

# Dynamic Programming:

<https://atcoder.jp/contests/dp/tasks>

$$f(n) = f(n-1) + f(n-2), \quad f(0) = 0, \quad f(1) = 1$$

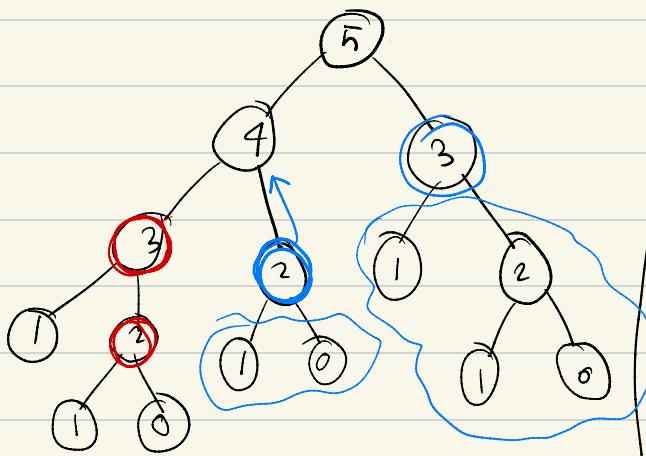
$f(n)$ :

if  $n \leq 1$ : return  $n$

else return  $f(n-1) + f(n-2)$

$$f(3) \approx$$

$$f(2) \approx$$



$$\rightarrow T(n) = T(n-1) + \overbrace{T(n-2)}^{} + 1$$

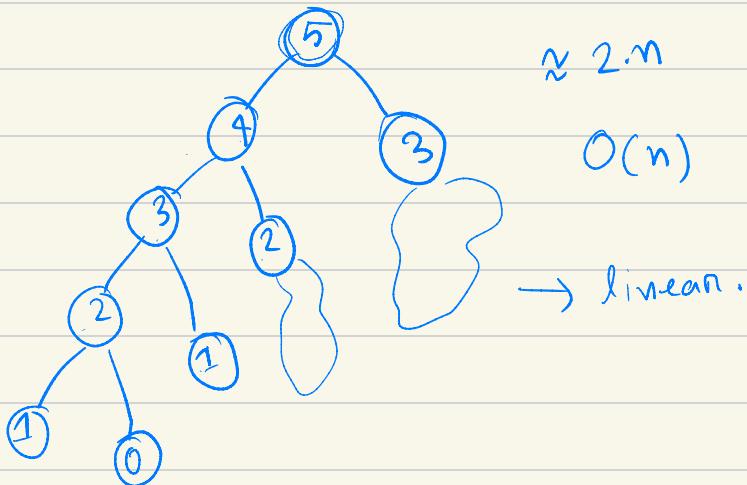
$$= T(n-1) + T(n-1) + 1$$

$$= 2T(n-1) + 1$$

$$= (2 \cdot T(n-2)) + (1 + 1 + 1 - \dots)$$

$$= 2^n + n$$

$$O(2^n) \rightarrow$$



$$\rightarrow f[0] = 0, f[1] = 1$$

$$f[i] = -1$$

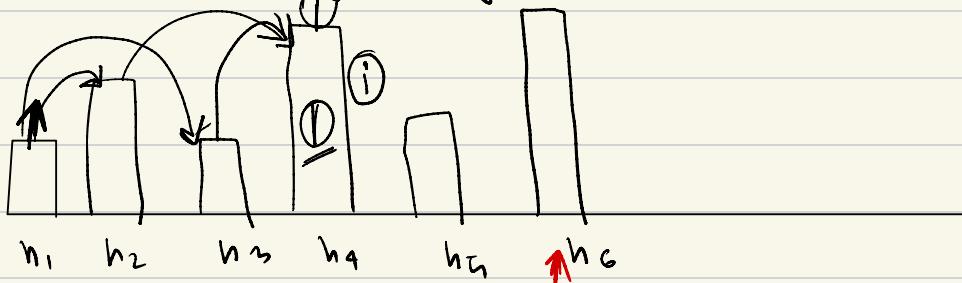
call( $n$ ):

if  $n \leq 1$ : return  $n$

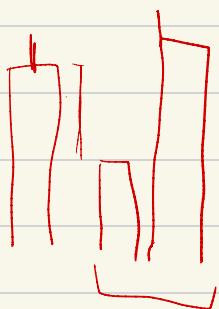
if  $f[n] \neq -1$ : return  $f[n]$

return  $f[n] = \text{call}(n-1) + \text{call}(n-2)$

# problem 1: jumping frogs:



$h_i \rightarrow h_{i+1} / h_{i+2}$  if you jump from  $h_i$  to  $h_j$   
then cost is  $|h_i - h_j|$



□ What is the base case?	$f(n)$
□ What is the recursion?	$f(1) = 0 \rightarrow$
→ $f(i)$	
$\min(f(i-1) +  h_i - h_{i-1} )$	
$f(i-2) +  h_i - h_{i-2} $	$1 \dots i-1$

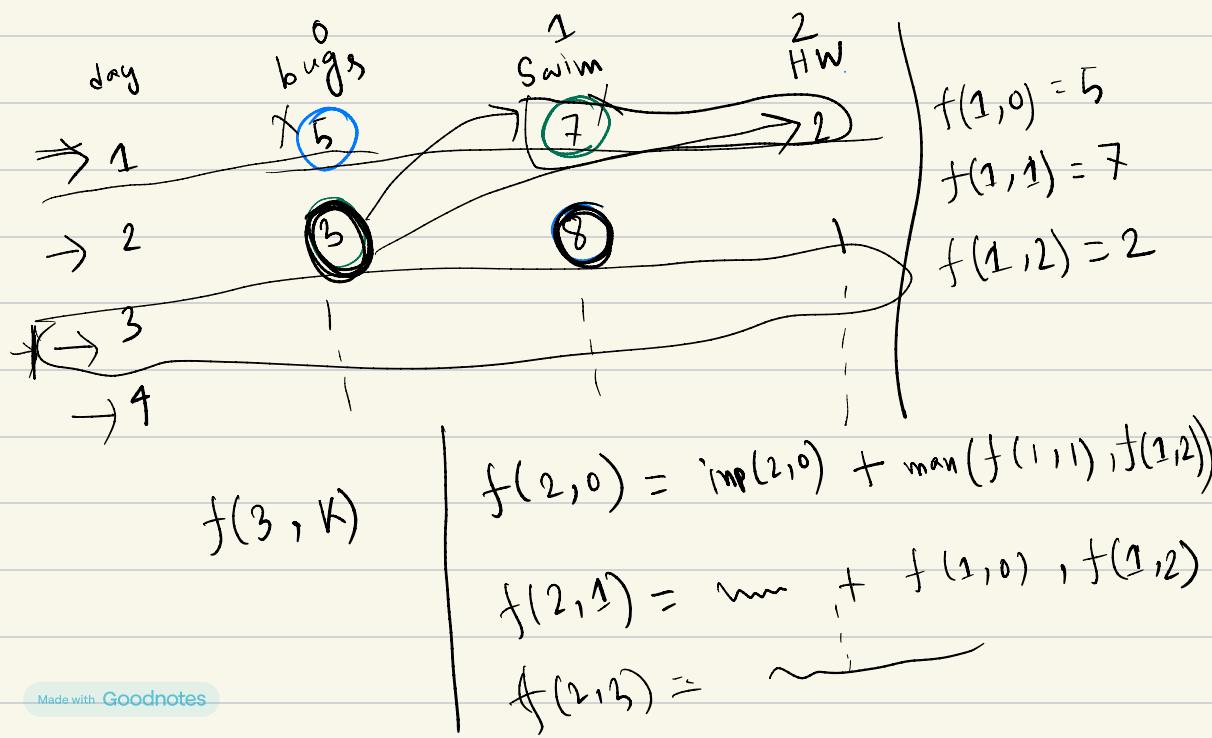
$$(n, m, k) \rightarrow (n', m', k')$$

$$dp[ ][ ][ ]$$

### problem 3: Vacation

n days of vacation:

$H \rightarrow c_1$	$c_2$	$c_3$			
$S \rightarrow a_1$	$a_2$	$a_3$	-	-	-
$b \rightarrow b_1$	$b_2$	$b_3$			
1	2	3	4	5	



$$\max (f(n,0), f(n,1), f(n,2))$$

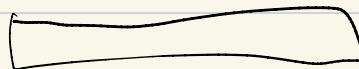
call ( $n, i$ ):

$$n = -1 : dp[n][i] = \underline{\underline{}}$$

if  $dp[n][i] \neq -1:$

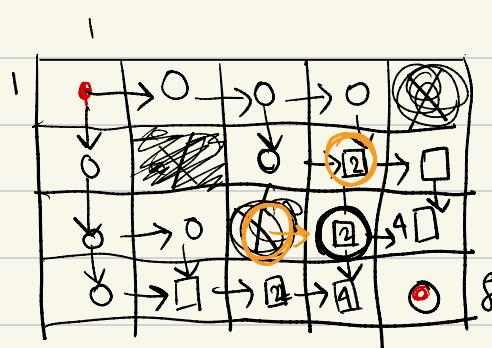
for  $j \rightarrow 0, 2$

$j = -1 :$



return  $\dots$

### Problem: Grid 1



$$(1,1) \rightarrow 1$$

$$(1,2) \rightarrow 1$$

$$dp(1,1) = 1$$

$dp(i,j) = dp(i-1,j) + dp(i,j-1)$

