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# Time and Space Complexity

## Time Complexity

Time complexity refers to the amount of time an algorithm takes to complete as a function of the input size. It helps us analyze the efficiency of an algorithm by estimating how the runtime increases as the input grows.

- **Big-O Notation** is used to express the upper bound of an algorithm's running time, describing the worst-case scenario.
- Common time complexities include:
  - **$O(1)$** : Constant time – the algorithm's runtime doesn't depend on input size.
  - **$O(n)$** : Linear time – the runtime grows directly with the input size.
  - **$O(\log n)$** : Logarithmic time – the runtime grows logarithmically with the input size.
  - **$O(n^2)$** : Quadratic time – the runtime grows quadratically, typically found in algorithms with nested loops.

## Space Complexity

Space complexity measures the amount of memory an algorithm needs to run, also as a function of input size. It's crucial for understanding the memory usage of algorithms, especially when working with large datasets or in memory-constrained environments.

- **Auxiliary Space**: The extra space or temporary space used by the algorithm, not including space used for input data.
- **Fixed vs. Variable Part**:
  - **Fixed Part**: Space used for constants, variables, and program code.
  - **Variable Part**: Space required for dynamic memory allocation, like recursion stacks or additional data structures.

### Key Concepts:

- **In-place Algorithms**: These algorithms use  $O(1)$  extra space, meaning they don't require additional memory for storing data structures.
- **Recursive Algorithms**: Recursive functions often have  $O(n)$  space complexity due to the function call stack.

## Why Time and Space Complexity Matter

Understanding time and space complexity is essential for:

- Writing **efficient algorithms** that scale well with increasing input sizes.
- Identifying and optimizing performance bottlenecks.

- Building systems that use resources (time and memory) wisely, especially for applications with large data or limited memory.

By analyzing both time and space complexity, you can make informed decisions on which algorithm to use for a particular problem based on the trade-off between speed and memory usage.