DKC³ 2015 Word Problems

Answer Key/Solutions

1. **Handshake**

   12

   Solution:

   In general, with n+1 people, the number of handshakes is the sum of the first n consecutive numbers: 1+2+3+ ... + n.
   Since this sum is n(n+1)/2, we need to solve the equation n(n+1)/2 = 66.
   This is the quadratic equation n2+ n -132 = 0. Solving for n, we obtain 11 as the answer and deduce that there were 12 people at the party.

   Since 66 is a relatively small number, you can also solve this problem with a hand calculator. Add 1 + 2 = + 3 = +... etc. until the total is 66. The last number that you entered (11) is n.

2. **Series**

   252

   Solution:

   The pattern that has been followed in the given series is:
   n^3 + n^2, where n = 1, 2, 3,.....
   1^3 + 1^2 = 1 + 1 = 2
   2^3 + 2^2 = 8 + 4 = 12
   3^3 + 3^2 = 27 + 9 = 36
   4^3 + 4^2 = 64 + 16 = 80
   5^3 + 5^2 = 125 + 25 = 150

   Following the same pattern, the last term is:
   6^3 + 6^2 = 216 + 36 = 252.

3. **Buckets**

3:0 5:5

3:3 5:2

3:0 5:2

3:2 5:0

3:2 5:5

3:3 5:4

Alternative Answer:

3:3 5:0

3:0 5:3

3:3 5:3

3:1 5:5

3:1 5:0

3:0 5:1

3:3 5:1

3:0 5:4

4. **Even Numbers**

4545

Solution:
Half of the 90 2-digit numbers are even and half of the 9000 4-digit numbers are even.
90 + 9000 = 9090 and 9090 / 2 = 4545.

5. **Cookies**

52

Solution:
The number of cookies must be a multiple of 3. It is easy to see that 13; 26; 39 are all impossible, but 52 = 2 * 21 + 10, so the answer is 52.

6. **Frodo**
   1 hour

7. **Plane**
   50% chance

   Solution:
   When the last passenger gets on seats 2 - 99 will be taken for sure. Each of these passengers would take their seats if they were available, so the only seat remaining at the end would be 1 or 100.

8. **School**
   They are equidistant to the school. They met in the same place.

9. **How Much Soil?**
   None, there is no dirt in a hole.

10. **Letter/Digit**
    A = 2
    B = 6
    C = 0
    D = 1
    E = 5
    F = 4
    G = 3

11. **Green House**
    House 1: Green, Lemonade, Swiss
    House 2: Red, Ice Tea, Danish
    House 3: Black, Water, Swede

12. **Plane Speed**
    2 hours at 105 mph and 3 hours at 115 mph

13. **6-digit Number**
    499849

14. **Acceleration**
    10 mph

    Solution:
    Since the given equation represents the speed (or velocity) of the car, we can find the acceleration curve by taking the derivative of the velocity. This leaves us with $a = -4t + 20$. Plugging in 10 for a, and solving for t gives us 2.5. So at 2.5 minutes into her trip, Mary was accelerating at exactly 10mph.

15. Find the Pattern

7	7	8	**13**	13	2	**14**	9	18
5	3	6	**7**	8	**1**	9	**7**	**9**
2	4	2	6	**5**	**1**	5	**2**	9
10	12	12	42	**40**	1	**45**	14	**81**

Solution:
The middle two rows in each column are the operands. The top row of each column is the sum of the operands; the bottom row is the product

16. Spousal Differences
Man 52 and Wife 39

17. One-Digit Number
4

18. Distance
9.22

19. Squeal Server
198 pigs

20. Packet Loss
1,971 Packets

Solution:
100 cups * 365 days = 36,500 cups * .6 = 21,900 cups with cream * .18 = 3,942 people who might steal a creamer packet * .5 = 1,971 Packets

21. The Universe and Everything
-7

22. Soccer Teams
15 cars in all

23. A Box
2/3

24. Albert and Isaac

2, 6, and 6

Solution:

If the product of his three children's ages is 72, there are the following possibilities:

```
1 *  1 * 72 = 72
1 *  2 * 36 = 72
1 *  3 * 24 = 72
1 *  4 * 18 = 72
1 *  6 * 12 = 72
1 *  8 *  9 = 72
2 *  2 * 18 = 72
2 *  3 * 12 = 72
2 *  4 *  9 = 72
2 *  6 *  6 = 72
3 *  3 *  8 = 72
3 *  4 *  6 = 72
```

Isaac later gives Albert the sum of their ages, but we don't know what number he says. We do, however, know that Albert can't figure it out from that information. So, we take the possibilities listed above and add them up:

```
1 +  1 + 72 = 74
1 +  2 + 36 = 39
1 +  3 + 24 = 28
1 +  4 + 18 = 23
1 +  6 + 12 = 19
1 +  8 +  9 = 18
2 +  2 + 18 = 22
2 +  3 + 12 = 17
2 +  4 +  9 = 15
2 +  6 +  6 = 14
3 +  3 +  8 = 14
3 +  4 +  6 = 13
```

The only way Albert wouldn't be able to figure out Isaac children's ages by knowing the sum is if the sum was 14, because there are two possibilities. So either the children's ages are 2, 6, and 6, or 3, 3, and 8. But Isaac points out that he has a youngest child. So the ages must be 2, 6, and 6.

25. Three Numbers

50, 94, 149