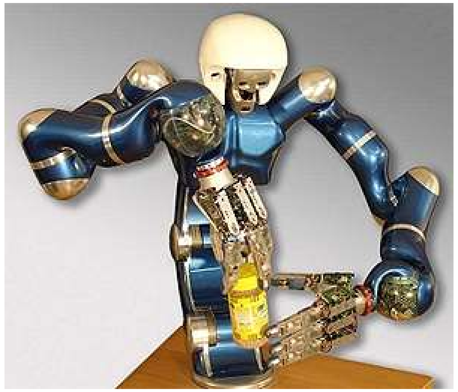
Control System Design for Automated Driving

Lecture 07

Simulink Model of Vehicle Lateral Dynamics

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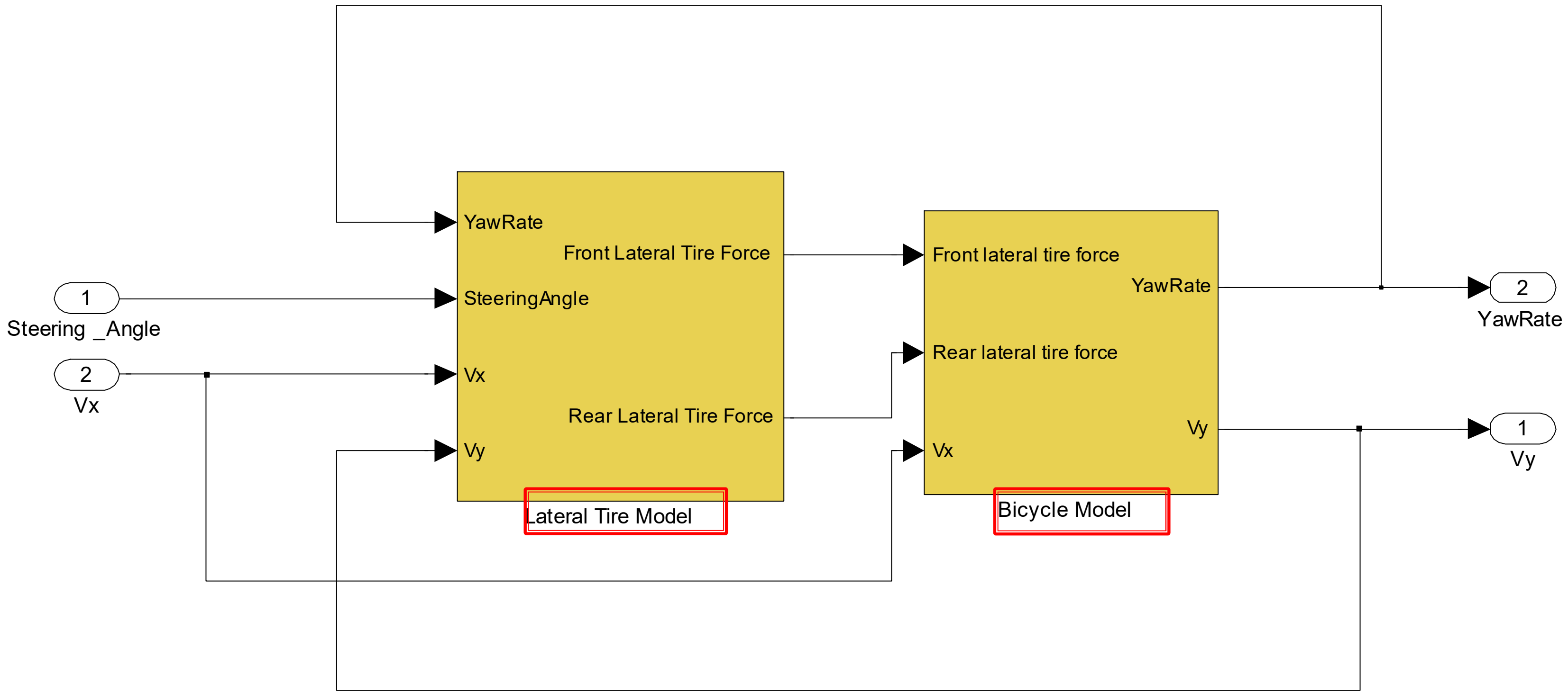
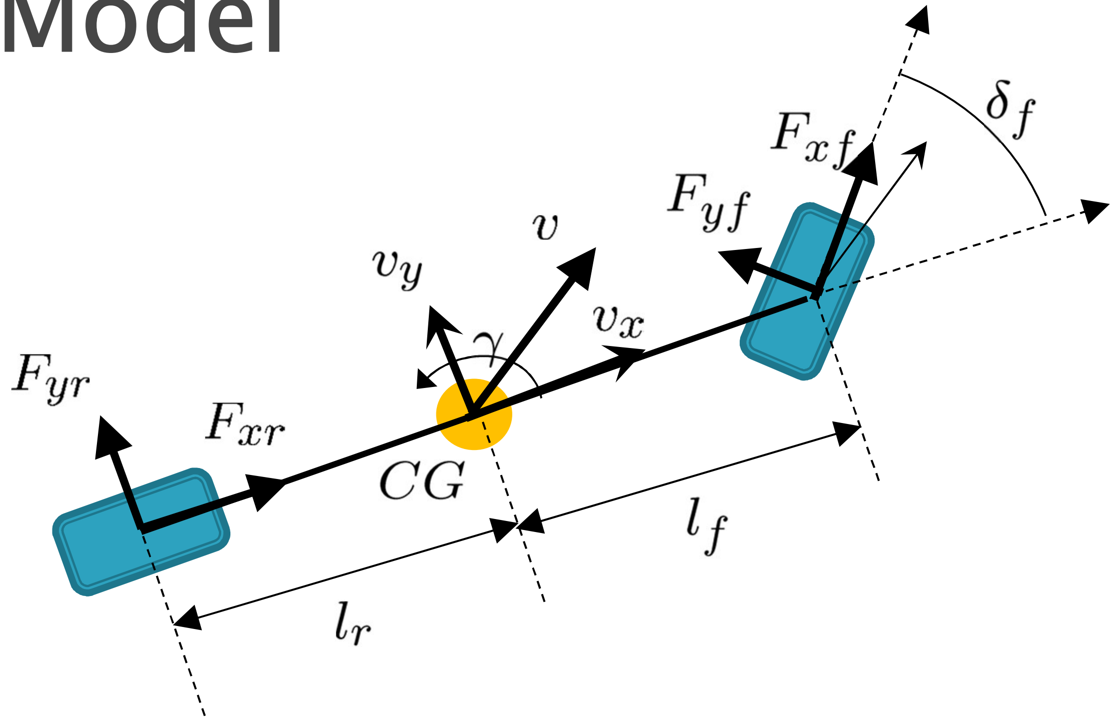


Lateral Vehicle Model

- Bicycle Model
  - δ_f Steering Angle
  - γ Yaw Rate
  - v_y Lateral Velocity



- Equations of motion (with assumption steering angle $<< 1$, $v_x =$ constant, symmetric about x axis)

$$m\dot{v}_y = F_{yf} + F_{yr} - m\frac{v_x^2}{r} \Rightarrow \dot{v}_y = \frac{F_{yf}+F_{yr}}{m} - v_x\gamma$$

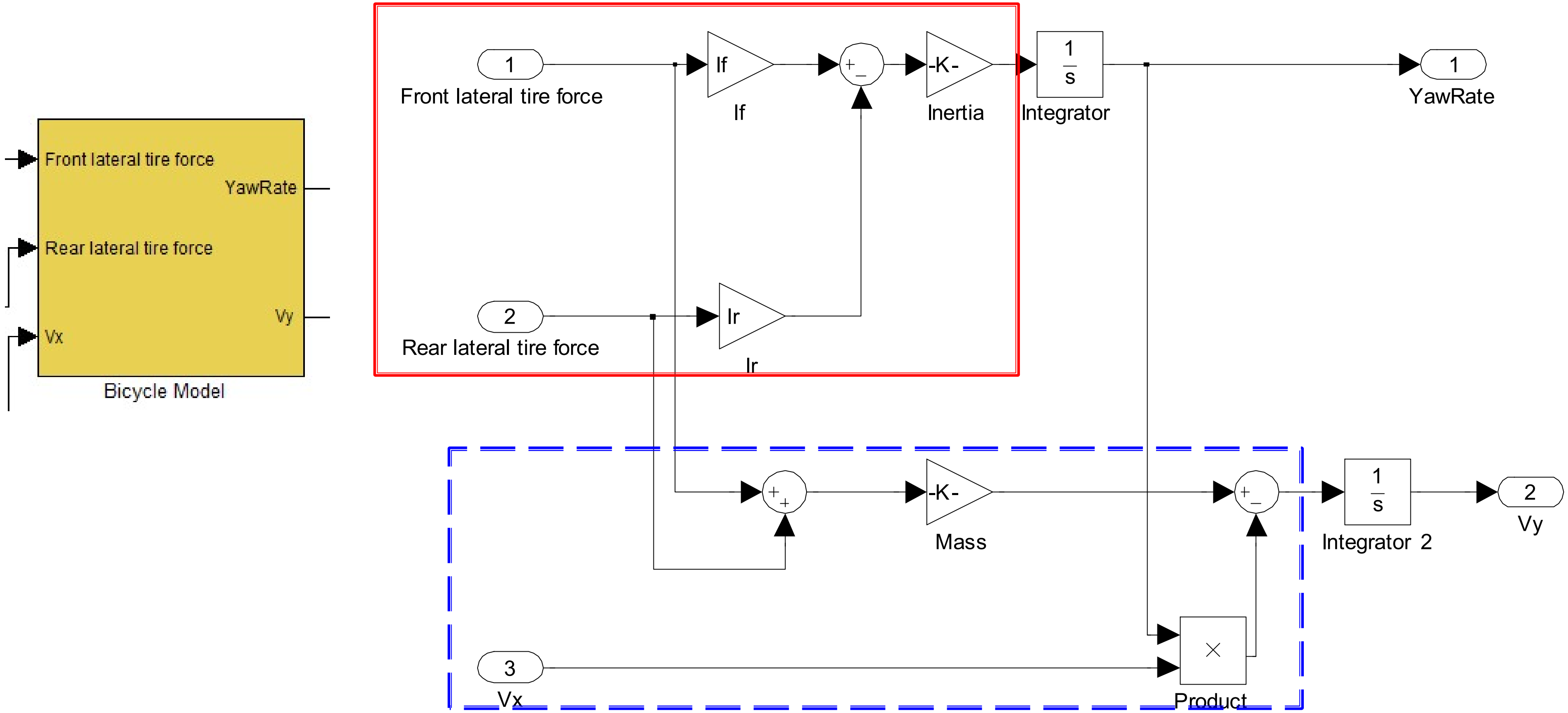
$$\dot{\gamma} = \frac{1}{I_{zz}}\left(l_f F_{yf} - l_r F_{yr}\right)$$

Bicycle Model

- Equations of motion (with assumption steering angle << 1)

$$\dot{v}_y = \frac{1}{m}(F_{yf} + F_{yr}) - v_x \gamma,$$

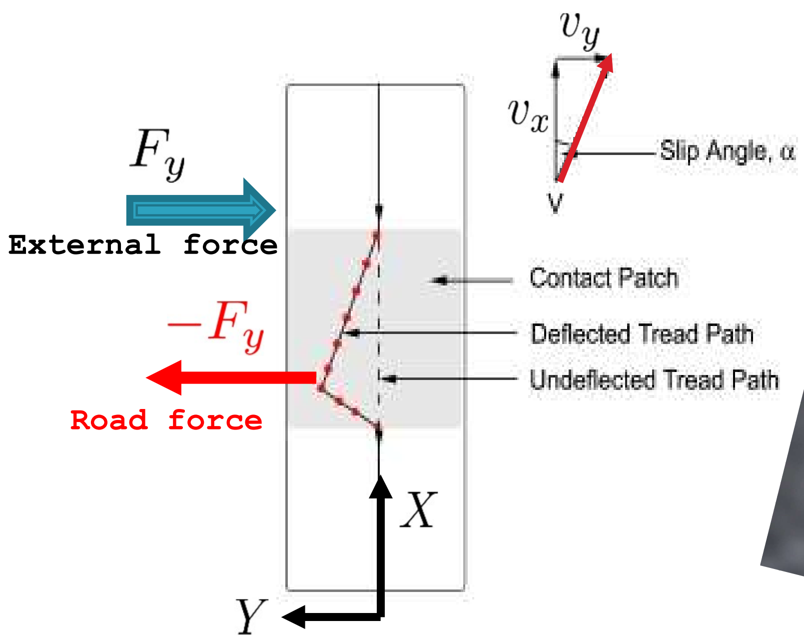
$$\dot{\gamma} = \frac{1}{I_{zz}}(l_f F_{yf} - l_r F_{yr})$$



Tire Model

- Tire Road Force F_{road} is usually modeled by empirical relationship between slip and force.

- Definition of Lateral Tire Slip (side slip angle) $\alpha = \tan^{-1}(\frac{v_y}{v_x})$

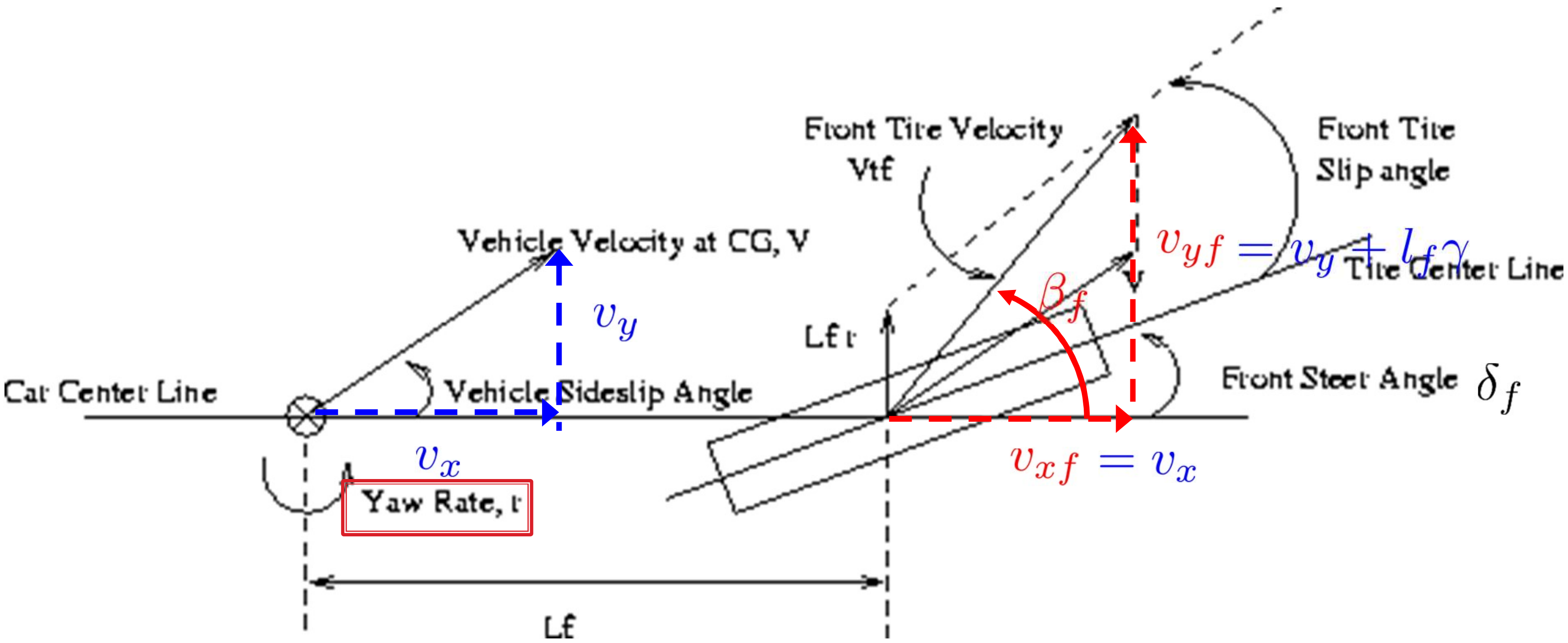


Tire Model

- Lateral Slip Angle of Tires

$$\alpha_f = \beta_f - \delta_f = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) - \delta_f, \qquad \alpha_r = \tan^{-1}\left(\frac{v_y - l_r\gamma}{v_x}\right)$$

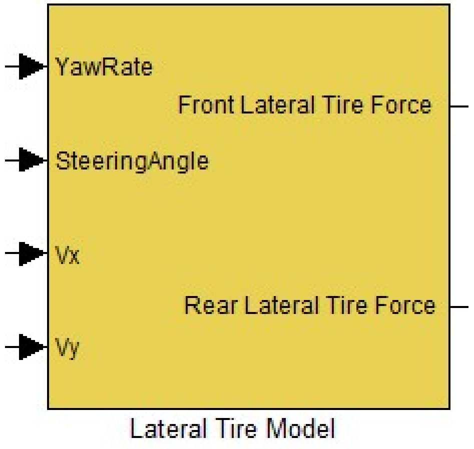
$$= \tan^{-1}\left(\frac{v_y + l_f\gamma}{v_x}\right) - \delta_f$$

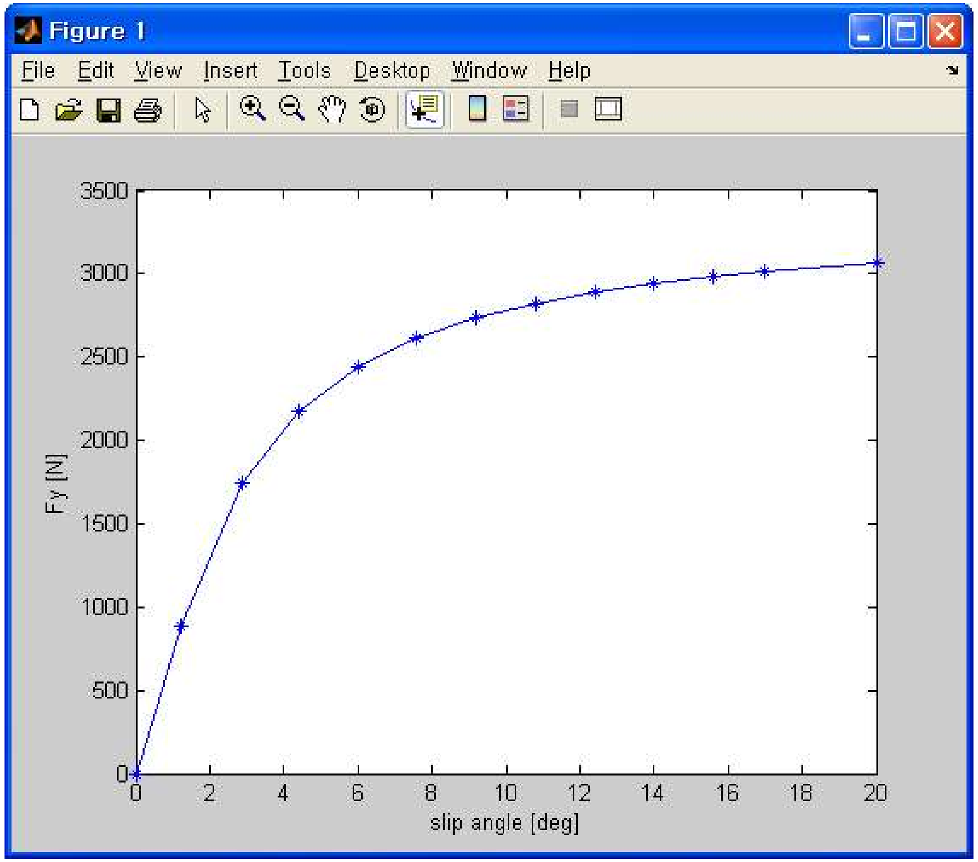


Lateral Tire Force

- Simple Saturation Tire Model

$$F_{tire} = C_{\alpha} \frac{\mu}{K} \tan^{-1}\left(\frac{K}{\mu} \alpha \right)$$

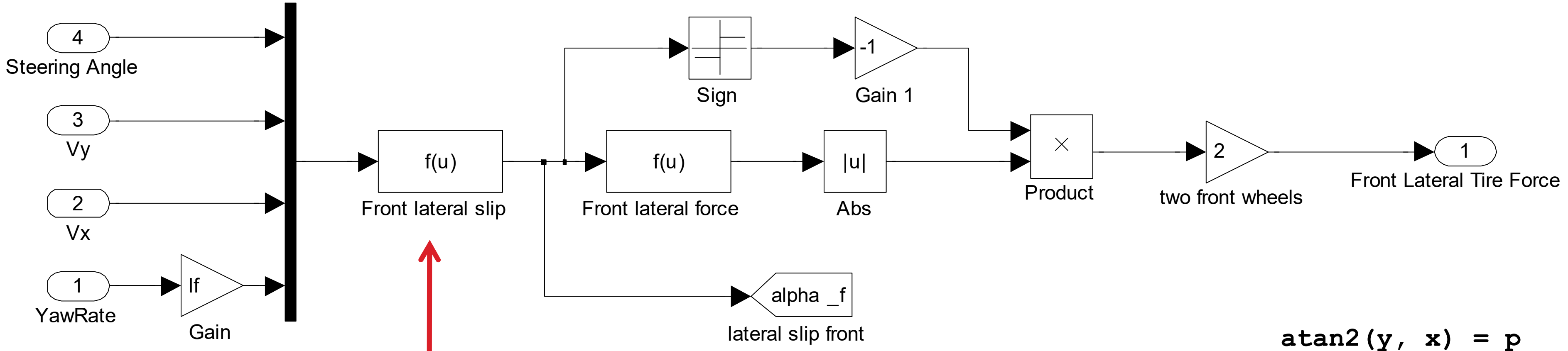


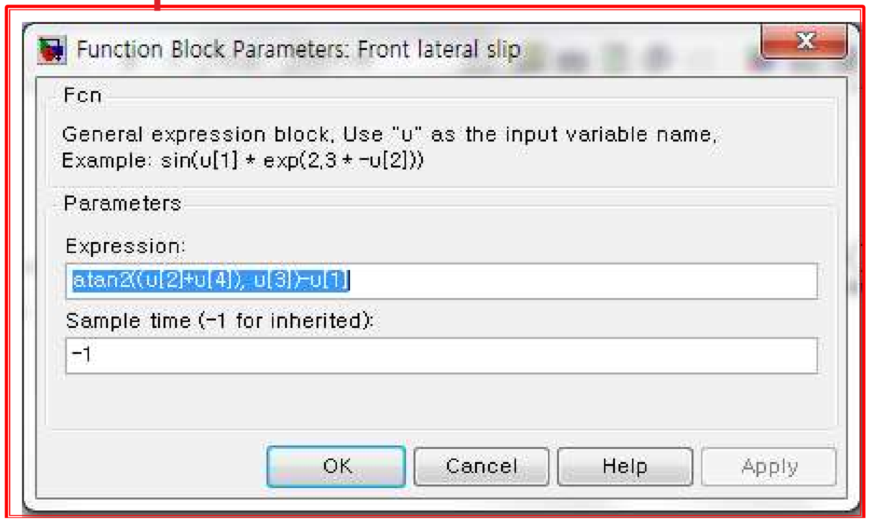


Tire Lateral Slip Angle Calculation

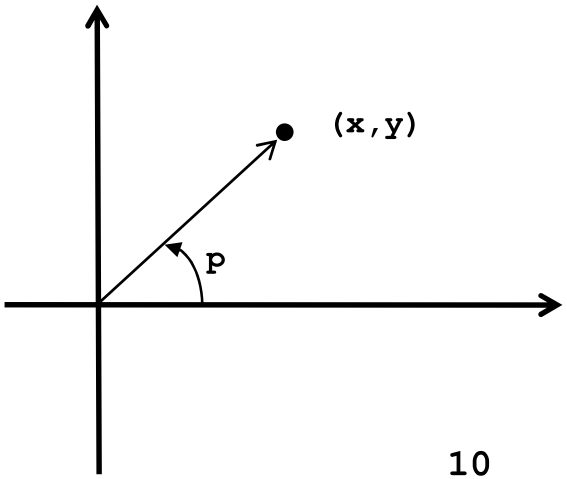
- Front Lateral Tire Slip Angle

$$\alpha_f = \tan^{-1}\left(\frac{v_y + l_f \gamma}{v_x}\right) - \delta_f$$



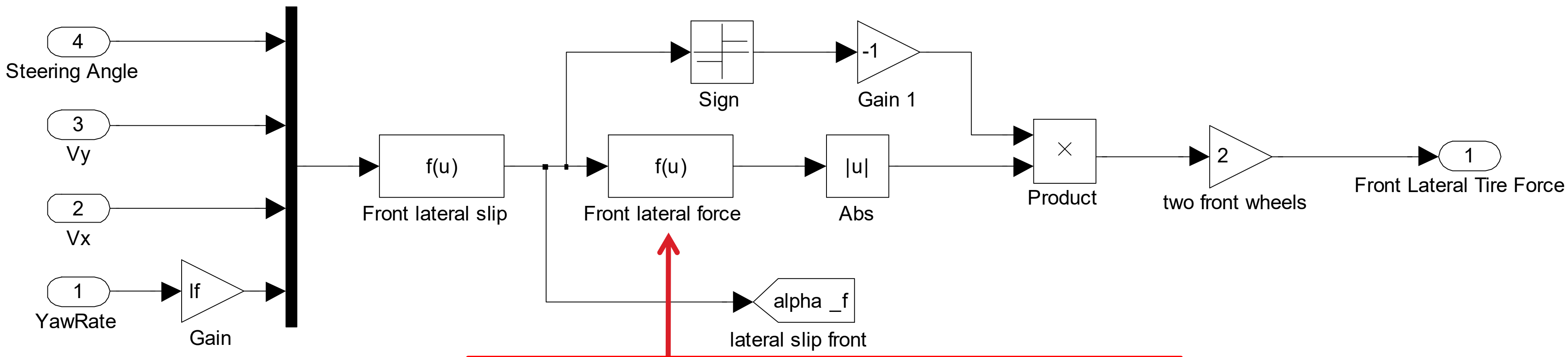


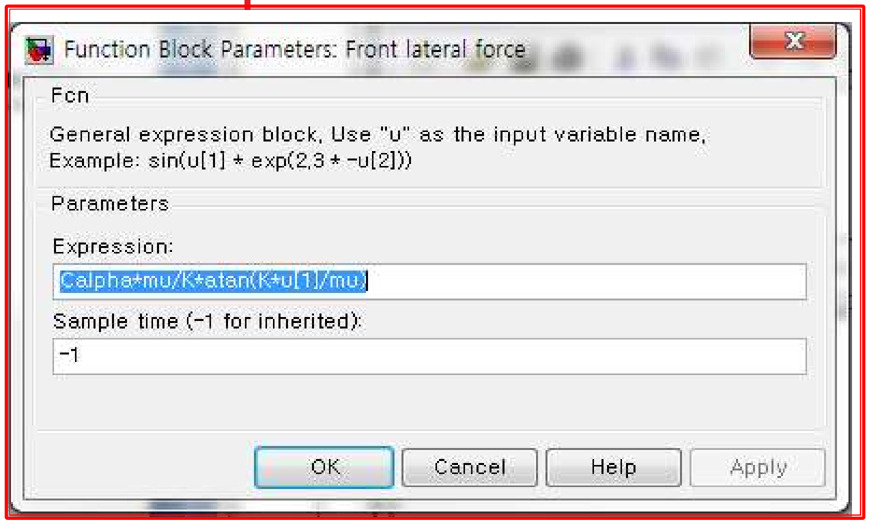
atan2(y, x) = p



Tire Lateral Force Calculation

- Front Lateral Tire Force $$F_{tire} = C_{\alpha}\frac{\mu}{K}\tan^{-1}\left(\frac{K}{\mu}\alpha\right)$$

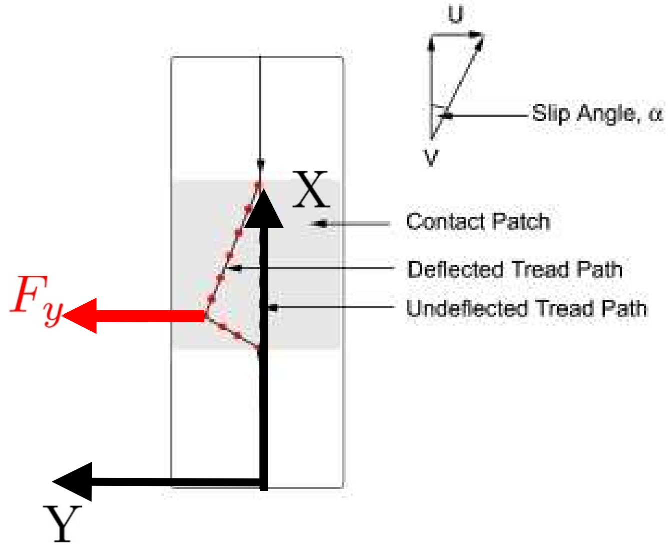


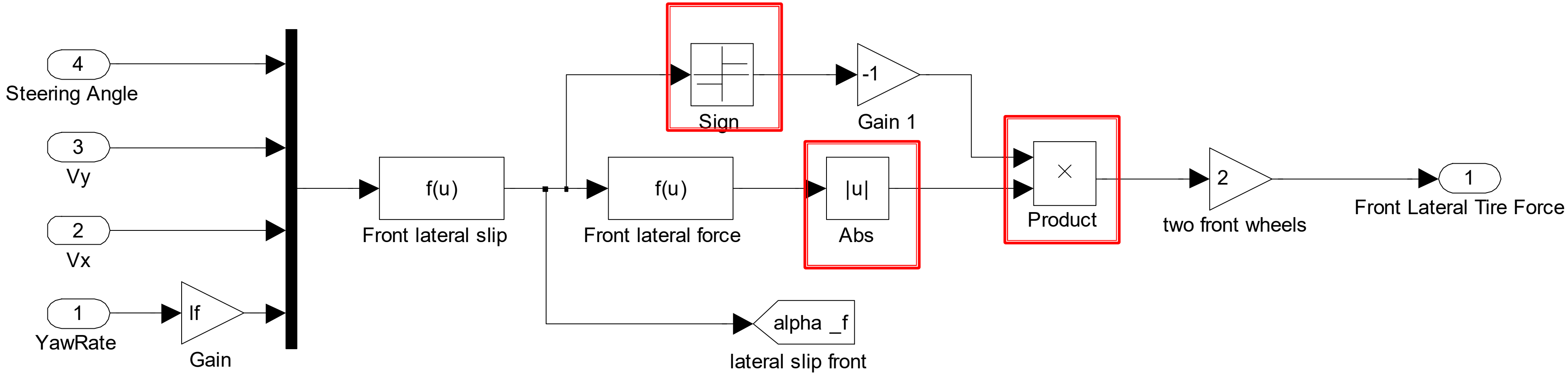


Tire Lateral Force Calculation

- Lateral Tire Force Direction Convention
  - Actual Tire Forces are opposite to the slip angle
  - Therefore,

$$\boxed{F_y = -sign(\alpha)\left|F_{tire}\right|}$$

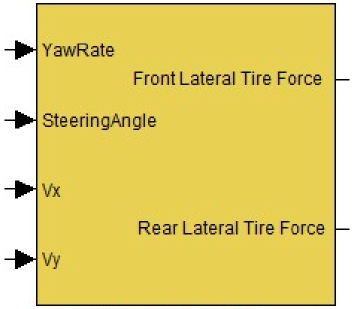




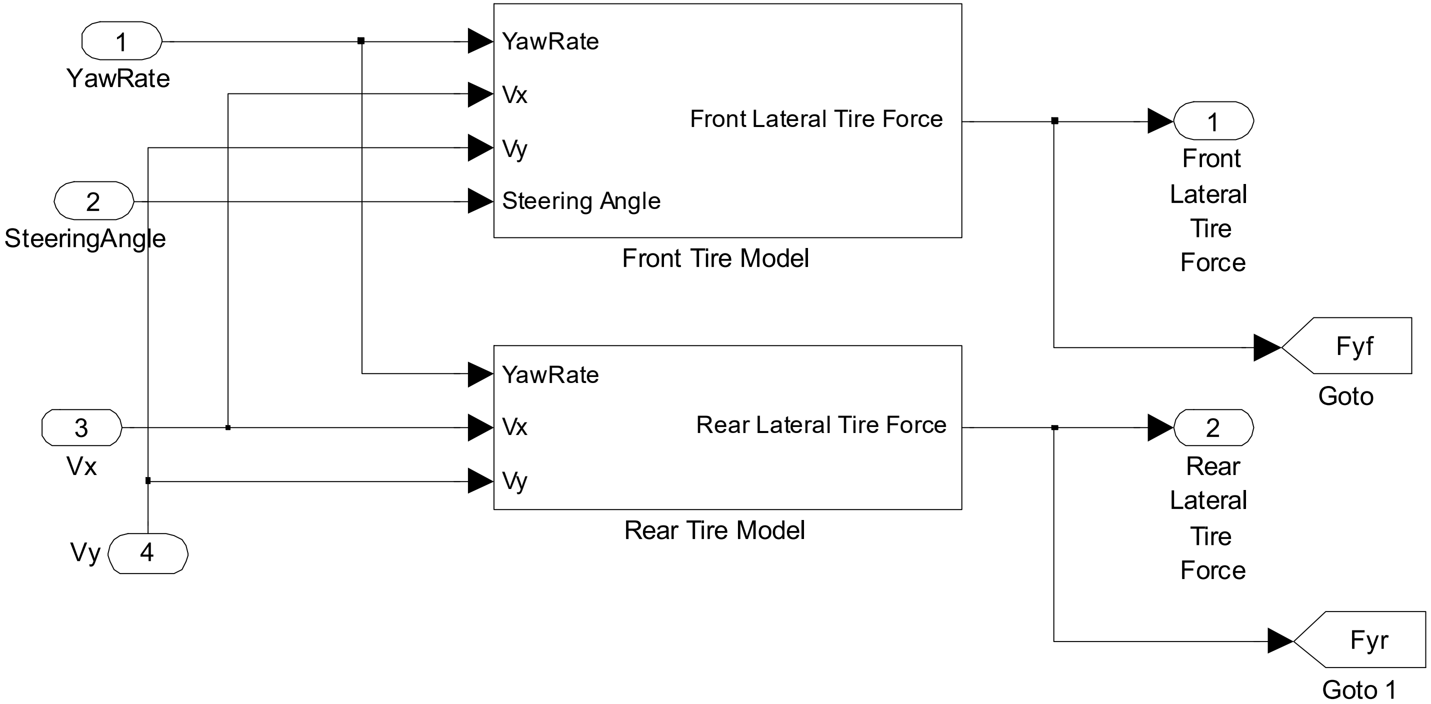
Tire Lateral Slip Angle Calculation

- Rear Lateral Tire Slip Angle

$$\alpha_r = \tan^{-1}\left(\frac{v_y - l_r\gamma}{v_x}\right)$$

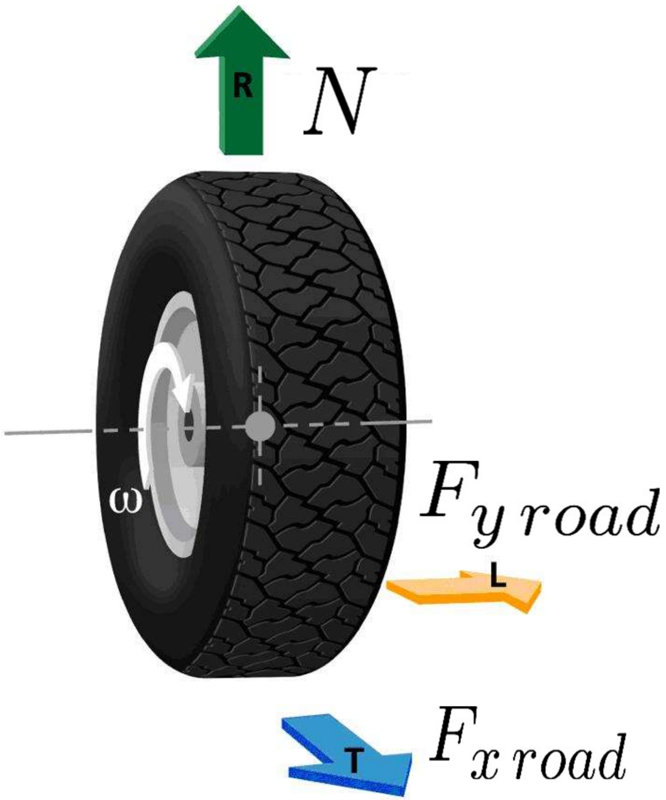


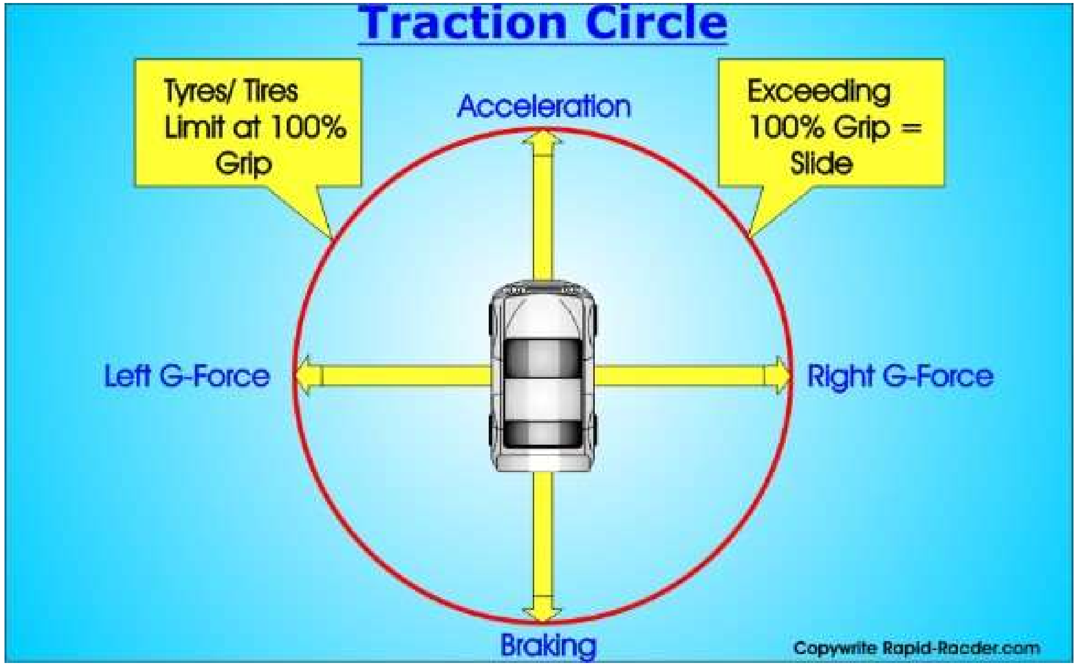
Lateral Tire Model



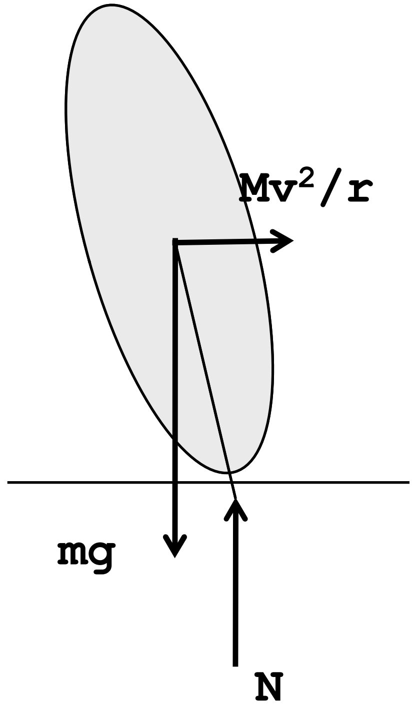
Tire Forces

$$F_{road} = \sqrt{F_{x\,road}^2 + F_{y\,road}^2} \leq \mu_{\max} N$$





Motor Cycle Dynamics



steer left to turn right !!!

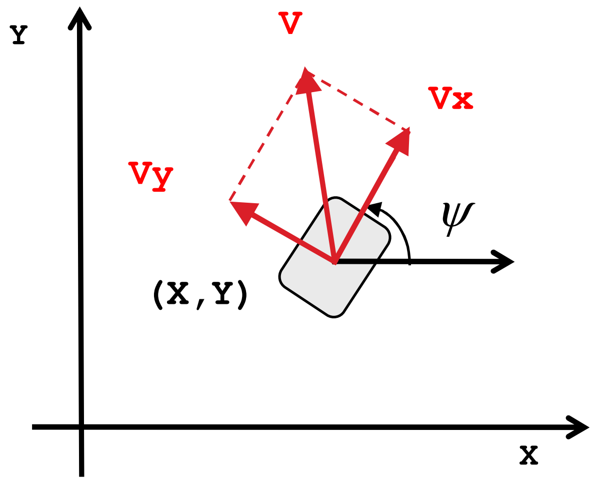
Kinematic Relations for Global X-Y Position

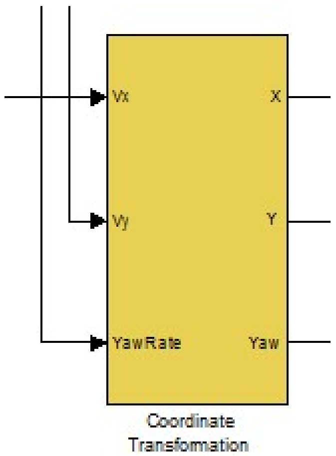
- Coordinate Transformation from the Vehicle's Body Fixed Coordinate to Global Coordinate System

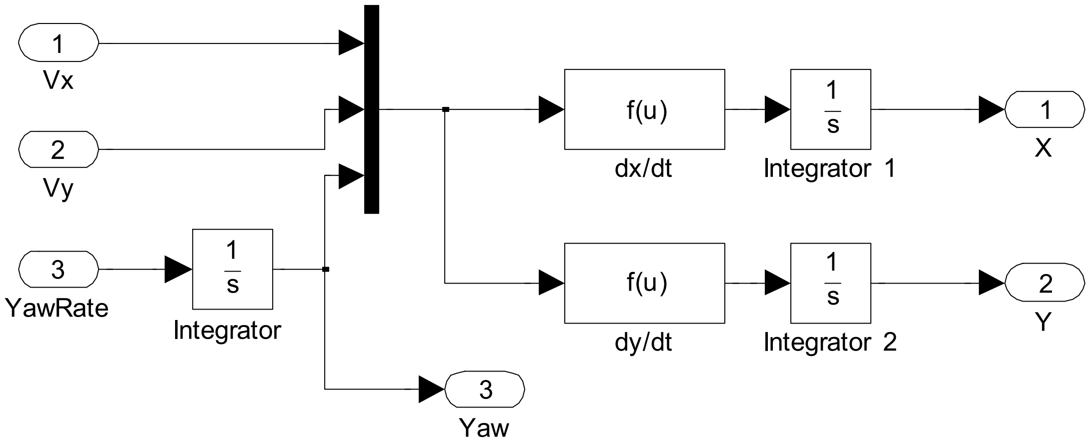
$$\dot{X} = v_x \cos(\psi) - v_y \sin(\psi)$$

$$\dot{Y} = v_x \sin(\psi) + v_y \cos(\psi)$$

$$\dot{\psi} = \gamma$$

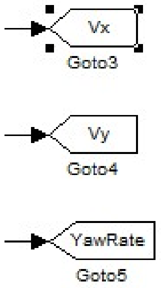


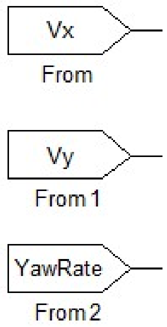


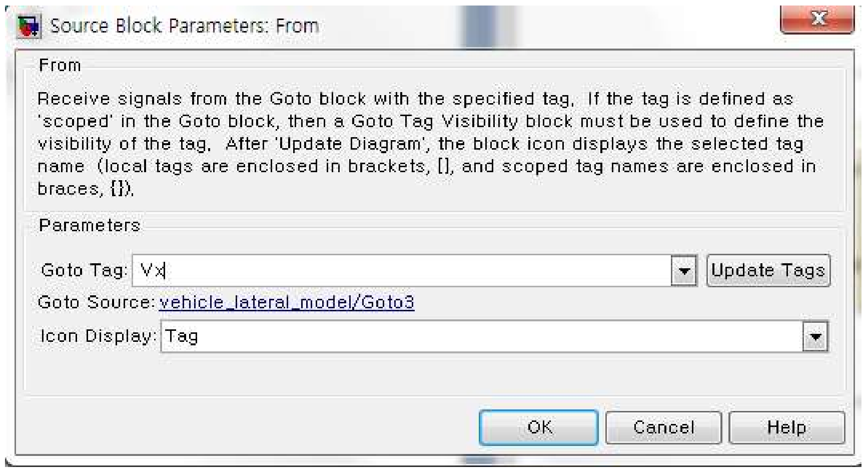


"Goto" blocks and "From" blocks

- "Goto" & "From" blocks to clear-up messy signal lines.
  - Once "Goto" block is created, "From" block can be placed anywhere in the simulink subsystems to deliver signals.
  - "Tag Visibility" determines the scope of the signal







Run Vehicle Lateral Model Simulation

- vehicle model parameters are in vehicle_lateral_model_setup.m file

```
% Simulation Time
Tfinal = 20;

% Tire Parameters
Calpha = 39000;
mu = 0.9;
K = 19;

% Vehicle parameters
m = 2045;
Izz = 5428;
lf = 1.488;
lr = 1.712;
l = lf + lr;
```

```
% Initial Values
yaw_rate0 = 0;
Vy0 = 0;
Yaw0 = 0;
X0 = 0;
Y0 = 0;

% Constant Values
SteerAngle = 3*pi/180;
Vx_const =   80/3.6;

% Controller Parameters
Ke = 6000;
Cf = 38925;
Cr = 38255;
```

Run Vehicle Lateral Model Simulation

- Run the simulation and use “result_plot.m” file to analyze the simulation result.
- Data will be saved through “To Workspace” blocks in Data Log block
- Front tire force greater than the Rear tire force, which generates turning moment of the vehicle.