

# Linear Regression

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## 1 Theory

1. LR Supervised Training Dataset:

(a) Coefficients for Closed-form Linear Regression:

$$X = \begin{bmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{bmatrix}, Y = \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

$$w = \frac{1}{N}(X^T X)^{-1} X^T Y \quad (1)$$

$$w = \left( \begin{bmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{bmatrix}^T \begin{bmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{bmatrix}^T \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix} \quad (2)$$

$$w = \left( \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -5 & -3 & 0 & -8 & -2 & 1 & 5 & -1 & 6 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{bmatrix}^T \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix} \quad (3)$$

$$w = \left( \begin{bmatrix} 10 & -9 \\ -9 & 169 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -5 & -3 & 0 & -8 & -2 & 1 & 5 & -1 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix} \quad (4)$$

$$w = \left( \begin{bmatrix} 0.10503418 & 0.00559354 \\ 0.00559354 & 0.00621504 \end{bmatrix} \right) \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -5 & -3 & 0 & -8 & -2 & 1 & 5 & -1 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix} \quad (5)$$

$$w = \begin{bmatrix} 0.10503418 & 0.00559354 \\ 0.00559354 & 0.00621504 \end{bmatrix} \begin{bmatrix} 14 \\ -79 \end{bmatrix} \quad (6)$$

$$w = \begin{bmatrix} 1.02858919 \\ -0.41267868 \end{bmatrix} \quad (7)$$

(b) Predictions,  $Y$  , for the Training Data:

$$Y_{pred} = \begin{bmatrix} 1.8539465 \\ 3.0919826 \\ 2.2666252 \\ 1.0285892 \\ 4.3300186 \\ 1.8539466 \\ 0.6159105 \\ -1.0348042 \\ 1.4412679 \\ -1.4474829 \end{bmatrix}$$

(c) RMSE:

```

1      def rmse(predictions , targets):
2          return np.sqrt(np.mean((predictions-targets)**2))
3
4      print("RMSE: ", rmse(Ypred , Y))
5

```

**RMSE:** 3.701325917666

2. Function Weights:

(a) Partial Gradients:

$$\begin{aligned}
 J &= (x_1 w_1 - 5x_2 w_2 - 2)^2 = (x_1 w_1 - 5x_2 w_2 - 2)(x_1 w_1 - 5x_2 w_2 - 2) \\
 J &= x_1^2 w_1^2 - 5x_1 x_2 w_1 w_2 - 2x_1 w_1 - 5x_1 x_2 w_1 w_2 + 25x_2^2 w_2^2 + 10x_2 w_2 - 2x_1 w_1 + 10x_2 w_2 + 4 \\
 J &= x_1^2 w_1^2 - 10x_1 x_2 w_1 w_2 - 4x_1 w_1 + 25x_2^2 w_2^2 + 20x_2 w_2 + 4
 \end{aligned}$$

$$\frac{\partial J}{\partial w_1} = 2x_1^2 w_1 - 10x_1 x_2 w_2 - 4x_1 \quad (8)$$

$$\frac{\partial J}{\partial w_2} = -10x_1 x_2 w_1 + 50x_2^2 w_2 + 20x_2 \quad (9)$$

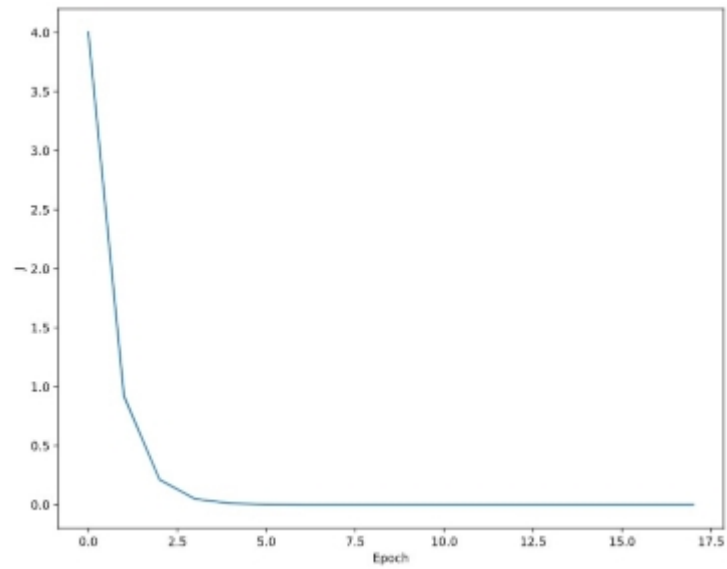
(b) Value of the Partial Gradients:

$$\begin{aligned}
 w_1 &= 0 & x_1 &= 1 \\
 w_2 &= 0 & x_2 &= 1
 \end{aligned}$$

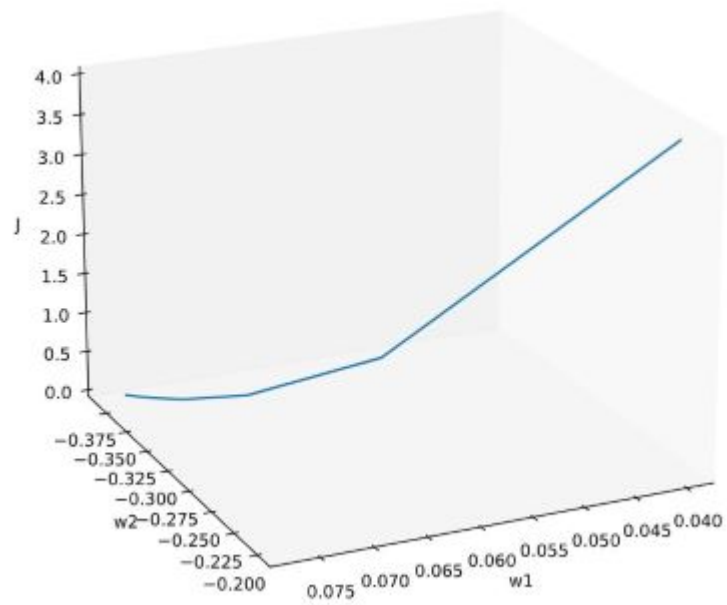
$$\begin{aligned}
 \frac{\partial J}{\partial w_1} &= 2 \times 0^2 \times 1 - 10 \times 1 \times 1 \times 0 - 4 \times 1 = -4 \\
 \frac{\partial J}{\partial w_2} &= -10 \times 1 \times 1 \times 0 + 50 \times 1^2 \times 0 + 20 \times 1 = 20
 \end{aligned}$$

## 2 Gradient Descent

1. Plot of epoch vs  $J$ :



2. 3D plot of  $w_1$  vs  $w_2$ , vs  $J$ :



3. Final learned values of  $w_1$ ,  $w_2$ , and  $J$ , and the number of epochs required to get there:

Converged at Epoch: 18

$$w_1 = 0.07692293618911292$$

$$w_2 = -0.3846146809455646$$

$$J = 5.8111496830407836^{-11}$$

### 3 Closed Form Linear Regression

1. RMSE (TRAINING - ENUM - NO BIAS): **151357.23969016664**  
RMSE (TESTING - ENUM - NO BIAS): **181121.86523476854**
2. RMSE (TRAINING - ENUM - BIAS): **5757.954440690525**  
RMSE (TESTING - ENUM - BIAS): **6519.373997851638**
3. RMSE (TRAINING - BIN - NO BIAS): **31707.42718070179**  
RMSE (TESTING - BIN - NO BIAS): **31434.144622561682**
4. RMSE (TRAINING - BIN - BIAS): **5763.9397804300825**  
RMSE (TESTING - BIN - BIAS): **6502.067257403235**