \phi_1 y_{t-2} + (1+\Phi_1)y_{t-4} \\ & - (1+\phi_1+\Phi_1+\phi_1\Phi_1)y_{t-5} + (\phi_1+\phi_1\Phi_1)y_{t-6} \\ & - \Phi_1 y_{t-8} + (\Phi_1+\phi_1\Phi_1)y_{t-9} - \phi_1\Phi_1 y_{t-10} \\ & + \varepsilon_t + \theta_1\varepsilon_{t-1} + \Theta_1\varepsilon_{t-4} + \theta_1\Theta_1\varepsilon_{t-5}. \end{aligned}$$

Seasonal ARIMA models

- How to interpret ACF/PACF plots with seasonal data?
- The seasonal part of an AR or MA model will be seen in the seasonal lags of the PACF and ACF.
- Examples:
  - ARIMA(0,0,0)(0,0,1)$_{12}$ will show:
    - ACF will show a spike at lag 12 but no other significant spikes.
    - PACF will show exponential decay in the seasonal lags; that is, at lags 12, 24, 36,…
  - ARIMA(0,0,0)(1,0,0)$_{12}$ will show:
    - ACF will show exponential decay in the seasonal lags.
    - PACF will show a single significant spike at lag 12.

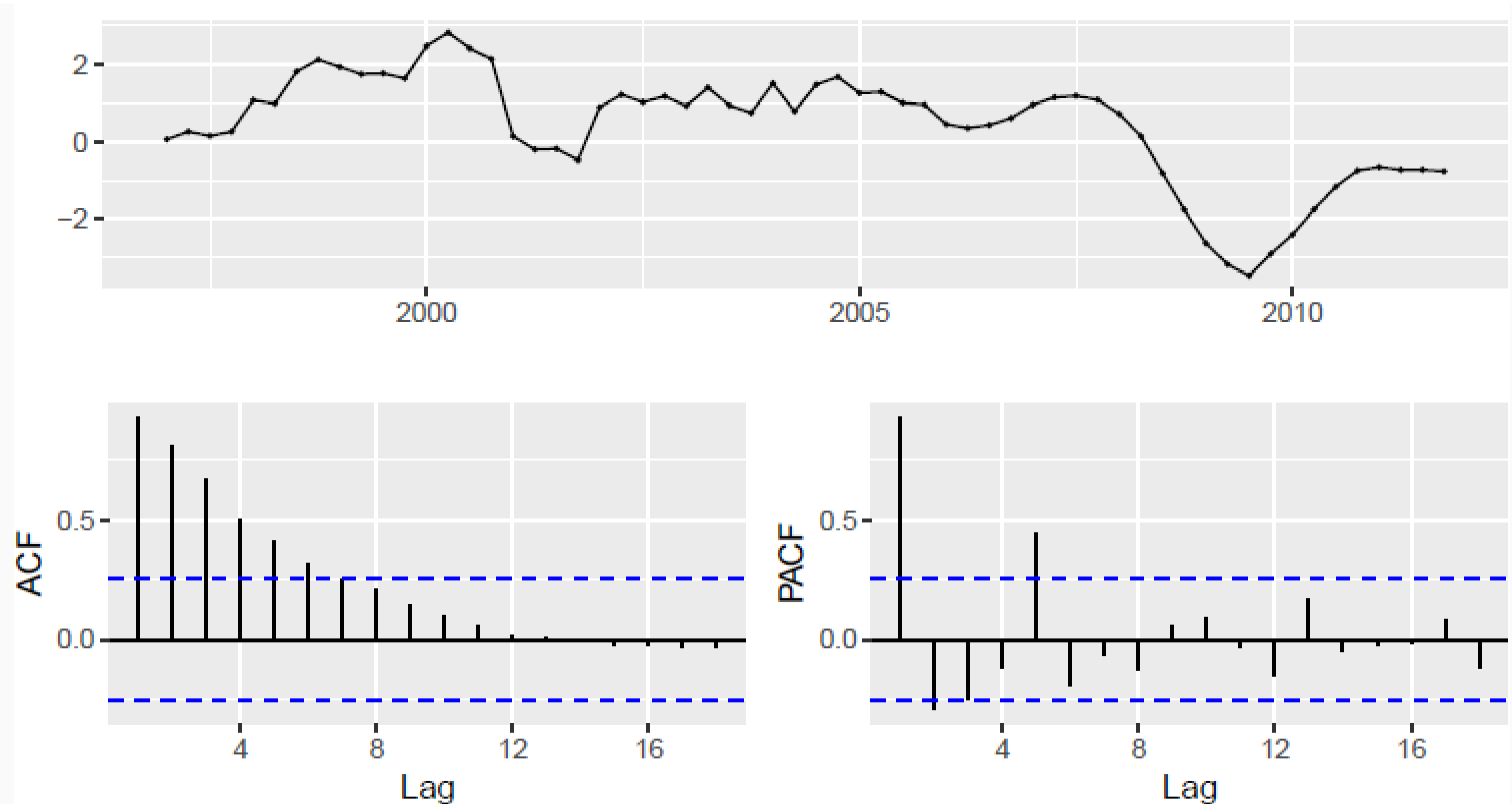
Seasonal ARIMA models

- Example: European quarterly retail trade index
  - (Index: 2005=1000)



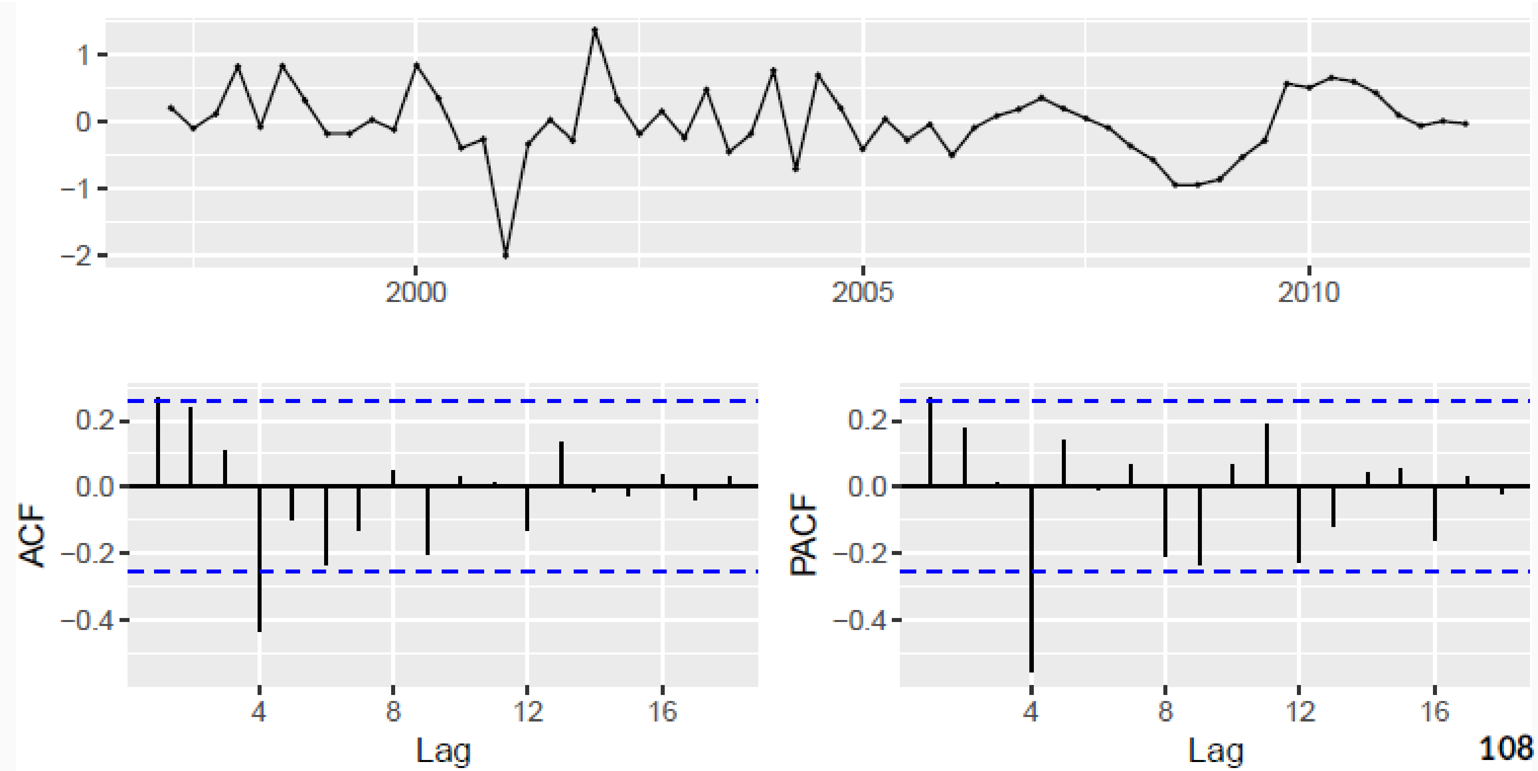
Seasonal ARIMA models

- Example: European quarterly retail trade index
  - Non-stationary with some seasonality
  - Let's first take a seasonal difference, i.e. $(1\text{-}B^4)y_t$



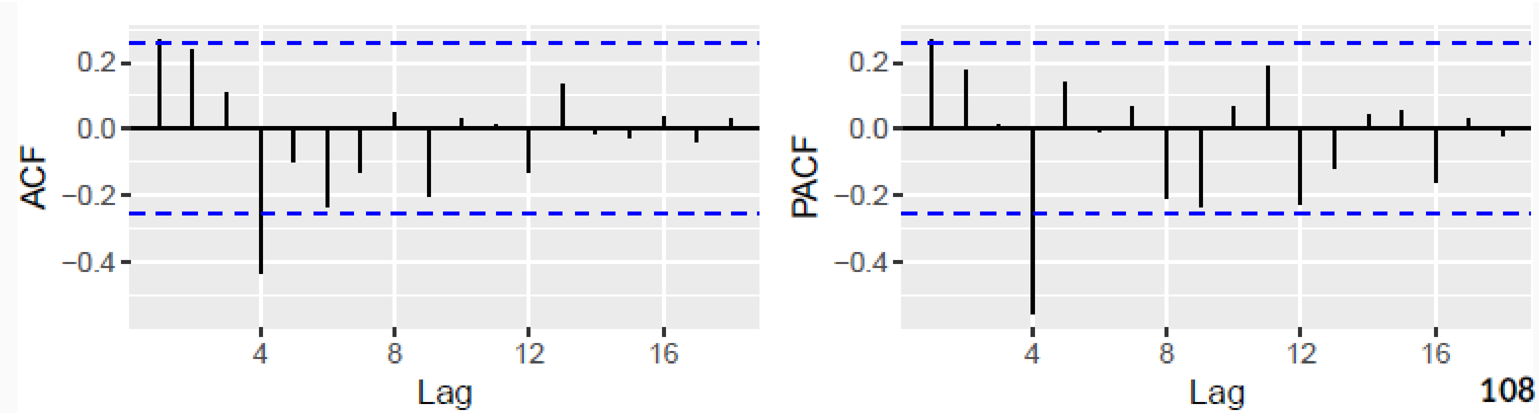
Seasonal ARIMA models

- Example: European quarterly retail trade index
  - Still non-stationary, so take another first difference, i.e. $(1\text{-}B)(1\text{-}B^4)y_t$



Seasonal ARIMA models

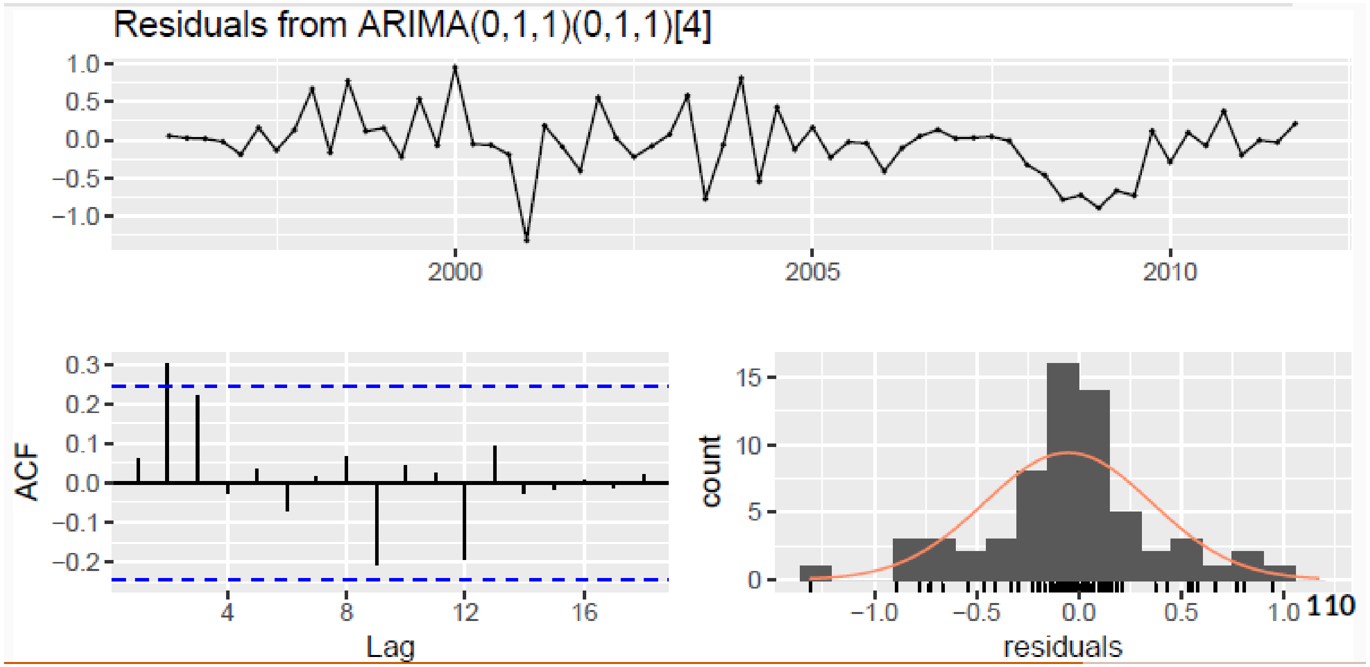
- Example: European quarterly retail trade index



- The significant spike at lag 1 in the ACF suggests a non-seasonal MA(1) component,
- The significant spike at lag 4 in the ACF suggests a seasonal MA(1) component.
- Thus start with ARIMA$(0,1,1)(0,1,1)_4$ (alternatively, you could select ARIMA$(1,1,0)(1,1,0)_4$)

Seasonal ARIMA models

- Example: European quarterly retail trade index
- Check the residuals from ARIMA(0,1,1)(0,1,1)$_4$ model



Seasonal ARIMA models

- Null hypothesis (residuals are not correlated) cannot be rejected at 5% level

```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(0,1,1)(0,1,1)[4]
## Q* = 10.654, df = 6, p-value = 0.09968
##
## Model df: 2.   Total lags used: 8
```

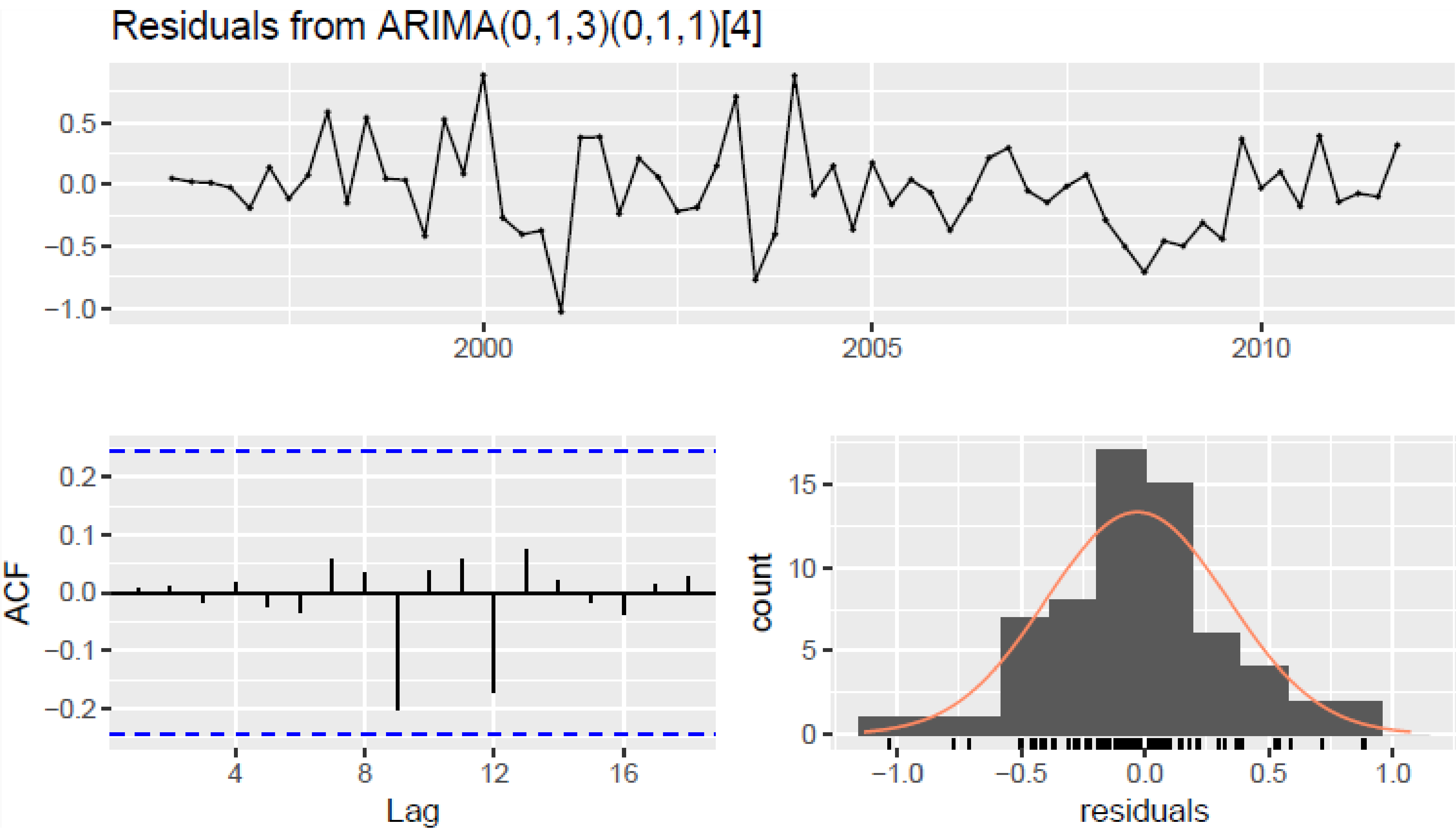
Seasonal ARIMA models

- ACF and PACF of residuals show significant spikes at lag 2, and maybe lag 3.
  - AICc of ARIMA(0,1,2)$(0,1,1)_4$ model is 74.27.
  - AICc of ARIMA(0,1,3)$(0,1,1)_4$ model is 68.39.

Seasonal ARIMA models

```
## Series: euretail
## ARIMA(0,1,3)(0,1,1)[4]
##
## Coefficients:
##          ma1     ma2     ma3     sma1
##       0.2630  0.3694  0.4200  -0.6636
## s.e.  0.1237  0.1255  0.1294   0.1545
##
## sigma^2 estimated as 0.156:  log likelihood=-28.63
## AIC=67.26   AICc=68.39   BIC=77.65
```

Seasonal ARIMA models



Seasonal ARIMA models

```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(0,1,3)(0,1,1)[4]
## Q* = 0.51128, df = 4, p-value = 0.9724
##
## Model df: 4.   Total lags used: 8
```

Seasonal ARIMA models

- Example: European quarterly retail trade index
- Forecasts for the next 3 years: