# CSE101 Homework 1

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**Question 1.**

(a) *False.* $3^n$ *grows way faster than* $2^n$*. Hence, it is slower. Therefore,* $2(3^n) \in \Omega(3(2^n))$

(b) *True. The highest power in both equation is 12.*

(c) *False.* $n^{10}$ *grows way faster than n. Hence, log(*$n^{10}$*) grows way faster than log(n). Hence, log(*$n^{10}$*)* $\in \Omega(log(n))$

(d) *False.* $\sum_{i=1}^{n} i^k = 1^k + 2^k + \cdots + n^k$*. The highest power term is* $n^k$*. Hence,* $\sum_{i=1}^{n} i^k \in O(n^{k+1})$

(e) *False. n! grows way faster than* $2^n$*. Hence, we should have* $n! \in \Omega(2^n)$

**Question 2.**

(a) *Base case: n=6 From the definition of Fibonacci numbers, we can calculate the following:*

$$F_2 = F_1 + F_0 = 1 + 0 = 1, F_3 = F_2 + F_1 = 1 + 1 = 2, F_4 = F_3 + F_2 = 1 + 2 = 3$$

$$F_5 = F_4 + F_3 = 2 + 3 = 5, F_6 = F_5 + F_4 = 8$$

*We know that* $2^{0.5*6} = 2^3 = 8$*. Hence, the base case is true.* $F_n \geqslant 2^{0.5n}$ *when* $n = 6$*. Assume* $F_n \geqslant 2^{0.5n}$ *for all* $n \geqslant 6$*. We need to show that this is true for the case* $n + 1$*. By the definition, we have the following:*

$$\begin{aligned}
F_{n+1} &= F_n + F_{n-1} && (1)\\
&\geqslant 2^{0.5n} + 2^{(0.5(n-1))} && (2)\\
&= 2^{0.5n} + 2^{0.5n-0.5} && (3)\\
&= 2^{0.5n} + \frac{2^{0.5n}}{2^{0.5}} && (4)\\
&= 2^{0.5n}(1 + 2^{-0.5}) && (5)
\end{aligned}$$

*Since* $(1 + 2^{-0.5}) > 2^{0.5n}$*, we get the following:*

$$\begin{aligned}
F_{n+1} &\geqslant 2^{0.5n}(1 + 2^{-0.5}) && (6)\\
&\geqslant 2^{0.5n}(2^{0.5}) && (7)\\
&\geqslant 2^{0.5(n+1)} && (8)
\end{aligned}$$

*Hence, we have shown that* $F_n \geqslant 2^{0.5n}$ *for all* $n \geqslant 6$

(b) *Base Case: n=0 From the definition of Fibonacci numbers, we can calculate the following:* $F_0 = 0$*,* $2^0 = 1$*. Hence, base case is true.* $F_n \leqslant 2^n$ *when n = 0. Assume* $F_n \leqslant 2^n$ *for all* $n \geqslant 0$*. We need to* *show that this is true for the case $n+1$. By the definition, we have the following:*

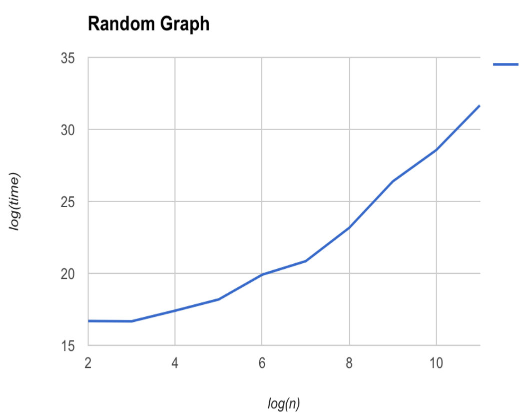
$$
\begin{aligned}
F_{n+1} &= F_n + F_{n-1} && (9)\\
&\leqslant 2^n + 2^{n-1} && (10)\\
&= 2^n + \frac{2^n}{2} && (11)\\
&= 2^n(1 + \frac{1}{2}) && (12)\\
&\leqslant 2(2^n) && (13)\\
&\leqslant 2^{(n+1)} && (14)
\end{aligned}
$$

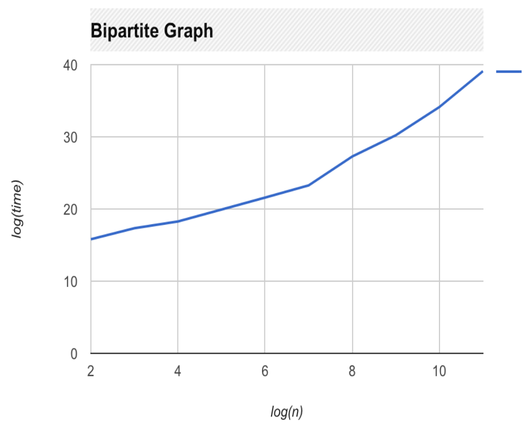
*Hence, we have shown that $F_n \leqslant 2^n$ for all $n \geqslant 0$*

(c) *We conclude that $F_n$ grows exponentially an it is between $\Omega(2^{0.5n})$ and $O(2^n)$.*

**Question 3.**

**Question 4.** *Below are the running time graphs*




*We see that the run time for bipartite graphs is longer than a randomly generated graph. This is because bipartite graphs represent the worse case in which we can never terminate the algorithm early because there are no triangles.*