

# CS419-Assignment 3

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## Implementation Details of the Descriptors

To develop a recognition solution, the first thing I tried was to try different methods to convert binary images into numerical descriptor vectors. Before doing that, I extracted the contours in the images. After that I get the contour that is biggest. In this way, I extracted the shape and I can easily use the functions of OpenCV.

### (a) Basic Shape Descriptors

#### (i) Area

Area is actually the surface area of the shape. But in the context of digital images, we can consider it as the count of the white pixels. However the image may contain some other small dots besides the main shape. So it is important to extract the biggest contour then find the area of it.

#### (ii) Perimeter

Perimeter is simply the number of border pixels of the shape in the context of digital binary images. Similarly, to find the perimeter, I extracted the biggest connected component and find the perimeter of it using *cv2.arcLength* function.

#### (iii) Convexity

Convexity indicates how close the shape is to its convex hull and calculated by dividing the area of the shape by the area of its convex hull. If the shape is convex, then the convexity of it should be 1. In my implementation, I found the convex hull of the extracted shapes and calculated the area of the convex hull. Since I calculated the area before, I simply divided the area by the area of the convex hull.

#### (iv) Circularity

Circularity can be calculated by taking the square of the perimeter and dividing it by the area of the shape. Perfect circles have the minimum circularity value which is  $4\pi$ . All other shapes has greater circularity value. Since I found both area and the perimeter, I easily calculated the circularity of the shapes.

#### (v) Rectangularity

It indicates how rectangular a shape is. To calculate it, we need to find the minimum bounding rectangle and divide the area of the shape by the area of that rectangle. This gives use how much it fills

the minimum bounding rectangle. To find the minimum bounding rectangle, I used *cv2.boundingRect* function of OpenCV.

#### (vi) Eccentricity

Eccentricity is the ratio of principal axes of inertia. To find the length of the major and minor axes, I calculated the eigenvalues of the covariance matrix of the boundary points of the shape. The larger eigenvalue is the length of the major axis and the smaller eigenvalue is the length of the minor axis. The ratio of them gives us the eccentricity of the shapes. I used Numpy functions to calculate the covariance matrix and the eigenvalues of it.

#### (b) Fourier Descriptor

To describe the shapes with their Fourier transform, I find the Fourier transforms of the shapes and take the first *n* frequency values. By changing the *n* parameter, later I will try to optimize the performance of the description.

#### (c) Shape Histogram

First I found the center of gravity of the shapes with the moments as:

$$(x, y) = (M[m10]/M[m00], M[m01]/M[m00])$$

After calculating the center of gravity of the shapes, I draw regularly spaced *n* concentric circles around it, up until the farthest point from the center. The descriptor is then constructed by measuring the amount of surface in each circular band. To make it rotation invariant, I normalized the surface in each circular band by dividing it by the area of the shape. By changing the *n* parameter, later I will try to optimize the performance of the description.

#### (d) Moment Invariant Descriptor

As shown in the class, Hu's 8 moments are translation, scale and rotation invariant. By using the formulas in our slides that are covered in the lecture, I calculated each Hu moment to describe the shape by numerical values.

## Distance Functions

#### (a) Euclidean Distance

The Euclidean distance between two vectors **X** and **Y** is given by the Euclidean norm:

$$\text{Euclidean Distance}(\mathbf{X}, \mathbf{Y}) = \|\mathbf{X} - \mathbf{Y}\| \quad (1)$$

This distance measure represents the straight-line distance between two points in the feature space.

#### (b) Manhattan Distance

The Manhattan distance, also known as the L1 norm or city block distance, between vectors **X** and **Y** is calculated as the sum of the absolute differences between corresponding elements:

$$\text{Manhattan Distance}(\mathbf{X}, \mathbf{Y}) = \sum_i |X_i - Y_i| \quad (2)$$

This distance is the sum of the absolute differences along each dimension.

### (c) Chi-squared Distance

The Chi-squared distance between vectors  $\mathbf{X}$  and  $\mathbf{Y}$  is defined by:

$$\text{Chi-squared Distance}(\mathbf{X}, \mathbf{Y}) = \sum_i \frac{(X_i - Y_i)^2}{X_i + Y_i} \quad (3)$$

This distance measure is commonly used for comparing histograms or frequency distributions.

### (d) Mahalanobis Distance

The Mahalanobis distance between vectors  $\mathbf{X}$  and  $\mathbf{Y}$  with the inverse of the covariance matrix  $\Sigma^{-1}$  is given by:

$$\text{Mahalanobis Distance}(\mathbf{X}, \mathbf{Y}, \Sigma^{-1}) = \sqrt{(\mathbf{X} - \mathbf{Y})^T \Sigma^{-1} (\mathbf{X} - \mathbf{Y})} \quad (4)$$

This distance accounts for the correlations between different dimensions by scaling the differences with the inverse covariance matrix.

## Optimizing the Parameters and the Combinations

To assess the recognition performance of the descriptors, I converted all shapes into numerical vectors by using different descriptor combinations and parameters. After that for every test sample I calculated the distance of the vectors between test vector and every descriptor vector of train shapes using different distance measures. I assigned the closest label to my test sample. After doing it for every test sample I count the correct predictions and divide it by the number of test shapes which is 700 to calculate the accuracy. By doing that, I can assess the performance of the descriptors and the distance functions. At first, I tried every descriptor isolated. Then I choose the best performing parameters and tried combination of those. You can see the isolated test results from Tables 1 to 5. For combined results, please check Table 6.

Table 1: Accuracy Results Only Using Fourier

Fourier with First N Frequencies	Euclidean	Manhattan	Chi-squared	Mahalanobis
0	1.43%	1.43%	1.43%	1.43%
2	39.71%	41.29%	2.71%	7.71%
3	44.71%	45.57%	2.29%	4.71%
4	46.29%	49.29%	2.14%	6.14%
5	47.71%	50.43%	2.43%	3.29%
7	49.00%	51.86%	2.71%	4.14%
10	49.29%	54.00%	2.86%	4.00%
20	48.86%	53.43%	2.71%	3.57%
30	48.86%	54.29%	2.86%	2.86%
40	48.86%	53.71%	3.29%	1.71%
50	49.00%	53.29%	3.14%	2.71%
100	48.86%	52.86%	2.43%	1.43%
250	44.43%	48.71%	2.71%	1.43%
500	30.43%	35.00%	2.86%	0.57%

Table 2: Accuracy Results Only Using Basic Shape Descriptors

Basic Shape Combination	Euclidean	Manhattan	Chi-squared	Mahalanobis
area, perimeter, convexity, circularity, rectangularity, eccentricity	28.71%	30.29%	34.71%	64.57%
area, perimeter, convexity, circularity, rectangularity	28.71%	30.14%	34.57%	56.14%
area, perimeter, convexity, circularity, eccentricity	28.71%	30.29%	34.29%	57.71%
area, perimeter, convexity, circularity	28.71%	30.14%	34.00%	48.14%
area, perimeter, convexity, rectangularity, eccentricity	28.71%	29.86%	33.86%	62.43%
area, perimeter, convexity, rectangularity	28.71%	29.86%	33.43%	53.86%
area, perimeter, convexity, eccentricity	28.71%	29.86%	33.57%	55.00%
area, perimeter, convexity	28.71%	29.86%	33.14%	44.86%
area, perimeter, circularity, rectangularity, eccentricity	28.71%	30.29%	34.71%	57.71%
area, perimeter, circularity, rectangularity	28.71%	30.14%	34.57%	48.00%
area, perimeter, circularity, eccentricity	28.71%	30.29%	34.29%	50.86%
area, perimeter, circularity	28.71%	30.14%	34.00%	32.57%
area, perimeter, rectangularity, eccentricity	28.71%	29.86%	33.86%	55.57%
area, perimeter, rectangularity	28.71%	29.86%	33.43%	44.57%
area, perimeter, eccentricity	28.71%	29.86%	33.57%	48.00%
area, perimeter	28.71%	29.86%	33.14%	32.71%
area, convexity, circularity, rectangularity, eccentricity	22.00%	22.57%	31.57%	64.00%
area, convexity, circularity, rectangularity	22.00%	22.57%	31.14%	56.71%
area, convexity, circularity, eccentricity	22.00%	22.57%	31.29%	57.43%
area, convexity, circularity	22.00%	22.57%	30.86%	47.14%
area, convexity, rectangularity, eccentricity	18.71%	18.71%	25.71%	58.57%
area, convexity, rectangularity	18.71%	18.71%	24.43%	48.57%
area, convexity, eccentricity	18.71%	18.71%	23.00%	50.57%
area, convexity	18.71%	18.71%	20.57%	37.71%
area, circularity, rectangularity, eccentricity	22.00%	22.57%	31.57%	58.29%
area, circularity, rectangularity	22.00%	22.57%	31.14%	46.86%
area, circularity, eccentricity	22.00%	22.57%	31.29%	48.00%
area, circularity	22.00%	22.57%	30.86%	33.71%
area, rectangularity, eccentricity	18.71%	18.71%	24.57%	47.14%
area, rectangularity	18.71%	18.71%	23.29%	37.86%
area, eccentricity	18.71%	18.71%	21.57%	30.71%
perimeter, convexity, circularity, rectangularity, eccentricity	28.71%	29.57%	40.00%	62.14%
perimeter, convexity, circularity, rectangularity	28.71%	29.57%	36.00%	54.14%
perimeter, convexity, circularity, eccentricity	28.71%	29.14%	37.86%	55.71%
perimeter, convexity, circularity	28.71%	29.00%	33.57%	46.29%
perimeter, convexity, rectangularity, eccentricity	16.00%	17.29%	33.00%	54.14%
perimeter, convexity, rectangularity	16.00%	16.71%	25.29%	42.43%
perimeter, convexity, eccentricity	15.00%	16.43%	29.57%	43.14%
perimeter, convexity	14.71%	15.57%	21.29%	31.86%
perimeter, circularity, rectangularity, eccentricity	28.71%	29.57%	40.00%	56.14%
perimeter, circularity, rectangularity	28.71%	29.57%	35.71%	46.71%
perimeter, circularity, eccentricity	28.71%	28.86%	37.43%	47.00%
perimeter, circularity	28.71%	28.86%	33.14%	31.43%
perimeter, rectangularity, eccentricity	15.86%	17.29%	31.86%	42.29%
perimeter, rectangularity	15.86%	16.71%	24.14%	29.43%
perimeter, eccentricity	14.71%	15.86%	25.14%	28.71%
convexity, circularity, rectangularity, eccentricity	26.29%	27.86%	40.14%	51.29%
convexity, circularity, rectangularity	21.43%	23.00%	30.43%	39.86%
convexity, circularity, eccentricity	19.57%	21.57%	28.00%	42.43%
convexity, circularity	16.14%	17.57%	20.00%	24.43%
convexity, rectangularity, eccentricity	40.00%	40.71%	43.00%	45.00%
convexity, rectangularity	26.71%	27.14%	28.14%	26.43%
convexity, eccentricity	29.14%	30.14%	29.86%	29.57%
circularity, rectangularity, eccentricity	25.86%	25.57%	38.00%	42.43%
circularity, rectangularity	21.29%	21.86%	26.29%	25.86%
circularity, eccentricity	17.86%	18.86%	24.43%	27.29%
rectangularity, eccentricity	25.29%	25.14%	26.57%	25.71%

Table 3: Accuracy Results Only Using Moment Invariants

Moment Combination	Euclidean	Manhattan	Chi-squared	Mahalanobis
1, 2, 3, 4, 5	43.86%	45.00%	54.00%	49.00%
1, 2, 3, 4	43.86%	45.00%	54.00%	48.29%
1, 2, 3, 4, 7	43.86%	45.00%	53.86%	49.00%
1, 2, 3, 4, 5, 7	43.86%	45.00%	53.86%	48.86%
1, 2, 3, 4, 8	43.86%	45.14%	51.43%	50.14%
1, 2, 3, 4, 5, 7, 8	43.86%	45.14%	51.43%	49.43%
1, 2, 3, 4, 5, 8	43.86%	45.14%	51.43%	49.14%
1, 2, 3, 4, 7, 8	43.86%	45.14%	51.29%	50.14%
1, 2, 3, 4, 5, 6, 7	43.86%	45.29%	50.71%	51.43%
1, 2, 3, 4, 6, 7	43.86%	45.29%	50.71%	50.57%
1, 2, 3, 4, 5, 6	43.86%	45.29%	50.57%	50.71%
1, 2, 3, 4, 6	43.86%	45.29%	50.57%	50.29%
1, 2, 3, 5	42.71%	43.29%	49.71%	44.71%
1, 2, 3	42.71%	43.29%	49.43%	45.43%
1, 2, 3, 7	42.71%	43.29%	49.29%	45.14%
1, 2, 3, 5, 7	42.71%	43.29%	49.29%	44.71%
1, 2, 3, 4, 5, 6, 7, 8	43.86%	45.43%	48.57%	51.86%
1, 2, 3, 4, 6, 7, 8	43.86%	45.43%	48.57%	51.00%
1, 2, 3, 4, 6, 8	43.86%	45.43%	48.43%	50.29%
1, 2, 3, 4, 5, 6, 8	43.86%	45.43%	48.43%	50.14%
1, 2, 3, 6, 7	42.86%	43.86%	47.43%	46.14%
1, 2, 3, 5, 6, 7	42.86%	43.86%	47.43%	45.86%
1, 2, 3, 6	42.86%	43.86%	47.29%	46.29%
1, 2, 3, 5, 6	42.86%	43.86%	47.29%	46.14%
1, 2, 4, 5	36.00%	37.57%	46.29%	42.00%
1, 2, 4	36.00%	37.57%	46.29%	40.57%
1, 2, 4, 5, 7	36.00%	37.57%	45.86%	42.57%
1, 2, 3, 5, 6, 7, 8	42.86%	44.14%	45.71%	45.71%
1, 2, 3, 6, 7, 8	42.86%	44.00%	45.57%	46.14%
1, 2, 3, 5, 6, 8	42.86%	44.14%	45.57%	45.71%
1, 2, 3, 5, 8	42.71%	43.29%	45.57%	44.29%
1, 2, 4, 7	36.00%	37.57%	45.57%	41.00%
1, 2, 3, 6, 8	42.86%	44.00%	45.43%	46.00%
1, 2, 3, 8	42.71%	43.29%	45.43%	45.57%
1, 2, 3, 5, 7, 8	42.71%	43.29%	45.43%	44.00%
1, 2, 3, 7, 8	42.71%	43.29%	45.14%	45.71%
1, 3, 4, 7	28.43%	31.29%	44.71%	41.86%
1, 3, 4, 5, 7	28.43%	31.29%	44.57%	44.14%
1, 3, 4, 5	28.43%	31.29%	44.43%	43.71%
1, 3, 4	28.43%	31.29%	44.43%	41.00%
1, 2, 4, 5, 8	36.00%	37.57%	43.57%	41.86%
1, 2, 4, 8	36.00%	37.57%	43.43%	40.29%
1, 2, 4, 5, 7, 8	36.00%	37.57%	43.14%	42.86%
1, 2, 4, 7, 8	36.00%	37.57%	43.00%	41.57%
2, 3, 4, 5	34.86%	35.86%	42.14%	41.57%
2, 3, 4	34.86%	35.86%	42.14%	41.00%
2, 3, 4, 5, 7	34.86%	35.86%	41.43%	42.29%
2, 3, 4, 7	34.86%	35.86%	41.43%	41.43%
1, 2, 4, 6, 7	36.29%	38.29%	41.14%	45.14%
1, 2, 4, 5, 6, 7	36.29%	38.29%	41.14%	44.00%
1, 2, 4, 5, 6	36.29%	38.29%	41.00%	44.57%
1, 2, 4, 6	36.29%	38.29%	40.86%	44.57%
1, 3, 4, 5, 7, 8	28.43%	31.43%	39.71%	44.57%
1, 3, 4, 7, 8	28.43%	31.29%	39.71%	43.71%
1, 3, 4, 5, 8	28.43%	31.43%	39.57%	44.71%
1, 3, 4, 8	28.43%	31.29%	39.57%	43.43%
2, 3, 4, 5, 8	34.86%	35.86%	39.29%	43.43%

Table 4: Accuracy Results Only Using Moment Invariants

Moment Combination	Euclidean	Manhattan	Chi-squared	Mahalanobis
1, 2, 4, 6, 7, 8	36.29%	38.14%	39.14%	44.86%
1, 2, 4, 6, 8	36.29%	38.14%	39.14%	44.29%
1, 2, 4, 5, 6, 7, 8	36.29%	38.14%	39.14%	44.14%
2, 3, 4, 8	34.86%	35.86%	39.14%	42.71%
1, 2, 4, 5, 6, 8	36.29%	38.14%	39.00%	44.29%
1, 2, 5	34.00%	34.29%	38.86%	33.57%
2, 3, 4, 5, 7, 8	34.86%	35.86%	38.71%	43.71%
2, 3, 4, 7, 8	34.86%	35.86%	38.57%	42.71%
1, 3, 4, 6, 7	28.71%	31.86%	38.43%	45.00%
1, 2, 5, 7	34.00%	34.29%	38.43%	35.86%
1, 3, 4, 5, 6, 7	28.71%	31.86%	38.29%	45.86%
1, 3, 4, 6	28.71%	31.86%	38.29%	45.29%
1, 3, 4, 5, 6	28.71%	31.86%	38.14%	45.57%
1, 2	33.86%	34.00%	37.43%	32.71%
1, 2, 7	33.86%	34.14%	36.43%	34.43%
1, 2, 5, 6, 7	34.71%	35.71%	35.86%	38.00%
1, 2, 6, 7	34.71%	35.71%	35.71%	37.43%
1, 2, 5, 6	34.71%	35.71%	35.71%	37.29%
1, 2, 6	34.71%	35.71%	35.71%	35.71%
2, 3, 4, 6	35.00%	35.57%	35.29%	43.14%
2, 3, 4, 5, 6	35.00%	35.57%	35.14%	43.14%
2, 3, 4, 6, 7	35.00%	35.57%	35.00%	43.29%
2, 3, 4, 5, 6, 7	35.00%	35.57%	34.86%	43.00%
1, 3, 4, 6, 7, 8	28.71%	32.14%	34.57%	45.86%
1, 3, 4, 5, 6, 7, 8	28.71%	32.14%	34.43%	46.57%
1, 3, 4, 6, 8	28.71%	32.14%	34.43%	45.57%
1, 3, 4, 5, 6, 8	28.71%	32.14%	34.29%	45.71%
1, 3, 5, 6, 7	26.57%	28.86%	33.86%	37.86%
1, 3, 6, 7	26.86%	28.86%	33.86%	35.71%
1, 4, 5, 7	21.57%	22.14%	33.71%	32.43%
1, 3, 6	26.86%	28.86%	33.57%	35.86%
1, 3, 5	26.43%	27.57%	33.57%	33.29%
1, 4	21.57%	22.00%	33.57%	29.43%
1, 2, 5, 6, 7, 8	34.71%	35.57%	33.43%	37.86%
1, 2, 5, 6, 8	34.71%	35.57%	33.43%	37.71%
1, 3, 5, 6	26.71%	28.86%	33.43%	37.71%
2, 3, 4, 6, 8	35.00%	35.86%	33.29%	43.71%
2, 3, 4, 5, 6, 8	35.00%	35.86%	33.29%	43.57%
1, 3, 5, 7	26.43%	27.57%	33.29%	33.00%
1, 4, 5	21.57%	22.00%	33.29%	31.43%
1, 4, 7	21.57%	22.00%	33.29%	30.43%
1, 2, 6, 7, 8	34.71%	35.57%	33.14%	37.57%
1, 2, 6, 8	34.71%	35.57%	33.14%	36.86%
2, 3, 4, 5, 6, 7, 8	35.00%	35.86%	33.00%	44.57%
2, 3, 4, 6, 7, 8	35.00%	35.86%	33.00%	43.71%
1, 3, 7	26.43%	27.43%	32.71%	32.29%
1, 3	26.43%	27.43%	32.57%	31.71%
2, 3, 5, 7	30.57%	31.00%	32.14%	30.14%
2, 3, 5	30.57%	31.00%	31.86%	30.71%
2, 3, 7	30.57%	30.86%	31.86%	30.57%
1, 2, 5, 8	34.00%	34.29%	31.57%	34.43%
1, 2, 5, 7, 8	34.00%	34.29%	31.43%	35.57%
2, 3	30.57%	30.86%	31.43%	30.57%
1, 2, 7, 8	33.86%	33.86%	30.57%	35.71%
1, 2, 8	33.86%	33.86%	30.43%	34.00%
1, 3, 6, 7, 8	26.86%	28.86%	29.86%	36.86%
1, 3, 6, 8	26.86%	28.86%	29.71%	36.86%

Table 5: Accuracy Results Only Using Shape Histogram

Shape Histogram - n Bins	Euclidean	Manhattan	Chi-squared	Mahalanobis
0	1.43%	1.43%	1.43%	1.43%
3	1.43%	1.43%	1.43%	1.43%
5	1.43%	1.43%	1.43%	1.43%
10	1.43%	1.43%	1.43%	1.43%
25	1.43%	1.43%	1.43%	1.43%
50	1.43%	1.43%	1.43%	1.43%
100	1.43%	1.43%	0.71%	1.43%
150	1.43%	1.43%	1.71%	1.43%
200	1.43%	1.43%	1.43%	1.43%

By looking at the results, I see that Basic Shape Descriptors are working well with Mahalanobis Distance. By using all Basic Shape Descriptors and Mahalanobis Distance, I was able to achieve an accuracy of 64.57%. For Fourier descriptor, Manhattan Distance seems working well. I obtained maximum accuracy of 54.29% with using first 30 frequencies of the Fourier Transform. I could not achieve large accuracy with shape histograms. Maximum I get was 1.71% accuracy using Manhattan distance, however it is greater than 1.43% which is 1/70 of random prediction. Lastly I could obtain 54% accuracy with moments [1, 2, 3, 4, 5] and [1, 2, 3, 4] by using Chi-squared distance.

After that, I choose high performing parameters and start trying their combinations. The maximum I could get was 71.86% accuracy by using all basic shape descriptors and all moment invariants with Mahalanobis distance. This result was quite promising but I wanted to obtain even higher accuracy results. At this point, I remember Ensemble Methods in Machine Learning. Particularly voting classifiers. I decided to implement 3 distinct models using 3 different parameters and distance measures. I took the best performing parameters I obtained during isolated tests of the descriptors for my voting classifier.

Model 1: All basic shape descriptors - Mahalanobis Distance

Model 2: Fourier descriptor with first 30 frequencies - Manhattan Distance

Model 3: First 5 moments - Chi-squared Distance

My classifier takes the predictions of the 3 models and find the dominant choice. If all are different, I choose to get the prediction of the first model since it has higher accuracy. The performance of the ensemble method was also quite promising. It achieved 67.29% accuracy.

Lastly I wanted to try to ensemble the parameters that performed best in combination tests in Table 6.

Model 1: All basic shape descriptors and all moments- Mahalanobis Distance

Model 2: Convexity, rectangularity, eccentricity and first 5 moments - Chi-squared Distance

Model 3: All basic shape, Fourier (n=10), all moments - Manhattan Distance

It works similar to the previous ensemble trial. The performance of this method gave me the highest accuracy particularly 72.14%.

Table 6: Accuracy Results of Combinations of Descriptors

Basic Shape	Fourier	Histogram	Moments	Euclidean	Manhattan	Chi-squared	Mahalanobis
area, perimeter, convexity, circularity, rectangularity, eccentricity	0	0	1, 2, 3, 4, 5, 6, 7, 8	28.71%	30.29%	34.43%	71.86%
perimeter, convexity, circularity, rectangularity, eccentricity	0	0	1, 2, 3, 4, 6, 7, 8	28.71%	29.71%	41.43%	71.71%
area, convexity, circularity, rectangularity, eccentricity	0	0	1, 2, 3, 4, 6, 7, 8	22.00%	22.57%	32.14%	71.71%
area, perimeter, convexity, circularity, rectangularity, eccentricity	0	0	1, 2, 3, 4	28.71%	30.29%	34.86%	71.57%
area, perimeter, convexity, circularity, rectangularity, eccentricity	0	0	1, 2, 3, 4, 5, 6, 7	28.71%	30.29%	34.43%	71.57%
convexity, rectangularity, eccentricity	0	0	1, 2, 3, 4, 5	48.71%	52.86%	59.29%	60.71%
convexity, rectangularity, eccentricity	0	0	1, 2, 3, 4	48.71%	52.86%	59.29%	60.43%
convexity, rectangularity, eccentricity	0	0	1, 2, 3, 4, 5, 6, 7	48.71%	53.00%	57.86%	61.00%
convexity, rectangularity, eccentricity	0	0	1, 2, 3, 4, 6, 7, 8	48.71%	53.00%	57.00%	60.29%
convexity, rectangularity, eccentricity	0	0	1, 2, 3, 4, 5, 6, 7, 8	48.71%	53.00%	56.86%	61.14%
area, perimeter, convexity, circularity, rectangularity, eccentricity	10	0	1, 2, 3, 4, 5, 6, 7, 8	56.00%	59.43%	4.00%	24.86%
area, perimeter, convexity, circularity, rectangularity, eccentricity	10	0	1, 2, 3, 4, 6, 7, 8	56.00%	59.43%	4.00%	24.14%
area, perimeter, convexity, circularity, rectangularity, eccentricity	10	0	1, 2, 3, 4, 5, 6, 7	56.00%	59.43%	3.86%	24.43%
area, perimeter, convexity, circularity, rectangularity, eccentricity	30	0	1, 2, 3, 4, 5, 6, 7, 8	56.00%	59.43%	3.43%	9.00%
area, perimeter, convexity, circularity, rectangularity, eccentricity	30	0	1, 2, 3, 4, 6, 7, 8	56.00%	59.43%	3.43%	8.86%