DKC^3 2011 WORD PROBLEMS

1) Painted Cube **5 Points**

A solid, four-inch cube of wood is coated with blue paint on all six sides. Then the cube is cut into smaller one-inch cubes. These new one-inch cubes will either have three blue sides, two blue sides, one blue side, or no blue sides. How many of each will there be?

Answer: There are 24 with one side colored, 8 with three sides colored, 24 with two sides colored, and 8 with no sides colored.

2) Gears **5 Points**

There are five gears connected in a row, the first one is connected to the second one, the second one is connected to the third one, and so on. If the first gear is rotating clockwise what direction is the fifth gear turning?

Answer: Clockwise**.**

3) Rock Band **5 Points**

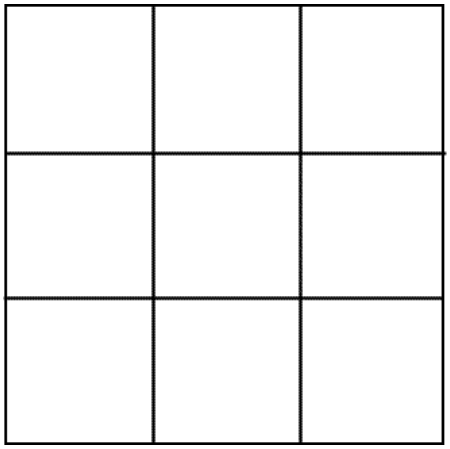
At the tryouts for the TV show “American Rock Band”, there are 80 guitar players, 32 keyboardists, 64 drummers, 112 bassists, and 48 singers. The show assigns the contestants to equal sized groups. Each group has an equivalent number of singers, and players of each instrument.

What is the largest number of groups possible? (no one is left without a group)

Answer: 16 (greatest common denominator).

4) Squares **5 Points**

How many squares can you find in the figure below?



Answer: 14

5) Diophantus **5 Points**

We know very little about the life of the mathematician Diophantus (often known as the 'father of algebra') except that he came from Alexandria and he lived around the year 250 AD.

However, there remains a riddle that describes the spans of Diophantus's life:

"This tomb hold Diophantus. Ah, what a marvel! And the tomb tells scientifically the measure of his life. God vouchsafed that he should be a boy for the sixth part of his life; when a twelfth was added, his cheeks acquired a beard; He kindled for him the light of marriage after a seventh, and in the fifth year after his marriage He granted him a son. Alas! late-begotten and miserable child, when he had reached the measure of half his father's life, the chill grave took him. After consoling his grief by this science of numbers for four years, he reached the end of his life."

In simpler English it says: Diophantus's youth lasted 1/6 of his life. He had the first beard in the next 1/12 of his life. At the end of the following 1/7 of his life Diophantus got married. Five years from then his son was born. His son lived exactly 1/2 of Diophantus's life. Diophantus died 4 years after the death of his son.

How long did Diophantus live?

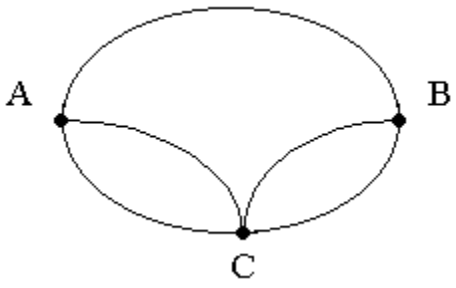
Answer: There is an equation to reflect the several ages of Diophantus:

$1/6x + 1/12x + 1/7x + 5 + 1/2x + 4 = x$

So the solution (x) is 84 years

6) Town Travel **5 Points**

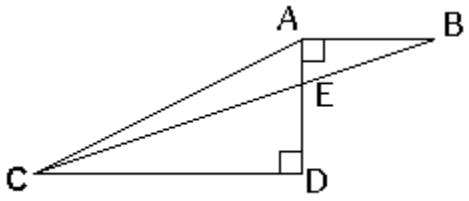
Amy has to visit towns B and C in any order. The roads connecting these towns with her home are shown on the diagram. How many different routes can she take starting from A and returning to A, going through both B and C (but not more than once through each) and not travelling any road twice on the same trip?



Answer: 8

7) Triangle Area **5 Points**

In the figure below, AD = 4, AB = 3 and CD = 9. What is the area of triangle AEC?



Answer: 4.5

8) Buses **5 Points**

Buses 1, 2, and 3 make one trip each day, and they are the only ones that riders A, B, C, D, E, F, and G take to work.

Neither E nor G takes bus 1 on a day when B does.
G does not take bus 2 on a day when D does.
When A and F take the same bus, it is always bus 3.
C always takes bus 3.

Traveling together to work, B, C, and G could take which of the same buses on a given day?

Answer: 3 Only.

9) Parking Lot **5 Points**

In a parking lot, there are 100 vehicles, 60 of which are cars, 30 are vans, and the remainder are trucks. If every vehicle is equally likely to leave, find the probability of a car leaving second after a van or truck had left first.

Answer: 60/99

10) Dice **5 Points**

Two dice are rolled. What is the probability that the sum is equal to 4?

Answer: 1/12

11) Acoustic Tile Holes **5 Points**

Jim was bored at work, and while he was staring at the ceiling started to wonder how many holes are in the acoustic ceiling tile throughout his square office building. He estimates that there are 23 holes in a random square inch of tile. He knows that each 24" x 48" tile is surrounded by one inch wide suspended rails. Every fourth row (narrow way) has a light fixture the same size as a tile and within a row there are two tiles (long way) between each light, with the first light starting in the second tile slot. He has also seen his company advertise that they have 10,000 sq. ft. of warehouse space, which he knows takes up the entire bottom floor (without a suspended ceiling) of their 4 floor building. Assuming the other floors are laid out the same as his, filled end to end with cubicles, how many holes are there?

Answer: 86,242,962 holes

To calculate:

- The ceiling is 100' x 100'. That means 1200" x 1200".
- The long way (48" side) I figure we'll need 24 tiles plus an extra 22" on the last tile. I figure this by 1" + (48" + 1")x24 + 22" + 1" = 1200"
- The short way (24" side) I figure we'll need 47 tiles plus an extra 23" on the last tile. I figure this by 1" + (24" + 1")x47 + 23" + 1" = 1200"
- So we'll need 1128 (24x47) full size tiles if you don't count light fixtures.
- I figure we'll need 88 light fixtures.
    - Since you need one fixture every 4 rows that means 11 rows will need lights if you measure the short way. Fixtures in rows 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44.
    - Each row with lights contains 8 of them. The lights start in column 2. Each light is separated by 2 tiles. Fixtures in columns 2, 5, 8, 11, 14, 17, 20, 23.
- That means we'll really only need 1128 – 88 = 1040 full size tiles. Area of full size tiles needed = 1040 x (24" x 48") = 1198080 sq. inches
- Now on to the partial tiles…
    - We'll need 24 tiles that are 23" x 48". Total surface area here is 26496 sq. inches.
    - We'll need 47 tiles that are 24" x 22". Total surface area here is 24816 sq. inches.
    - We'll need one corner tile that is 23" x 22". Total surface area of this is 506 sq. inches.
- Total surface area of tiles is 1198080 + 26496 + 24816 + 506 = 1249898 sq. inches.
- Total number of holes per level is 1249898 sq. inches/floor x 23 holes/sq. inch = 28747654 holes/floor
- **Total number of holes in the building is 28747654 holes/floor x 3 floors = 86,242,962 holes.**

12) Wig Maker **5 Points**

Jim, having lost his job at the office, has decided to get in to the wig making business. His sister Joann runs a beauty parlor, so she will supply him with cut hair. She said she will charge him $15 per individual that she takes the hair from. He plans to sell three varieties – a full wig for $95, a 2/3 partial for $70, and a ¼ patch for $35. He knows that the average human head has 100,000 hairs. Assuming he gets all the cut hairs from an individual, and is able to make use of ½ of them (due to size and quality), how much money will he make on the pending order of three fulls, a partial, and two patches?

Answer: $290

13) Cereal Counter **5 Points**

Sitting at the breakfast table, trying to come up with his next great scheme, Jim becomes distracted by how many pieces of cereal are left in his favorite cereal box. He counts out 462 full pieces in a single serving. The box tells him there are 12 servings per box. However, Jim knows that the last serving always contains all of the broken pieces in the bottom of the box. Half of the last serving consists of cereal split into 2 pieces and the rest of the serving consists of cereal split into 3 pieces. Jim is currently eating his third serving of the box, so how many pieces (full and broken combined) are left in the box.

Answer: 4851 full and broken pieces remain in the box.

14) Steps to the Mall **5 Points**

Jim has noticed that when he walks he travels 3 feet with each step with his right foot. Curious about this, he notices that he only travels 2.5 feet with each step with his left foot. According to the online map, he notices it is 1.56 miles to the mall. He plans to walk there. Assuming he walks normal, alternating each foot, how many steps will he take and if he starts with his right foot, which foot will he end with?

Answer: 2996 steps ending with his left foot. Note that the last step is only .3 feet.

15) Stepping on the Lines **5 Points**

While he was walking, Jim noticed that the lines in the concrete were spaced 4 feet apart. Considering his abnormal gait – he travels 3 feet with each step of the right foot and 2.5 feet with each step of the left foot, he started to wonder how many lines he would step on in an average block. Assuming he starts with the middle of his foot on a line at the beginning of the block, and always steps off first with his right foot, how many additional lines will he step on with the middle of his foot before coming to the end of the 900 foot block?

Answer: 40 lines

16) Confused Bank Teller **5 Points**

A confused bank teller transposed the dollars and cents when he cashed a check for Ms. Smith, giving her dollars instead of cents and cents instead of dollars. After buying a newspaper for 50 cents, Ms. Smith noticed that she had left exactly three times as much as the original check. What was the amount of the check?

Answer: The check was for $18.56.

17) 1000 Divisors **5 Points**

Find the smallest natural number greater than 1 billion that has exactly 1000 positive divisors. (The term divisor includes 1 and the number itself. So, for example, 9 has three positive divisors.)

Answer: The smallest natural number greater than 1 billion that has exactly 1000 positive divisors is 1,060,290,000.

18) Car Journey **5 Points**

A car travels downhill at 72 mph (miles per hour), on the level at 63 mph, and uphill at only 56 mph. The car takes 4 hours to travel from town A to town B. The return trip takes 4 hours and 40 minutes. Find the distance between the two towns.

Answer: The distance between the two towns is 273 miles.

19) Horse Race **5 Points**

In how many ways, counting ties, can eight horses cross the finishing line?
(For example, two horses, A and B, can finish in three ways: A wins, B wins, A and B tie.)

Answer: Counting ties, eight horses can cross the finishing line in 545835 ways.

20) Basketball **5 Points**

Yolanda scored 10 points in a basketball game. She could have scored with one-point free throws, two-point field goals, or three-point field goals. In how many different ways could she have scored her 10 points?

Answer: 14 ways.

21) Apartment Numbers **5 Points**

Ronnie's apartment number plus the square of Harry's apartment number equal 2240, which happens to be the room number of Harry's girlfriend. Harry's apartment plus the square of Ronnie's apartment equal 1008, which happens to be the dorm number of Ronnie's boyfriend. What are the apartment numbers of Ronnie and Harry?

Answer: Ronnie: 31, Harry: 47.

22) Checking Accounts **5 Points**

Javier received a letter from his bank concerning his checking account. Under his current plan, each check he writes costs 15 cents and be pays a monthly fee of $1.60. Under the new plan, each check he writes will cost 12 cents and there will be a monthly fee of $2.75. What is the minimum amount of checks Javier must write monthly so that the new plan will cost him less than the current plan?

Answer: 39 checks.

23) Clocks **5 Points**

Imagine a digital clock. How many times will the clock display three or four of the same number in a row over the course of one day? Assume that the clock is on a 12 hour scale, not on military time.

Answer: 34

24) Farmer Bob **5 Points**

Land values and crop prices are up and Bob is trying to decide whether it would be worth it to improve his land by installing drain tile. It will cost Bob $230,000 to hire a contractor to tile his best section (1 section is 640 acres). Bob estimates he will increase his average corn yield by 30 bushels per acre and boost his soybean yield by 6 bushels per acre with the tile. His current average yield is 98 bushels per acre for corn and 32 bushels per acre for soybeans. Assuming a bushel of corn is worth $4.50, a bushel of soybeans is worth $8.00, and Bob switches crops every year starting with corn next year, how many years will it take him to pay for the tile?

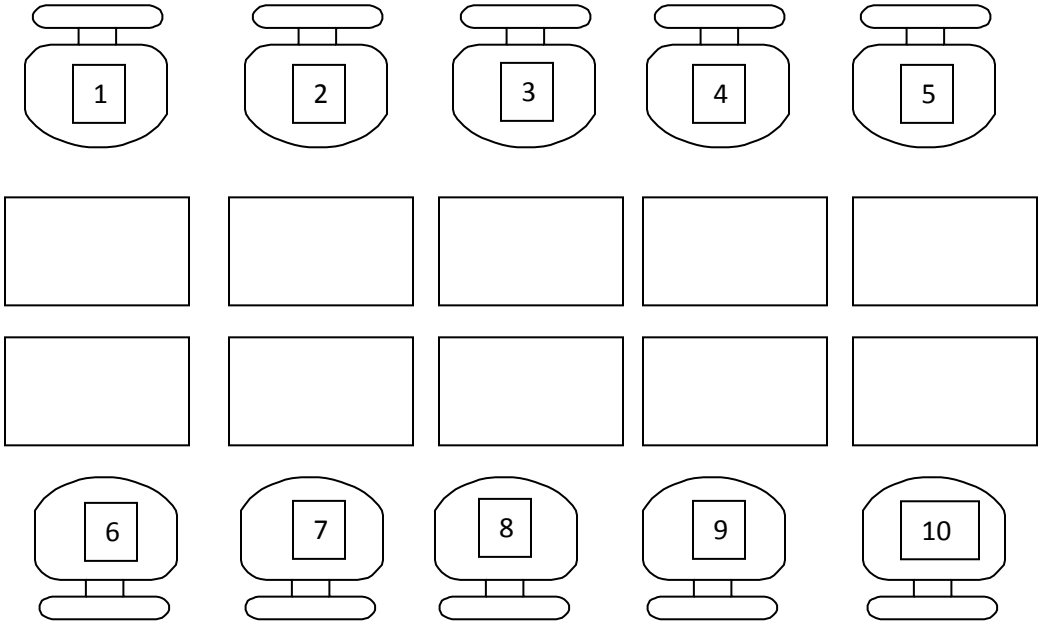
Answer: 4 Years

We are only interested in the extra money made by adding the tile. When corn is planted, this should be 30 bushels per acre * 640 acres * $4.50 per bushel = $86,400. When soybeans are planted, this should be 6 bushels per acre * 640 acres * $8.00 per bushel = $30,720. We then just add these up until the total exceeds the cost of the tile:

Year	Crop	Added Value	Total
1	Corn	$86,400	$86,400
2	Soybeans	$30,720	$117,120
3	Corn	$86,400	$203,520
4	Soybeans	$30,720	$234,240 > $230,000

25) Sitting Pretty **5 Points**

At Xavier's School for Gifted Youngsters the boys sit on chairs numbered 1-5 and the girls sit opposite them on chairs numbered 6-10. Given the clues below, list who sits on which chair.



1. The girl sitting next to the girl opposite no. 1 is Jean.
2. Jean is three desks away from Storm.
3. Shadowcat is opposite Cyclops.
4. Iceman is opposite the girl next to Shadowcat.
5. If Cyclops is not central than Angel is.
6. Beast is next to Banshee.
7. Banshee is three desks away from Cyclops.
8. If Jean is not central then Rogue is.
9. Shadowcat is three desks away from Emma.
10. Beast is opposite Storm.
11. The girl sitting next to the girl opposite Angel is Emma.
12. Cyclops is not at desk no. 5.
13. Emma is not at desk no. 10.

Answer:

1. Cyclops
2. Iceman
3. Angel
4. Banshee
5. Beast
6. Shadowcat
7. Jean
8. Rogue
9. Emma
10. Storm