

Mathematics:

Permutation → Combinatorics
 → GCD → Number theory
 [Geometry], Modular Arithmetic

Number theory:

$$15, 20, 3 \times \rightarrow 10 \text{ based numbers.}$$

2 based → $\begin{matrix} \rightarrow \text{on} \\ \rightarrow \text{off} \end{matrix}$

$n = \boxed{5439}$
 $= 5 \cdot 10^3 + 4 \cdot 10^2 + 3 \cdot 10^1 + 9 \cdot 10^0$
 $= 5400 + 30 + 9$

2-based number

$$\boxed{101} \Rightarrow [1, 0, 1, 1]$$

↑↑↑↑

$$\sum_{i=0}^{k-1} a_i \cdot 2^i$$

$$\Rightarrow 1 + 4 + 8 = 13$$

$$(13)_{10} = (1101)_2$$

$$5 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$$

$$\Rightarrow \sum_{i=0}^{k-1} a_i \cdot 10^i$$

[9, 3, 4, 5]

$$a_0 \cdot 10^0 + a_1 \cdot 10^1 + a_2 \cdot 10^2 + a_3 \cdot 10^3$$

$$\Rightarrow 9 + 3 \times 10 + 4 \times 10^2 + 5 \times 10^3$$

$$= 5439$$

$$n = 17$$

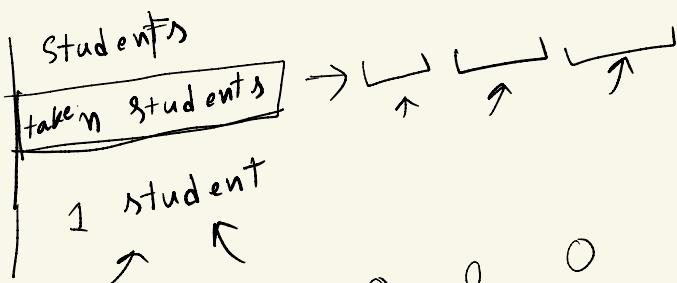
$$\begin{array}{r} 2 \mid 17 \\ 2 \mid 8 - 1 \\ 2 \mid 9 - 0 \\ 2 \mid 2 - 0 \\ 2 \mid 1 + 0 \\ 0 + 1 \end{array}$$

$$(10001)_2 = (17)_{10}$$

decimal \rightarrow binary

1. GCD, LCM

\downarrow
 35
 cookies
 21
 juices



take 7 students,

5	5	5	5	5	5	5
3	3	3	3	3	3	3

$$n | K \Rightarrow [K \% n = 0]$$

\hookrightarrow n divides K

$$K | 35, K | 21$$

\hookrightarrow K is a common divisor of 35 and 21

3	0	0	0	0

\rightarrow 35 has to be divisible by K

\rightarrow 21 has to - - - by K

GCD = greatest common divisor

$$48 = 4 \times 12$$

$$= 2 \times 2 \times 3 \times 2 \times 2$$

$$= 2^4 \times 3^1$$

→ prime factorization

Theorem: Any natural number $n > 1$ can be written
as a product of some prime numbers.

$$24 = 2 \times 12$$
$$= 2 \times 3 \times 4 = 2 \times 3 \times 2^2 = 2^3 \times 3$$

$$n_1 = p_0^{a_0} \cdot p_1^{a_1} \cdot p_2^{a_2} \cdots p_n^{a_n}$$

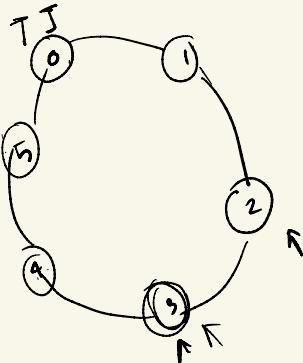
$$n_2 = p_0^{b_0} \cdot p_1^{b_1} \cdot p_2^{b_2} \cdots p_n^{b_n}$$
$$(n_1, n_2) = p_0^{\min\{a_0, b_0\}} \cdot p_1^{\min\{a_1, b_1\}} \cdots p_n^{\min\{a_n, b_n\}}$$

$$15 = 3^1 \times 5^1 = 2^0 \cdot 3^1 \cdot 5^1$$
$$12 = 2^2 \cdot 3^1 = 2^2 \cdot 3^1 \cdot 5^0$$
$$\text{GCD} \Rightarrow 2^0 \cdot 3^1 \cdot 5^0 = 3$$

L.C.M = Least common multiple

$$3, 5 \Rightarrow 15 \Rightarrow 3|15, 5|15$$

Tom, Jenny



K jumps
↑

4

2, 3

\Rightarrow 6 12 18, 24
↓
1/2, 3

2 | 9, 0

3 | 0, 3

9 | 2, 0

5 | 9, 3

6 | 0, 0

19, 6

Lcm: 42

$$19 = 2 \times 7 + 1 \quad 2 \times 3 = 6 \quad 2 \times 3 \times 7 = 42$$

$$6 = 2 \times 3 = 2 \times 3 \times 7 = 42$$

Lcm: $2^1 \cdot 3^1 \cdot 7^1$

$$n_1 = p_0^{a_0} \cdot p_1^{a_1} \cdots p_K^{a_K}$$

$$n_2 = p_0^{b_0} \cdot p_1^{b_1} \cdots p_K^{b_K}$$

$$\text{lcm} = p_0^{\max\{a_0, b_0\}} p_1^{\max\{a_1, b_1\}} \cdots p_K^{\max\{a_K, b_K\}}$$



$$\downarrow a, \downarrow b$$

$$c = \min(a, b)$$

$$d = \max(a, b)$$

$$a \times b = c \times d$$

$$\gcd = p_0^{\min\{a_0, b_0\}} p_1^{\min\{a_1, b_1\}} \cdots p_k^{\min\{a_k, b_k\}}$$

$$\text{lcm} = p_0^{\max\{a_0, b_0\}} p_1^{\max\{a_1, b_1\}} \cdots p_k^{\max\{a_k, b_k\}}$$

$$\gcd \times \text{lcm} = p_0^{(a_0+b_0)} \cdot p_1^{(a_1+b_1)} \cdots p_k^{(a_k+b_k)}$$

$$= n_1 \times n_2$$

$$n_1 = p_0^{a_0} p_1^{a_1} \cdots p_k^{a_k}$$

$$n_2 = p_0^{b_0} p_1^{b_1} \cdots p_k^{b_k}$$

$$= p_0^{a_0} p_0^{b_0} \cdot p_1^{a_1} p_1^{b_1} \cdots p_k^{a_k} p_k^{b_k}$$

$$= p_0^{a_0+a_1} \cdots p_k^{a_k} \cdot p_0^{b_0+b_1} \cdots p_k^{b_k}$$

$$= n_1 \cdot n_2$$

$\boxed{\gcd \cdot \boxed{\text{lcm}}} = n_1 \times n_2 \rightarrow \text{cool}$

$$\text{lcm} = \frac{n_1 \times n_2}{\gcd}$$

19, 6

$$\gcd = 2 \quad \text{lcm} = 42$$

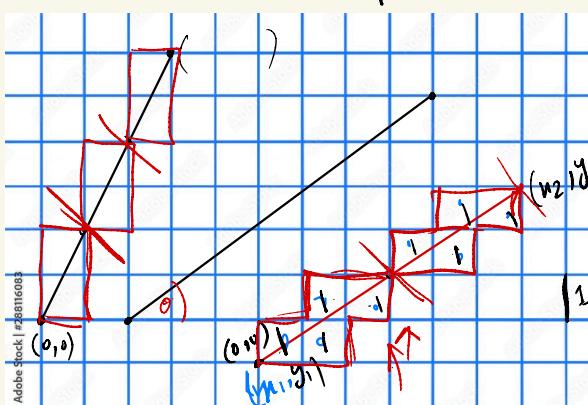
$$19 \times 6 = 8^4$$

$$\boxed{(1, 2)} = 2$$

$$42 \times 2 = 8^4$$

$$(3, 6) = \boxed{3}$$

$$(2, 4)$$



8 boxes

$$b+4 = 10 - 2$$

$$(6, 4) = \boxed{2}$$

$$(3, 2) = 4$$

$$\frac{(u_1 + y_1) - 1}{\gcd(g^{cd})} \left(\frac{\frac{u_2}{g^{cd}} + \frac{y_2}{g^{cd}} - 1}{\gcd(g^{cd})} \right)$$