BILKENT UNIVERSITY

Analysis of motion of elastic beam with moving base in 3D
Me-571 Project Report

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1 Introduction

1.1 System To Be Modeled

In this project, I will analyze the motion of an elastic beam with moving base, shown in 1. In the figure, İ and J define the motion of the rigid base body, w(x,t) and v(x,t) define the displacement of elastic beam from the center of the base body. As the base moves, the elastic rod will bend and crate the w(x,t) and v(x,t) displacements.

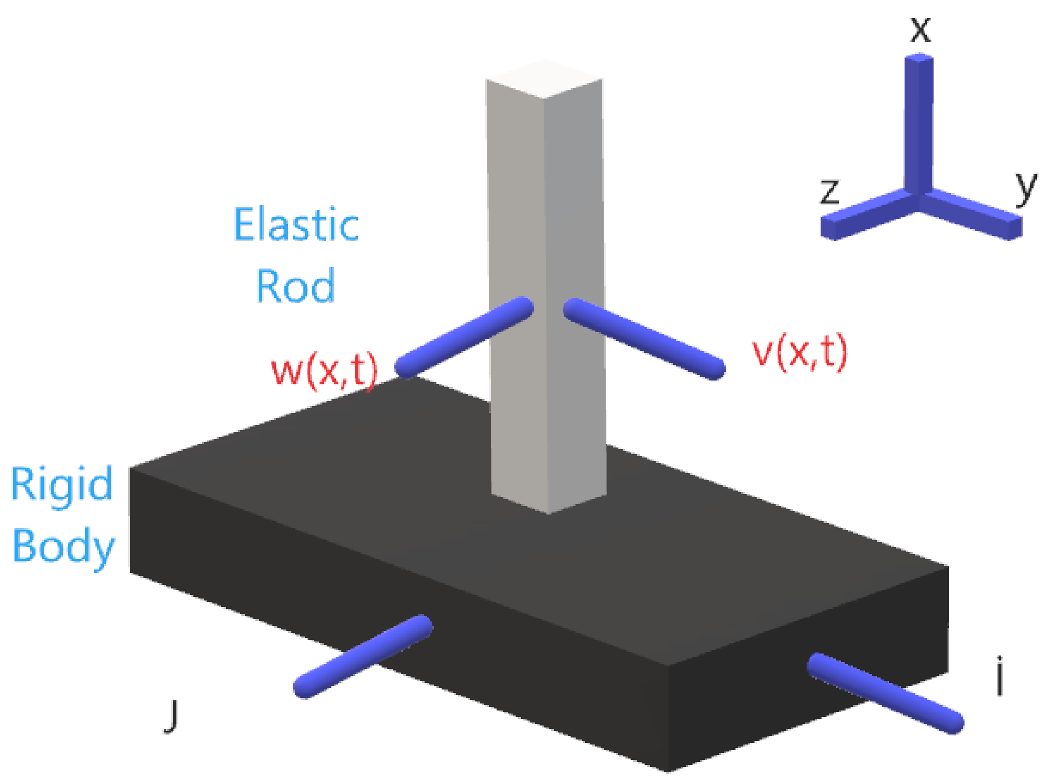


Figure 1: System to be analyzed

1.2 Physics Informed Neural Network

Traditional deep learning neural networks rely on data to capture the underlying information of the modeled system [4]. This generally requires huge amount of data which is not always accessible. A different approach to deep learning is to give the known information about the system to neural network before training the network. Such deep networks are called physics informed neural networks. The known information about the system is given to the network as loss function. For example, a differential equation can be given to the network as a loss function. Network makes its predictions such that the predicted data fits the given differential equation. This networks are much more data efficient, they require much less data then traditional neural networks to capture the underlying information [4].

2 Related Work

This simple model can be utilized in many different modeling purposes and therefore studied before by some. Stoykov studies the vibration of such systems under forced conditions [1], while Sayyad studies the transverse displacement, axial bending stress, transverse shear stress and natural frequencies of simply supported systems [2]. Pham et. al. has studied the vibration control of cantilever beams in 3D [3]. While their method and results are very promising, they have omitted some important terms in their governing equations, mainly the effect of lateral and horizontal displacement in vertical strain, and they have also studied the system with method of virtual work. These conditions discern the study that is done by author from the work done by Pham et. al..

3 Mathematical Model

In order to obtain the mathematical model, Hamilton's equations will be utilized.We begin with some assumptions, the first one is that the beam does not shear. The second one is that there are no stresses in y and z directions. Third one is that rigid base body does not have mass, it is only used in order to model the system better.

3.1 Potential Energy

With this assumptions, we can define our displacement in x direction as:

$$u = -z * \frac{\partial w}{\partial x} - y * \frac{\partial v}{\partial x} \tag{1}$$

Potential energy stored in elastic beam can be formulated as:

$$PE_{Elastic} = \int_V \frac{1}{2} * \sigma * \epsilon * dV \tag{2}$$

Where w and v are the displacements in z and y axis from figure 1.Our assumptions from before ensures that we only have strain and stress in x direction. The strain in x direction can be written as:

$$\epsilon_x = \frac{\partial u}{\partial x} = -z * \frac{\partial^2 w}{\partial x^2} - y * \frac{\partial^2 v}{\partial x^2} \tag{3}$$

From this strain, elastic potential energy can be calculated as:

$$PE_{Elastic} = \int_V \frac{1}{2} * E_x * \epsilon_x^2 * dV \tag{4}$$

Gravitational potential energy can be formulated as:

$$PE_G = \int_V \rho * g * (x - y * \frac{\partial v}{\partial x} - z * \frac{\partial w}{\partial x}) * dV \tag{5}$$

3.2 Kinetic Energy

Kinetic energy comes from motion of the elastic body.

$$KE = \int_V \frac{1}{2} * \rho * ((\dot{I} + \dot{v})^2 + (\dot{J} + \dot{w})^2 + (z * \frac{\partial^2 w}{\partial x \partial t} + y * \frac{\partial^2 v}{\partial x \partial t})^2 dV \tag{6}$$

3.3 Hamilton's Method

With the energies calculated Lagrangian can be written as:

$$L = \int_V \left(\frac{1}{2} * \rho * ((\dot{I} + \dot{v})^2 + (\dot{J} + \dot{w})^2 + (z * \frac{\partial^2 w}{\partial x \partial t} + y * \frac{\partial^2 v}{\partial x \partial t})^2 - \frac{1}{2} * E_x * \epsilon_x^2 - \rho * g * (x - y * \frac{\partial v}{\partial x} - z * \frac{\partial w}{\partial x}) \right) * dV \tag{7}$$

with Lagrangian defined as $\int_V g(u, \dot{u}, u', u'', \dot{u'}) dV$, virtual lagrangian can be found as:

$$\delta L = \int_V \left(\frac{\partial g}{\partial u} * \delta u + \frac{\partial g}{\partial \dot{u}} * \delta \dot{u} + \frac{\partial g}{\partial u'} \delta u' + \frac{\partial g}{\partial u''} \delta u'' + \frac{\partial g}{\partial \dot{u'}} \delta \dot{u'} \right) \tag{8}$$

Hamiltons principle suggests that:

$$\int_{t=0}^{t} \delta L dt = 0 \tag{9}$$

Our virtual lagrangian becomes after taking the integrals in y and z directions:

$$\delta L = \int_0^L (\rho * (\dot{w} + \dot{I}) * A * \delta \dot{w} + \rho * g * Q * \delta w' + E_x * (-I_{yz} * v'' - I_{zz} * w'') * \delta w'' - \rho * (-I_{yz} * \dot{v'} - I_{zz} * \dot{w'}) * \delta \dot{w}' \\ + \rho * (\dot{v} + \dot{J}) * A * \delta \dot{v} + \rho * g * Q * \delta v' + E_x * (-I_{yz} * v'' - I_{zz} * w'') * \delta v'' - \rho * (-I_{yz} * \dot{v'} - I_{zz} * \dot{v'}) * \delta \dot{v}') * dx \tag{10}$$

Taking the time integral and applying integration by parts, grouping terms under δv and δw, we obtain the following governing equations:

$$\begin{aligned}\rho * A * \frac{\partial^2 I}{\partial t^2} + \rho * A * \frac{\partial^2 w}{\partial t^2} + E_x * (I_{yz} * \frac{\partial^4 v}{\partial x^4} + I_{zz} * \frac{\partial^4 w}{\partial x^4}) - \rho * (I_{yz} * \frac{\partial^4 v}{\partial t^2 \partial x^2} + I_{zz} * \frac{\partial^4 w}{\partial t^2 \partial x^2}) = 0 \\ \rho * A * \frac{\partial^2 J}{\partial t^2} + \rho * A * \frac{\partial^2 v}{\partial t^2} + E_x * (I_{yz} * \frac{\partial^4 v}{\partial x^4} + I_{zz} * \frac{\partial^4 w}{\partial x^4}) - \rho * (I_{yz} * \frac{\partial^4 v}{\partial t^2 \partial x^2} + I_{zz} * \frac{\partial^4 w}{\partial t^2 \partial x^2}) = 0\end{aligned} \tag{11}$$

Subjected to the boundary conditions: $w(0,t) = 0, w'(0,t) = 0, w''(L,t) = 0, w'''(l,t) = 0$
$v(0,t) = 0, v'(0,t) = 0, v''(L,t) = 0, v'''(l,t) = 0$

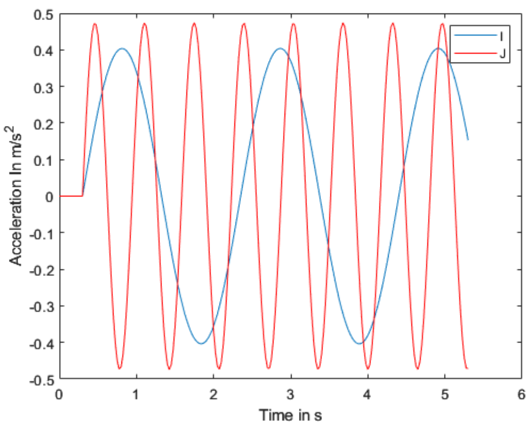
4 Physics Informed Neural Network Solution

To find solution to the governing equations, a physics informed neural network is employed. A simple fully connected network consisting of 3 deep layers with 32 neurons is trained for the problem. The network takes the motion of the base body as input and predicts the motion of the elastic rod. The governing differential equation and the boundary and initial conditions are given as the loss function. The network predicts the displacements from the centerline and calculates the derivatives with respect to time and x. The calculated derivatives and the displacements are then substituted into the differential equation and residual is calculated. The network takes the residual of the differential equation, the residual of the boundary and initial conditions then uses them as the loss function. As the network minimizes the loss function, the predicted displacements comes close to the correct values. With this network, any given base motion, the displacement fields can be calculated.

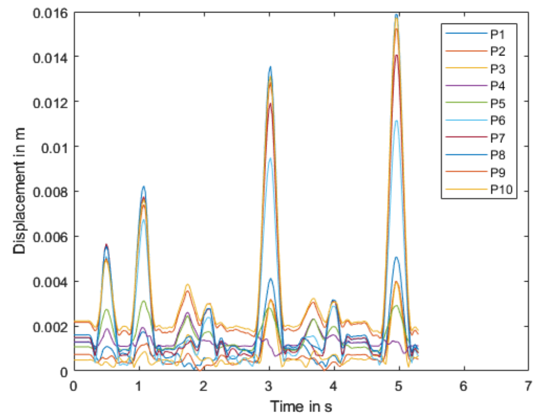
To train the network an example system is created with a TPU rod that has density of 1.26 g/cm^3, Young's modulus of 0.621 GPa, width and length of 5 mm and height of 1 meter. A dataset that contains 1536 random sinusoidal base motion with amplitudes varying between 0-1, angular velocities varying between 0-4π is created and separated into training, validation and test sets. The network is then trained with full batch training with ADAM optimizer. Matlab code can be accessed from https://github.com/ahmtkeles27/ME-571-Project.git. The results are given in the table below. The loss is calculated as the summation of boundary conditions, initial conditions and residuals of the differential equation.

Error	**Res1**	**Res2**	**BC1**	**BC2**	**IC1**	**IC2**	**Total**
Training	0.0058	0.0058	0.144	0.1094	0	0	0.2655
Validation	0.0057	0.0057	0.1419	0.1159	0	0	0.2692
Test	0.0061	0.0061	0.1505	0.1158	0	0	0.2796

An example case from the test set is given below where base motion is show in figure 2 a) and resulting displacement from the centerline calculated as $\sqrt{w^2+v^2}$ for 10 equally distanced points on the rod is given in figure 2 b).



(a)



(b)

Figure 2: a)Input Acceleration b)Displacement

5 Conculusion

In this project, I have modelled the dynamics of elastic beam with moving base and designed a deep neural network to find the solution from the system dynamics. The results of the deep neural network can be further improved with deeper network with more data and more training. A deep learning based controller can be designed based on the trained network at this project as a further work.

References

[1] S. Stoykov, P. Ribeiro Nonlinear forced vibrations and static deformations of 3D beams with rectangular cross section: The influence of warping, shear deformation and longitudinal displacements, *International Journal of Mechanical Sciences*, Volume 52, Issue 11, 2010, Pages 1505-1521, ISSN 0020-7403, https://doi.org/10.1016/j.ijmecsci.2010.06.011

[2] Sayyad, Atteshamuddin. (2011). Comparison of various refined beam theories for the bending and free vibration analysis of thick beams. *Applied and Computational Mechanics.* 5. 217-230.

[3] Pham, PT., Nguyen, Q.C., Yoon, M. et al. Vibration control of a nonlinear cantilever beam operating in the 3D space. Sci Rep 12, 13811 (2022). https://doi.org/10.1038/s41598-022-16973-y

[4] M. Raissi, P. Perdikaris, G.E. Karniadakis, Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations, Journal of Computational Physics, Volume 378, 2019, Pages 686-707, ISSN 0021-9991, https://doi.org/10.1016/j.jcp.2018.10.045.