Lecture 1:

- Complexity analysis (Time and space)
- Binary Search

Section 1: Complexity Analysis

A determines efficiency of given sol^n.

$$f(n) = \begin{cases} 1 & \text{if } n = 1 \\ n\, f(n-1) \end{cases} \Rightarrow \begin{aligned} f(5) &= 5 \times f(4) \\ &= 5 \times 4 \times f(3) \\ &\vdots \\ &= 5 \times 4 \times 3 \times 2 \times 1 = 5! \end{aligned}$$

$\boxed{O(\cancel{n^2})} \Rightarrow O(n)$

$T(n)$ = the number of iterations to calc $f(n)$

$$f(n) = n \overbrace{f(n-1)}^{P} \quad = nP$$

$$\boxed{T(n) = 1 + T(n-1)}$$

$$= 1 + 1 + T(n-2)$$

$$= 1 + 1 + 1 + T(n-3)$$

$$\vdots$$

$$= 1 + 1 + \ldots\ldots + 1 \text{ (n times)}$$

$$= n$$

$$T(n) = n \Downarrow O(n)$$

$$f(n) = \overset{\uparrow}{n}\, f(n-1) = O(n)$$

$1 +$

$O(n^2 + \underbrace{3n + 17}_{\Rightarrow n})$ $= O(n^2)$

1s $\Downarrow$ $O(n^2)$

$O(n^2 + n^3) = O(n^3)$

$$f(n) = \begin{cases} n & \text{if } n < 2 \\ \underbrace{f(n-1)}_{} + \underbrace{f(n-2)}_{} \end{cases}$$

$f(0) = 0,\ f(1) = 1$

$f(2) = f(1) + f(0)$

$\vdots$

$O(?)$ $(n+n)$ [off by 1]

$O(2^n)$

$O(2n)$
$\|$
$O(n) \approx O(n+n)$

$\hookrightarrow O(cn) = O(n)$

$T(n)$ - . . . . . .

$f(n) = \underbrace{f(n-1)}_{p} + \underbrace{f(n-2)}_{a}$

$= p + q$

$\Rightarrow$

$T(n) = 1 + T(n-1) + T(n-2)$

$= 1 + T(n-1) + T(n-1)$

$= 1 + 2T(n-1)$

$= 1 + 2(1 + T(n-2) + T(n-3))$

$= \cancel{1} + 2(\cancel{1} + 2T(n-2))$

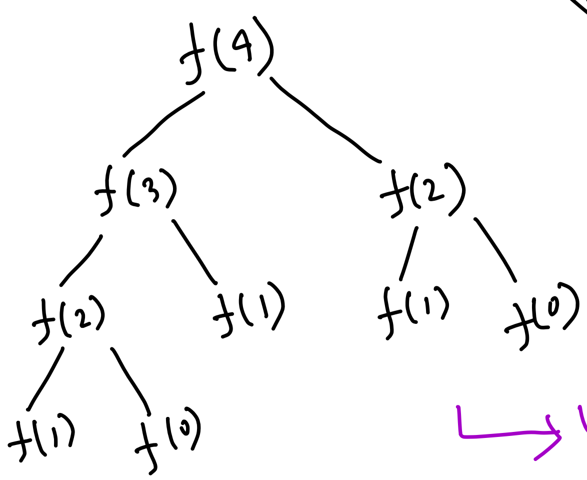
$= 2 \cdot 2 \cdot T(n-2)$

$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot \ldots 2$ (n times)

9

$\boxed{2^4 = 16}$

$O(2^n) \approx 2^n$



$\boxed{\text{kb mb}}$

↳ Upper bound

$O(n^2)$

$f(n) = f(n-1) + f(n-2) + f(n-3)$

$O(3^n)$

Binary Search:

1 2 3 7 8 9 . . .

$K = 8$

1 2 0 5 7 8 9 ~~4~~ $K = \cancel{8}\ 4$

$5 < 8$

$\log_2(n)$ $(n/2)/2 \ldots$

□ given x, find $\sqrt{x}$ without library functions

$x/x = 1$ $\boxed{\sqrt{x}}$ $\Rightarrow$ p s.t. $\boxed{p^2 = x}$

$\sqrt{p^2} = \sqrt{x}$

$p = \sqrt{x}$

$x = 100$

$l = 1,\ r = 100$

[1, 2, 3, 4, ... 10, 11, ... 100]

$m = (l+r)/2$

$m^2 = x$, return m 1 ..... 50 100

$m^2 > x \Rightarrow r = m - 1$

$m^2 < x \Rightarrow l = m + 1$

$6^2 = 36 < 100$

[1, 100] = 50

[1, 49] $m = 25$

[1, 24] $m = 12$

[1, 11] $m = 6$

[7, 11] $m = 8$

[9, 11] $m = 10$

[11, 11]

$p^2 = 100$

1 ..... 6 11

$p^2 < 100$

$10^2 = 100$

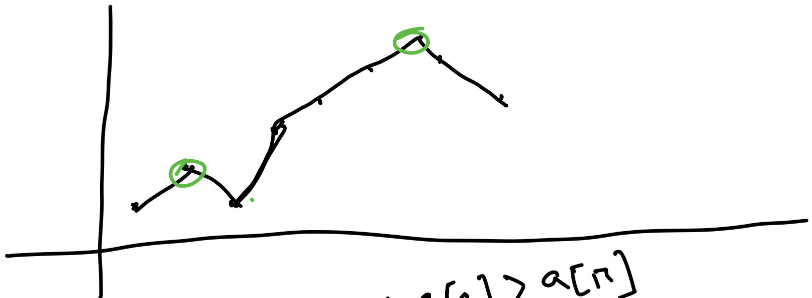
$x = 101$ $\boxed{p^2 \le 100}$

for the maximum p

10 $11^2 \Rightarrow 121$

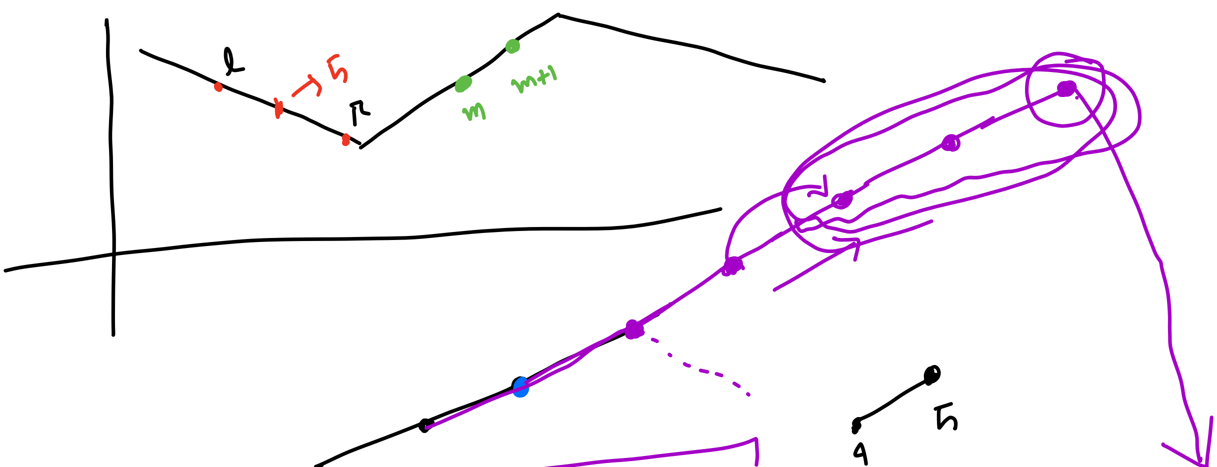
10 8

-1 | 1 2 1 3 4 7 9 8 5 | nth.
-∞ | | -∞



$a[l] < a[c] > a[r]$

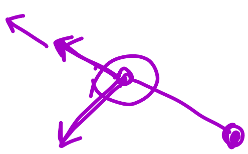
$a[m] \le \overbrace{a[m+1]}$



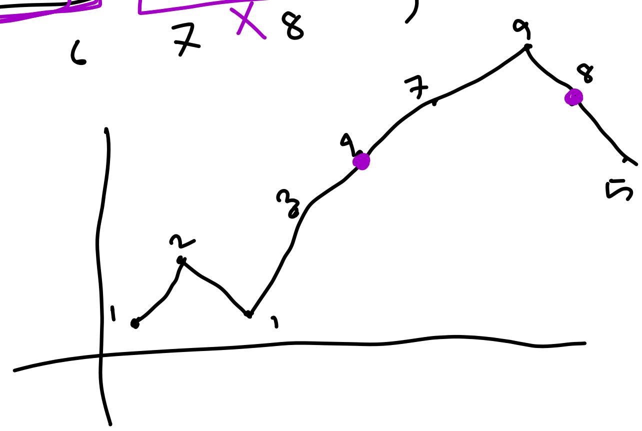
m-1 m < m+1

-∞ 1 2 1 3 4 7 9 8 5 -∞

-1 0 1 2 3 4 5 6 7 8 9



7 9



[a, b, c ... ___]   xyz

                    assume   10000

a summation