SA CCR Allocation under consideration of ISDA-SIMM, Master Thesis

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Abstract

This will be the abstract.

1 Introduction

Here goes some text

2 Motivation

Here goes some more text

3 Establishing Background

3.1 Bilateral initial margin

3.1.1 Reducing counterparty credit risk with initial margin

Counterparty credit risk refers to the risk of a default of the counterparty of a derivative. Derivatives are contracts between two institutions which give raise to future cash flows dependent of the performance of its underlying. These future cash flows may be at risk if the counterparty defaults during the lifetime of the derivative.

Over the past decades several measures were established in the derivatives markets to mitigate counterparty credit risk.

The most impactful measure is close-out netting. Close-out netting is a contractual agreement of two counterparties how their bilateral derivative contracts have to be settled if one of the counterparties defaults. With close-out netting, in case one of the two counterparties defaults all derivatives which are yet to mature are immediately closed out based on the current market value. The market values of the individual derivatives are summed up and the netted amount needs to be paid by whichever party is trailing across the portfolio. In case of a default this close-out netting procedure takes priority over all other claims of creditors against the defaulted counterparty. Close-out netting has two major advantages. First, the non-defaulting counterparty only suffers a counterparty credit loss, if it is ahead across the entire portfolio of currently running derivatives with the defaulted party. Just having a positive market value on a few derivatives does not necessarily result in a counterparty credit loss. Secondly, the immediate close-out of the open derivatives of the defaulted counterparty greatly facilitates unwinding its portfolio. A disadvantage of close-out netting is, that it may proof difficult to find an objective market value of the derivatives that have to be closed out - especially in a stressed market environment, which is likely to be present if e.g. a large investment bank defaults. The contractual obligation to perform close-out netting is agreed upon in a master agreement, which was introduced to the derivatives market by ISDA in 1985. More details on close-out netting may be found in [3, Chapter 5].

The second most effective measure in mitigating counterparty credit risk is posting variation margin. If the obligation to post variation margin is agreed as part of a master agreement the accrued mark-to-market of the derivative portfolio has to be collateralized by the trailing counterparty. This measure effectively resets counterparty credit risk to zero for both parties every time a variation margin payment is made or the exchanged variation margin is adjusted to the current market value of the portfolio. The exchange of variation margin was common but not a given in the inter-bank market before the financial crisis of 2008. After the crisis it has become commonplace in the interbank market and recently has even been mandated by regulators[1]. Non-financial counterparties oftentimes do not collaterialize their derivatives since they are not mandated to do so, shy away from the operational burden and have a harder time funding the significant amount of cash necessary to cover the current mark-to-market value of their entire derivatives portfolio. Collateralizing a derivatives portfolio not only significantly reduces CCR but also significantly alters how the remaining CCR behaves. The CCR of a collateralized portfolio may rather be driven by the terms of the CSA[2] or residual phenomenons such as collateral spikes[3] than by the underlying instruments.

Close-out Netting Variation margin Initial margin

A more comprehensive introduction to counterparty credit risk and its reduction through netting and margining may be found in chapters four through six of [3].

3.1.2 Market structure and regulatory background

The derivative market is divided into exchange traded derivatives, cleared OTC derivatives and uncleared OTC derivatives. According to [3, Figure 3.2] based on notional 9% of derivatives are exchange traded, 55% are cleared OTC derivatives and

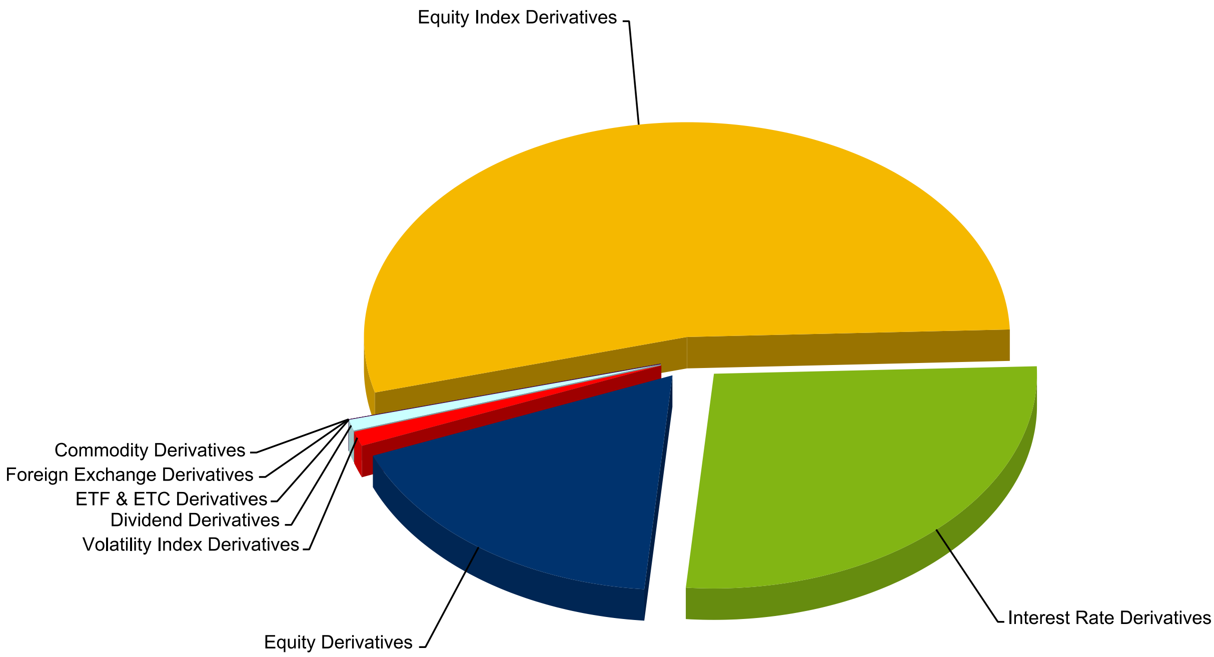
[1] In the European Union the exchange of Initial Margin for inter bank bilateral OTC derivatives is compulsory since September of 2016 for large banks or March of 2017 for smaller banks.

[2] Explain what a CSA is

[3] Quote something regarding collateral Spikes

36% are uncleared OTC derivatives. It has to be noted that these figures are from 2014 and it can be assumed, that the fraction of cleared OTC derivatives has increased since then at the expense of the fraction of uncleared OTC derivatives. The reason for this is, that regulators have incentivised or even mandated the clearing of simpler OTC derivatives over the course of the last years. In connection with this development the large CCPs such as Eurex or the LCH have extended the product range for which they offer OTC clearing in recent years.

Exchange traded or listed derivatives are usually futures or options on a limited scope of underlyings which can be directly traded at a clearing house at a bid and ask quoted by the clearing house. As seen in figure XX usual underlyings are single name equities, equity indices, commodities or baskets of liquid government bonds.



The clearing house acts as a mediator to match market participants which are willing to buy and sell and market participants will not be informed with whom they have been matched. All market participants exchange variation margin with the CCP and have to post a unilateral initial margin to the CCP. This initial margin is calculated by the CCP and is usually calculated using a portfolio based historical simulation VaR model. If a market participants defaults to CCP inherits its positions and needs to auction them to other market participants. The purpose of the initial margin is to cover mark-to-market losses which the portfolio suffers until it can be fully auctioned or otherwise closed by the CCP.

Cleared OTC derivatives are initially bilaterally agreed upon by two market participants. If the parametrization of the derivative is sufficiently common CCPs may offer to clear it and the market participants may choose or even be forced by regulation to clear it using a qualifying CCP. OTC clearing is especially common for simple interest rate derivatives such as interest rate swaps, forward rate agreements or cross currency swaps and for some credit derivatives. Similarly to exchange traded derivatives the CCP will charge both participants an initial margin and will also regularly determine the derivatives present value and execute the exchange of variation margin between the two market participants. If one of the market participants defaults, the CCP inherits the positions of the defaulted party and can again use the initial margin posted by the defaulted party to cover mark-to-market losses until it has auctioned or closed the inherited positions [1].

Assuming that the CCP cannot default itself, cleared OTC derivatives and listed derivatives bear no counterparty credit risk for the market participants.

As of December 2018 common derivatives that are still uncleared are structured equity derivatives, swaptions and many other interest rate derivatives with optionality, derivatives on securitizations, swaps with unusual cashflow structures or in uncommon currencies and most FX derivatives [5].

What are uncleared OTC derivatives

3.1.3 Regulatory requirements for an internal initial margin model

After the 2008 financial crisis the G20 agreed to reduce systematic, counterparty and operational risk and as a result of this commitment has been put into effect by regulators worldwide. In Europe the European Market Infrastructure Regulation (EMIR) came into force in August 2012 and focused on promoting or mandating central clearing as the primary measure to reduce counterparty risk. EMIR also mant

3.1.4 The ISDA-SIMM model

In December of 2013 the International Swap and Derviatives Association (ISDA) published a motivation and basic methodological outline of a common internal initial margin model called Standard Initial Margin Model (SIMM™)[10]. The goal of the model is to meet the model requirements to an internal model of all regulators, while being among others easy to replicate, quick to calculate and relatively cheap to operate, implement and validate.

SIMM is a Delta-Gamma VaR model using Delta and Vega sensitivities calculated by the banks themselves and risk weights and correlations provided and recalibrated annually by ISDA. ISDA provides member with a methodological paper [9] and a paper describing the input format of sensitivities [8].

3.2 Capital requirements for counterparty credit risk

Counterparty credit risk is considered to be a part of credit risk by the regulator. Risk weighted assets have to be calculated and need to be backed by own capital. The three main inputs for calculating credit risk are the probability of default (PD) the loss given default (LGD) and the exposure at default (EAD). Assuming the default of a counterparty over the course of the next year, the EAD is the current estimation of money indebted by the counterparty to the bank at the time of default. Estimating EAD for traditional credit instruments s.a. loans, credit cards, mortgages or bonds is relatively simple. Such instruments do often times have deterministic payment schedules making it easy to predict the exposure in one years time. Credit lines or credit cards behave less deterministic but it is still simple to determine an upper bound to the future exposure by assuming that the entire credit line is exhausted. The counterparty credit risk incurred by derivatives has first been regarded in regulatory capital calculation in Basel II [6]. Due to the stochastic nature of derivatives EAD calculation for counterparty credit risk has always been regulated separately ever since To calculate the counterparty credit risk associated with derivatives a different approach to calculating

3.2.1 The SA-CCR

For analysis we create an SA CCR object that implements SA CCR as specified in [7]

When using *SA-CCR* the exposure at default (EAD) has to be calculated as:

$$EAD = \alpha * (RC + PFE)$$

$$\begin{aligned}
\text{where} \qquad \alpha &= 1.4 \\
RC &: \text{Replacement Cost} \\
PFE &: \text{Potential Future Exposure}
\end{aligned}$$

[In]:
```
SA_CCR.calculate_sa_ccr_ead(rc = 10, pfe = 20)
```

[Out]:
```
42.0
```

Relation of RC and PFE The purpose of the RC is to assess the imidiate loss suffered by the default of a counterparty. It is based on the current MtM of the derivative less the accessible collateral. If a bank has posted collateral to non-segregated accounts of a counterparty this collateral is also assumed to be lost in case of a default which increases the replacement cost.

The potential future exposure (PFE) on the other hand assesses how the RC might develop in the future. The future being defined as during the next year. If the RC today is 0 but is likely to be larger than 0 in the near future the estimated EAD should take this expected increase in RC into account.

See also Paragraph 130 and 131 of [7]

Paragraph 130 - case without margining:

> For unmargined transactions, the *RC* intends to caputre the loss that would occur if a counterparty were to default and were closed out of its transactions immediately. The *PFE* add-on represents a potential conversative increase in expousre over a one-year time horizon from the present date (i.e. the calculation date).

Paragraph 131 - case with margining:

> For margined trades, the *RC* intends to capture the loss that would occur if a counterparty were to default at the present or at a future time, assuming that the closeout and replacement of transactions occur instantaneously. However, there may be a period (the margin period of risk) between the last exchange of collateral before default and replacement of the trades in the market. The *PFE* add-on represents the potential change in value of the trades during this time period.

Definition of Potential Future Exposure (PFE)

$$PFE = \text{multiplier} * AddOn^{\text{aggregate}}$$

$$\text{where} \qquad AddOn^{\text{aggregate}} : \text{aggregate add-on component}$$
$$\text{multiplier} : f(V, C, AddOn^{\text{aggregate}})$$

$AddOn$ is calculated differently for each asset a class. Since no netting is allowed between asset classes the aggregate is calculated as:

$$AddOn^{\text{aggregate}} = \sum_a AddOn^a$$

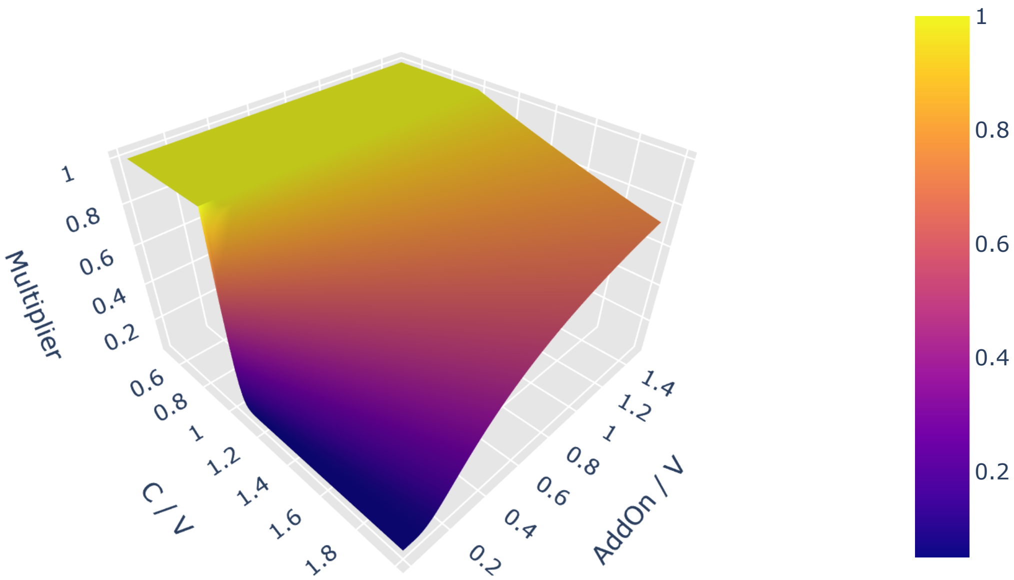
Collateralization is taken into account of the PFE calculation through the multiplier that uses the collateral held as an input. As overcollateralization e.g. through IM increases, the multiplier decreases. However, the multiplier is floored at 5%.

$$\text{multiplier} = \min\left\{1; Floor + (1 - Floor)\exp\left(\frac{V - C}{2(1 - Floor)AddOn^{\text{aggregate}}}\right)\right\}$$

where $\quad Floor = 5\%$

It is important to note that the multiplier can only be below 1 if $C > V$, i.e. if the portfolio is overcollateralized. If the portfolio is overcollateralized, the *AddOn* comes into play. The idea behind the *AddOn* is related to the idea of value at risk. The higher the *AddOn* the faster the SA-CCR model expects the positions to lose in value. Therefore, the higher the value, the higher the multiplier.

In the example below the current NPV of the portfolio is 30 and the received collateral (*IM* + *VM*) is 37. The portfolio is overcollateralized as it should be when initial margin is used. On the other hand, as collateralization decreases e.g. V=C since only VM is exchanged or C=0 in the case of an uncollateralized portfolio the multiplier increases.



4 AddOn calculation

Most of the SA-CCR logic is hidden inside the AddOn calculation. At first it is important to define the following four data parameters:

M_i

Maturity of the derivative contract. If the underlying of a derivative is another derivative - e.g. in the case of a swaption the maturity date of the underlying needs to be chosen.

S_i

For interest rate and credit derivatives the start date of the time periodreferenced by an interst rate or credit contract. If the derivtives underlying is another interest rate or credit intsrument (eg swaption or bond option) S_i is the start date of the underlying instead.

E_i

Defined as S_i but referencing the end date instead of the start date.

T_i

For options across all asset classes this is the latest contractual exercise date.

4.1 Trade level adjusted notional

Each trade i has a trade level adjusted notional d_i^a assigned to it. This is calculated differently for the different asset classes.

Interest rate and credit derivatives The notional of the trade is usually a well defined value in domestic currency for interest rate and credit derivatives. It is multiplied by a supervisory duration factor. The basic idea is, that the value of the derivative can change more the longer the remaining

$$d_i = \text{Notional}_i * SD_i$$

$$\text{where} \qquad SD_i = \frac{\exp\left(-0.05 * S_i\right) - \exp\left(-0.05 * E_i\right)}{0.05}$$

FX derivatives While the wording in the BCBS paper is a bit more specific we will just assume that every FX traded derivative has a USD leg and set the notional equal the to USD notional.

Equity and commodity derivatives The notional is defined as the price of the underlying. Therefore, it fluctuates over time.

Notional of exotic derivatives For more exotic derivatives which do have adjustable notionals, resetting notionals etc. detailed handling of the notional is defined in paragraph 158.

Within this thesis we investigate only equity and interest rate derivatives. For these we can make a few exemplary calculations of the trade level adjusted notional.

For equity trades determining the trade level adjusted notional is trivial as it always is the spot price of the underlying. As an example consider the two trades defined below:

[In]:
```
#When the strike is not set explicitly an at the money option is␣
↪created with K = S(t0)
eqOption1 = EquityOption(maturity = ql.Period(1, ql.Years),
                         tradeType= TradeType.CALL,
                         tradeDirection= TradeDirection.LONG,
                         underlying= Stock.ADS)

eqOption2 = EquityOption(maturity = ql.Period(1,   ql.Years),
                         tradeType= TradeType.PUT,
                         tradeDirection= TradeDirection.SHORT,
                         underlying= Stock.ADS,
                         strike = 60)
```

Let the spot price of Adidas stock be 42. Then, the adjusted notional of `eqOption1`, an at the money call on Adidas, is 42 and the adjusted notional of `eqOption2`, a short in the money put on Adidas, is also 42.

For interest rate derivatives such as interest rate swaps or swaptions on the other hand, the notional is adjusted by the supervisory duration factor. As the supervisory duration depends on S and E it is important to understand how these are determined for the different interest rate derivatives.

Trade Type	S	E
Interest Rate Swap	Current date	Maturity date
Forward starting IRS	Start date of the underlying swap	Maturity date of the underlying swap
Swaption	Start date of the underlying swap	Maturity data of the underlying swap

4.2 Supervisory delta adjustments: δ_i

For linear derivatives δ is 1 for long derivatives and -1 for short derivatives.

For options δ is defined as under Black-Scholes:

$$\delta_{\text{long Call}} = +\Phi\left(\frac{\ln\left(P_i/K_i\right) + 0.5 * \sigma_i^2 * T_i}{\sigma_i * \sqrt{T_i}}\right)$$

where Φ : standard normal cdf

σ_i : supervisory volatility as defined in Table 2 in paragraph 183

This delta is multiplied by -1 in case of a long Put option or a short Call option. This formula is used for both, equity options and swaptions.

No detail is given at this point on the delta calculation of CDO tranches as these are not in the scope of this thesis.

In the case of an european equity option the parametrization is quite straight forward.

σ_i: 1.2 is the supervisory volatility for a single stock option

K_i: The strike of the option

P_i: The spot price of the underlying stock

T_i: The maturity of the option

A swaption on the other hand is parametrized as follows for calculation of its supervisory delta:

σ_i: 0.5 is the supervisory volatility for any interst rate option.

K_i: The strike of the option is the fixed rate of the underlying swap

P_i: Is the current par rate of the underlying (forward starting) swap

T_i: The maturity of the option. Please note the difference to E_i used for calculation of the adjusted notional, which is the maturity of the underlying swap.

SA-CCR uses the same Black-Scholes based formula for Swaps as it uses for Equities. It differentiates options in two dimensions. Whether they are *bought* or *sold* and whether they are *Call* or *Put* options (Compare paragraph 159).

SA-CCR defines an option as a call option if it rises in value as the underlying rises in value. A fixed payer swap rises in value as the underlying interest rate rises in value. Therefore, an option to buy a fixed payer swap at a predetermined strike also rises in value as the underlying interest rate rises in value. Therefore, a swaption on a payer swap is considered a *Call* under SA-CCR, while a swaption on a receiver swap is considered a *Put*.

For the at the money option `eqOption1` that was set up above we yield a supervisory delta adjustment of 0.7257.

For an examplary short european swaption that has a par swap as underlying (i.e. the NPV of the swap is 0) that is set up as follows:

[In]:
```
swap = IRS(notional=100,
           timeToSwapStart=ql.Period(1, ql.Years),
           timeToSwapEnd=ql.Period(3, ql.Years),
           swapDirection=SwapDirection.PAYER,
           index=InterestRateIndex.EURIBOR6M
          )

swaption = Swaption(underlyingSwap=swap,
                    optionMaturity=ql.Period(1, ql.Years),
                    tradeDirection=TradeDirection.SHORT)

SA_CCR.calculate_sa_ccr_delta(swaption)
```

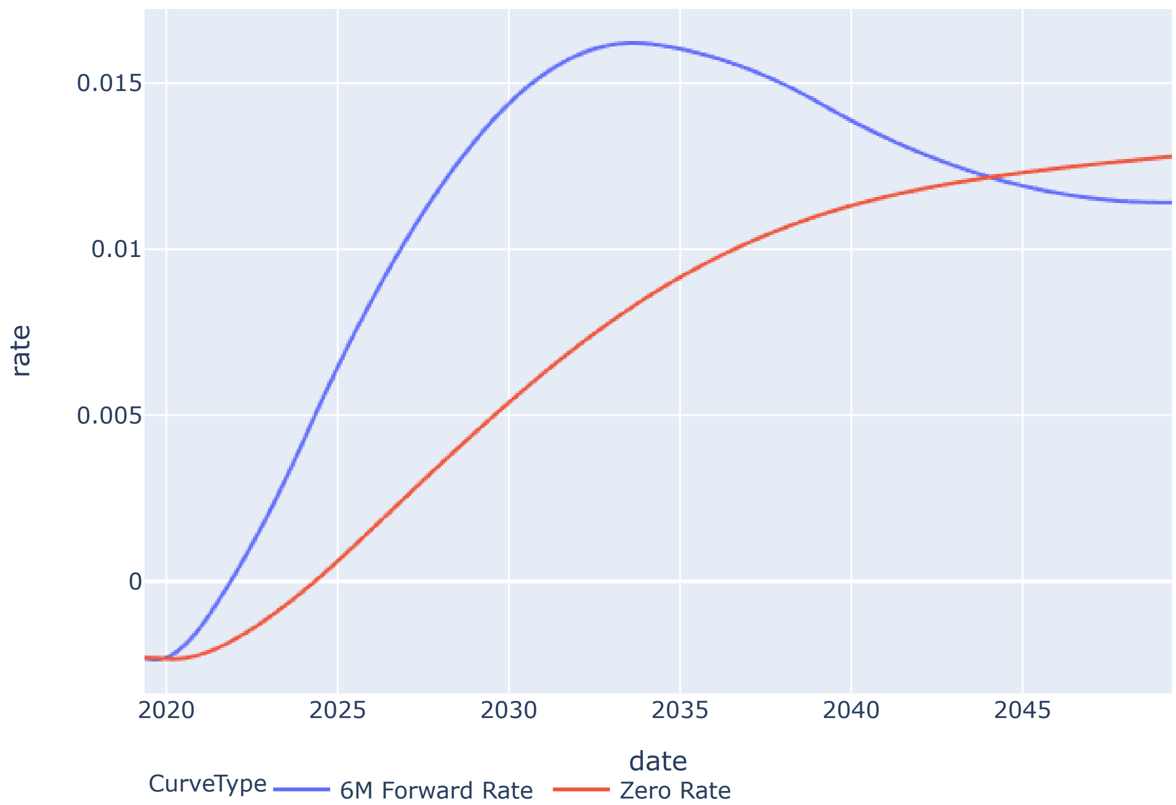
[Out]:
```
-0.5987063256828626
```

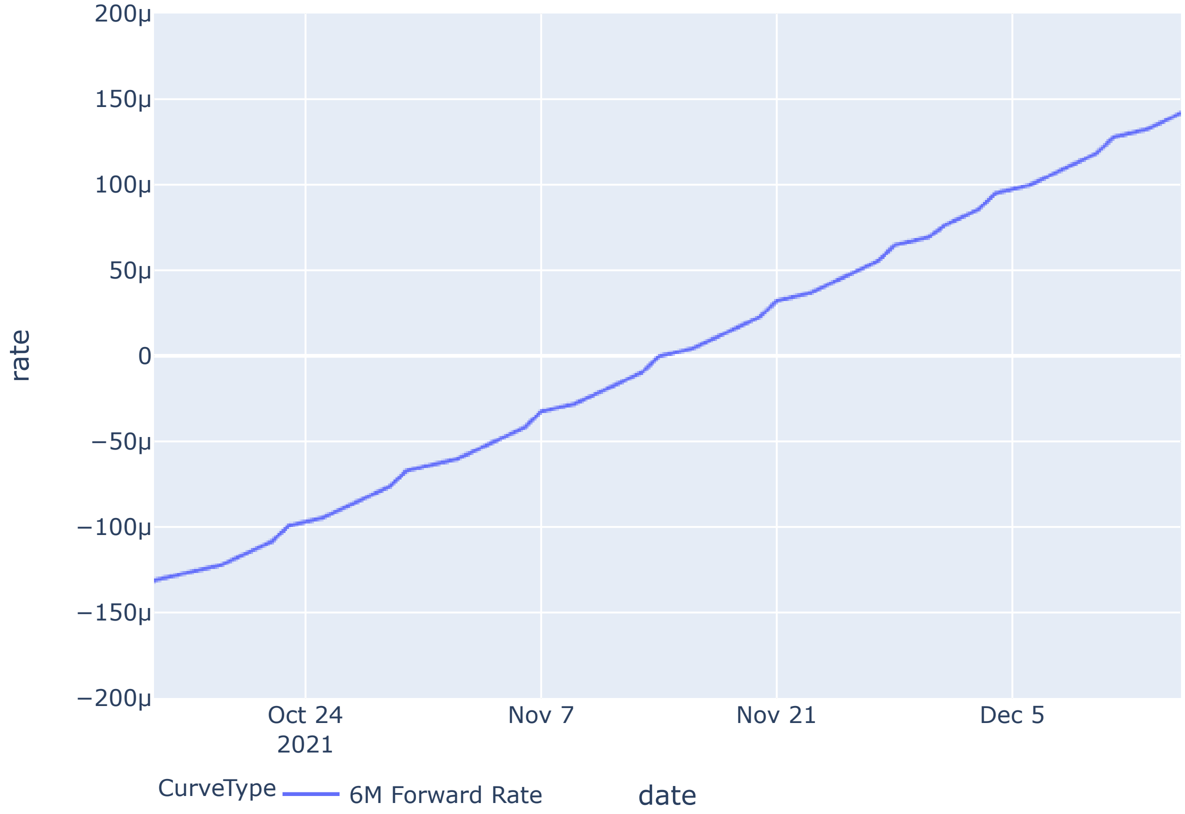
we yield a regulatory delta of -0.5987.

However, the approach of SA-CCR to use the Black-Scholes for interest rate derivatives does have a significant flaw.

Obviously, $\ln(P_i/K_i)$ is not defined, if P_i/K_i is negative. This has, however been very commonplace in recent years, especially for Euro swaptions. One can always set the fixed rate of the underlying swap and therefore K_i as a negative value. But based on the market data of the 10th of May 2019 it is even possible to yield negative par rates for the underlying swap.

As an example we are plotting the zero and the 6M forward curve for the EURIBOR 6M index below. When zooming further in, we can see that that the 6M forward curve crosses 0 on the 15th of November 2021.





To highlight the relationship between the 6M forward curve and the par rate P_i of forward starting swap we can set up the following two swaps depicted below. Since the $EndDate - StartDate$ for these two swaps is six months their par rate or fair price can directly be read from the 6M Forward curve.

[In]:
```
print('StartDate: ' + str(today+ql.Period(30,ql.Months)))
swap1 = IRS(notional=100,
            timeToSwapStart=ql.Period(30,ql.Months),
            timeToSwapEnd=ql.Period(36,ql.Months),
            swapDirection=SwapDirection.PAYER,
            index=InterestRateIndex.EURIBOR6M,
            fixed_rate=0.01
           )
print('ParRate: %.6f%%' % (swap1.get_par_rate()*100))
```

```
StartDate: November 10th, 2021
ParRate: -0.002346%
```

```
[In]: print('StartDate: ' + str(today+ql.Period(31,ql.Months)))
swap2 = IRS(notional=100,
            timeToSwapStart=ql.Period(31,ql.Months),
            timeToSwapEnd=ql.Period(37, ql.Months),
            swapDirection=SwapDirection.PAYER,
            index=InterestRateIndex.EURIBOR6M,
            fixed_rate=0.01
            )
print('ParRate: %.6f%%' % (swap2.get_par_rate()*100))
```

```
StartDate: December 10th, 2021
ParRate: 0.011976%
```

4.2.1 Risk Horizon

For unmargined transaction the margining factor is

$$MF_i^{\text{unmargined}} = \sqrt{\frac{\min\left(M_i; 1 \text{ year}\right)}{1 \text{ year}}}$$

This factor can be used to scale down a risk weight calibrated for a 1 year horizon to a shorter period.

With margining the margin period of risk (MPOR) is:

- 10 business days for small, uncleared OTC portfolios
- 5 business days for cleared derivatives
- 20 business days for netting sets with more than 5000 transactions that are not with a central counterparty
- and doubling this period for portfolios with outstanding disputes

The margining factor is then

$$MF_i^{\text{margined}} = \frac{3}{2}\sqrt{\frac{MPOR_i}{1 \text{ year}}}$$

At this point we need to introduce a collateral agreement object. For simplicities sake we will not differentiate between collateral and netting sets in this thesis. All trades that are covered by the same collateral agreement are also admissible for netting with each other. (Also refer to the introduction of close out netting above). To take into account the different parameters determining the risk horizon a couple of parameters are required to create a collateral agreement. As an example, below we are setting up a collateral agreement for uncleared derivatives without exchange of variation margin or initial margin.

[In]:
```
ca = CollateralAgreement(
        margining=Margining.UNMARGINED,
        clearing=Clearing.UNCLEARED,
        tradecount=Tradecount.UNDER_FIVE_THOUSAND,
        dispute=Dispute.NO_OUTSTANDING_DISPUTES,
        threshold=0.0,       #Threshold to trigger a margin call
        mta=0.0,             #Minimum transfer amount for a margin call
        vm=0.0,              #Variation margin balance
        posted_im=0.0,       #posted initial margin
        received_im=0.0      #received initial margin
        )
```

With this collateral set object we can define a function for calculation the margining factor:

For trades of differing maturity let's compare the margining factor for the three most common scenarios:

1. No margining
2. Bilateral margining
3. Centrally cleared

[Out]:

	Three days	Two weeks	Six months	One year	Ten years
No margining	0.2000	0.2000	0.7071	1.0000	1.0000
Bilateral margining	0.3000	0.3000	0.3000	0.3000	0.3000
Centrally cleared	0.2121	0.2121	0.2121	0.2121	0.2121

[In]:
```
#import cell
import QuantLib as ql
from IPython.core.display import display, Markdown
from scipy import optimize
from collateralAgreement import CollateralAgreement
from instruments.interestRateInstrument.irs import IRS
from instruments.interestRateInstrument.swaption import Swaption
from jupyterUtils import export
from marketdata.fx_spot import FxSpot
from marketdata.interestRateIndices import InterestRateIndex
```

```
from sa_ccr.sa_ccr import SA_CCR
from utilities.Enums import SwapDirection, TradeDirection
asdf =1
```

4.3 AddOn for interest rate derivatives

Step 1 - calculation of effective notional D_{jk}^{IR}

$$D_{jk}^{IR} = \sum_{i \in \{Ccy_j, MB_k\}} \delta_i * d_i^{IR} * MF_i$$

Here, the notation $i \in \{Ccy_j, MB_k\}$ refers to trades whose underlying is the interest rate of a common currency j and which mature in a common maturity bucket k

We can test our implementation of the effective notional against a small exemplary portfolio in Annex 4a Example 1 of the SA_CCR paper. It consists of the following trades:

Trade #	Nature	Residual Maturity	Base Ccy	Notional (tsd)	Pay Leg	Receive Leg	Market value (tsd)
1	Interest rate swap	10 years	USD	10000	Fixed	Floating	30
2	Interest rate swap	4 years	USD	10000	Floating	Fixed	-20
3	European Swaption	1 into 10 years	EUR	5000	Floating	Fixed	50

To set up this exemplary portfolio we need to find fixed rates for the swaps and underlying swaps to match the desired market values.

[In]:
```
def find_1(fixed_rate):
    target_value = 30000
    swap = IRS(notional = 10000000,
               timeToSwapStart=ql.Period(2, ql.Days),
               timeToSwapEnd=ql.Period(10, ql.Years),
               swapDirection=SwapDirection.PAYER,
```

```
                index = InterestRateIndex.USDLIBOR3M,
                fixed_rate=fixed_rate[0]
                )
    return abs(swap.get_price()-target_value)

def find_2(fixed_rate):
    target_value = -20000
    swap = IRS(notional = 10000000,
                timeToSwapStart=ql.Period(2, ql.Days),
                timeToSwapEnd=ql.Period(4, ql.Years),
                swapDirection=SwapDirection.RECEIVER,
                index = InterestRateIndex.USDLIBOR3M,
                fixed_rate=fixed_rate[0]
                )
    return abs(swap.get_price()-target_value)

def find_3(fixed_rate):
    target_value = 50000
    notional = 5000000*FxSpot.USDEUR.value
    swap = IRS(notional = notional,
                timeToSwapStart=ql.Period(1, ql.Years),
                timeToSwapEnd=ql.Period(10, ql.Years),
                swapDirection=SwapDirection.RECEIVER,
                index = InterestRateIndex.EURIBOR6M,
                fixed_rate=fixed_rate[0]
                )
    swaption = Swaption(underlyingSwap=swap,
                         optionMaturity=ql.Period(1, ql.Years),
                         tradeDirection=TradeDirection.LONG)
    price = swaption.get_price()*FxSpot.EURUSD.value
    return abs(price-target_value)

def find_4(fixed_rate):
    target_value = -0.27
    notional = 5000000*FxSpot.USDEUR.value
    swap = IRS(notional = notional,
                timeToSwapStart=ql.Period(1, ql.Years),
                timeToSwapEnd=ql.Period(10, ql.Years),
                swapDirection=SwapDirection.RECEIVER,
```

```
                index = InterestRateIndex.EURIBOR6M,
                fixed_rate=fixed_rate[0]
                )
    swaption = Swaption(underlyingSwap=swap,
                         optionMaturity=ql.Period(1, ql.Years),
                         tradeDirection=TradeDirection.LONG)
    delta = SA_CCR.calculate_sa_ccr_delta(swaption)
    return abs(delta-target_value)

result = optimize.minimize(find_1, x0=0.01, tol=0.0000000001)
fixed_rate_trade_1 = result.x[0]
result = optimize.minimize(find_2, x0=0.01, tol=0.0000000001)
fixed_rate_trade_2 = result.x[0]
result = optimize.minimize(find_3, x0=0.01, constraints={'type':
 ↪'ineq','fun': lambda x: x[0]+0.02}, tol=0.0000000001)
fixed_rate_trade_3 = result.x[0]
result = optimize.minimize(find_4, x0=0.01, constraints={'type':
 ↪'ineq','fun': lambda x: x[0]}, tol=0.0000000001)
fixed_rate_trade_4 = result.x[0]
```

Through optimization and using the market data of the 10th of May 2019 the fixed rates to match the market values in Example 1 were identified.

[In]:
```
trade_1 = IRS(notional = 10000000,
             timeToSwapStart=ql.Period(2, ql.Days),
             timeToSwapEnd=ql.Period(10, ql.Years),
             swapDirection=SwapDirection.PAYER,
             index = InterestRateIndex.USDLIBOR3M,
             fixed_rate=fixed_rate_trade_1
             )

trade_2 = IRS(notional = 10000000,
              timeToSwapStart=ql.Period(2, ql.Days),
              timeToSwapEnd=ql.Period(4, ql.Years),
              swapDirection=SwapDirection.RECEIVER,
              index = InterestRateIndex.USDLIBOR3M,
              fixed_rate=fixed_rate_trade_2
              )

notional = 5000000*FxSpot.USDEUR.value
```

```
ul_swap = IRS(notional = notional,
              timeToSwapStart=ql.Period(1, ql.Years),
              timeToSwapEnd=ql.Period(10, ql.Years),
              swapDirection=SwapDirection.RECEIVER,
              index = InterestRateIndex.EURIBOR6M,
              fixed_rate=fixed_rate_trade_3
              )

trade_3 = Swaption(underlyingSwap=ul_swap,
                   optionMaturity=ql.Period(1, ql.Years),
                   tradeDirection=TradeDirection.LONG)

test_portfolio = [trade_1, trade_2, trade_3]
```

[In]:
```
display(Markdown('For trade 1 the matching fixed rate is %.4f%%, for␣
 ↪trade 1 it is %.4f%% and for the underlying swap of trade 3 it is %.
 ↪4f%%' %␣
 ↪((fixed_rate_trade_1*100),(fixed_rate_trade_2*100),(fixed_rate_trade_3*100))))
```

For trade 1 the matching fixed rate is 2.3754%, for trade 1 it is 2.2108% and for the underlying swap of trade 3 it is 0.1610%

[In]:
```
ca=CollateralAgreement()

print(SA_CCR.calculate_sa_ccr_delta(trade_1))
print(SA_CCR.trade_level_adjusted_notional(trade_1))
print(SA_CCR.calculate_sa_ccr_delta(trade_2))
print(SA_CCR.trade_level_adjusted_notional(trade_2))
print(SA_CCR.calculate_sa_ccr_delta(trade_3))
print(SA_CCR.trade_level_adjusted_notional(trade_3))

SA_CCR.interest_rate_addOn(test_portfolio,ca)
```

```
1
78638320.21725275
-1
36198301.54418307
-0.00343382383244269
31712286.360503413
```

[Out]:
```
296732.72980308026
```

```
[In]: export("SA_CCR_ird_addon.ipynb")
```

4.4 Approaches to allocation

With increasing sophistication of risk, own capital and margining models the need for equally sophisticated tools for attributing these measures rises aswell. Allocating the variation margin or the current exposure method (CEM) to individual trades is trivial as these measures may just be calculated for an individual trade and then added up across all trades to obtain the correct aggregate value. For measures which take portfolio effects into account such as a state of the art VaR model, ISDA-SIMM or SA-CCR however, this approach is not possible. The advent of portfolio based models for internal risk measurement in the late 1990s and for regulatory risk measurement in the late 2000s sparked research into how such measures should be reallocated. Gregory [3, Chapter 10.7] states that three approaches are used in practice:

- Incremental allocation
- Marginal allocation
- Pro rata allocation

Additionally [4] Eigenschaften von Allokationen

- Nativ additiv
- Risk sensitivity
- Unabhaengig von Portfoliozusammensetzung
- Stable through time

4.4.1 Incremental allocation

Incremental allocation can only be applied when observing the development of a portfolio through time. Given a pre-existing portfolio P consisting of n trades t_1 through t_n and a portfolio-based measure M the incremental contribution of the first and second additional trade may be calculated as:

$$\begin{aligned} M_{\text{inc},t_{n+1}} &= M\left(t_1 \ldots t_{n+1}\right) - M\left(t_1 \ldots t_n\right) \\ M_{\text{inc},t_{n+2}} &= M\left(t_1 \ldots t_{n+2}\right) - M\left(t_1 \ldots t_{n+1}\right) \end{aligned}$$

It can be easily seen that this approach yields a natively additive allocation since it forms a telescoping sum[4] :

$$
\begin{aligned}
M_{\text{inc},t_1} &= M(t_1) \\
M_{\text{inc},t_i} &= M(t_i) - M(t_{i-1}) \\
M_{\text{inc},t_n} &= M(t_n) - M(t_{n-1}) \\
\sum_{i=1}^{n} M_{\text{inc},i} &= M(t_1) - M(t_1) + \cdots + M(t_{n-1}) - M(t_{n-1}) + M(t_n) = M(t_n)
\end{aligned}
$$

The incremental allocation can be calculated as or before a new trade is added to the portfolio. It is a risk sensitive value when it is calculated as it accurately reflect how the additional trade changes the risk measure. If the trade is mitigating risk at the time of its inception according to M its incremental allocation M_{inc} is negative. If it increases the risk its M_{inc} is positive. However, M_{inc} does not adapt over time and is likely to loose its accurate risk depiction as additional trade are added to the portfolio. As a portfolio develops it may well be possible, that a trade for which a negative M_{inc} was calculated at its inception may loose its risk mitigation. Due to this property M_{inc} of a given trade should ideally only be used at or before trade inception. One such use case is the PnL calculation of a new trade to determine the performance of the trading desk or trader which initiated the trade. Another would be to use it prior to an investment decision [11]. It can however not be used to analyse an existing portfolio to e.g. identify trades which drive risk or determine how increases or decreases in a given position would impact the portfolio measure. It also cant be calculated deterministically a posteriori for a portfolio without knowing its composition through time.

In the past, some academic work focussed on approximating the incremental VaR as it requires a recalculation of the VaR for the entire portfolio. An overview of these works and their potential pitfalls may be found in [11]. Since this work will rather focus on marginal than incremental allocation further details may be found in the referred paper. In the empirical analysis the incremental allocation will be calculated exactly.

4.4.2 Marginal allocation

Next, we are setting up a function to calculate the *AddOn* for derivatives from the FX asset class. The approach is a little simpler than that for the IR asset class as no differentiaten between time buckets is necessary. All derivatives on a common currency pair can be set off against each other.

[4]For brevity in Notation let $M(t_i)$ be equivalent to $M(t_1 \ldots t_i)$

$$
\begin{aligned}
AddOn^{FX} &= \sum_j AddOn^{FX}_{HS_j} \\
AddOn^{FX}_{HS_j} &= SF^{FX}_j * |\text{EffectiveNotional}^{FX}_j| \\
\text{EffectiveNotional}^{FX}_j &= \sum_{i \in HS_j} \delta_i * d^{FX}_i * MF^{type}_i
\end{aligned}
$$

With the hedging sets $i \in HS$ referencing all derivatives on a common currency pair.

In the following example we will create a swap and display its price.

[2]:
```
# Creating a swap



irs = IRS(notional = 100,
    timeToSwapStart=ql.Period(2, ql.Days),
    timeToSwapEnd=ql.Period(10, ql.Years),
    swapDirection=SwapDirection.PAYER,
    index=InterestRateIndex.EURIBOR6M,
    fixed_rate=0.02
    )

irs.get_price()
```

[2]: -15.11727999472697

[3]:
```
# Let us also create an OIS

ois = OIS(notional = 1000,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          swapDirection=SwapDirection.PAYER,
          index=InterestRateIndex.FEDFUNDS,
          fixed_rate=0.04)

ois.get_price()
```

[3]: -165.77717223100274

Now we shall have a final example of this glorious integration of Jupyter and LaTeX. For this I will create an equity swap and print its price.

Well, I do think that I do need to change a tiny bit here to be honest.

[5]:
```
eqopt = EquityOption(notional = 20,
                     maturity = ql.Period(1, ql.Years),
                     tradeType= TradeType.PUT,
                     underlying=Stock.DBK,
                     strike= 60)

eqopt.get_price()
```

[5]:
```
1024.5398107359952
```

This is a markdown cell that I want to see.

[7]:

	0	1	2
0	1	2	3
1	2	3	4

4.4.3 Euler allocation

4.4.4 Shapley allocation

4.5 Allocation of SA-CCR

4.5.1 Allocation without margining

4.5.2 Allocation under VM collateralization

4.5.3 Allocation under VM and IM collateralization

4.6 Comparison of allocation approaches

4.7 Calculating the Euler allocation

5 Outlook

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