SA-CCR Allocation under consideration of margining

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Abstract

This will be the abstract.

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Chapter 1

Introduction

1.1 Motivation

With increasing sophistication of risk models, own capital models and margining models the need for equally sophisticated tools for allocating these measures rises as well. For any risk metric that considers portfolio effects calculating the contribution to the risk measure of individual trades is a challenge. As part of the Basel 3 reform, regulators have updated the standardized models for market risk own capital requirement and credit risk own capital requirements. The new standardized model for market risk own capital requirements is the FRTB-SA and the new standardized model for credit risk own capital requirements is the SA-CCR model . Both of these models are portfolio based risk models. Gregory [4, Chapter 10.7] states that three allocation approaches are used in practice:

Add appropriate citation to Basel 3 or CRD 2

Citation to FRTB-SA needed

Enter Citation of SA-CCR

- Incremental allocation for pre trade risk checks and for front office incentivization
- Marginal allocation for risk analytics of existing portfolios
- Pro rata allocation if trade contributions must not be negative, if risk sensitivity is not required or if the allocated risk measure does not take portfolio effects into account

These allocation approaches and their advantages and disadvantages are analyzed in further detail in section 2.5. Calculation of pro rata and incremental allocation is fairly straightforward and can generally be performed under any circumstances. Marginal allocation on the other hand is more challenging. This thesis will also include results for incremental and pro rata allocations for the sake of completeness but the main focus will be the analysis of marginal allocation.

Schulze [10] has analytically calculated the marginal allocation for the FRTB-SA. However, an approach to marginal allocation of SA-CCR has not been published yet. This thesis intends to close this gap by showcasing a numerical marginal allocation approach for SA-CCR. A particular challenge in the allocation of SA-CCR is its consideration of margining. This makes the allocation of SA-CCR dependent on margin models which can themselves be significantly more complex than the SA-CCR model itself.

The goal of this thesis is to find an approach for allocating SA-CCR while considering margining. Particular focus is put on the allocation of SA-CCR under consideration of variation margin an an internal initial margin model.

1.2 Literature review

1.3 Structure of the thesis

Throughout this document a small but diverse selection of derivatives is used for exemplary calculations and to explore edge cases of the different models. Section 2.1 does briefly introduce these instruments and the financial models and market data that is used to price them.

Since margining is an integral part of the SA-CCR model, section 2.2 will afterwards introduce different margin types and typical counterparty relations and margin models observed in the market. Section 2.2 will also establish why out of the different margin models, the ISDA SIMM model needs to be investigated the closest for the purpose of this thesis.

Section 2.3 introduces the ISDA SIMM which is the most commonly used model for initial margin calculation of uncleared derivatives. As the ISDA SIMM is based on first order sensitivities, section 2.3 also describes how to calculate ISDA SIMM compliant sensitivities for the financial instruments introduced in section 2.1.

After the different margin components have been introduced, section 2.4 presents the SA-CCR model that is used for the calculation of the EAD of derivatives. Special emphasis will be put on the inclusion of margining.

Section 2.5 presents literature results regarding the allocation of risk measures. Additionally, the theoretical foundation for the Euler allocation is laid out as the investigation if an when an Euler allocation is possible is the main subject for the analysis section of the thesis.

At this point all relevant concepts, models and financial instruments have been introduced to conduct the analysis chapter of this thesis. The main goal is to establish a numerical allocation approach for SA-CCR.

As the received initial margin is a subcomponent of SA-CCR it might be necessary to also allocate initial margin. Due to the complexity of initial margin models allocation of this subcomponent is investigated separately. Section 3.1 establishes numerical approaches for marginal and incremental allocation of initial margin figures.

Finally, section 3.2 uses the results of all previous sections as the basis to analyze the marginal and incremental allocation of SA-CCR. The prerequisites for marginal allocation are higher than those for incremental allocation. Therefore, the section investigates in detail under which circumstances marginal allocation of SA-CCR is possible and examines portfolios that represent edge cases.

Chapter 4 provides an outlook to adjacent topics and other use cases of SA-CCR allocation which might yield different challenges. The chapter also contains the conclusion of the thesis.

For the interested reader, Appendix **??** showcases an architecture blueprint for the implementation of SA-CCR and initial margin allocation as presented in this thesis.

Chapter 2

Applied models and methods

2.1 Instruments, pricing and market data

For the analysis presented in chapter 3 a small but diverse set of financial instruments is required. Due to the structure of the ISDA SIMM and the SA-CCR model the set of financial instruments should meet the following criteria:

1. The instruments should range across multiple asset classes
2. Non-linear instruments should be included
3. The instruments should range across multiple currencies
4. The instruments should be commonly traded as bilateral, uncleared derivatives to be relevant for ISDA SIMM
5. Pricing and sensitivity calculation should be possible without implementation of simulation approaches
6. Inferring market data objects required for pricing from market quotes of traded instruments must be simple

Items 4 and 5 of the above list are slightly conflicting. Bilaterally traded derivatives are usually more complex than cleared derivatives. Due to this increased complexity many of them have to be priced with a Monte Carlo simulation since an analytical solution is not possible.

Item 6 rises from the requirement of the ISDA SIMM model to calculate all sensitivities against market quotes. This means for example, that interest rate sensitivities mustn't be calculated with regard to a movement of the interest rate curve used as a pricing input but with regard to the price of the traded instrument that is used to build the interest rate curve in the first place. In the case of interest rate curves the process to build an interest rate curve is commonly referred to as *bootstrapping* and has to be performed again whenever a sensitivity is calculated to be compliant with

ISDA SIMM. Designing a pricing framework that can handle this required interdependence of market quotes, market data objects such as curves and priced instruments is a steep task even for deceptively simple instruments such as plain vanilla interest rate swaps. For this reason the implementation is based on QuantLib which offers an excellent and proven framework to monitor these interdepencies with ease. Calculation of ISDA SIMM compliant sensitivities of the instruments introduced in this section is the topic of section 2.3.2.2.

Careful consideration of the criteria listed above and the available market data lead to the following set of financial instruments that will be used for analysis:

- European equity options
- Overnight indexed swaps
- Interest rate swaps
- Swaptions

2.1.1 European Equity Option

2.1.1.1 Market data

2.1.1.2 Pricing

2.1.2 Overnight indexed swap

2.1.2.1 Market data

2.1.2.2 Pricing

2.1.3 Interest rate swap

2.1.3.1 Market data

2.1.3.2 Pricing

2.1.4 Swaption

2.1.4.1 Market data

2.1.4.2 Pricing

2.2 Margining

In the context of derivatives, margining refers to the process of posting and receiving collateral in the form of cash or securities to reduce the counterparty credit risk associated with the derivatives.

Counterparty credit risk refers to the risk of a default of the counterparty of a derivative. Derivatives are contracts between two institutions which give raise to future cash flows dependent of the performance of its underlying. These future cash flows may be at risk if the counterparty defaults during the lifetime of the derivative.

Over the past decades several measures were established in the derivatives markets to mitigate counterparty credit risk.

The most impactful measure is close-out netting. Close-out netting is a contractual agreement of two counterparties how their bilateral derivative contracts have to be settled if one of the counterparties defaults. With close-out netting, in case one of the two counterparties defaults all derivatives which are yet to mature are immediately closed out based on the current market value. The market values of the individual derivatives are summed up and the netted amount needs to be paid by whichever party is trailing across the portfolio. In case of a default this close-out netting procedure takes priority over all other claims of creditors against the defaulted counterparty. Close-out netting has two major advantages. First, the non-defaulting counterparty only suffers a counterparty credit loss, if it is ahead across the entire portfolio of currently running derivatives with the defaulted party. Just having a positive market value on a few derivatives does not necessarily result in a counterparty credit loss. Secondly, the immediate close-out of the open derivatives of the defaulted counterparty greatly facilitates unwinding its portfolio. A disadvantage of close-out netting is, that it may proof difficult to find an objective market value of the derivatives that have to be closed out - especially in a stressed market environment, which is likely to be present if e.g. a large investment bank defaults. The contractual obligation to perform close-out netting is agreed upon in a master agreement, which was introduced to the derivatives market by ISDA in 1985. Currently, close out netting is in effect for almost all traded derivatives and it will implicitly always be assumed to be in effect throughout this thesis. More details on close-out netting may be found in [4, Chapter 5].

The second most effective measure in mitigating counterparty credit risk is the exchange of variation margin. If the obligation to post variation margin is agreed as part of a master agreement the accrued mark-to-market of the derivative portfolio has to be collateralized by the trailing counterparty. This measure effectively resets counterparty credit risk to zero for both parties every time a variation margin payment is made or the exchanged variation margin is adjusted to the current market value of the portfolio. The exchange of variation margin was common but not a given in the inter-bank market before the financial crisis of 2008. After the crisis it has become commonplace in the interbank market and recently has even been mandated by regulators[1]. Non-financial counterparties oftentimes do not collateralize their derivatives since they are not mandated to do so, shy away from the operational burden and have a harder time funding the significant amount of cash necessary to cover the current mark-to-market value of their entire derivatives portfolio. Collateralizing a derivatives portfolio not only significantly reduces CCR but also significantly alters how the remaining CCR behaves. The CCR of a collateralized portfolio may rather be driven by the terms of the CSA or residual phenomenon such as collateral spikes than by the underlying instruments.

Explain what a CSA is

Quote something regarding collateral Spikes

As a final measure, initial margin can be exchanged. Historically, initial margin was a collateral amount that was calculated and exchanged once at the inception of a new derivative and held until maturity - hence the name *initial* margin. One common formulation used w.r.t. initial margin, which has also found its way into regulatory documents is that initial margin is collateral, that - in contrast to variation margin - is not based on the MtM of a portfolio or derivative. The idea behind initial margin is that it secures the counterparties against losses that can incur between the last time variation margin has been exchanged prior to a default until the original position has

cite location in regulatory document that does this

[1] In the European Union the exchange of Variation Margin for inter bank bilateral OTC derivatives is compulsory since September of 2016 for large banks or March of 2017 for smaller banks.

been restored. This time period is referred to as the margin period of risk and this time period results as the sum of

1. The contractually agreed regular frequency of variation margin exchanges
2. The time it takes from a counterparty not complying with a margin call to ascertain that the counterparty has indeed defaulted
3. The necessary time to reopen the defaulted derivatives with new counterparties to rehedge the bank and thereby restoring the risk profile held prior to the default of the counterparty

Initial margin should cover the gap between the MtM of the derivatives with the defaulted counterparty when variation margin was last exchanged to the price for which the derivatives are reopened in step 3 under the assumption that the market has moved unfavorably during the MPoR. In the context of CCR, *unfavorable* means that the banks position would have increased in value throughout the margin period of risk and therefore the market price for which the bank repurchases the derivatives is higher than the value at which variation margin was exchanged last.

A more comprehensive introduction to counterparty credit risk and its reduction through netting and margining may be found in chapters four through six of [4].

2.2.1 Market structure and associated margining approaches

The derivative market is divided into exchange traded derivatives, cleared OTC derivatives and uncleared bilateral derivatives. Uncleared bilateral derivatives can either be uncollateralized, collateralized with VM or collateralized with IM and VM. Below, these five counterparty relations are briefly introduced. They are ordered w.r.t. their associated counterparty credit risk.

Uncollateralized bilateral derivatives Derivatives are arranged between two counterparties without involvement of a third party. No collateral is exchanged at any point, only the contractual cashflows of the derivatives are exchanged. The CCR is very high as the entire MtM of the portfolio is at risk. Since no margin is posted at all, the MPoR is the maturity of the traded derivatives and can therefore easily eclipse multiple years. IM posted, IM received and VM are zero at all times.

Bilateral derivatives collateralized with VM The CCR is still significant. When the counterparty defaults the bank can suffer unmitigated losses for a couple of days until it can rebuild its position. The MtM of the portfolio is collateralized with VM. VM exchange is subject to contract parameters such as the threshold, minimum transfer amount or the exchange frequency. Values of these parameters impact how well CCR is mitigated.

Bilateral derivatives collateralized with VM and IM Counterparty credit risk is low. Only in edge cases is it possible that the counterparties credit losses surpass the available collateral. An MPoR of at least 10 days is mandated by the regulator. The IM is calculated with an internal or standardized bilateral initial margin model. Posted and received IM are recalculated daily. VM exchange obeys the same mechanics as for Bilateral derivatives collateralized with VM.

Cleared OTC derivatives Derivatives are initially arranged bilaterally between two counterparties and then cleared by a CCP. The CCP takes over positions in case of a default of either party mitigating any CCR in the traditional sense. The bank has no direct counterparty credit risk. It may however suffer losses to its clearing fund contribution if another Member of the CCP defaults. The MPoR is mandated by the regulator to be five days. The initial margin that is posted by the bank to the CCP is calculated by the CCP with his proprietary internal initial margin model. In line with the assumption that the CCP can not default, the CCP does not post IM to its clearing members. Therefore, the IM received from the perspective of the bank is always zero. The daily PnL of the portfolio is exchanged as VM between the CCP and the Bank.

Exchange traded derivatives Banks enter positions in exchange traded derivatives listed by a CCP. Positions are matched by the CCP and the counterparties of a transaction remain anonymous to each other. Associated CCR and margining is largely the same as for Cleared OTC derivatives but the MPoR is generally below five days since it is assumed that positions in exchange traded derivatives can be closed faster than in cleared OTC derivatives. The used internal initial margin model may differ e.g. since the regulator requires coverage of a 99% quantile instead of the 99.5% mandated for cleared OTC derivatives.

According to [4, Figure 3.2] based on notional 9% of derivatives are exchange traded, 55% are cleared OTC derivatives and 36% are uncleared OTC derivatives. It has to be noted that these figures are from 2014 and it can be assumed, that the fraction of cleared OTC derivatives has increased since then at the expense of the fraction of uncleared OTC derivatives. The reason for this is, that regulators have incentivized or even mandated the clearing of simpler OTC derivatives over the course of the last years. In connection with this development the large CCPs such as Eurex or the LCH have extended the product range for which they offer OTC clearing in recent years.

2.3 Bilateral initial margin

After the 2008 financial crisis the G20 agreed to reduce systematic, counterparty and operational risk and as a result of this commitment has been put into effect by regulators worldwide. In Europe the European Market Infrastructure Regulation (EMIR) came into force in August 2012 and focused on promoting or mandating central clearing as the primary measure to reduce counterparty risk.

2.3.1 The standard approach

The standard approach to calculate bilateral initial margin has been proposed by the bank for international settlement in and has been implemented in European law in . It is a schedule based approach that calculates an IM contribution on a trade by trade basis multiplying the trades notional with a regulatory factor based on the asset class and term to maturity of the trade. The resulting sum may be reduced by up to 60% through the so called net-gross ratio, if the portfolio has a negative present value from the perspective of the calculating bank . For a detailed specification of the aggregation the reader may refer to . The implementation of this approach is addressed in 2.3.2.1

Is it called standard approach or standardized approach?

Insert paragraph and paper of the IM standard approach

Section in CRD II that mandates the standard approach

Check if negative is correct

insert again BIS and CRD definition of standardized approach

2.3.2 The ISDA-SIMM model

In December of 2013 the International Swap and Derivatives Association (ISDA) published a motivation and basic methodological outline of a common internal initial margin model called Standard Initial Margin Model (SIMM™)[14]. The goal of the model is to meet the model requirements to an internal model of all regulators, while being among others easy to replicate, quick to calculate and relatively cheap to operate, implement and validate.

SIMM is a Delta-Gamma VaR model using Delta and Vega sensitivities calculated by the banks themselves and risk weights and correlations provided and recalibrated annually by ISDA. ISDA provides member with a methodological paper [13] and a paper describing the input format of sensitivities [12]. Additionally, the authors of ISDA SIMM have provided a technical paper [15] that lays out the mathematical foundation of the model. The core idea of the model is to multiply sensitivities with risk weights and aggregate them with nested variance-covariance computations.

2.3.2.1 Implementation

As already pointed out, ISDA-SIMM is standardized despite being an internal model. Therefore, all market participants using an internal model for bilateral initial margin calculation are forced to calculate ISDA-SIMM compliant sensitivities, convert them into the CRIF format and to aggregate them to an initial margin figure using the ISDA-SIMM aggregation. The process to create ISDA-SIMM compliant sensitivities is individual to each bank. Many vendor solutions for trading and risk have incorporated the creation of ISDA-SIMM compliant sensitivities and a CRIF into their products but the most suitable way to produce a CRIF still needs to be established on a bank to bank basis.

Aggregation on the other hand is absolutely standardized. It uses a single file, the CRIF, as input does not need any auxiliary market data and returns a single value, the

IM. Considering this, Acadiasoft[2] decided to provide an open source implementation of the ISDA-SIMM aggregation [1]. Acadiasoft is an ISDA-affiliated company who also offers a dispute resolution platform for bilateral initial margin that has become the market standard. For the analysis shown in chapter 3, this open source library was used for aggregation. Therefore, only the ISDA SIMM compliant sensitivities needed to be calculated and parsed into a CRIF entry minimizing potential sources of error and necessary testing effort. The open source library by Acadiasoft also offers functionality to calculate bilateral initial margin according to the standard approach presented in section 2.3.1 using an extended CRIF standard.

2.3.2.2 Calculation of compliant sensitivities

2.4 SA-CCR

Counterparty credit risk is considered to be a part of credit risk by the regulator. Risk weighted assets have to be calculated and need to be backed by own capital. The three main inputs for calculating credit risk are the probability of default (PD) the loss given default (LGD) and the exposure at default (EAD). Assuming the default of a counterparty over the course of the next year, the EAD is the current estimation of money indebted by the counterparty to the bank at the time of default. Estimating EAD for traditional credit instruments s.a. loans, credit cards, mortgages or bonds is relatively simple. Such instruments do often times have deterministic payment schedules making it easy to predict the exposure in one years time. Credit lines or credit cards behave less deterministic but it is still simple to determine an upper bound to the future exposure by assuming that the entire credit line is exhausted. The counterparty credit risk incurred by derivatives has first been regarded in regulatory capital calculation in Basel II [8]. Due to the stochastic nature of derivatives EAD calculation for counterparty credit risk has always been regulated separately ever since. Consideration of CCR in regulatory capital was overhauled as part of Basel III . The regulation for the internal margin model (IMM) approach was adjusted and the current exposure model (CEM) that was introduced with Basel II as the standard approach for CCR EAD calculation was replaced with the SA-CCR model and the simplified SA-CCR model. .

proper citation needed

Include timeline of SA-CCR regulatory documents

The SA-CCR model was implemented for the analysis section of this thesis but will not be presented in a comprehensive fashion here. Instead the reader may refer to the latest regulatory documents and or the library developed for this thesis [5]. This section will highlight the aspects of the SA-CCR model and the simplified SA-CCR model that are of special interest within the scope of this thesis such as the consideration of margin.

CRD2 document

latest EBA update

[2] https://acadiasoft.com/

2.4.1 Consideration of margining in SA-CCR

When using SA-CCR the exposure at default has to be calculated as:

$$
\begin{aligned}
& EAD = \alpha * (RC + PFE) \\
\text{where} \quad & \alpha = 1.4 \\
& RC : \text{Replacement Cost} \\
& PFE : \text{Potential Future Exposure}
\end{aligned}
\tag{2.4.1}
$$

The purpose of the RC is to assess the immediate loss suffered by the default of a counterparty. It is based on the current MtM of the derivative less the accessible collateral. If a bank has posted collateral to non-segregated accounts of a counterparty this collateral is also assumed to be lost in case of a default which increases the replacement cost. The potential future exposure (PFE) on the other hand assesses how the RC might develop in the future. The future being defined as during the next year. If the RC today is 0 but is likely to be larger than 0 in the near future the estimated EAD should take this expected increase in RC into account.

See also Paragraph 130 and 131 of [9]

Paragraph 130 - case without margining:

> For unmargined transactions, the *RC* intends to capture the loss that would occur if a counterparty were to default and were closed out of its transactions immediately. The *PFE* add-on represents a potential conservative increase in exposure over a one-year time horizon from the present date (i.e. the calculation date).

Paragraph 131 - case with margining:

> For margined trades, the *RC* intends to capture the loss that would occur if a counterparty were to default at the present or at a future time, assuming that the closeout and replacement of transactions occur instantaneously. However, there may be a period (the margin period of risk) between the last exchange of collateral before default and replacement of the trades in the market. The *PFE* add-on represents the potential change in value of the trades during this time period.

The PFE is defined as

$$
\begin{aligned}
& PFE = \text{multiplier} * AddOn^{\text{agg}} \\
\text{where} \quad & AddOn^{\text{agg}} : \text{aggregate add-on component} \\
& \text{multiplier} : f(V, C, AddOn^{\text{agg}})
\end{aligned}
\tag{2.4.2}
$$

AddOn is calculated differently for each asset a class. Since no netting is allowed between asset classes the aggregate is calculated as:

$$AddOn^{\text{agg}} = \sum_a AddOn^a$$

Collateralization is taken into account of the PFE calculation through the multiplier that uses the collateral held as an input. As overcollateralization increases, the multiplier decreases. The most important source of overcollateralization is initial margin. However, the multiplier is floored at 5%.

$$\begin{aligned} \text{multiplier} &= \min\left\{1; Floor + (1 - Floor)\exp\left(\frac{V - C}{2(1 - Floor)AddOn^{\text{agg}}}\right)\right\} \\ &\text{where } Floor = 5\% \end{aligned} \tag{2.4.3}$$

The RC is defined as

$$RC = \max\{V - C; TH + MTA - NICA; 0\}$$

$$\begin{aligned} \text{where} \qquad V &: \text{Current portfolio value} \\ C &: \text{Net collateral held} \\ TH &: \text{Threshold} \\ MTA &: \text{Minimum Transfer Amount} \\ NICA &: \text{Net Independent Collateral Amount} \end{aligned} \tag{2.4.4}$$

C is defined according to the $NICA$ definition, which in accordance with paragraph 143 of [9]. By making assumption 1

Could add reasoning to make this assumption

Assumption 1. *Variation margin is posted in unsegregated accounts, initial margin is posted in segregated accounts and initial margin is the only form of overcollateralization.*

the calculation of $NICA$ and C simplifies to:

$$\begin{aligned} C &= \text{Variation Margin balance} + NICA \\ NICA &= \text{Received initial margin} \end{aligned} \tag{2.4.5}$$

Assuming also

Assumption 2. $IM_{received} > TH + MTA \mid IM_{received} > 0$

This assumption is wrong because of the way threshold actually works...

	NICA	C_{calc}	RC
Uncollateralized bilateral derivatives	0	0	V
Bilateral derivatives collateralized with VM	0	VM	TH_{VM}+MTA
Bilateral derivatives collateralized with VM and IM	IM_received	VM+IM_received	0
Cleared OTC derivatives	0	VM	Unclear
Exchange traded derivatives	0	VM	Unclear

Table 2.1: Calculation of NICA, C and RC under different margining approaches

we do yield the results for the five counterparty relation introduced in 2.2 that are displayed in table 2.1. Based on the calculated C in this table, the MTA and threshold need to be taken into account when calculating the collateral that is actually received.

Clarify RC under Clearing, need to tidy up subscripts, 0 for RC und bilateral margin is wrong, clarify what TH_{IM} is.

Regarding the incorporation of a threshold and a MTA we will assume that the mechanics are the following:

1. No threshold for the exchange of variation margin
2. A threshold exists for the exchange of initial margin
3. If IM_{calc} > TH + MTA then IM_{calc} − TH is posted as collateral to cover the initial margin
4. The MTA applies for the combined change of VM and IM

These are the usual rules in place in margin agreements applicable for the case of bilateral derivatives collateralized with VM and IM and are in line with the minimum requirements by the regulators. In line with these rules the received collateral C can be calculated as follows:

Reference these minimum requirements

$$
\begin{aligned}
VM &= \sum_t P(t) \\
IM_{rec} &= \max\left(0, IM_{\text{calc}} - \text{TH}\right) \\
C_{calc} &= VM + IM_{rec} \\
C_t &= \begin{cases} C_{t-1} & \text{if } |C_t - C_{\text{calc}}| < MTA \\ C_{calc} & \text{else} \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\text{where} \qquad P(t) &: \text{Present value of trade } t \\
IM_{\text{calc}} &: \text{calculated IM to be received} \\
\text{TH} &: \text{Threshold} \\
MTA &: \text{Minimum transfer amount} \\
C_{t-1} &: C \text{ calculated in last time period}
\end{aligned}
$$

The case that is analyzed the most in this thesis is Bilateral derivatives collateralized with VM and IM. It is important to note that *RC* is always floored at zero in this case and a change in VM or IM then only impacts the SA-CCR EAD through the use of C in the multiplier calculation of equation 2.4.3. The multiplier is therefore the central point of focus when analyzing the interaction between SA-CCR and margin. The multiplier function is plotted in figure 2.1. The multiplier is ceiled at one if $C > V$, i.e. if the portfolio is overcollateralized which under assumption 1 is the case when the bank receives IM. With increasing overcollateralization the multiplier drops and approaches its floor of 5%. The other factor that drives the multiplier is the portfolios *AddOn*.

This is wrong RC can be larger than 0

The *AddOn* is a portfolio metric that is supposed to represent how quickly the value of the portfolio can rise within the MPoR. The underlying idea is similar to a value at risk and to *AddOn* is designed to be easy to compute while still being portfolio based and taking optionalities into account. Margining does not impact the calculated *AddOn*. Therefore, *AddOn* calculation for SA-CCR is not presented in great detail at this point. The reader is referred to and the library that was implemented for the purpose of this thesis [5].

Cite AddOn Section of SA-CCR

2.5 Allocation of Risk Measures

With increasing sophistication of risk, own capital and margining models the need for equally sophisticated tools for attributing these measures rises as well. Allocating the variation margin or models that disregard portfolio effect entirely such as the current exposure method (CEM) to individual trades is trivial as these measures may just be calculated for an individual trade and then added up across all trades to

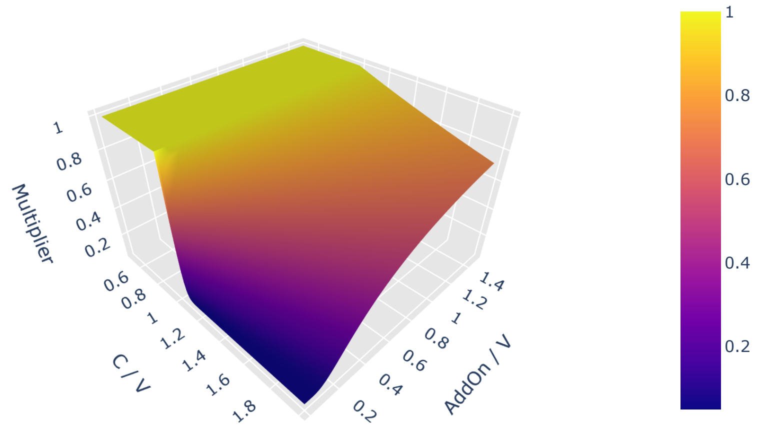


Figure 2.1:

obtain the correct aggregate value. For measures which take portfolio effects into account such as a VaR model, ISDA SIMM or SA-CCR however, this approach is not possible. The advent of portfolio based models for internal risk measurement in the late 1990s and for regulatory risk measurement in the late 2000s sparked research into how such measures should be reallocated. Gregory [4, Chapter 10.7] states that three approaches are used in practice:

1. Incremental allocation
2. Marginal allocation which will be called Euler allocation in this thesis
3. Pro rata allocation

Based on the paper of Koyluoglu and Stoker [6] the list of approaches can be complemented by:

4. Discrete marginal allocation
5. Shapley value

Unfortunately, naming conventions for the different allocation approaches are not consistent between the different publications. Therefore, a definition of the five approaches is following based on the notation used by Tasche [16]. In the following we will always assume that $X_1, \dots, X_n$ are real valued random variables that are representing the profits and losses of the trades in a portfolio. $1, \dots, n$ represents the order in which the trades have been added to the portfolio. X denotes the portfolio-wide PnL, s.t.

$$X = \sum_{i=1}^{n} X_i. \tag{2.5.1}$$

$\rho(X)$ is a risk measure that is supposed to estimate the profit or loss of the portfolio at a certain quantile for a certain time period. Both, the ISDA SIMM model and the SA-CCR model are in their core such risk measures.

The allocation or contribution of trade i to risk measure $\rho(X)$ is denoted as $\rho\left(X_i|X\right)$. Position sizes in the portfolio can be notated through a vector $\mathbf{u} = (u_1, \ldots, u_n)$:

$$X(u) = X(u_1, \ldots, u_n) = \sum_{i=1}^{n} u_i X_i \tag{2.5.2}$$

To make it more convenient to analyze changes to $\mathbf{u}$ we also introduce the function

$$f_{\rho,X}\left(\mathbf{u}\right) \tag{2.5.3}$$

Then, with $\mathbf{1}$ being a vector of ones, $\rho\left(X\left(\mathbf{1}\right)\right) = \rho\left(X\right)$. $\mathbf{u} = \mathbf{1}$ indicates the initial state of the portfolio when calculating an allocation - it does not imply that the notional of each position is 1.

Definition 2.5.4. *Assuming that $\rho\left(X\right)$ is a risk measure, the **incremental allocation** of trade n can be calculated as*

$$\begin{gathered} \text{with } u_{i\neq n} = 1 \text{ and } u_n = 0 \\ \rho_{inc}\left(X_n|X\right) = \rho\left(X\right) - \rho\left(X\left(u\right)\right) \end{gathered} \tag{2.5.5}$$

The incremental allocation can only be calculated for trade n.

Definition 2.5.6 (). *Assuming that $\rho\left(X\right)$ is a risk measure that is homogeneous of degree one and continuously differentiable, the **Euler allocation** of an arbitrary trade i can be calculated assume*

$$\rho_{Euler}\left(X_i|X\right) = \frac{d\rho}{dh}\left(X + hX_i\right)|_{h\to 0} = 1\frac{\partial f_\rho}{\partial u_i} \tag{2.5.7}$$

Definition 2.5.8 (). *Assuming that $\rho\left(X\right)$ is a risk measure, the **pro rata allocation** of an arbitrary trade i can be calculated as*

$$\text{with } u_i = 1 \text{ and } u_{\neq i} = 0 \tag{2.5.9}$$

$$\rho_{ProRata}\left(X_i|X\right) = \frac{\rho(X(u))}{\rho(X)} \tag{2.5.10}$$

Definition 2.5.11. *Assuming that $\rho(X)$ is a risk measure, the **discrete marginal allocation** of an arbitrary trade i can be calculated as*

$$\begin{aligned} &\textit{with } u_i = 0 \textit{ and } u_{\neq i} = 1 \\ &\rho_{discrete}\left(X_n|X\right) = \rho\left(X\right) - \rho\left(X\left(u\right)\right) \end{aligned} \tag{2.5.12}$$

To calculate the Shapley allocation of a portfolio one needs to iterate through all permutations how the trades in the portfolio could be ordered. For a given trade i the Shapley allocation is the average of the amount by which the risk measure changes when adding trade i to the portfolio in each of the permutations.

Definition 2.5.13.

Need to put Shapley in a formula...

The usefulness of any of the five allocation approaches listed above is dependent on the individual application of the allocation. Criteria by which the allocation approach is judged are also highly dependent of the application. However, the two criteria

1. Native additivity
2. Risk sensitivity

are usually regarded to be the most important one. They are for example the criteria by which Koyluoglu and Stoker [6] compare the different allocation approaches.

A allocation algorithm *alloc* exhibits native additivity if equation 2.5.14 holds.

$$\sum_{i=1}^{n} \rho_{alloc}\left(X|X_i\right) = \rho\left(X\right) \tag{2.5.14}$$

Risk sensitivity means that $\rho_{alloc}\left(X|X_i\right)$ should indicate how the trade i impacts the overall risk $\rho\left(X\right)$. No mathematical definition is available for when an allocation is considered to be risk sensitive. A sensible criteria could be that a trade that reduces the risk of the portfolio, i.e. a hedge trade should have a negative contribution to the risk measure.

Depending on the application of the allocation other criteria might be important such as

- Non-negativity of allocations
- The value allocated to a trade must not change through time
- The allocated value needs to be independent from the order in which trades are entered

Generally, such auxiliary requirements raise through operational or technical limitations. Some of the allocation algorithms presented above comply with them, while others do not. Such requirements might be the reason that allocation algorithms that are dismissed as inappropriate in this thesis still find application in the field.

The incremental allocation excels for use at the trading desk. It is well suited as an input when making investment decisions or for calculating the remuneration of traders and trading desks after entering a new trade. Both, Gregory [4] and Koyluoglu and Stoker [6] state that incremental allocation is the best suited allocation for these purposes. It does, however, perform poorly for risk analysis of an existing portfolio. Incremental allocation is further investigated in section 2.5.1.

Euler allocation on the other hand is well suited for analysis of an existing portfolio. It can identify concentration risk within an portfolio or be used for portfolio optimization. In the literature, Euler allocation is generally regarded as the best allocation approach for such purposes as it exhibits native additivity and risk sensitivity and can be calculated for all trades.

Pro rata allocation is generally not risk sensitive for risk measures that take portfolio effects into account. It is, however, very inexpensive to compute, suitable for models that are based on trade contributions such as the CEM or the standardized approach for initial margin (see 2.3.1) and may circumvent some operational or technical issues as trade contributions are always positive. Due to its simplicity and packing risk sensitivity for the models analyzed in this thesis, pro rata allocation will not be analyzed in further detail in this thesis.

While being a very intuitive approach, performance of discrete marginal allocation is relatively poor. The approach does not exhibit native additivity as Tasche [16] shows that

$$\rho_{discrete}\left(X_i|X\right) \leq \rho_{Euler}\left(X_i|X\right)$$

for ρ that are continuously differentiable, sub-additive and homogeneous of degree 1. Koyluoglu and Stoker [6] mention that "it could be argued that discrete marginal allocation is wholly dominated by the Euler allocation".

Check if this is an appropriate way to cite directly

Finally, the Shapley method introduced in [11], like the pro rata model, can not result in negative contributions but exhibits native additivity without the necessity of being normalized by division through $\rho(X)$. With no negative contributions being possible it can be argued, that the Shapley allocation is not risk sensitive. It is certainly considered by authors to be less risk sensitive that the Euler allocation. Since it exhibits natural additivity the Shapley allocation might be considered superior to the pro rata allocation. However, its computation is much more time consuming than any other allocation presented. Therefore it can only be realistically used for very small portfolios or to calculate allocations of subportfolios, e.g. the subportfolios of certain departments. Koyluoglu and Stoker [6] compare Euler and Shapley allocation and find that Shapley allocation is a more robust measure as it does not require differentiability of ρ. The relatively rigorous requirements against ρ to use Euler allocation are introduced in detail in section 2.5.2 and chapter 3 investigates under which circumstances the ISDA SIMM and SA-CCR model comply with these requirements. Overall, Koyluoglu and Stoker suggest to only use Shapley allocation over the Euler allocation for calculating the contribution of few subportfolios if the political cost or confusion caused by negative contributions is considered to be too high.

Non-negativity of Shapley allocation contradicts the results by FIS - investigate

2.5.1 Incremental allocation

Incremental allocation can only be applied when observing the development of a portfolio through time. Given a pre-existing portfolio P consisting of n trades t_1 through t_n and a portfolio-based measure M the incremental contribution of the first and second additional trade may be calculated as:

$$M_{\text{inc},t_{n+1}} = M\left(t_1 \ldots t_{n+1}\right) - M\left(t_1 \ldots t_n\right)$$
$$M_{\text{inc},t_{n+2}} = M\left(t_1 \ldots t_{n+2}\right) - M\left(t_1 \ldots t_{n+1}\right)$$

It can be easily seen that this approach yields a natively additive allocation since it forms a telescoping sum[3] :

$$\begin{aligned} M_{\text{inc},t_1} &= M(t_1) \\ M_{\text{inc},t_i} &= M(t_i) - M(t_{i-1}) \\ M_{\text{inc},t_n} &= M(t_n) - M(t_{n-1}) \\ \sum_{i=1}^{n} M_{\text{inc},i} &= M(t_1) - M(t_1) + \cdots + M(t_{n-1}) - M(t_{n-1}) + M(t_n) = M(t_n) \end{aligned}$$

The incremental allocation can be calculated as or before a new trade is added to the portfolio. It is a risk sensitive value when it is calculated as it accurately reflect how the additional trade changes the risk measure. If the trade is mitigating risk at the time of its inception according to M its incremental allocation M_{inc} is negative. If it increases the risk its M_{inc} is positive. However, M_{inc} does not adapt over time and is likely to loose its accurate risk depiction as additional trades are added to the portfolio. As a portfolio develops it may well be possible, that a trade for which a negative M_{inc} was calculated at its inception may loose its risk mitigation. Due to this property M_{inc} of a given trade should ideally only be used at or before trade inception. One such use case is the PnL calculation of a new trade to determine the performance of the trading desk or trader which initiated the trade. Another would be to use it prior to an investment decision [17]. It can however not be used to analyze an existing portfolio to e.g. identify trades which drive risk or determine how increases or decreases in a given position would impact the portfolio measure. It also cant be calculated deterministically a posteriori for a portfolio without knowing its composition through time.

2.5.2 Euler allocation

The idea of Euler allocation is based on Euler's homogeneous functions theorem.

Definition 2.5.15. *A function f is a positive homogeneous function to a degree of k if*

$$f\left(\alpha\mathbf{x}\right) = \alpha^k f\left(\mathbf{x}\right) \tag{2.5.16}$$
$$\textit{for } \alpha > 0 \tag{2.5.17}$$

[3]For brevity in Notation let $M(t_i)$ be equivalent to $M(t_1 \ldots t_i)$

A function would be homogeneous rather than just *positive* homogeneous if equation 2.5.16 would also hold for $\alpha < 0$. Risk measures can only exhibit positive homgenity. Many risk measures do have the property that doubling position size does double the measured risk. However, inverting the postion, e.g. having a short instead of a long position does not result in a negative risk estimate.

Euler's homogeneous functions theorem states

Theorem 2.5.18. *Let $f(\mathbf{x})$ be a homogeneous function of degree k, then*

$$x_i \frac{\partial f}{\partial x_i} = kf(\mathbf{x}) \tag{2.5.19}$$

With if we assume that $k = 1$, use our risk measure $\rho(\mathbf{u})$ as a function of invested position size with $\mathbf{u} = \mathbf{1}$ being the current position size we yield

$$1 \frac{\partial \rho(\mathbf{u})}{\partial u_i} = \rho(\mathbf{u}) \tag{2.5.20}$$

which is what is stated in definition 2.5.6.

While $u = \mathbf{1}$ is defined as the current position size we can also define it as the notional in USD invested in the individual trades, i.e. $\mathbf{n} = (notional_1, \ldots, notional_n)$.

The Euler allocation w.r.t trade i would then be calculated as

$$notional_i \frac{\partial \rho(\mathbf{n})}{\partial notional_i} = \rho(\mathbf{n}) \tag{2.5.21}$$

As any partial derivative, $\frac{\partial \rho((u))}{\partial u_i}$ may be approximated as a finite difference.

Need to investigate if investment of 1 is right or investment of notional. Shouldn't one be equivalent to a relative bump and the other be equivalent to a relative bump?

$$\text{with } \mathbf{h} = (h_0, \ldots, h_n) \text{ and } h_i = \epsilon > 0 \text{ and } h_{\neq i} = 0$$

$$\frac{\partial \rho(\mathbf{u})}{\partial u_i} = \frac{\rho(\mathbf{u}+\mathbf{h}) - \rho(\mathbf{u}-\mathbf{h})}{2\epsilon} + \mathcal{O}(\epsilon^2) \qquad \text{(central difference)} \tag{2.5.22}$$

$$\frac{\partial \rho(\mathbf{u})}{\partial u_i} = \frac{\rho(\mathbf{u}+\mathbf{h}) - \rho(\mathbf{u})}{\epsilon} + \mathcal{O}(\epsilon) \qquad \text{(forward difference)} \tag{2.5.23}$$

$$\frac{\partial \rho(\mathbf{u})}{\partial u_i} = \frac{\rho(\mathbf{u}) - \rho(\mathbf{u}-\mathbf{h})}{\epsilon} + \mathcal{O}(\epsilon) \qquad \text{(backward difference)} \tag{2.5.24}$$

Closed form formulas for contribution derived for standard deviation based models, VaR models, conditional VaR models.

Could show this for a single variance covariance submatrix of ISDA SIMM?!

2.5.3 Shapley allocation

Inclusion of this section optional

Chapter 3

Results

3.1 Allocation of initial margin

The goal of this section is to investigate if a numerical Euler allocation of ISDA SIMM is possible.

As pointed out in section 2.5.2 a risk measure needs to exhibit positive homogeneity of degree 1 to be able to perform an Euler allocation. In a first step we can investigate by calculating ISDA SIMM for a single trade whether ISDA SIMM does exhibit positive homogeneity for a minimal example.

For this we set up an USD Libor IRS with ten years time to maturity and a notional of 200 billion USD. This is our initial portfolio $\mathbf{u}$. ISDA SIMM would fulfill the required positive homogeneity condition if $a\rho(\mathbf{u}) = \rho(a\mathbf{u}$ for $a > 0$. In figure 3.1 $\rho(a\mathbf{u})$ is plotted for $0 < a \leq 2$ in blue. The function exhibits homogeneity for $0 < a < 1.4$ but not for higher a. The reason for this is, that at this point the concentration risk charge of ISDA SIMM does kick in. The concentration risk for interest rate risks for our minimal example is defined as [13, Article 7.b]

find the exact point where homogeneity breaks

$$CR = \max\left(1, \left(\frac{|\sum s|}{T}\right)^{1/2}\right)$$

with s being the sensitivities against USD interest rate risk and T being 230Mn USD as specified in [13, Article 74]. Due to subsequent variance-covariance aggregation the concentration risk impacts the calculated IM as

$$IM_{\text{with conc. risk}} = CR^2 \cdot IM_{\text{without conc. risk}}$$

This causes the change in slope and implied loss of homogeneity visible in figure 3.1. If the portfolio would consist of a more diverse set of risk factors than the minimal example displayed in figure 3.1 the associated concentration risk would kick in at different levels of a. The slope of the function would increase with each additional concentration risk not being floored at one any more.

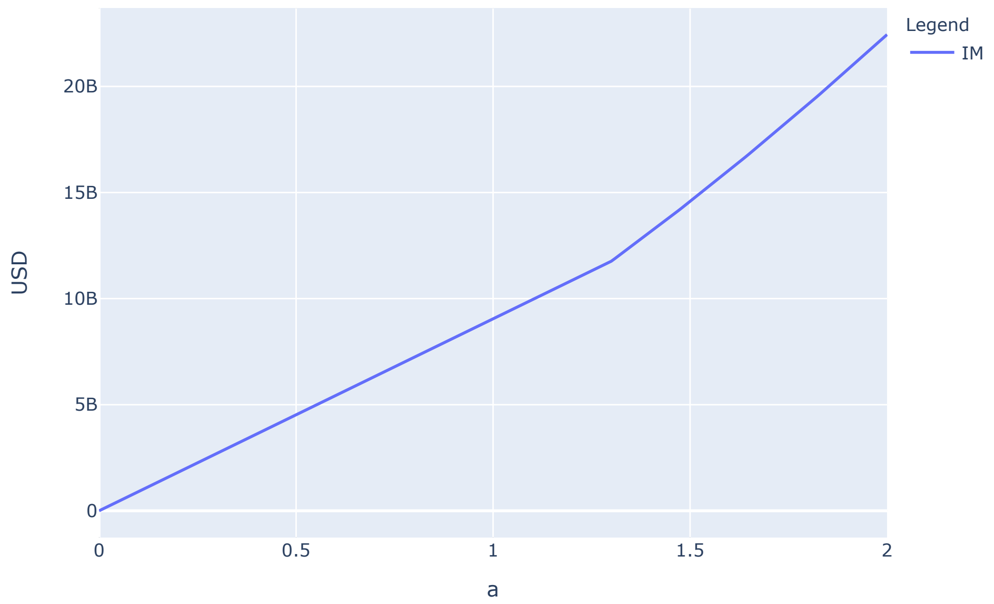


Figure 3.1:

It is important to note that as soon as the sensitivity against a single risk factor in the portfolio is above the concentration threshold the ISDA SIMM risk measure does not exhibit homogeneity anymore.

Even a trivial example with just one trade is sufficient to show that Euler allocation does not work in the inhomogeneous part of the ISDA SIMM equation. For this, we compare two sample portfolios one consisting of one USD IRS with 200 bn notional and one consisting of one USD IRS with 400 bn notional. Critically, the second portfolio is penalized by the model since its USD IRS risk is too large. We calculate the Euler calculation with a forward finite difference approach as displayed in equation 2.5.23.

Assuming that we calculate the finite difference with an $\epsilon = 0.0001$ this means that we calculate the ISDA SIMM of an IRS with 200Bn notional ($SIMM_{200Bn}$) and the ISDA SIMM of an equivalent IRS with 200.02 Bn notional ($SIMM_{200.02Bn}$) and this yields an Euler allocation to this trade as

$$\frac{SIMM_{200.02Bn} - SIMM_{200Bn}}{0.0001} = 9.04Bn$$

We can see in figure 3.1 that this value is both, the slope and the IM value at $a = 1$. The portfolios IM was correctly fully allocated to the single trade of which it consists.

However, performing the same calculation for an equivalent IRS with 400Bn notional yields

$$\frac{SIMM_{400.04Bn} - SIMM_{400Bn}}{0.0001} = 33.67Bn$$

again, we can refer to figure 3.1 to check if this is a reasonable allocation result. As $a = 1$ represents the IM charge for investing 200Bn of notional in the IRS, $a = 2$ represents an investment of 400Bn notional. The associated IM is just 22.44Bn - allocating 33.67Bn of the risk measure to the only trade in the portfolio is therefore clearly wrong. The Euler allocation of 33.67Bn can also be read off figure 3.1 - it is the slope at $a = 2$ times two.

3.2 Allocation of SA-CCR

3.2.1 Homogeneity of SA-CCR

3.2.1.1 Homogeneity of C

To allocate SA-CCR under consideration of margining, the available collateral C is of special interest. As pointed out in table 2.1 depending on the margining approach, C can be calculated as $C = \text{VM}$ or $C = \text{VM} + \text{IM}_{\text{received}}$.

The actually exchanged collateral, however has to be calculated under consideration of the threshold and minimum transfer amount as displayed in equation **??** . With this consideration of threshold and minimum transfer amount C is not a homogeneous function. Fix reference

This can exemplary seen in figures 3.2 and 3.3. These figures display C for an at the money 10Y USD interest swap and the same swap with a lower fixed rate making it an in the money swap. Again, a very high notional of 200 Bn is chosen to showcase the concentration risk charge of ISDA SIMM and the threshold and minimum transfer amount have also been chosen to be very high at 2Bn and 1Bn USD. Figure 3.3 also showcases the trivial result that the Variation Margin is a homogeneous function - the value of the trade scales linearly with the notional.

To be able to calculate an Euler allocation of SA-CCR one has to calculate C for use in 2.4.3 without recognition of the minimum transfer amount as the C_{calc} as defined in equation 3.2.1.

$$C_{calc} = VM + IM_{rec} \tag{3.2.1}$$

If only a MTA but no threshold is in place, no further adjustment is necessary. This can exemplary been seen in the next example. Again, similar to the exemplary calculation in section 3.1 it can be shown with a trivial example, that Euler allocation

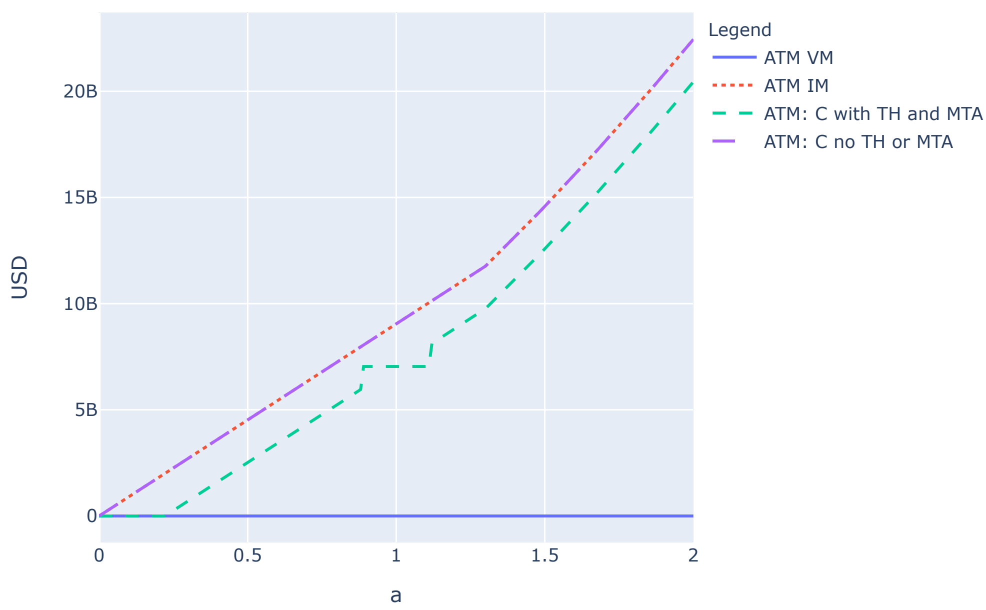


Figure 3.2: VM, IM, C and C under consideration of TH and MTA for a portfolio consisting of a single at the money interest rate swap. Values are calculated based on different notionals invested in the IRS with $a = 1$ referring to a notional of 200Bn USD. More details on creation can be found in Appendix B.2.

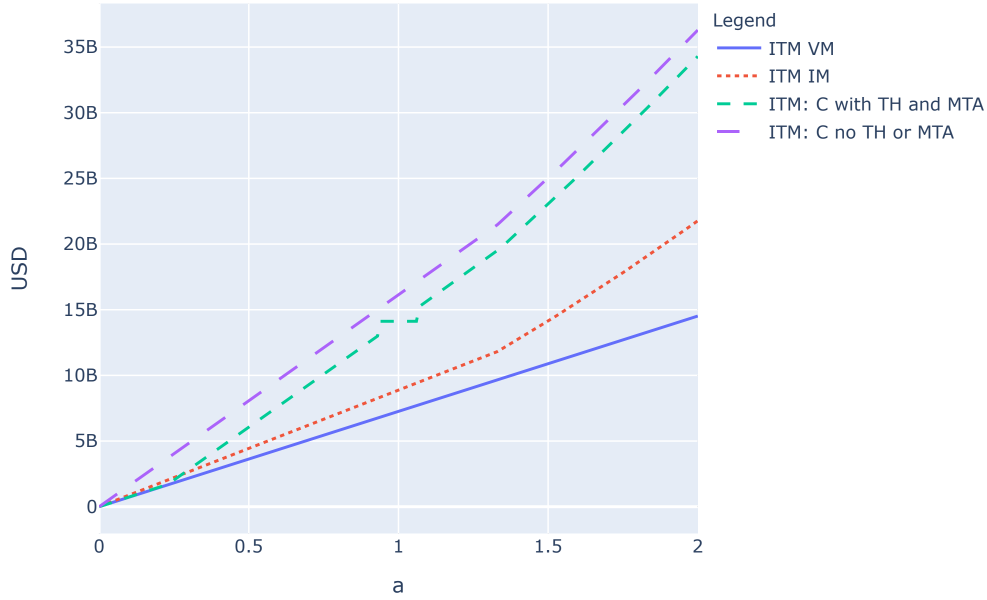


Figure 3.3: Same as figure 3.2 but for an in the money trade with an payer interest rate at 2% which is below par.

of SA-CCR is not possible under recognition of a MTA. For this we consider the same 200Bn IRS as in section 3.1. We assume that the currently posted margin is 9.04Bn which is the calculated ISDA-SIMM margin. Setting C at 9.04Bn in 2.4.3 and then calculating the SA-CCR EAD for this single IRS yields an regulatory EAD of 582.8Mn USD. Any natively additive allocation should allocate this full amount to the IRS. The VM is zero as the IRS is struck at par. In table 3.2.1.1 we assume that the initially received collateral is the currently calculated collateral. When calculating the Euler allocation with a forward difference in line with 2.5.23 the received collateral when rising the notional to 200.02Bn increases when no MTA is assumed while it remains unchanged with consideration of an MTA. Ultimately, this difference leads to a correct allocation of the entire EAD to the single IRS with the *No MTA* approach while the *MTA* approach obviously yields a wrong result by allocating 244% of the portfolios EAD to its only trade.

as specified in …

Critically, this result can also be transferred to any SA-CCR allocation approach that would treat C as an externally given constant value as locally, treating C as a constant value is the same as consideration of a MTA. In both cases the slope of C is zero.

Euler allocation of SA-CCR also does not work if the calculated C does not exhibit homogeneity, i.e. if a concentration risk threshold of the ISDA-SIMM model is exceeded. Again using the 400Bn IRS from 3.1 we calculate an $\text{EAD}_{400\text{Bn IRS}}$ of 843.5Mn but, despite not applying an MTA, Euler allocation yields a vastly different amount

	$\text{SA-CCR}_{\text{MTA}}$	$\text{SA-CCR}_{\text{No MTA}}$
Initial C	9038.2Mn	9038.2Mn
$\text{EAD}_{\text{200Bn IRS}}$	582.8Mn	582.8Mn
$\text{C}_{\text{200Bn IRS}}$	9038.2Mn	9038.2Mn
$\text{C}_{\text{200.02Bn IRS}}$	9038.2Mn	9039.1Mn
$\text{EAD}_{\text{200.02Bn IRS}}$	583.0Mn	582.9Mn
$\frac{\text{EAD}_{\text{200.02Bn IRS}}-\text{EAD}_{\text{200Bn IRS}}}{0.0001}$	1425.3Mn	582.8Mn

Table 3.1: Numerical Euler allocation of SA-CCR with and without consideration of a minimum transfer amount for an example of a portfolio with a single 200Bn notional IRS. Euler allocation is only successful if the MTA is not considered for the recalculation of the received margin C. A threshold of 0 is assumed.

of $\frac{\text{EAD}_{\text{200.02Bn IRS}}-\text{EAD}_{\text{200Bn IRS}}}{0.0001} = 201.0Mn$. If for a given portfolio C does not exhibit homogeneity, neither does SA-CCR and therefore Euler allocation of SA-CCR is not possible for such portfolios.

Finally, consideration of the threshold is challenging. When calculating C with a threshold, C does not exhibit homogeneity and an allocation will fail. On the other hand, when calculating C without threshold the considered margin is to high and the sum of allocated EAD will be too low. One solution to this is to first allocate the SA-CCR assuming the threshold is 0 and then scaling up the allocation by $EAD_{TH=0}$ EAD where $EAD_{TH=0}$ is the calculated SA-CCR EAD assuming a zero threshold and EAD is the actual SA-CCR EAD.

This implies the following important result:

Euler allocation of SA-CCR is not possible if C is assumed to be constant $\neq 0$. C has to be a homogeneous function w.r.t. the position size and a possible minimum transfer amount has to disregarded for the calculation of the Euler allocation.

Unlike the MTA, the threshold should be taken into consideration when calculating an Euler allocation. Inclusion of the threshold does not break the calculation of the Euler allocation, since both cases of equation **??** exhibit homogeneity in line with its definition 2.5.16 for a certain range around a. Numerical calculation of the Euler allocation only malfunctions due to inclusion of the threshold if $C(1) = 0$ but $C(1+\epsilon) = C_{\text{calc}}$.

notation (1), (1+epsilon) needs to be refinded

3.2.1.2 Euler allocation of hedge trades

As pointed out in 2.5 Euler allocation is a risk sensitive allocation and as such does generally attribute negative contributions to trades that are decreasing the risk of the portfolio. If we consider a portfolio of a 200Mn payer IRS, an equivalent 100Mn receiver IRS and 1Mn long stock call options we yield the result depicted in 3.2.1.2

when calculating the Euler allocation numerically with a forward difference. The

	SA-CCR	ISDA SIMM
$\text{IRS}_{\text{100Mn Rec}}$	-246k	-4.52Mn
$\text{IRS}_{\text{200Mn Pay}}$	493k	9.04Mn
Equity Option	1.30Mn	6.95Mn
Sum of allocations	1.54Mn	11.47Mn
Portfolio risk measure	1.54Mn	11.47Mn

Table 3.2:

100Mn receiver IRS partially hedges the risk induced by the 200Mn payer IRS. Both, the ISDA SIMM and the SA-CCR model do not recognize any hedge effects across asset classes and therefore the risk associated with the equity option is completely independent from the two IRS trades. Appropriately, a negative IM and EAD is allocated to the smaller IRS trade and the allocation exhibits native additivity as the sum of the allocation of the three trades coincides with the risk measures of the portfolio.

However, an allocation

	Forward	Central	Backward
$\text{IRS}_{\text{200Mn Rec}}$	188k	0k	-188k
$\text{IRS}_{\text{200Mn Pay}}$	188k	0k	-188k
Equity Option	1.32Mn	1.32Mn	1.32Mn
Sum of allocations	1.69Mn	1.32Mn	944k
Portfolio risk measure		1.32Mn	

Table 3.3:

	Forward	Central	Backward
$\text{IRS}_{\text{200Mn Rec}}$	4.52Mn	0.00Mn	-4.52Mn
$\text{IRS}_{\text{200Mn Pay}}$	4.52Mn	0.00Mn	-4.52Mn
Equity Option	6.95Mn	6.95Mn	6.95Mn
Sum of allocations	15.99Mn	6.95Mn	-2.09Mn
Portfolio risk measure		6.95Mn	

Table 3.4:

3.2.2 Allocation without margining

3.2.3 Allocation under VM collateralization

3.2.4 Allocation under VM and IM collateralization

Chapter 4

Outlook and conclusion

4.1 Outlook

4.2 Conclusion

Remove 2cm Margin from preamble

Appendix A

Architectural blueprint for SA-CCR allocation

Appendix B

Detailed documentation of results

The purpose of this section is to document how the results presented primarily in section 3 were computed. For the purpose of this thesis a library has been implemented in Python and Java that can be found at [5]. This implemented library has a very similar architecture to the one presented in A. Using this library, results for this thesis were computed within Jupyter Notebooks. These Jupyter notebooks are presented in this section. Some, more technical notebook cells are not displayed for brevity.

B.1 ISDA SIMM homogeneity for portfolio of a single trade

We want to showcase that the concentration risk addOn breaks homogeneity of the ISDA SIMM risk measure. The concentration threshold for USD interest rate risk is 230Mn USD per Basis Point change. Considering that IRS trades roughly have a delta of one against the interest rate this means that a trade with a notional of $\frac{230\text{Mn}}{0.0001} = 2300\text{Bn}$ and a maturity of 1 year would incur a risk above the threshold. If the maturity increases to 10 years a notional of roughly 230Bn should be enough to exceed the concentration threshold.

[In]:
```
notional = 200000000000

irs = IRS(notional = notional,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          swapDirection=SwapDirection.PAYER,
          index = InterestRateIndex.USDLIBOR3M)

simm_sensis = irs.get_simm_sensis()
```

```
ir_delta = sum([float(entry['amountUSD']) for entry in simm_sensis␣
↪if entry['riskType'] == 'Risk_IRCurve'])
ir_delta
```

[Out]: 177293785.56303406

The trade has an aggregated delta sensitivity against a 1BP move of the USD interest rate of 177,293,786 USD. With the available market data the delta of the IRS appears to be slightly lower than one.

We create a collateral agreement with associated ISDA SIMM and SA CCR model and load the irs in the portfolio. The collateral agreement uses ISDA SIMM for IM calculation.

[In]:
```
ca = CollateralAgreement(initialMargining=InitialMargining.SIMM,
                         margin_currency=Currency.USD)
ca.link_sa_ccr_instance(SA_CCR(ca))
ca.add_trades(irs)
```

We now want to investigate if the ISDA SIMM exhibits homogeneity for this single trade portfolio. To do so we test if

$$
\begin{gathered}
f(\alpha \mathbf{x}) = \alpha^k f(\mathbf{x}) \\
\text{for } \alpha > 0
\end{gathered}
$$

holds. We test for $0 < \alpha \leq 2$ with an increment size of 0.01.

[In]:
```
bumps = arange(0,2.01,0.01)
resultDataframe = pd.DataFrame(columns = ['USD','k','Legend'])
```

We create a utility function that supports IM, VM and Collateral although we are just exploring IM right now.

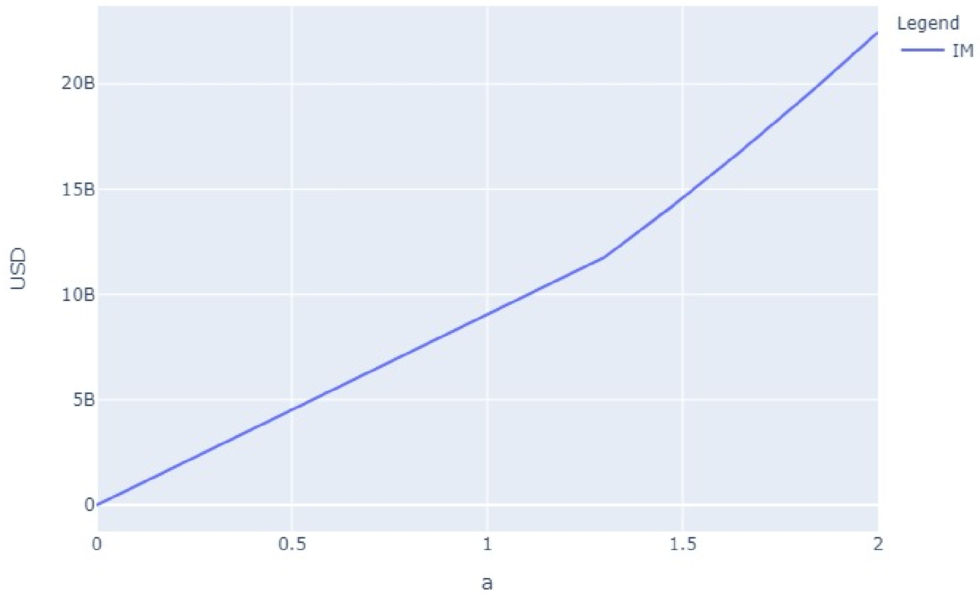
[In]:
```
def bump_and_get_results(bumpsize, trade, collateralagreement):
    record = {}
    record['Bumpsize']=bumpsize
    bumped_copy = trade.get_bumped_copy(rel_bump_size=bumpsize-1)
    collateralagreement.remove_all_trades()
    collateralagreement.add_trades(bumped_copy)
    record['IM'] = collateralagreement.get_im_model().
↪get_risk_measure()
    record['VM'] = collateralagreement.get_vm_model().
↪get_risk_measure()
    record['Collateral'] = collateralagreement.get_C()
    collateralagreement.remove_all_trades()
```

```
    collateralagreement.add_trades(trade)
    return record
```

[In]:
```
for bump in bumps:
    result = bump_and_get_results(bump, irs, ca)
    im_record = {'X': result['Bumpsize'],
                 'Y': result['IM'],
                 'Legend': 'IM'}
    resultDataframe = resultDataframe.append(im_record,␣
 ↪ignore_index=True)
```

[In]:
```
fig = px.line(resultDataframe[resultDataframe.Legend == 'IM'],␣
 ↪x='X', y='Y', color='Legend', line_dash='Legend')
fig.update_layout(xaxis_title = 'a', yaxis_title = 'USD')
img_bytes = fig.to_image(format='jpeg')
Image(img_bytes)
```

[Out]:



As can be seen, ISDA SIMM does not exhibit homogeneity. Further discussion of this in section 3.1.

B.1.1 SA-CCR and ISDA SIMM Euler allocation under a perfect hedge

The goal is to create a portfolio with a perfect hedge and see if and under which circumstances EAD allocation is still possible. We load two perfectly offsetting IRS (one payer, one receiver). To avoid the unrealistic case of a zero IM and EAD portfolio we add an unrelated equity option into the portfolio.

[In]:
```
IRS_pay = IRS(notional=100000000,
              timeToSwapStart=ql.Period(2, ql.Days),
              timeToSwapEnd=ql.Period(10, ql.Years),
              swapDirection=SwapDirection.PAYER,
              index=InterestRateIndex.USDLIBOR3M)

IRS_rec = IRS(notional=100000000,
              timeToSwapStart=ql.Period(2, ql.Days),
              timeToSwapEnd=ql.Period(10, ql.Years),
              swapDirection=SwapDirection.RECEIVER,
              index=InterestRateIndex.USDLIBOR3M)

eqOpt = EquityOption(notional = 1000000,
                     underlying=Stock.ADS,
                     tradeType = TradeType.CALL,
                     tradeDirection = TradeDirection.LONG,
                     maturity = ql.Period(1, ql.Years))

ca = CollateralAgreement()
ca.link_sa_ccr_instance(SA_CCR(ca))
```

[In]:
```
ca.add_trades([IRS_pay, IRS_rec, eqOpt])
print(ca.get_sa_ccr_model().get_risk_measure())
print(ca.get_im_model().get_risk_measure())
```

```
1319277.1719911932
6952717.387110085
```

We now create an Euler allocator which can be used to perform a numerical Euler allocation of the ISDA-SIMM IM or the SA-CCR EAD risk measure. The allocator can be set to use forward, backward or central differentiation. We will see that the differentiation approach makes a big difference for this perfectly hedged portfolio.h

[In]:
```
eulerAllocator = EulerAllocator(ca)
im_alloc_forward = eulerAllocator.allocate_im()
saccr_alloc_forward = eulerAllocator.allocate_ead()
```

```
eulerAllocator.fdApproach2=FdApproach2.Central
im_alloc_central = eulerAllocator.allocate_im()
saccr_alloc_central = eulerAllocator.allocate_ead()

eulerAllocator.fdApproach2=FdApproach2.Backward
im_alloc_backward = eulerAllocator.allocate_im()
saccr_alloc_backward = eulerAllocator.allocate_ead()
```

Below the resulting allocation for the IM is displayed. The allocation only exhibits nativ additivity when using the central difference appraoch since then the allocated values sum up to the IM value of 6952717.39 USD.

[In]:
```
display_table(im_alloc_forward, im_alloc_central, im_alloc_backward)
```

[Out]:

	Backward	Central	Forward
IRS_Long	-4.518969e+06	-2.421718e-01	4.518969e+06
IRS_Short	-4.519001e+06	-6.323463e+01	4.518937e+06
EquityOption_Long	6.952717e+06	6.952717e+06	6.952717e+06
Sum	-2.085253e+06	6.952654e+06	1.599062e+07

Below the resulting allocation for the EAD is displayed. The allocation only exhibits nativ additivity when using the central difference appraoch since then the allocated values sum up to the IM value of 1319277.17 USD.

[In]:
```
display_table(saccr_alloc_forward, saccr_alloc_central,␣
↪saccr_alloc_backward)
```

[Out]:

	Backward	Central	Forward
IRS_Long	-1.877916e+05	4.854053e-02	1.877916e+05
IRS_Short	-1.877853e+05	1.267496e+01	1.877979e+05
EquityOption_Long	1.319277e+06	1.319277e+06	1.319277e+06
Sum	9.437004e+05	1.319290e+06	1.694867e+06

The reason for the Euler allocation not working is that the SA-CCR is not differentiable in case of a perfect hedge. This can be shown by plotting the function SA-CCR w.r.t. the position size in the three trades.

[In]:
```
bumps = arange(-0.05, 0.06, 0.01)
```

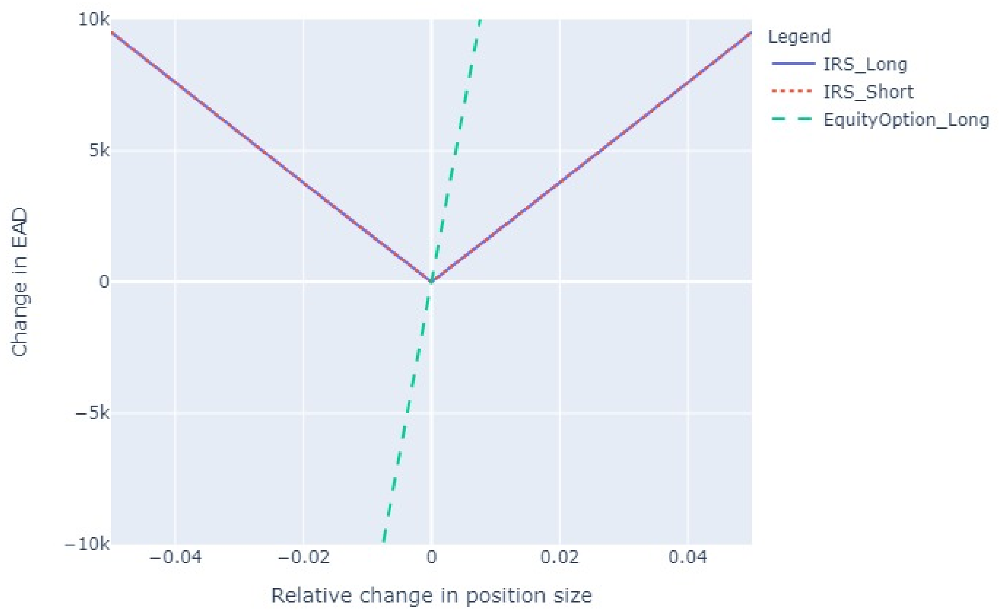
[In]:
```
def bump_one_trade_and_return_diff(bump, trade: Trade, ca:␣
↪CollateralAgreement, method):
    base = method()
    ca.remove_trades(trade)
    bumped_trade = trade.get_bumped_copy(rel_bump_size=bump)
    ca.add_trades(bumped_trade)
```

```
    result = method()
    ca.remove_trades(bumped_trade)
    ca.add_trades(trade)
    return result-base
```

[In]:
```
result_df = pd.DataFrame()

for t in ca.trades:
    for bump in bumps:
        record = {'Relative change in position size': bump,
                  'Change in EAD':␣
↪bump_one_trade_and_return_diff(bump, t, ca, ca.get_sa_ccr_model().
↪get_risk_measure),
                  'Legend': ast.literal_eval(str(t))['Instrument'] +␣
↪'_' + ast.literal_eval(str(t))['TradeDirection']}
        result_df = result_df.append(record, ignore_index=True)
```

Displaying result_df yields:

[Out]:



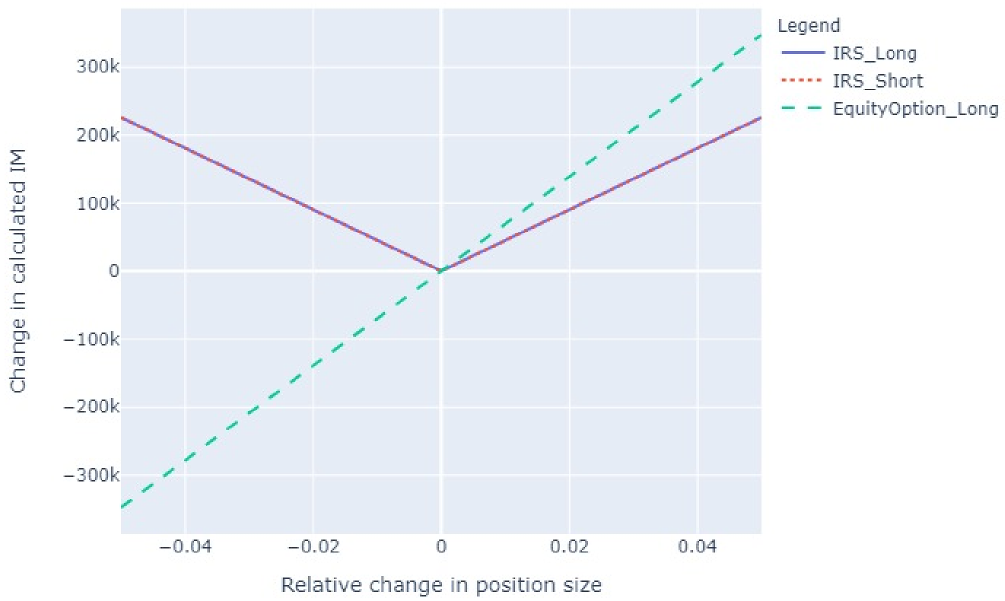
Do the same for the IM:

[In]:
```
result_df = pd.DataFrame()
```

```
for t in ca.trades:
    for bump in bumps:
        record = {'Relative change in position size': bump,
                  'Change in calculated IM':␣
↪bump_one_trade_and_return_diff(bump, t, ca, ca.get_im_model().
↪get_risk_measure),
                  'Legend': ast.literal_eval(str(t))['Instrument'] +␣
↪'_' + ast.literal_eval(str(t))['TradeDirection']}
        result_df = result_df.append(record, ignore_index=True)
```

Displaying `result_df` yields:

[Out]:



The same phenomenon does not appear for if a hedge is not perfect i.e. if the hedge size can be increased to further diminish the risk metric.

[In]:
```
IRS_pay = IRS(notional=200000000,
              timeToSwapStart=ql.Period(2, ql.Days),
              timeToSwapEnd=ql.Period(10, ql.Years),
              swapDirection=SwapDirection.PAYER,
              index=InterestRateIndex.USDLIBOR3M)

IRS_rec = IRS(notional=100000000,
              timeToSwapStart=ql.Period(2, ql.Days),
```

```
                timeToSwapEnd=ql.Period(10, ql.Years),
                swapDirection=SwapDirection.RECEIVER,
                index=InterestRateIndex.USDLIBOR3M)

eqOpt = EquityOption(notional = 1000000)
```

The IM for this portfolio is 11471795.91 USD and the EAD is 1542260.63 USD.

For the IM the allocation yields:

```
[Out]:
                          Backward       Central       Forward
IRS_Short            -4.519079e+06 -4.519078e+06 -4.519079e+06
IRS_Long              9.038157e+06  9.038157e+06  9.038157e+06
EquityOption_Long     6.952717e+06  6.952717e+06  6.952717e+06
Sum                   1.147180e+07  1.147180e+07  1.147180e+07
```

and for the EAD the different differentiation approaches yield:

```
[Out]:
                          Backward       Central       Forward
IRS_Short            -2.463114e+05 -2.463099e+05 -2.463083e+05
IRS_Long              4.926136e+05  4.926198e+05  4.926259e+05
EquityOption_Long     1.295949e+06  1.295951e+06  1.295952e+06
Sum                   1.542251e+06  1.542261e+06  1.542270e+06
```

B.2 Homogeneity of C for a single trade portfolio

We want to investigate under which circumstances the C representing the received collateral in the calculation of SA-CCR exhibits homogeneity. For this we consider a portfolio consisting of a single IRS as follows

```
[In]:
notional = 200000000000
irs = IRS(notional = notional,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          swapDirection=SwapDirection.PAYER,
          index = InterestRateIndex.USDLIBOR3M)
```

Since we have not explicitly set a fixed rate the IRS is struck at par and should have a present value of close to 0.

```
[In]:
print('Fixed rate: %f' %irs.get_fixed_rate())
print('Present value: %.2f' %irs.get_price())
```

```
Fixed rate: 0.024093
Present value: 0.00
```

We also set up a fixed payer IRS that is in the money as it has a fixed rate of 2%.

[In]:
```
irs_itm = IRS(notional = notional,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          swapDirection=SwapDirection.PAYER,
          index = InterestRateIndex.USDLIBOR3M,
          fixed_rate=0.02)

print('Fixed rate: %f' %irs_itm.get_fixed_rate())
print('Present value: %.2f' %irs_itm.get_price())
```

```
Fixed rate: 0.020000
Present value: 7258788031.38
```

Additionally, we set up a collateral agreement exchanging IM in accordance with ISDA-SIMM with a minimum transfer amount of 1Bn and a threshold of 2Bn. For technical reasons we need to first create the collateral agreement and afterwards link it to an instance of the SA-CCR model. We put each of the two trades created above in a separate portfolio and collateral agreement.

[In]:
```
ca = CollateralAgreement(mta = 1000000000,
                         threshold= 2000000000,
                         initialMargining=InitialMargining.SIMM,
                         margin_currency=Currency.USD)
ca.link_sa_ccr_instance(SA_CCR(ca))
```

Again we explore homogeneity at this example by exploring whether

$$f(\alpha \mathbf{x}) = \alpha^k f(\mathbf{x})$$
$$\text{for } \alpha > 0$$

holds based on our initial portfolio for a range $0 < \alpha \leq 2$

Next, we calculate IM, VM, and C for the two IRS. C is once calculated with consideration of MTA and threshold and once without.

To use the MTA we need to set a current margin amount. We will set this as the currently calculated C. With the MTA in place C will afterwards only be updated if the sum of VM and IM differ from the current margin amount by more than the MTA.

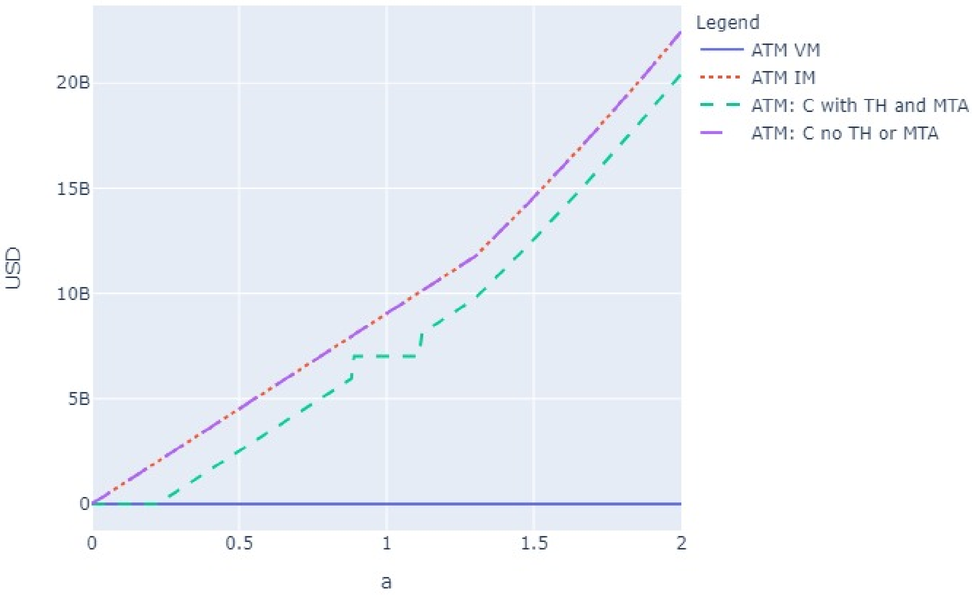
[In]:
```
bumps = arange(0,2.01,0.01)
resultDataframe = pd.DataFrame(columns = ['X','Y','Legend'])
```

[In]:
```
# At the money IRS, with threshold and mta
ca.add_trades(irs)
ca.set_start_collateral_amount(ca.get_C())
for bump in bumps:
    result = bump_and_get_results(bump, irs, ca)
    result_to_record('ATM VM', 'VM', result)
    result_to_record('ATM IM', 'IM', result)
    result_to_record('ATM: C with TH and MTA', 'Collateral', result)

# In the money IRS, with threshold and mta
ca.remove_all_trades()
ca.add_trades(irs_itm)
ca.set_start_collateral_amount(ca.get_C())
for bump in bumps:
    result = bump_and_get_results(bump, irs_itm, ca)
    result_to_record('ITM VM', 'VM', result)
    result_to_record('ITM IM', 'IM', result)
    result_to_record('ITM: C with TH and MTA', 'Collateral', result)

# In the money IRS without threshold or mta
ca.threshold = 0
ca.mta = 0

for bump in bumps:
    result = bump_and_get_results(bump, irs_itm, ca)
    result_to_record('ITM: C no TH or MTA', 'Collateral', result)

# At the money IRS without threshold or mta
ca.remove_all_trades()
ca.add_trades(irs)
ca.set_start_collateral_amount(ca.get_C())
for bump in bumps:
    result = bump_and_get_results(bump, irs, ca)
    result_to_record('ATM: C no TH or MTA', 'Collateral', result)
```

Displaying the result for the at the money IRS it can be seen that the VM is flat at zero since the IRS is at par and therefore has a PV of 0. The IM shows the behaviour described in B.1. C with threshold and minimum transfer amount is not a homogeneous at all, while C without threshold and MTA is partially. This is further discussed in section

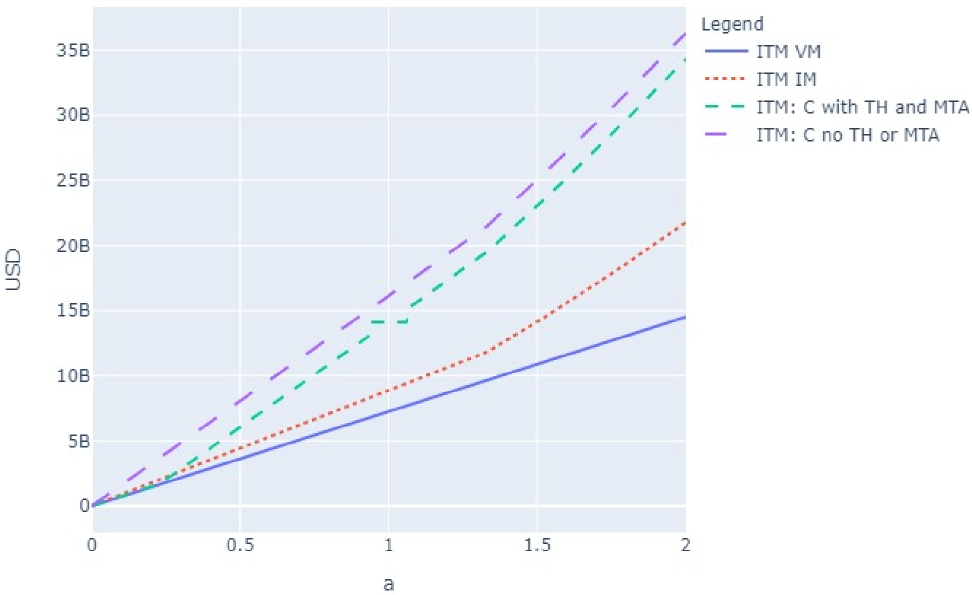
include reference

[Out]:



Results are the same for the in the money IRS with the exception, that the VM is not 0 but VM is still a homogeneous function.

[Out]:



B.3 Exemplary SA-CCR allocation under consideration of an initial margin threshold

Our goal is to perform an Euler allocation for the minimal example of a one trade portfolio. We use the same 200Bn IRS as in previous examples.

[In]:
```
irs = IRS(notional=200000000000,
          index=InterestRateIndex.USDLIBOR3M,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years))
```

[In]:
```
ca = CollateralAgreement(threshold=2000000000,
                         mta=0)
ca.link_sa_ccr_instance(SA_CCR(ca))
ca.add_trades(irs)
```

The inclusion of the threshold raises the ead since it lowers the available overcollataralization.

[In]:
```
ead_with_threshold = ca.get_sa_ccr_model().get_risk_measure()
print(ead_with_threshold)
ca.threshold = 0
ead_no_threshold = ca.get_sa_ccr_model().get_risk_measure()
print(ead_no_threshold)
ca.threshold = 2000000000
```

```
817798882.3982577
582881953.4648591
```

The EAD with threshold is 817798882.40 while the EAD without threshold is 582881953.46

When trying to allocate with threshold we realize that the allocation is not working due to the missing homogeneity of C.

[In]:
```
eulerAllocator = EulerAllocator(ca)
allocated_value = eulerAllocator.allocate_ead()[irs]
print(allocated_value)
```

```
526604164.2713547
```

The value of 526604164.27 that has been allocated to the single trade in the portfolio is far off from the portfolios EAD of 817798882.40

If we instead allocate without threshold, the allocation works.

[In]:
```
ca.threshold = 0
allocated_value=eulerAllocator.allocate_ead()[irs]
print(allocated_value)
```

```
582887839.6499157
```

The value of 582887839.65 that has been allocated to the single trade in the portfolio equals the portfolios EAD without threshold of 582881953.46

A reasonable approach to allocate an SA-CCR EAD under consideration of a threshold could be to allocate without threshold and then scale accordingly:

$$X_{t,\ \mathrm{TH}} = X_{t,\ \mathrm{no\ TH}} \frac{EAD_{\mathrm{TH}}}{EAD_{\mathrm{no\ TH}}} \tag{B.3.1}$$

[In]:
```
print(allocated_value * (ead_with_threshold/ead_no_threshold))
```

```
817807140.8724346
```

However, this approach does coincide with a loss of precision. If the C_{calc} is below the threshold then C is 0 and exhibits homogeneity, even when taking the threshold into account. We can construct an example that shows, that the approach in B.3.1 does not yield the correct allocation which can be calculated when taking the threshold into account since the IM is below the IM threshold.

We set up an IRS and an Equity option that have a similar EAD on their own. Here we calculate with a threshold of 50Mn which is a common value as it is the maximum amount permitted by the regulator.

Cite source for this

[In]:
```
irs = IRS(notional=100000000,
          index = InterestRateIndex.USDLIBOR3M,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years))

ca = CollateralAgreement(threshold=50000000)
ca.link_sa_ccr_instance(SA_CCR(ca))
ca.add_trades(irs)
```

[In]:
```
print(ca.get_im_model().get_risk_measure())
print(ca.get_sa_ccr_model().get_risk_measure())
```

```
4519078.535740282
1651404.7245623078
```

Through optimization it can be identified that for a one year long call option on Adidas an underlying stock count of 403106 stocks results in the same EAD as for the IRS above.

[In]:
```
eqOpt = EquityOption(notional = 403106,
                     maturity = ql.Period(1, ql.Years),
                     underlying = Stock.ADS,
                     tradeType = TradeType.CALL,
                     tradeDirection = TradeDirection.LONG)
```

[In]:
```
eqOpt = EquityOption(notional = eqOptNot.x[0])
ca2 = CollateralAgreement(threshold = 50000000)
ca2.link_sa_ccr_instance(SA_CCR(ca2))
ca2.add_trades(eqOpt)
```

The initial margin of this single trade is also far below the 50Mn threshold but differs significantly from the IM of the IRS.

[In]:
```
print('EAD: ' + str(ca2.get_sa_ccr_model().get_risk_measure()))
print('IM:  ' + str(ca2.get_im_model().get_risk_measure()))
```

```
EAD: 1651404.7245622796
IM:  2802685.461885011
```

Given the market data, difference in model, risk horizon etc. the SA CCR EAD model calculates the same risk for the two trades when calculated individually, while the ISDA SIMM IM model evaluates the IRS to be 61% riskier.

When putting both trades in a common portfolio we observe, that the EAD and the IM of this joint portfolio is the sum of the two separate portfolios. This is not surprising since both, the SA CCR and ISDA SIMM model do not recognize any hedge effect between different asset classes.

[In]:
```
ca3 = CollateralAgreement(threshold=50000000)
ca3.link_sa_ccr_instance(SA_CCR(ca3))
ca3.add_trades([irs, eqOpt])
```

The EAD of the combined portfolio is 3302809 USD. The calculated IM of the combined portfolio is 7321763 USD.

[In]:
```
eulerAllocator = EulerAllocator(ca3)
ca3.threshold = 50000000
allocation = eulerAllocator.allocate_ead()
```

Euler allocation allocates 1651404 USD of the EAD to the IRS.Euler allocation allocates 1651404 USD of the EAD to the equity option.The sum of these two allocations is 3302809 USD, Which is close to the EAD calculated for the portfolio of 3302809 USD.

Due to the high threshold, no IM is exchanged. Only VM is exchanged which is not overcollateralization and therefore only reduces the RC in formula to 0 but does not impact the PFE.

reference RC+PFE formula

The available collateral is 1450296 USD. Due to the hight threshold this consists only of the VM which is 1450296 USD.

The 50/50 allocation is certainly the correct result since there are no hedge effects between the two trades and they both have the same stand alone EAD. However, when applying formula B.3.1 we yield a different result.

[In]:
```
ead_with_threshold = ca3.get_sa_ccr_model().get_risk_measure()
ca3.threshold=0
ead_no_threshold = ca3.get_sa_ccr_model().get_risk_measure()
allocation = eulerAllocator.allocate_ead()
factor = ead_with_threshold/ead_no_threshold
print(allocation[irs]*factor)
ca3.threshold = 50000000
```

```
1153317.0534241588
```

Euler allocation allocates 1153317 USD of the EAD to the IRS. Euler allocation allocates 2149501 USD of the EAD to the equity option. The sum of these two allocations is 3302819 USD, which is close to the EAD calculated for the portfolio of 3302809 USD.

Therefore, the approximation of B.3.1 should only be used if the $IM_{calc} > TH_{IM}$.

B.4 Exemplary SA-CCR allocation under consideration of a minimum transfer amount.

The result of Appendix B.2 shows, that inclusion of the MTA results in a local plateau of C.

In this section we want to investigate if inclusion of the MTA breaks homogeneity of the SA-CCR EAD function and what can be done to mitigate this effect.

We initialize a 200Bn IRS, a collateral agreement with a 0 threshold and a 1Bn MTA.

[In]:
```
irs = IRS(notional = 200000000000,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          index = InterestRateIndex.USDLIBOR3M)
```

```
ca = CollateralAgreement(threshold=0,
                         mta=1000000000)
ca.link_sa_ccr_instance(SA_CCR(ca))
ca.add_trades(irs)

ca.set_start_collateral_amount(ca.get_C())
```

The starting collateral C_{t-1} is set to 9038157077 USD which is also the calculated IM since the VM of this par IRS is 0.

The EAD is:

[In]:
```
original_ead = ca.get_sa_ccr_model().get_risk_measure()
original_ead
```

[Out]: 582881953.4866074

When bumping the notional of the irs by 0.01%, we can see that the collateral of the portfolio does not change due to the MTA.

[In]:
```
ca.remove_all_trades()
ca.add_trades(irs.get_bumped_copy(rel_bump_size=0.0001))
ead_bumped_mta = ca.get_sa_ccr_model().get_risk_measure()
print('EAD:  %d' %ead_bumped_mta)
print('C:   %d' %ca.get_C())
```

```
EAD:  583024482
C:   9038157077
```

[In]:
```
ead_bumped_mta = ca.get_sa_ccr_model().get_risk_measure()
```

When temporarily disabling the MTA the resulting EAD and C differ.

[In]:
```
ca.mta = 0
ead_bumped_no_mta = ca.get_sa_ccr_model().get_risk_measure()
print('EAD:  %d' %ead_bumped_no_mta)
print('C:   %d' %ca.get_C())
```

```
EAD:  582940242
C:   9039060887
```

Calculating the forward difference in line with equation with and without consideration of the MTA yields

reference forward dif eq

[In]:
```
print('With MTA:    %d' %((ead_bumped_mta-original_ead)/0.0001))
print('Without MTA:  %d' %((ead_bumped_no_mta-original_ead)/0.0001))
```

```
With MTA:     1425289375
Without MTA:   582887602
```

B.4.0.1 Impact of the minimum transfer amount on RC

the MTA also impacts RC as displayed in table 2.1. Since IM reduces the RC the most relevant case is when the calculated IM is below the threshold.

In an example we try to allocate the EAD of a portfolio consisting of a single 100Mn IRS. The associated collateral agreement has a threshold of 50Mn and a minimum transfer amount of 2Mn.

[In]:
```python
irs = IRS(notional=100000000,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          index=InterestRateIndex.USDLIBOR3M)

ca = CollateralAgreement(threshold=50000000,
                         mta = 2000000)
ca.link_sa_ccr_instance(SA_CCR(ca))
ca.add_trades(irs)

print('RC:      %d USD' %ca.get_sa_ccr_model().get_rc())
print('RC*1.4:  %d USD' %(ca.get_sa_ccr_model().get_rc()*1.4))
print('PFE:     %d USD' %(ca.get_sa_ccr_model().get_pfe()))
print('PFE*1.4: %d USD' %(ca.get_sa_ccr_model().get_pfe()*1.4))
print('EAD:     %d USD' %ca.get_sa_ccr_model().get_risk_measure())
```

```
RC:      2000000 USD
RC*1.4:  2800000 USD
PFE:     1179574 USD
PFE*1.4: 1651404 USD
EAD:     4451404 USD
```

The EAD is the sum of the RC and the PFE component time the α factor of 1.4. In all previous examples, the RC has always been floored at 0 since the received IM was higher than the MTA or since the MTA was 0.

Again, Euler allocation is not possible, because the EAD is a sum of the PFE, which is a function of the portfolio notional and the RC which, at least locally, is a constant. Similar to the issue with threshold described in B.3 one can only allocate without MTA and then allocate the remainder $EAD_{\text{MTA}} - EAD_{\text{no MTA}}$ according to some rule.

Below, we allocate the ead assuming a mta of 0.

[In]:
```
ca.mta = 0
eulerAllocator = EulerAllocator(ca)
allocation = eulerAllocator.allocate_ead()
ca.mta = 2000000
allocation[irs]
```

[Out]:
```
1651404.724563472
```

As we can see the result equals $PFE * 1.4$ of the entire portfolio but the RC has not been allocated.

The RC is also > 0, if $TH < IM_{calc} < MTA$. Based on the available marketdata this is e.g. the case for an IRS with a notional of 1130Mn USD.

[In]:
```
irs2 = IRS(notional=1130000000,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          index=InterestRateIndex.USDLIBOR3M)

ca.remove_all_trades()
ca.add_trades(irs2)

print('RC:       %d USD' %ca.get_sa_ccr_model().get_rc())
print('PFE:     %d USD' %(ca.get_sa_ccr_model().get_pfe()))
print('EAD/1.4: %d USD' %(ca.get_sa_ccr_model().get_risk_measure()/
 ↪1.4))
print('EAD:     %d USD' %ca.get_sa_ccr_model().get_risk_measure())
```

```
RC:        934412 USD
PFE:     13329195 USD
EAD/1.4: 14263607 USD
EAD:     19969050 USD
```

This observation is discussed further in .

Reference appropriate section in Results

Glossary

CCP	Central counterparty
CCR	Counterparty credit risk
CEM	Current exposure method
CRD2	Capital requirements directives two of the european union
CRIF	Common Risk Interchange Format - standardized file format for inputs for the ISDA SIMM model
CSA	Credit support annex
EAD	Exposure at default
IM	Initial margin
IMM	Internal model method
IRS	Interest Rate Swap
ISDA	International Swaps and Derivatives Association
ISDA SIMM	Internal initial margin model for uncleared derivatives developed by ISDA and used by most market participants
MPoR	Margin period of risk
MTA	Minimum transfer amount of a collateral agreement
MtM	Mark to market - current market value of a portfolio or a financial instrument
OTC	Over the counter derivatives as opposed to exchange trades derivatives
PnL	Profit and loss
SA-CCR	Standard approach for counterparty credit risk EAD calculation under CRD2
VaR	Value at risk
VM	Variation Margin

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