1 Allocation of Risk Measures

With increasing sophistication of risk, own capital and margining models the need for equally sophisticated tools for attributing these measures rises as well. Allocating the variation margin or the current exposure method (CEM) to individual trades is trivial as these measures may just be calculated for an individual trade and then added up across all trades to obtain the correct aggregate value. For measures which take portfolio effects into account such as a state of the art VaR model, ISDA-SIMM or SA-CCR however, this approach is not possible. The advent of portfolio based models for internal risk measurement in the late 1990s and for regulatory risk measurement in the late 2000s sparked research into how such measures should be reallocated. Gregory (**?**, Chapter 10.7) states that three approaches are used in practice:

- Incremental allocation

- Marginal allocation

- Pro rata allocation

Additionally (**?**) Allocation properties

- Native additivity

- Risk sensitivity

- Independent of portfolio constellation

- Stable through time

1.1 Incremental allocation

Incremental allocation can only be applied when observing the development of a portfolio through time. Given a pre-existing portfolio P consisting of n trades t_1 through t_n and a portfolio-based measure M the incremental contribution of the first and second additional trade may be calculated as:

$$M_{\text{inc},t_{n+1}} = M\left(t_1 \ldots t_{n+1}\right) - M\left(t_1 \ldots t_n\right)$$
$$M_{\text{inc},t_{n+2}} = M\left(t_1 \ldots t_{n+2}\right) - M\left(t_1 \ldots t_{n+1}\right)$$

It can be easily seen that this approach yields a natively additive allocation since it forms a telescoping sum[1] :

$$M_{\text{inc},t_1} = M(t_1)$$

$$M_{\text{inc},t_i} = M(t_i) - M(t_{i-1})$$

$$M_{\text{inc},t_n} = M(t_n) - M(t_{n-1})$$

$$\sum_{i=1}^{n} M_{\text{inc},i} = M(t_1) - M(t_1) + \cdots + M(t_{n-1}) - M(t_{n-1}) + M(t_n) = M(t_n)$$

The incremental allocation can be calculated as or before a new trade is added to the portfolio. It is a risk sensitive value when it is calculated as it accurately reflect how the additional trade changes the risk measure. If the trade is mitigating risk at the time of its inception according to M its incremental allocation M_{inc} is negative. If it increases the risk its M_{inc} is positive. However, M_{inc} does not adapt over time and is likely to loose its accurate risk depiction as additional trade are added to the portfolio. As a portfolio develops it may well be possible, that a trade for which a negative M_{inc} was calculated at its inception may loose its risk mitigation. Due to this property M_{inc} of a given trade should ideally only be used at or before trade inception. One such use case is the PnL calculation of a new trade to determine the performance of the trading desk or trader which initiated the trade. Another would be to use it prior to an investment decision **?**. It can however not be used to analyze an existing portfolio to e.g. identify trades which drive risk or determine how increases or decreases in a given position would impact the portfolio measure. It also cant be calculated deterministically a posteriori for a portfolio without knowing its composition through time.

In the past, some academic work focused on approximating the incremental VaR as it requires a recalculation of the VaR for the entire portfolio. An overview of these works and their potential pitfalls may be found in **?**. Since this work will rather focus on marginal than incremental allocation further details may be found in the referred paper. In the empirical analysis the incremental allocation will be calculated exactly.

1.2   Marginal allocation

1.3   Euler allocation

1.4   Shapley allocation

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[1]For brevity in Notation let $M(t_i)$ be equivalent to $M(t_1 \ldots t_i)$