Compression Terms for the Settling

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The one-dimensional Buger-Diehl equation for the activated solid settling may be written as[1, 2], ignoring the feed mechanism,

$$\frac{\partial X}{\partial t} = -\frac{\partial(XV_{hs}(X))}{\partial z} - \frac{\partial(XV_{conv}(z,t))}{\partial z} + \frac{\partial}{\partial z}\left(d_{comp}(X)\frac{\partial X}{\partial z}\right) + \frac{\partial}{\partial z}\left(d_{disp}(Z, Qin(t))\frac{\partial X}{\partial z}\right). \tag{1}$$

Here,

1. X = concentration
2. V_{hs} = hindered settling velocity
3. V_{conv} = flow velocity
4. d_{comp} = compression coefficient
5. d_{disp} = dispersion coefficient (due to turbulence effect).

Based on the above mentioned transport equation, the volume fraction equation may be written as,

$$\frac{\partial \alpha_d}{\partial t} = -\nabla.(\alpha_d \phi) - \nabla.(\alpha_d V_{hs}) + \nabla.(d_{comp}\nabla\alpha_d) + \nabla.(\Gamma\alpha_d). \tag{2}$$

The compression term may written as,

$$d_{comp} = \begin{cases} 0 & \text{when } 0 \le X < X_{cr} \\ \frac{\rho_d}{g(\rho_d - \rho_c)} V_{hs}(X)\sigma'_e(X) & \text{when } X \ge X_{cr}. \end{cases}$$

Here,

1. X_{cr} = critical concentration
2. σ'_e = derivative of the effective solid stress.

The effective solid stress may be expressed as [3, 4],

$$\sigma_e = \begin{cases} 0 & \text{when } 0 \le X < X_{cr} \\ \lambda ln\frac{(X - X_{cr} + \beta)}{\beta} & \text{when } X \ge X_{cr}. \end{cases}$$

Here, λ, and β are the constants, which needs to be calibrated based on the experiment. Since in OpenFOAM, the volume fraction (α) is used as non-dimensional, the derivative of the stress may be expressed as,

$$\sigma'_e = \begin{cases} 0 & \text{when } 0 \le X < X_{cr} \\ \frac{\lambda}{\beta + (\alpha_d - \alpha_{cr}) * \rho_d} & \text{when } X \ge X_{cr}. \end{cases}$$

Input parameters:

1. λ ($\mathrm{kgm^{-1}s^{-2}}$)
2. β ($\mathrm{kgm^{-3}}$)
3. $\alpha_{\mathrm{cr}} = \mathrm{X_{cr}}/\rho_{\mathrm{d}}$

References

[1] R. Burger and etal. A consistent modelling methodology for secondary settling tanks: a reliable numerical method. *Water Science Technology*, 68(1):192–, 2013.

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[4] D.J. Clercq and etal. Extending and calibrating a mechanistic hindered and compression settling model for activated sludge using in-depth batch experiments. *Water Research*, 42(3):781–791, 2008.