Effect of Rheology Models on the Hydrodynamics

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Abstract

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Keywords: `elsarticle.cls`, LaTeX, Elsevier, template

2010 MSC: 00-01, 99-00

1. Introduction

2. Rheology

Newtonian flow is characterised as the dynamic viscosity μ is independent of the strain, which yields an linear relationship between the shear-stress and strain-rate in the rheological equation as,

$$\tau_{ij} = 2\mu\dot{s}_{ij}. \tag{1}$$

Here, τ_{ij} is the deviatoric stress tesnor, μ is the viscosity, and $\dot{\mathrm{s}}_{\mathrm{ij}}$ is the strain rate tensor. The strain rate tensor $\dot{\mathrm{s}}_{\mathrm{ij}}$ can be written as,

$$\dot{s}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{2}$$

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The viscosity for the Newtonian fluid can be expressed as,

$$\mu = \frac{|\tau|}{|\dot{\gamma}|}. \tag{3}$$

Here, $\dot{\gamma} = \sqrt{\frac{1}{2}\dot{\mathrm{s}}_{\mathrm{ij}}\dot{\mathrm{s}}_{\mathrm{ij}}}$ is the effective strain rate. Unlike the Newtonian fluid, the viscosity of the fluid shows the shear dependency through increasing or decreasing with it in the case of Non-Newtonian fluid. Here, the Non-Newtonian fluids are categorised as the pseudo-plastics, i.e. shear-thinning where $\frac{\partial \mu}{\partial \dot{\gamma}} < 0$, and dilatants, i.e. shear-thickening where $\frac{\partial \mu}{\partial \dot{\gamma}} > 0$.

2.1. Bingham Plastic Model

The Bingham fluids represent the viscoplastic nature where the fluid processed with the yield stress, or stress threshold, below which there is no flow. Upon yielding, the stress varies linearly with the magnitude of the strain-rate, $\dot{\gamma}$, similar to the Newtonian flow [1]. The Bingham plastic model may be written as,

$$\mu = \frac{\tau_y}{\dot{\gamma}} + \mu_p, \tag{4}$$

where, μ is the effective viscosity, τ_{y} is the yield stress, $\dot{\gamma}$ is the shear-strain rate, and μ_{p} is the plastic viscosity.

Note:

1. Yield stress is defined as the minimum stress required for producing the shear flow
2. Upon yielding, the flow properties show time dependency, which indicates the degradation of fluid with continued shear

The yield stress and plastic viscosity vary with the solid concentration. A number of experimental studies have proposed these variations with the solid concentrations as exponential as [2],

$$\Phi = aC^{b\alpha}, \tag{5}$$

where, Φ is the physical property to be evaluated, a and b are the constants, C is the exponent term which typically considered as the logarithm base e or the base 10, and α is the phase fraction of the solids. In OpenFOAM, the yield stress is calculated as,

$$\tau_y = a10^{b\alpha}. \tag{6}$$

Here, the values of a and b are taken from the works of Brennan [2], Dahl [3], de Clercq [4], and Liu *et al.* [5] as a = 1.1×10^{-4} kg/ms^2 and b = 0.98.

2.1.1. Plastic viscosity

The plastic viscosity in OpenFOAM is calculated as [5],

$$\mu_p = \mu_c + a_p(10^{b_p\alpha} - 1), \tag{7}$$

where, μ_p is the plastic viscosity, $\mathrm{a_p}$ and $\mathrm{b_p}$ are the plastic coefficients, and μ_c is the continuous phase viscosity. Based on the works of Brennan [2] and Dahl [3], the values of plastic coefficients are taken as $\mathrm{a_p} = 2.3143 \times 10^{-4}$ kg/ms and $\mathrm{b_p} = 179.26$.

Apparent viscosity calculated in OpenFOAM as,

$$\mu = \frac{\tau_y}{\dot{\gamma} + 1 \times 10^{-4}(\frac{\tau_y}{\mu_p})} + \mu_p. \tag{8}$$

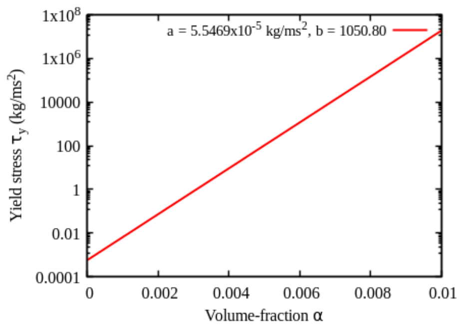
Bokil and Bewtra *et al.* [6] provided the expression for plastic viscosity as a function of solid concentration as,

$$\mu_p = 0.00327 \times 10^{0.132C}, \tag{9}$$

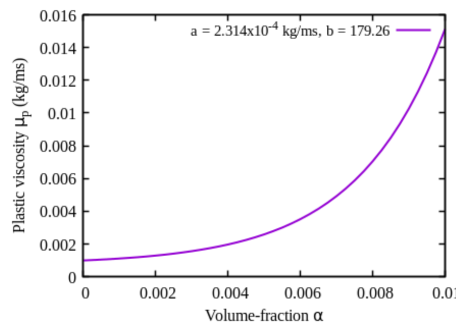
where, C is the solid concentrations. The validity of the mentioned function holds when the solid concentration is above 0.7 kg/m^3. Dahl [3] suggested the plastic viscosity as well as the yield stress as a function of solid concentration, depending on the inlet concentration of 4 kg/m^3, as,

$$\mu_p = \mu + C_{pl}C^2, \tag{10}$$

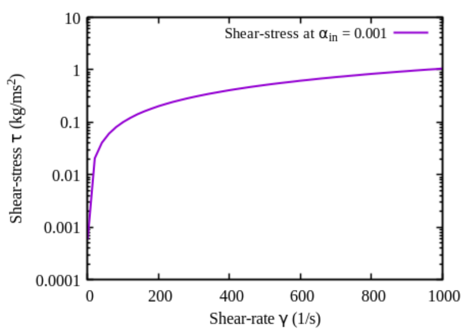
$$C_{pl} = 2.473 \times 10^{-4}. \tag{11}$$



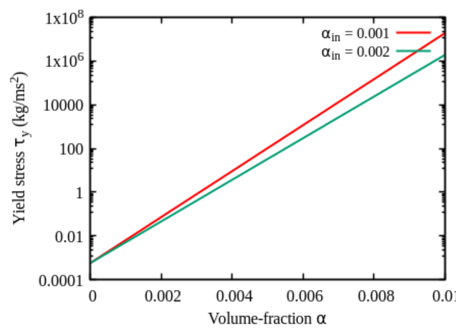
(a) Yield stress at $\alpha_{\text{in}} = 0.001$



(b) Plastic viscosity at $\alpha_{\text{in}} = 0.001$



(c) Shear stress at $\alpha_{\text{in}} = 0.001$



(d) Yield stresses at $\alpha_{\text{in}} = 0.001$ and 0.002

Figure 1: (a) Yield stress, (b) plastic viscosity and (c) shear stress at the inlet volume fraction of $\alpha_{\text{in}} = 0.001$, and (d) comparison of yield stresses at $\alpha_{\text{in}} = 0.001$ and 0.002, predicted by Brennan [2].

2.2. Herschel-Bulkley (HB) Model

The Herschel-Bulkley model can be expresses,

$$\tau = \tau_y + K\dot{\gamma}^n, \tag{12}$$

where, K is the consistency coefficient (Pa.s^n) and n is the flow behaviour index. The apparent viscosity may be expressed as,

$$\frac{\tau}{\dot{\gamma}} = \frac{\tau_y}{\dot{\gamma}} + K\frac{\dot{\gamma}^n}{\dot{\gamma}}, \tag{13}$$

or,

$$\mu = \frac{\tau_y}{\dot{\gamma}} + K\dot{\gamma}^{n-1}. \tag{14}$$

As can be seen in eq. 14, the viscosity becomes infinite at the low strain rates, i.e. $\dot{\gamma} \to 0$, and eventually causes the numerical error. With this in mind, the function is bounded at the low strain rates as,

$$\mu = \begin{cases} \frac{\tau_y}{\dot{\gamma}} + K\dot{\gamma}^{n-1} & \text{if } \dot{\gamma} > \dot{\gamma}_{cr} \\ \mu_0 & \dot{\gamma} < \dot{\gamma}_{cr}, \end{cases}$$

where, $\dot{\gamma}_{\mathrm{cr}}$ is the critical strain rate, i.e. strain rate limit to be used at calculation.

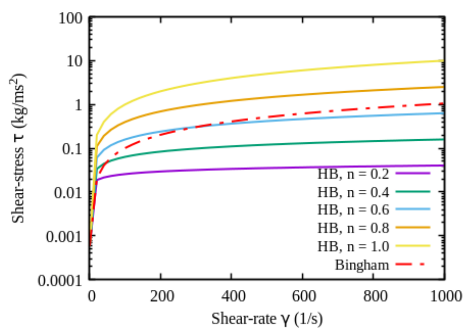


Figure 2: Shear stress as a function of shear-rate using the Herschel-Bulkley and Bingham plastic models at the inlet volume fraction of $\alpha_{\mathrm{in}} = 0.001$. Here, in the case of Herschel-Bulkley model the parameters are taken as K = 0.01 Pa.s^n, $\tau_{\mathrm{y}} = 6{\times}10^{-4}$ kg/ms, and n = 0.2, 0.4, 0.6, 0.8 and 1.0.

The regularisation used in implementing the Herschel-Bulkley model are,

1. The viscosity may be calculated as de-Clercq [4, 7],

$$\mu = \begin{cases} \mu_y & \text{if } \dot{\gamma} < \frac{\mu_y}{\tau_y} \\ \frac{\tau_y}{\dot{\gamma}} + K\dot{\gamma}^{n-1} & \text{otherwise}, \end{cases}$$

2. The method suggested by Papanastasiou [8],

$$\mu = (1 - e^{-m\dot{\gamma}})\frac{\tau_y + K\dot{\gamma}^n}{\dot{\gamma}}, \tag{15}$$

where, m is a constant.

3. Small perturbation may be added to avoid the numerical inconsistency as de-Clercq [4],

$$\mu = \frac{\tau_y + K\dot{\gamma}^n}{\dot{\gamma} + \epsilon}, \tag{16}$$

where, ϵ is the small number.

2.2.1. Concentration dependent yield stress and consistency coefficient

The solid volume fraction calculated by the experimental group as,

1. Total solid (TS):

$$TS = \frac{Driedwt. - Emptywt.}{Wetwt. - Emptywt.} \tag{17}$$

2. Total suspended solid (TSS):

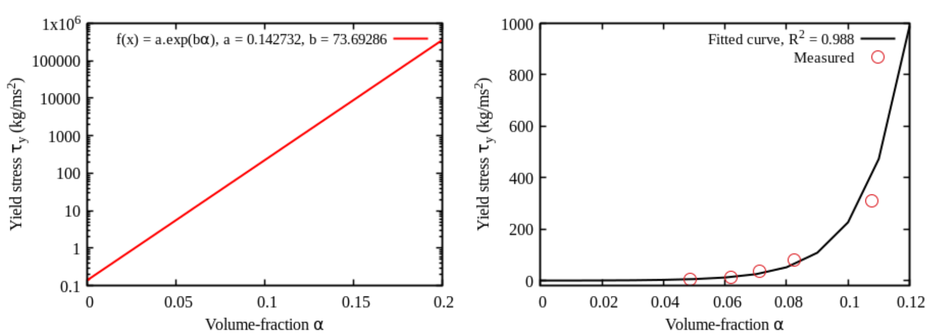
$$\frac{TS - TDS}{1 - TDS}, \tag{18}$$

where, TDS is the total dilute solid

3. Volume fraction:

$$\alpha = \frac{\frac{TSS}{\rho_d}}{\frac{1-TSS}{\rho_c} + \frac{TSS}{\rho_d}} \tag{19}$$

where, ρ_d and ρ_c are the densities of water and solids.



(a) Yield stress as a function of α (b) Measured data and fitted curve

Figure 3: (a) Yield stress as a function of volume fraction of the solid, and (b) measured and fitted curves of the yield stress. Here, the fitted function is $\tau_\mathrm{y} = \mathrm{a.exp(b\alpha)}$; a = 0.142732 and b = 73.69286, and $\mathrm{R}^2 = 0.988$.

2.2.2. *Herschel-Bulkley (HB) based on Bingham plastic flow*

The Herschel-bulkley (HB) model based on the Bingham plastic properties may be calculated as,

$$\tau(\alpha) = \tau_y(\alpha) + K(\alpha)\dot{\gamma}^n. \tag{20}$$

Here, the yield stress τ_{y} and consistency coefficient K are the function of solid volume fractions α, which may be taken as the power-law variation similar to the Bingham plastic fluid as,

$$\tau_y = a_\tau.C^{b_\tau \alpha} \quad \text{and} \quad K = a_k.C^{b_k \alpha}. \tag{21}$$

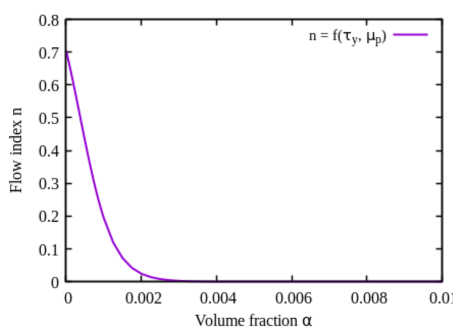
Here, a and b are the constants, and C is the exponential/log function. The apparent viscosity of the mixture can be written as,

$$\mu(\dot{\gamma}, \alpha) = \frac{\tau_y(\alpha)}{\dot{\gamma}} + K(\alpha)\dot{\gamma}^{n-1}. \tag{22}$$

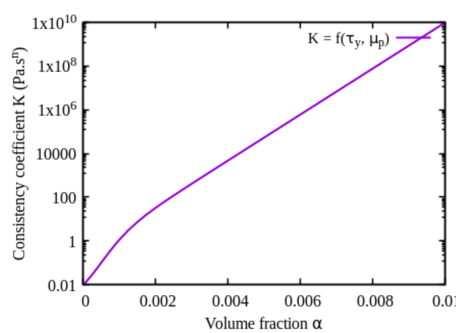
The consistency coefficient K and the flow behaviour index n may be calculated using the Bingham yield stress and plastic viscosity as follows [9],

$$n = 3.322 \times log\left(\frac{2\mu_p(\alpha) + \tau_y(\alpha)}{\mu_p(\alpha) + \tau_y(\alpha)}\right), \tag{23}$$

$$K = 511^{(1-n)} \times \left(\mu_p(\alpha) + \tau_y(\alpha)\right). \tag{24}$$



(a) n as a function of α



(b) K as a function of α

Figure 4: (a) Flow index n and (b) consistency coefficient K as a function of volume fraction α. Here, the yield stress $\tau_{\mathrm{y}}(\alpha)$ and the plastic viscosity $\mu_{\mathrm{p}}(\alpha)$ are taken from the Bingham plastic model the inlet $\alpha_{\mathrm{in}} = 0.001$.

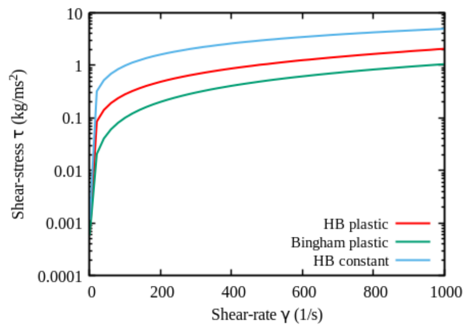


Figure 5: Shear stress as a function of shear-rate using the different functions. Note that, the Bingham plastic is the Bingham model at inlet solid fraction $\alpha = 0.001$ and the corresponding parameters are used for evaluating the HB plastic model, and HB constant is the typical Herschel-Bulkley model using the constant values of yield stress $\tau_{\rm y}$ and consistency coefficient K. Here, in the case of HB constant model, the values of yield stress and consistency coefficient are taken from the average values of the HB plastic model, i.e. $\overline{\tau_{\rm y}} = 1.15{\times}10^{-3}$ kg/ms^2, $\overline{\rm K}$ = 0.0394 Pa.s$^{\rm n}$, and n = 0.7.

3. Test Cases

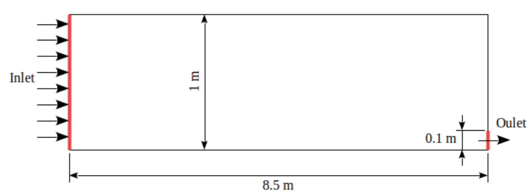
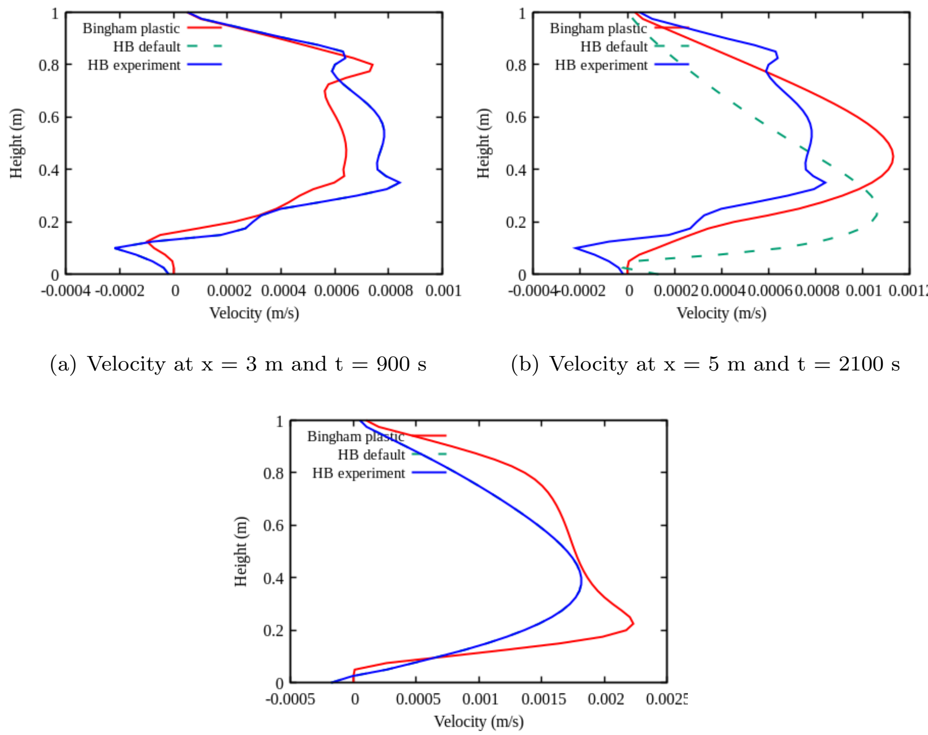


Figure 6: Schematic of the problem used in simulations. Note that the problem is used similar to the case conducted by Dahl [3].



(a) Velocity at x = 3 m and t = 900 s

(b) Velocity at x = 5 m and t = 2100 s

(c) velocity at x = 5 m and t = 3000 s

Figure 7: Velocity at (a) x = 3 m and t = 900 s, (b) x = 5 m and t = 2100 s, and (c) x = 7 m and t = 3000 s using the rheology models of HB constant, Bingham plastics, and HB model with experimentally fitted data.

4. MULES algorithm

4.1. Flux correcting technique (FCT)

5. Modelling Turbulence

6. Conclusions

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