SA-CCR Allocation under consideration of margining

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Abstract

The SA-CCR model has replaced the current exposure method (CEM) as the standard approach for counterparty credit risk capital requirement calculations. SA-CCR is a more risk sensitive model that, unlike its predecessor, takes portfolio effects and available collateral into account. Therefore, the SA-CCR more accurately estimates the counterparty credit risk taken by the bank but it is also more necessary and challenging to analyze the SA-CCR risk metric and allocate it on single trades of a portfolio than was the case for the CEM model. This thesis investigates, how the SA-CCR exposure at default of a bilateral derivative portfolio should be allocated to the individual trades of said portfolio. The incremental allocation approach and the marginal allocation (also known as Euler allocation) were identified as the most promising approaches that also complement each other well. The incremental allocation approach can and should be applied for the SA-CCR for use cases such as pre-trade analysis or PnL allocation among trading desks. Whether the Euler allocation which should be utilized for tasks such as portfolio analysis or optimization can also be calculated for SA-CCR was analyzed.

It has been found that calculation of an Euler allocation for bilateral portfolios is generally possible and a numerical calculation approach appears to be most suitable. If the portfolio is margined with only variation margin or variation margin and initial margin, an Euler allocation of these margin models has to be part of the Euler allocation of SA-CCR.

However, circumstances have been identified under which an Euler allocation of SA-CCR is not possible since SA-CCR does not exhibit the properties of a positive homogeneous function under these circumstances. The most notable of those circumstances is if an initial margin threshold is present and is exceeded. For this case, which has high practical relevance, no unambiguous, risk sensitive allocation could be established but just upper and lower bounds for the amount allocated to each trade could be established.

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Chapter 1

Introduction

1.1 Motivation

With increasing sophistication of risk models, own capital models and margining models, the need for equally sophisticated tools for allocating these measures rises as well. For any risk metric that considers portfolio effects, calculating the contribution to the risk measure of individual trades is a challenge. As part of the Basel 3 reform, regulators have updated the standardized models for market risk own capital requirement and credit risk own capital requirements. The new standardized model for market risk own capital requirements is the FRTB-SA and the new standardized model for credit risk own capital requirements is the SA-CCR model. Both of these models are portfolio based risk models and as such allocation approaches to allocate these models on single trades will be required for different purposes within a bank.

Gregory [18, Chapter 10.7] states that the three allocation approaches are used most in practice are:

- Incremental allocation for pre trade risk checks and for front office incentivization
- Marginal allocation for risk analytics of existing portfolios
- Pro rata allocation if trade contributions must not be negative, if risk sensitivity is not required or if the allocated risk measure does not take portfolio effects into account

Calculation of pro rata and incremental allocation is fairly straightforward and can generally be performed under any circumstances. Marginal allocation on the other hand is more challenging. This thesis will investigate if and how a marginal allocation of the SA-CCR model is possible.

Schulze [32] has analytically calculated the marginal allocation for the FRTB-SA model. However, an approach to marginal allocation of SA-CCR has not been published yet. This thesis intends to close this gap by showcasing a numerical marginal allocation approach for SA-CCR. A particular challenge in the allocation of SA-CCR is its consideration of margining. This makes the allocation of SA-CCR dependent on margin models which can themselves be significantly more complex than the SA-CCR model itself.

The goal of this thesis is to find an approach for allocating SA-CCR while considering bilateral margining. As such, the allocation under consideration of variation margining and bilateral initial margin - especially under the use of an internal initial margin model - will be the focus.

Unlike the analytical approach of Schulze [32] this thesis will instead apply a numerical approach to calculating a marginal allocation.

1.2 Structure of the thesis

The remainder of the thesis is segmented in four chapters. Chapter 2 introduces applied methods and models. This includes used financial instruments, risk models, an introduction of the concept of margining and approaches to risk measure allocation. Next, chapter 3 proposes a numerical approach to Euler allocation of the SA-CCR model and highlights some of the strengths and weaknesses of this approach by means of small exemplary portfolios. These results are discussed in chapter 4 where the results of chapter 3 are summarized and the theoretical and practical usefulness of the numerical allocation of SA-CCR is assessed. Finally, chapter 5 provides an outlook to adjacent topics and concludes the thesis.

Due to the scope of chapter 2 and 3 their structure is presented in additional detail below.

1.2.1 Structure of chapter 2

Since margining is an integral part of the SA-CCR model, section 2.1 will afterwards introduce different margin types and typical counterparty relations and margin models observed in the market. Section 2.1 will also establish why out of the different margin models, the ISDA SIMM model is investigated the closest for the purpose of this thesis. Section 2.2 introduces the ISDA SIMM which is the most commonly used model for initial margin calculation of uncleared derivatives.

After the different margin components have been introduced, section 2.3 presents the SA-CCR model that is used for the calculation of the EAD of derivatives. Special emphasis will be put on the inclusion of margining.

Section 2.4 presents literature results regarding the allocation of risk measures. Additionally, the theoretical foundation for the Euler allocation is laid out as the investigation if and when an Euler allocation is possible is the main subject for the analysis section of the thesis.

In chapter 3 a small but diverse selection of derivatives is used for exemplary calculations and to explore edge cases of the different models. Section 2.5 does briefly introduce these instruments and the financial models and market data that is used to price them. Additionally, it describes, how ISDA SIMM compliant sensitivities are calculated for these instruments.

At this point all relevant concepts, models and financial instruments have been introduced to perform the analysis that is then presented in the results chapter of the thesis.

1.2.2 Structure of chapter 3

The main goal of the thesis is to establish a numerical allocation approach for SA-CCR.

An approach to numerical allocation of SA-CCR is introduced with the aid of two exemplary portfolios in chapters 3.1.1 and 3.1.2. In these chapters both, ISDA SIMM and SA-CCR are allocated and the portfolios are used to showcase analytical insights that an Euler allocation may provide. Finally, section 3.1.3 showcases the value of an Euler allocation not only for an analysis on trade level but also on subportfolios.

In contrast to the encouraging results of section 3.1, section 3.2 analyzes when and why Euler allocation of SA-CCR is not possible in certain edge cases. A property of the ISDA SIMM model that may prevent risk-sensitive allocation is discussed in section 3.2.1. On the other hand, contractual agreements governing the exchange of margin may interfere with the allocability of the SA-CCR model which is discussed in section 3.2.2. Finally, section 3.2.3 discusses a possible, specific hedge constellation that infringes a necessary precondition for an Euler allocation.

Chapter 2

Applied models and methods

2.1 Margining

In the context of derivatives, margining refers to the process of posting and receiving collateral in the form of cash or securities to reduce the counterparty credit risk associated with the derivatives.

Counterparty credit risk refers to the risk of a default of the counterparty of a derivative. Derivatives are contracts between two institutions which give raise to future cash flows dependent of the performance of its underlying. These future cash flows may be at risk if the counterparty defaults during the lifetime of the derivative.

Over the past decades several measures were established in the derivatives markets to mitigate counterparty credit risk.

The most impactful measure is close-out netting. Close-out netting is a contractual agreement of two counterparties how their bilateral derivative contracts have to be settled if one of the counterparties defaults. With close-out netting, in case one of the two counterparties defaults all derivatives which are yet to mature are immediately closed out based on their current market value. The market values of the individual derivatives are summed up and the netted amount needs to be paid by whichever party is trailing across the portfolio. In case of a default this close-out netting procedure takes priority over all other claims of creditors against the defaulted counterparty.

Close-out netting has two major advantages. First, the non-defaulting counterparty only suffers a counterparty credit loss, if it is ahead across the entire portfolio of currently running derivatives with the defaulted party. Just having a positive market value on a few derivatives does not necessarily result in a counterparty credit loss. Secondly, the immediate close-out of the open derivatives of the defaulted counterparty greatly facilitates unwinding its portfolio.

A disadvantage of close-out netting is, that it may proof difficult to find an objective market value of the derivatives that have to be closed out - especially in a stressed market environment, which is likely to be present if e.g. a large investment bank defaults. The contractual obligation to perform close-out netting is agreed upon in a master agreement, which was introduced to the derivatives market by ISDA in 1985. Currently, close out netting is in effect for almost all traded derivatives and it will implicitly always be assumed to be in effect throughout this thesis. More details on close-out netting may be found in [18, Chapter 5].

The second most effective measure in mitigating counterparty credit risk is the exchange of variation margin. If the obligation to post variation margin is agreed as part of a master agreement, the accrued mark-to-market of the derivative portfolio has to be collateralized by the trailing counterparty. This measure effectively resets counterparty credit risk to zero for both parties every time a variation margin payment is made or the exchanged variation margin is adjusted to the current market value of the portfolio. The exchange of variation margin was common but not a given in the inter-bank market before the financial crisis of 2008. After the crisis it has become commonplace in the interbank market and recently has even been mandated by regulators[1]. Non-financial counterparties oftentimes do not collateralize their derivatives since they are not mandated to do so, shy away from the operational burden and have a harder time funding the significant amount of cash necessary to cover the current mark-to-market value of their entire derivatives portfolio. Collateralizing a derivatives portfolio not only significantly reduces CCR but also significantly alters how the remaining CCR behaves. The CCR of a collateralized portfolio may rather be driven by the terms of the collateral support annex or residual phenomenon such as collateral spikes than by the underlying instruments [3].

As a final measure, initial margin can be exchanged. Historically, initial margin was a collateral amount that was calculated and exchanged once at the inception of a new derivative and held until maturity - hence the name *initial* margin. One common formulation used w.r.t. initial margin, which has also found its way into regulatory documents (e.g. [27, Article 52.16]) is that initial margin is collateral, that - in contrast to variation margin - is not based on the MtM of a portfolio or derivative.

The idea behind initial margin is that it secures the counterparties against losses that can incur between the last time variation margin has been exchanged prior to a default until the original position has been restored. This time period is referred to as the margin period of risk and this time period results as the sum of

1. The contractually agreed regular frequency of variation margin exchanges
2. The time it takes from a counterparty not complying with a margin call to ascertain that the counterparty has indeed defaulted
3. The necessary time to reopen the defaulted derivatives with new counterparties to rehedge the bank and thereby restoring the risk profile held prior to the default of the counterparty.

[1] In the European Union the exchange of Variation Margin for inter bank bilateral OTC derivatives is compulsory since September of 2016 for large banks or March of 2017 for smaller banks.

Initial margin should cover the gap between the MtM of the derivatives with the defaulted counterparty when variation margin was last exchanged to the price for which the derivatives are reopened in step 3 under the assumption that the market has moved unfavorably during the MPoR. In the context of CCR, *unfavorable* means that the banks position would have increased in value throughout the margin period of risk and therefore the market price for which the bank repurchases the derivatives is higher than the value at which variation margin was exchanged last.

The use of these different measures to mitigate counterparty credit risk between two parties and their parametrization is set in the credit support annex (CSA). A credit support annex is a legal document arranged between derivative counterparties that, among others, regulates the exchange of margin and the proceedings in case of a default of one of the counterparties. Generally, negotiated CSAs are based on some template provided by ISDA and therefore have a homogeneous structure but contract parameters have to be agreed bilaterally and can show some variety.

A range of parameters of a CSA is of special interest within the scope of this thesis and therefore warrant a more detailed inspection. In the following we only focus on bilateral contracts regarding uncleared derivatives, i.e. contracts that do not involve a CCP as a counterparty.

Collateralization with variation margin The counterparties decide whether the portfolio has to be collateralized with variation margin. If so, the calculated variation margin is the sum of the PV of all trades within the portfolio. The counterparties need to agree on a common valuation of a portfolio. This can be a complex process, especially if differences in pricing need to be resolved. Once a calculated variation margin has been determined it depends on the minimum transfer amount, margin call frequency and variation margin threshold how much of the calculated variation margin is actually exchanged.

Variation margin threshold If a variation margin threshold is in place, no variation margin is exchanged before the absolute calculated variation margin does exceed this threshold. The purpose of the threshold is mainly to reduce operational efforts related to very small derivative portfolios. In line with focus on operational impact and in contrast to the initial margin threshold, the calculated variation margin amount is usually exchanged in full once the threshold has been exceeded. Variation margin thresholds are banned by the regulator for any portfolio for which bilateral initial margin needs to be exchanged [28, Requirement 2.1].

Collateralization with initial margin In a bilateral contract, the counterparties need to agree whether they collateralize their portfolio with initial margin and how they will calculate the initial margin. If initial margin should be exchanged it can be calculated with an internal model or a standardized model. It is also possible to calculate the IM of parts of the portfolio under the internal model while others are calculated under the standardized model. Bilateral initial margin models are further discussed in section 2.2.

Initial margin threshold If an initial margin threshold is in place and the calculated initial margin does not exceed this threshold, no initial margin is exchanged. Additionally, the regulator allows that even if the threshold is exceeded it may be deducted from the calculated initial margin resulting in the exchanged initial margin being the calculated IM minus the IM threshold. The regulator allows an IM threshold of up to 50Mn USD or EUR [28, Requirement 2.2] which is quite a considerable amount in comparison to the calculated IM of average derivative portfolios [21].

Minimum transfer amount If a minimum transfer amount is in place, margin is only exchanged if the margin amount that would need to be exchanged based on current calculations differs by more than the minimum transfer amount from the margin that has already been exchanged. Details on how exactly the MTA applies may vary but within the scope of this thesis we assume that a common MTA may be in place for both initial and variation margin amounts. If bilateral initial margin is exchanged the minimum transfer amount may not exceed 500000EUR according to European regulation [28, Requirement 2.3].

Margin frequency The margin frequency determines with which frequency margins need to be recalculated and - assuming that the MTA is exceeded - adjusted. Margin frequencies may differ between VM and IM calculations of the same CSA and are subject to regulation. The regulator e.g. expects that the margin frequency for variation margin is daily [28, Requirement 3.14].

Having a CSA in place is a prerequisite for two financial or non-financial counterparties to enter any derivative contract with each other.

A more comprehensive introduction to counterparty credit risk and its reduction through netting and margining may be found in chapters four through six of [18].

2.1.1 Market structure and associated margining approaches

The derivative market is divided into exchange traded derivatives, cleared OTC derivatives and uncleared bilateral derivatives. Uncleared bilateral derivatives can either be uncollateralized, collateralized with VM or collateralized with IM and VM. Below, these five counterparty relations are briefly introduced. They are ordered w.r.t. their associated counterparty credit risk.

Uncollateralized bilateral derivatives Derivatives are arranged between two counterparties without involvement of a third party. No collateral is exchanged at any point, only the contractual cashflows of the derivatives are exchanged. The CCR is very high as the entire MtM of the portfolio is at risk. Since no margin is posted at all, the MPoR is the maturity of the traded derivatives and can therefore easily eclipse multiple years. IM posted, IM received and VM are zero at all times.

Bilateral derivatives collateralized with VM The CCR is still significant. When the counterparty defaults the bank can suffer unmitigated losses for a couple of days until it can rebuild its position. The MtM of the portfolio is collateralized with VM. VM exchange is subject to contract parameters such as the threshold, minimum transfer amount or the exchange frequency. Values of these parameters impact how well CCR is mitigated.

Bilateral derivatives collateralized with VM and IM Counterparty credit risk is low. Only in edge cases is it possible that the counterparties credit losses surpass the available collateral. An MPoR of at least 10 days is mandated by the regulator [28, Requirement 3.1]. The IM is calculated with an internal or standardized bilateral initial margin model. Posted and received IM are recalculated daily. VM exchange obeys the same mechanics as for Bilateral derivatives collateralized with VM.

Cleared OTC derivatives Derivatives are initially arranged bilaterally between two counterparties and then cleared by a CCP. The CCP takes over positions in case of a default of either party mitigating any CCR in the traditional sense. The bank has no direct counterparty credit risk. It may however suffer losses to its clearing fund contribution if another Member of the CCP defaults. The MPoR is mandated by the regulator to be five days [29, Article 26]. The initial margin that is posted by the bank to the CCP is calculated by the CCP with his proprietary internal initial margin model. In line with the assumption that the CCP can not default, the CCP does not post IM to its clearing members. Therefore, the IM received from the perspective of the bank is always zero. The daily PnL of the portfolio is exchanged as VM between the CCP and the Bank.

Exchange traded derivatives Banks enter positions in exchange traded derivatives listed by a CCP. Positions are matched by the CCP and the counterparties of a transaction remain anonymous to each other. Associated CCR and margining is largely the same as for Cleared OTC derivatives but the MPoR is generally below five days since it is assumed that positions in exchange traded derivatives can be closed faster than in cleared OTC derivatives. The used internal initial margin model may differ e.g. since the regulator requires coverage of a 99% quantile instead of the 99.5% mandated for cleared OTC derivatives [29, Article 24].

According to [18, Figure 3.2] based on notional 9% of derivatives are exchange traded, 55% are cleared OTC derivatives and 36% are uncleared OTC derivatives. It has to be noted that these figures are from 2014 and it can be assumed, that the fraction of cleared OTC derivatives has increased since then at the expense of the fraction of uncleared OTC derivatives. The reason for this is, that regulators have incentivized or even mandated the clearing of simpler OTC derivatives over the course of the last years. In connection with this development the large CCPs such as Eurex or the LCH have extended the product range for which they offer OTC clearing in recent years.

Within the scope of this thesis we will only investigate the allocation of the first three cases listed above, i.e. uncleared derivatives.

2.2 Bilateral initial margin

After the 2008 financial crisis the G20 agreed to reduce systematic, counterparty and operational risk and as a result of this commitment regulations have been put into effect by regulators worldwide. In Europe, the European Market Infrastructure Regulation (EMIR) came into force in August 2012 and focused on promoting or mandating central clearing as the primary measure to reduce counterparty risk.

2.2.1 The standardized approach

The standardised approach to calculate bilateral initial margin has been proposed by the bank for international settlement in [28, Requirement 3.5 and 3.6] and has been implemented in European law in [30]. It is a schedule based approach that calculates an IM contribution on a trade by trade basis multiplying the trades notional with a regulatory factor based on the asset class and term to maturity of the trade. The resulting sum may be reduced by up to 60% through the so called net-gross ratio, if the portfolio has a negative present value from the perspective of the calculating bank. For a detailed specification of the aggregation the reader may refer to [28] and [30]. The implementation of this approach is addressed in 2.2.2.1.

2.2.2 The ISDA SIMM model

In December of 2013 the International Swap and Derivatives Association (ISDA) published a motivation and basic methodological outline of a common internal initial margin model called Standard Initial Margin Model (SIMM™)[36]. The goal of the model is to meet the model requirements to an internal model of all regulators, while being, among others, easy to replicate, quick to calculate and relatively cheap to operate, implement and validate.

SIMM is a Delta-Gamma VaR model using Delta and Vega sensitivities calculated by the banks themselves and risk weights and correlations provided and recalibrated annually by ISDA. ISDA provides member with a methodological paper [35] and a paper describing the input format of sensitivities [34]. Additionally, the authors of ISDA SIMM have provided a technical paper [37] that lays out the mathematical foundation of the model. The core idea of the model is to multiply sensitivities with risk weights and aggregate them with nested variance-covariance computations. For a detailed introduction to the structure, requirements against input data and underlying mathematical concepts of ISDA SIMM the reader may please refer to sources [35], [34] and [37].

2.2.2.1 Implementation

As already pointed out, ISDA SIMM is standardized despite being an internal model. Therefore, all market participants using an internal model for bilateral initial margin calculation are forced to calculate ISDA SIMM compliant sensitivities, convert them into the so-called CRIF format and to aggregate them to an initial margin figure using the ISDA SIMM aggregation. The process to create ISDA SIMM compliant sensitivities is individual to each bank. Many vendor solutions for trading and risk have incorporated the creation of ISDA SIMM compliant sensitivities and a CRIF into their products but the most suitable way to produce a CRIF still needs to be established on a bank to bank basis.

Aggregation on the other hand is absolutely standardized. It uses a single file, the CRIF, as input, does not need any auxiliary market data and returns a single value, the IM. Considering this, Acadiasoft[2] decided to provide an open source implementation of the ISDA SIMM aggregation [1]. Acadiasoft is an ISDA-affiliated company who also offers a dispute resolution platform for bilateral initial margin that has become the market standard. For the analysis shown in chapter 3, this open source library was used for aggregation of ISDA SIMM. Therefore, only the ISDA SIMM compliant sensitivities needed to be calculated and parsed into a CRIF entry minimizing potential sources of error and necessary testing effort. The open source library by Acadiasoft also offers functionality to calculate bilateral initial margin according to the standardized approach presented in section 2.2.1 using an extended CRIF standard.

2.3 SA-CCR

Counterparty credit risk is considered to be a part of credit risk by the regulator. Risk weighted assets have to be calculated and need to be backed by own capital. The three main inputs for calculating credit risk are the probability of default (PD) the loss given default (LGD) and the exposure at default (EAD). Assuming the default of a counterparty over the course of the next year, the EAD is the current estimation of money indebted by the counterparty to the bank at the time of default. Estimating EAD for traditional credit instruments s.a. loans, credit cards, mortgages or bonds is relatively simple. Such instruments do often times have deterministic payment schedules making it easy to predict the exposure in one years time. Credit lines or credit cards behave less deterministic but it is still simple to determine an upper bound to the future exposure by assuming that the entire credit line is exhausted. The counterparty credit risk incurred by derivatives has first been regarded in regulatory capital calculation in Basel II [24]. Due to the stochastic nature of derivatives, EAD calculation for counterparty credit risk has been regulated separately ever since. Consideration of CCR in regulatory capital was overhauled as part of Basel III [26].

[2] `https://acadiasoft.com/`

The regulation for the internal margin model (IMM) approach was adjusted and the current exposure model (CEM) that was introduced with Basel II as the standard approach for CCR EAD calculation was replaced with the SA-CCR model and the simplified SA-CCR model.

For this thesis, the SA-CCR model was implemented to yield the results presented in chapter 3 but the structure of the SA-CCR model will not be presented in a comprehensive fashion here. Instead the reader may refer to the latest regulatory documents [31, Article 274 and following] and [6] or the library developed for this thesis [19]. This section will highlight the aspects of the SA-CCR model that are of special interest within the scope of this thesis such as the consideration of margin.

2.3.1 Consideration of margining in SA-CCR

When using SA-CCR, the exposure at default has to be calculated as:

$$EAD = \alpha * (RC + PFE)$$

$$\begin{aligned} \text{where} \qquad & \alpha = 1.4 \\ & RC : \text{Replacement Cost} \\ & PFE : \text{Potential Future Exposure} \end{aligned} \tag{2.3.1}$$

The purpose of the RC is to assess the immediate loss suffered by the default of a counterparty. It is based on the current MtM of the derivative less the accessible collateral. If a bank has posted collateral to non-segregated accounts of a counterparty this collateral is also assumed to be lost in case of a default which increases the replacement cost. The potential future exposure (PFE) on the other hand assesses how the RC might develop in the future. The future being defined as during the next year. If the RC today is zero but is likely to be larger than zero in the near future the estimated EAD should take this expected increase in RC into account.

See also Paragraph 130 and 131 of [25]

Paragraph 130 - case without margining:

> For unmargined transactions, the RC intends to capture the loss that would occur if a counterparty were to default and were closed out of its transactions immediately. The PFE add-on represents a potential conservative increase in exposure over a one-year time horizon from the present date (i.e. the calculation date).

Paragraph 131 - case with margining:

> For margined trades, the RC intends to capture the loss that would occur if a counterparty were to default at the present or at a future time,

assuming that the closeout and replacement of transactions occur instantaneously. However, there may be a period (the margin period of risk) between the last exchange of collateral before default and replacement of the trades in the market. The *PFE* add-on represents the potential change in value of the trades during this time period.

The PFE is defined as

$$\begin{aligned} PFE &= \text{multiplier} * AddOn^{\text{agg}} \\ \text{where} \qquad AddOn^{\text{agg}} &: \text{aggregate add-on component} \\ \text{multiplier} &: f(V, C, AddOn^{\text{agg}}) \end{aligned} \tag{2.3.2}$$

AddOn is calculated differently for each asset class a. Since no netting is allowed between asset classes the aggregate is calculated as:

$$AddOn^{\text{agg}} = \sum_a AddOn^a$$

Collateralization is taken into account of the PFE calculation through the multiplier that uses the collateral held as an input. As overcollateralization increases, the multiplier decreases. The most important source of overcollateralization is initial margin. However, the multiplier is floored at 5%.

$$\begin{aligned} \text{multiplier} &= \min\left\{1; Floor + (1 - Floor)\exp\left(\frac{V - C}{2(1 - Floor)AddOn^{\text{agg}}}\right)\right\} \\ \text{where } Floor &= 5\% \end{aligned} \tag{2.3.3}$$

The RC is defined as

$$\begin{aligned} RC &= \max\{V - C; TH + MTA - NICA; 0\} \\ \text{where} \qquad V &: \text{Current portfolio value} \\ C &: \text{Net collateral held} \\ TH &: \text{Threshold} \\ MTA &: \text{Minimum Transfer Amount} \\ NICA &: \text{Net Independent Collateral Amount} \end{aligned} \tag{2.3.4}$$

C is defined according to the *NICA* definition, which is defined in paragraph 143 of [25]. For the purpose of this thesis we simplify the *NICA* definition by making assumption 1.

Assumption 1. *Variation margin is posted in unsegregated accounts, initial margin is posted in segregated accounts and initial margin is the only form of overcollateralization.*

Under assumption 1, the calculation of $NICA$ and C simplifies to:

$$\begin{aligned} C &= \text{Variation Margin balance} + NICA \\ NICA &= \text{Received initial margin} \end{aligned} \tag{2.3.5}$$

Regarding the incorporation of a variation margin and initial margin threshold and a MTA we will assume that the mechanics are the following:

1. A threshold $\mathrm{TH}_{\mathrm{VM}}$ may exist if only VM is exchanged but no IM
2. No threshold for the exchange of variation margin $\mathrm{TH}_{\mathrm{VM}}$ can be in place if initial margin is exchanged
3. A threshold $\mathrm{TH}_{\mathrm{IM}}$ for initial margin may exist
4. If $IM_{calc} > \mathrm{TH}_{\mathrm{IM}} + \mathrm{MTA}$ then $IM_{calc} - \mathrm{TH}_{\mathrm{IM}}$ is posted as collateral to cover the initial margin
5. A possible MTA applies for the combined change of VM and IM

These are the usual rules in place in margin agreements applicable for the case of bilateral derivatives collateralized with VM and IM and are in line with the minimum requirements by the regulators [28]. In line with these rules the received collateral C can be calculated as follows:

$$\begin{aligned} VM &= \begin{cases} \sum_t P(t) & \text{if} \quad |\sum_t P(t)| > \mathrm{TH}_{\mathrm{VM}} \\ 0 & \text{else} \end{cases} \\ IM_{rec} &= \max\left(0, IM_{\mathrm{calc}} - \mathrm{TH}_{\mathrm{IM}}\right) \\ C_{calc} &= VM + IM_{rec} \\ C_t &= \begin{cases} C_{t-1} & \text{if} \ |C_t - C_{\mathrm{calc}}| < MTA \\ C_{calc} & \text{else} \end{cases} \end{aligned} \tag{2.3.6}$$

$$\begin{aligned} \text{where} \qquad P(t) &: \text{Present value of trade } t \\ \mathrm{IM}_{\mathrm{calc}} &: \text{calculated IM to be received} \\ \mathrm{TH}_{\mathrm{VM}} &: \text{Variation Margin Threshold} \\ \mathrm{TH}_{\mathrm{IM}} &: \text{Initial Margin Threshold} \\ MTA &: \text{Minimum transfer amount} \\ C_{t-1} &: C \text{ calculated in last time period} \end{aligned}$$

Considering thresholds and a possible minimum transfer amount we yield the formulas depicted in table 2.1 for some key figures of the SA-CCR model.

	NICA	C_{calc}	RC
Uncollateralized bilateral derivatives	0	0	$\sum_t P(t)$
Bilateral derivatives collateralized with VM	0	VM	TH_{VM}+MTA
Bilateral derivatives collateralized with VM and IM	IM_{rec}	VM+IM_{rec}	$\max\left(0, \text{MTA} - \text{IM}_{\text{rec}}\right)$

Table 2.1: Calculation of NICA, C and RC under different, uncleared margining approaches

The case that is analyzed the most in this thesis is Bilateral derivatives collateralized with VM and IM. It is important to note that assuming the absence of a minimum transfer amount, the *RC* is always floored at zero in this case and a change in VM or IM then only impacts the SA-CCR EAD through the use of C in the multiplier calculation of equation 2.3.3.

The multiplier is therefore the central point of focus when analyzing the interaction between SA-CCR and margin. The multiplier function is plotted in figure 2.1. The multiplier is ceiled at one if $C > V$, i.e. if the portfolio is overcollateralized which under assumption 1 is the case when the bank receives IM. With increasing overcollateralization the multiplier drops and approaches its floor of 5%. The other factor that drives the multiplier is the portfolios *AddOn*.

The *AddOn* is a portfolio metric that is supposed to represent how quickly the value of the portfolio can rise within the MPoR. The underlying idea is similar to a value at risk and the *AddOn* is designed to be easy to compute while still being portfolio based and taking optionalities into account. Margining does not impact the calculated *AddOn*. Therefore, *AddOn* calculation for SA-CCR is not presented in great detail at this point. The reader is referred to [31, Article 280] and the library that was implemented for the purpose of this thesis [19].

2.4 Allocation of Risk Measures

With increasing sophistication of risk, own capital and margining models the need for equally sophisticated tools for attributing these measures rises as well. Allocating the variation margin or models that disregard portfolio effects entirely such as the current exposure method (CEM) to individual trades is trivial as these measures may just be calculated for an individual trade and then added up across all trades to obtain the correct aggregate value. For measures which take portfolio effects into

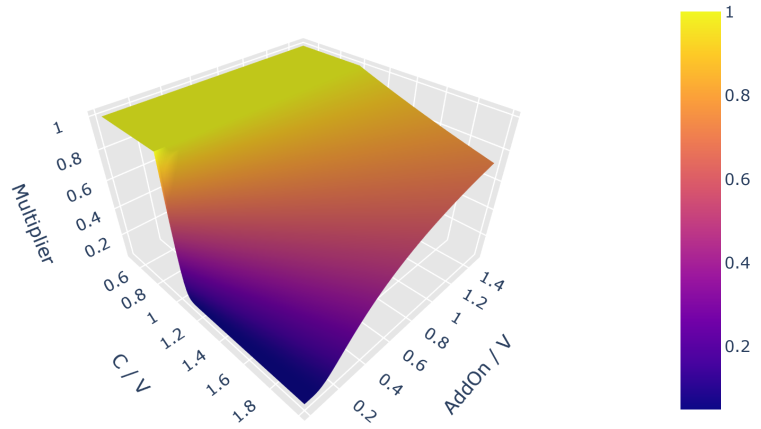


Figure 2.1: The multiplier function of the SA-CCR model that is part of the potential future calculation (PFE) calculation. Overcollateralization, i.e. posted IM reduces SA-CCR through reducing the multiplier.

account such as a VaR model, ISDA SIMM or SA-CCR however, this approach is not possible. The advent of portfolio based models for internal risk measurement in the late 1990s and for regulatory risk measurement in the late 2000s sparked research into how such measures should be reallocated. Gregory [18, Chapter 10.7] states that three approaches are used in practice:

1. Incremental allocation
2. Marginal allocation which will be called Euler allocation for the remainder of this thesis
3. Pro rata allocation

Based on the paper of Koyluoglu and Stoker [22] the list of approaches can be complemented by:

4. Discrete marginal allocation
5. Shapley value

Unfortunately, naming conventions for the different allocation approaches are not consistent across different publications. Therefore, a definition of the five approaches is following based on the notation used by Tasche [38]. In the following we will always assume that $X_1, \ldots, X_n$ are real valued random variables that are representing the profits and losses of the trades in a portfolio. $1, \ldots, n$ represents the order in which the trades have been added to the portfolio. X denotes the portfolio-wide PnL, s.t.

$$X = \sum_{i=1}^{n} X_i. \tag{2.4.1}$$

$\rho(X)$ is a risk measure that is supposed to estimate the profit or loss of the portfolio at a certain quantile for a certain time period. Both, the ISDA SIMM model and the SA-CCR model are in their core such risk measures.

The allocation or contribution of trade i to risk measure $\rho(X)$ is denoted as $\rho\left(X_i|X\right)$. Position sizes in the portfolio can be notated through a vector $\mathbf{u} = (u_1, \ldots, u_n)$:

$$X(\mathbf{u}) = X(u_1, \ldots, u_n) = \sum_{i=1}^{n} u_i X_i \tag{2.4.2}$$

To make it more convenient to analyze changes to $\mathbf{u}$ we also introduce the function

$$f_{\rho,X}\left(\mathbf{u}\right) \tag{2.4.3}$$

Then, with $\mathbf{1}$ being a vector of ones, $\rho\left(X\left(\mathbf{1}\right)\right) = \rho\left(X\right)$. $\mathbf{u} = \mathbf{1}$ indicates the initial state of the portfolio when calculating an allocation - it does not imply that the notional of each position is 1.

Definition 2.4.4 (Incremental allocation)**.** *Assuming that $\rho\left(X\right)$ is a risk measure, the incremental allocation of trade n can be calculated as*

$$\begin{gathered} \text{with } u_{i\neq n} = 1 \text{ and } u_n = 0 \\ \rho_{inc}\left(X_n|X\right) = \rho\left(X\right) - \rho\left(X\left(\mathbf{u}\right)\right)\ . \end{gathered} \tag{2.4.5}$$

The incremental allocation can only be calculated for trade n.

Definition 2.4.6 (Euler allocation)**.** *Assuming that $\rho\left(X\right)$ is a risk measure that is homogeneous of degree one and continuously differentiable, the Euler allocation of an arbitrary trade i can be calculated as*

$$\rho_{Euler}\left(X_i|X\right) = \frac{d\rho}{dh}\left(X + hX_i\right)|_{h\to 0} = 1\frac{\partial f_\rho}{\partial u_i}\ . \tag{2.4.7}$$

Definition 2.4.8 (Pro rata allocation)**.** *Assuming that $\rho\left(X\right)$ is a risk measure, the pro rata allocation of an arbitrary trade i can be calculated as*

$$\begin{gathered} \text{with } u_i = 1 \text{ and } u_{\neq i} = 0 \\ \rho_{ProRata}\left(X_i|X\right) = \frac{\rho(X(\mathbf{u}))}{\rho(X)}\ . \end{gathered} \tag{2.4.9}$$

Definition 2.4.10 (Discrete marginal allocation)**.** *Assuming that $\rho(X)$ is a risk measure, the discrete marginal allocation of an arbitrary trade i can be calculated as*

$$\begin{aligned} &\textit{with } u_i = 0 \textit{ and } u_{\neq i} = 1 \\ \rho_{discrete}\left(X_n|X\right) &= \rho\left(X\right) - \rho\left(X\left(\mathbf{u}\right)\right) \end{aligned} \tag{2.4.11}$$

Definition 2.4.12 (Shapley allocation)**.** *To calculate the Shapley allocation of a portfolio one needs to iterate through all permutations how the trades in the portfolio could be ordered. For a given trade i the Shapley allocation is the average of the amount by which the risk measure changes when adding trade i to the portfolio in each of the permutations.*

The usefulness of any of the five allocation approaches listed above is dependent on the individual application of the allocation. Criteria by which the allocation approach is judged are also highly dependent of the application. However, the two criteria

1. Native additivity
2. Risk sensitivity

are usually regarded to be the most important ones. They are for example the criteria by which Koyluoglu and Stoker [22] compare the different allocation approaches.

A allocation algorithm *alloc* exhibits **native additivity** if equation 2.4.13 holds.

$$\sum_{i=1}^{n} \rho_{alloc}\left(X|X_i\right) = \rho\left(X\right) \tag{2.4.13}$$

Risk sensitivity means that $\rho_{alloc}\left(X|X_i\right)$ should indicate how the trade i impacts the overall risk $\rho\left(X\right)$. No mathematical definition is available to define when an allocation is considered to be risk sensitive. A sensible criteria could be that a trade that reduces the risk of the portfolio, i.e. a hedge trade should have a negative contribution to the risk measure.

Depending on the application of the allocation other criteria might be important such as

- Non-negativity of allocations
- The value allocated to a trade must not change through time
- The allocated value needs to be independent from the order in which trades are entered

Generally, such auxiliary requirements raise through operational or technical limitations. Some of the allocation algorithms presented above comply with them, while others do not. Such requirements might be the reason that allocation algorithms that are dismissed as inappropriate in this thesis still find application in the field.

The incremental allocation excels for use at the trading desk. It is well suited as an input when making investment decisions or for calculating the remuneration of traders and trading desks after entering a new trade. Both, Gregory [18] and Koyluoglu and Stoker [22] state that incremental allocation is the best suited allocation for these purposes. It does, however, perform poorly for risk analysis of an existing portfolio. Incremental allocation is further investigated in section 2.4.1.

Euler allocation on the other hand is well suited for analysis of an existing portfolio. It can identify concentration risk within a portfolio or be used for portfolio optimization. In the literature, Euler allocation is generally regarded as the best allocation approach for such purposes as it exhibits native additivity and risk sensitivity and can be calculated for all trades. Euler allocation is further investigated in section 2.4.2.

Pro rata allocation is generally not risk sensitive for risk measures that take portfolio effects into account. It is, however, very inexpensive to compute and suitable for models that are based on trade contributions such as the CEM or the standardized approach for initial margin (see 2.2.1). Usage of a pro rata allocation may circumvent some operational or technical issues as trade contributions are always positive. Due to its simplicity and lacking risk sensitivity for the models analyzed in this thesis, pro rata allocation will not be analyzed in further detail in this thesis.

While being a very intuitive approach, performance of discrete marginal allocation is relatively poor. The approach does not exhibit native additivity as Tasche [38] shows that

$$\rho_{discrete}\left(X_i|X\right) \leq \rho_{Euler}\left(X_i|X\right)$$

for ρ that are continuously differentiable, sub-additive and homogeneous of degree 1. Koyluoglu and Stoker [22] note that "it could be argued that discrete marginal allocation is wholly dominated by the Euler allocation". For these reasons discrete marginal allocation is not analyzed in further detail in this thesis.

Finally, the Shapley method introduced in [33] exhibits native additivity and risk sensitivity. However, its computation is much more time consuming than any other allocation presented. Therefore it can realistically only be used for very small portfolios or to calculate allocations of subportfolios, e.g. the subportfolios of certain departments. Koyluoglu and Stoker [22] compare Euler and Shapley allocation and find that Shapley allocation is a more robust measure as it does not require differentiability of ρ. The relatively rigorous requirements against ρ to use Euler allocation are introduced in detail in section 2.4.2 and chapter 3 investigates under which circumstances the ISDA SIMM and SA-CCR model comply with these requirements. Overall, Koyluoglu and Stoker suggest to only use Shapley allocation over Euler allocation for calculating the contribution of few subportfolios. Due to the poor computational performance and the resulting small practical relevance, Shapley allocation is not investigated further in this thesis.

2.4.1 Incremental allocation

Incremental allocation can only be applied when observing the development of a portfolio through time. Given a pre-existing portfolio P consisting of n trades t_1 through t_n and a portfolio-based measure M, the incremental contribution of the first and second additional trade may be calculated as:

$$\begin{aligned} M_{\text{inc},t_{n+1}} &= M\left(t_1 \dots t_{n+1}\right) - M\left(t_1 \dots t_n\right) \\ M_{\text{inc},t_{n+2}} &= M\left(t_1 \dots t_{n+2}\right) - M\left(t_1 \dots t_{n+1}\right) \end{aligned}$$

It can be easily seen that this approach yields a natively additive allocation since it forms a telescoping sum[3] :

$$\begin{aligned} M_{\text{inc},t_1} &= M(t_1) \\ M_{\text{inc},t_i} &= M(t_i) - M(t_{i-1}) \\ M_{\text{inc},t_n} &= M(t_n) - M(t_{n-1}) \\ \sum_{i=1}^{n} M_{\text{inc},i} &= M(t_1) - M(t_1) + \cdots + M(t_{n-1}) - M(t_{n-1}) + M(t_n) = M(t_n) \end{aligned}$$

The incremental allocation can be calculated as or before a new trade is added to the portfolio. It is a risk sensitive value when it is calculated as it accurately reflects how the additional trade changes the risk measure. If the trade is mitigating risk at the time of its inception according to M its incremental allocation M_{inc} is negative. If it increases the risk its M_{inc} is positive. However, M_{inc} does not adapt over time and is likely to loose its accurate risk depiction as additional trades are added to the portfolio. As a portfolio develops it may well be possible, that a trade for which a negative M_{inc} was calculated at its inception may loose its risk mitigation. Due to this property M_{inc} of a given trade should ideally only be used at or before trade inception. One such use case is the PnL calculation of a new trade to determine the performance of the trading desk or trader which initiated the trade. Another would be to use it prior to an investment decision [39]. It can however not be used to analyze an existing portfolio to e.g. identify trades which drive risk or determine how increases or decreases in a given position would impact the portfolio measure. It also cant be calculated deterministically a posteriori for a portfolio without knowing its composition through time.

Unlike the Euler allocation that will be discussed in detail in the next chapter, the risk measure or portfolio composition does not need to fulfill any requisites for an incremental allocation. The only, very minor constraint is that no two trades can be added to portfolio simultaneously. This may actually be a limitation in practice if e.g. the incremental allocation is calculated based on the last EOD portfolio and different trading desks add trades to a common portfolio throughout the day.

However, it is clear that the incremental allocation may be applied to the SA-CCR model and the ISDA SIMM model without any restrictions and practitioners should

[3]For brevity in Notation let $M(t_i)$ be equivalent to $M(t_1 \dots t_i)$

therefore use this approach for the use cases in the front office at which is excels as pointed out in section 2.4.

As no further analysis whether incremental allocation is suitable for SA-CCR allocation is necessary, it will not be discussed further in this thesis.

2.4.2 Euler allocation

The idea of Euler allocation is based on Euler's homogeneous functions theorem.

Definition 2.4.14. *A function f is a positive homogeneous function to a degree of k if*

$$f(\alpha \mathbf{x}) = \alpha^k f(\mathbf{x}) \tag{2.4.15}$$

$$\textit{for } \alpha > 0 \tag{2.4.16}$$

A function would be homogeneous rather than just *positive* homogeneous if equation 2.4.15 would also hold for $\alpha < 0$. Risk measures can only exhibit positive homgenity. Many risk measures do have the property that doubling position size does double the measured risk. However, inverting the postion, e.g. having a short instead of a long position does not result in a negative risk estimate.

Euler's homogeneous functions theorem states

Theorem 2.4.17. *Let $f(\mathbf{x})$ be a (positive) homogeneous function of degree k, then*

$$\sum_i x_i \frac{\partial f}{\partial x_i} = k f(\mathbf{x}) \tag{2.4.18}$$

If we assume that $k = 1$ and use the risk measure $\rho(\mathbf{u})$ as a function of invested position size with $\mathbf{u} = \mathbf{1}$ being the current position equation 2.4.18 transforms to

$$\mathbf{1} \frac{\partial \rho(\mathbf{u})}{\partial u_i} = \rho(\mathbf{u}) \tag{2.4.19}$$

which is what is stated in as the definition of Euler allocation in definition 2.4.6.

While $u = \mathbf{1}$ is defined as the current position size we can also define it as the notional in USD invested in the individual trades, i.e. $\mathbf{n} = (\text{notional}_1, \dots, \text{notional}_n)$.

The Euler allocation w.r.t trade i would then be calculated as

$$\mathbf{n} \frac{\partial \rho(\mathbf{n})}{\partial \text{notional}_i} = \rho(\mathbf{n}) \tag{2.4.20}$$

As any partial derivative, $\frac{\partial \rho((u))}{\partial u_i}$ may be approximated as a finite difference.

$$\text{with } \mathbf{h} = (h_0, \dots, h_n) \text{ and } h_i = \epsilon > 0 \text{ and } h_{\neq i} = 0$$

$$\frac{\partial \rho(\mathbf{u})}{\partial u_i} = \frac{\rho(\mathbf{u}+\mathbf{h}) - \rho(\mathbf{u}-\mathbf{h})}{2\epsilon} + \mathcal{O}(\epsilon^2) \qquad \text{(central difference)} \qquad (2.4.21)$$

$$\frac{\partial \rho(\mathbf{u})}{\partial u_i} = \frac{\rho(\mathbf{u}+\mathbf{h}) - \rho(\mathbf{u})}{\epsilon} + \mathcal{O}(\epsilon) \qquad \text{(forward difference)} \qquad (2.4.22)$$

$$\frac{\partial \rho(\mathbf{u})}{\partial u_i} = \frac{\rho(\mathbf{u}) - \rho(\mathbf{u}-\mathbf{h})}{\epsilon} + \mathcal{O}(\epsilon) \qquad \text{(backward difference)} \qquad (2.4.23)$$

Throughout this thesis we will work with formula 2.4.19. However, one could instead also work with formula 2.4.20 and yield the same results. Calculating the partial derivatives of equation 2.4.19 through a finite difference approach is equivalent to a relative bump of the notional while applying a finite difference approach on equation 2.4.20 would result in an absolute bump of the notional of each trade.

2.5 Instruments, pricing and market data

For the analysis whose results are presented in chapter 3, a small but diverse set of financial instruments is required. Due to the structure of the ISDA SIMM and the SA-CCR model the set of financial instruments should meet the following criteria:

1. The instruments should range across multiple asset classes
2. Non-linear instruments should be included
3. The instruments should range across multiple currencies
4. The instruments should be commonly traded as bilateral, uncleared derivatives to be relevant for ISDA SIMM
5. Pricing and sensitivity calculation should be possible without implementation of simulation approaches
6. Inferring market data objects required for pricing from market quotes of traded instruments must be simple

Items 4 and 5 of the above list are slightly conflicting. Bilaterally traded derivatives are usually more complex than cleared derivatives. Due to this increased complexity many of them have to be priced with a Monte Carlo simulation since an analytical solution is not possible.

Item 6 rises from the requirement of the ISDA SIMM model to calculate all sensitivities against market quotes. This means for example, that interest rate sensitivities must not be calculated with regard to a movement of the interest rate curve used as a pricing input but with regard to the price of the traded instrument that is used to build the interest rate curve in the first place. In the case of interest rate curves the process to build an interest rate curve is commonly referred to as *bootstrapping* and has to be performed again whenever a sensitivity is calculated to be compliant with

ISDA SIMM. Designing a pricing framework that can handle this required interdependence of market quotes, market data objects such as curves and priced instruments is a steep task even for deceptively simple instruments such as plain vanilla interest rate swaps. For this reason the implementation is based on QuantLib [14] which offers an excellent and proven framework to monitor these interdepecncies with ease.

Careful consideration of the criteria listed above and the available market data lead to the following set of financial instruments that will be used for analysis:

- Overnight indexed swaps
- Fixed-float interest rate swaps
- European equity options
- Swaptions

In the following, we will outline how these instruments are priced, what market data is used for this and how ISDA SIMM compliant sensitivities are calculated for each instrument.

The instruments just serve as a means to perform the analysis of the topics on which the thesis is focused which are the SA-CCR model, margining and risk measure allocation. Accordingly, pricing algorithms and required market data have been chosen rather pragmatically and do not claim to represent the currently established state-of-the-art in pricing these derivatives. In line with the focus of the thesis the following explanations on derivative pricing, market data and sensitivity calculation are kept brief.

A basic familiarity of the reader with interest rate derivative markets and equity derivative markets is assumed. A basic introduction to both may be found in [20]. A comprehensive approach to interest rate derivative markets may be found in [9] or [4].

2.5.1 Overnight indexed swaps

An overnight indexed swap (OIS) is a fixed-float swap whose floating leg underlying are the daily fixings of an overnight index. It can be priced using the so called OIS-curve, whose construction will be discussed below. Usually, only a single OIS curve exists per currency which is used both as a forward curve for estimating future cashflows on the floating leg of an OIS swap and as a *'risk free'* discounting curve for all cash flows of the given currency.

The net present value (NPV) of an OIS may be calculated like that of any other swap by estimating future cashflows of the floating leg with the appropriate interest rate curve and discounting all cashflows of the swap with the currencies' discount curve. Accordingly, an OIS Swap may be priced using only the OIS curve of the swaps currency.

Since the OIS curve also serves as the discount curve it does not require any other interest rate curve to be constructed. For the purpose of this thesis, EUR and USD OIS curves were built from the par rates of OIS swaps as they were quoted on the 10th of May 2019.[4] Conveniently, QuantLib does offer a built in functionality to infer an OIS curve from quoted OIS par rates, namely *OISRateHelper*. Essentially, the OIS par curve is built by interpolating between the quotes. This par curve may then be transformed into other interest curve representations such as a zero curve, forward curve or a discount curve under the assumptions of a single curve universe. How this can be done is e.g. pointed out in [20, Chapter 4].

An OIS changes its NPV, if the OIS curve that it used to price it moves. This delta sensitivity against points on the OIS curve is the basis for the ISDA SIMM calculation of an OIS. ISDA SIMM requires interest rate delta sensitivities of an OIS to be calculated as a finite difference against movements of the quoted market par rates used to build the OIS-curve in the first place. This means that delta sensitivities of an OIS are calculated by raising a market quote that was used to build the OIS-curve by one basis point, rebuilding the OIS-curve as described above and repricing the OIS for which the delta is being calculated. A definition of ISDA SIMM compliant interest rate deltas may be found in [35, Point 22] and [34, Section 2.2].

Additionally, an FX delta needs to be calculated which is used if the trade currency does not coincide with the ISDA SIMM calculation currency that was agreed upon in the CSA. This delta may be approximated as $0.01 * NPV$ if the trades underlying is not an exchange rate [34, Section 2.7].

In fact, such an FX delta is required for all of the following derivatives and as none of them are FX derivatives can always be calculated as $0.01 * NPV$.

2.5.2 Interest rate swaps

In a narrow definition, interest rate swaps or IRS are swaps of a single currency, which have a floating and a fixed leg and whose floating leg references not an OIS index but rather an index of a larger tenor such as the six month EURIBOR index.

For this thesis, IRS on the six months EURIBOR and three month USD LIBOR index are set up for analysis. These tenors have been chosen as they are the reference tenors for their respective currencies. This indicates that IRS on this tenor are very liquid whereas other tenors are rather traded through float-float basis swaps between the reference tenor and another tenor of the same currency.

As for an OIS, an IRS is priced by estimating future payments of the floating leg through the tenor curve and then discounting all cash flows whether they are estimated or fixed. However, unlike the OIS the curve used for forecasting future

[4] To build an arbitrage free OIS curve especially the short end should be based on more than just OIS quotes as is for example pointed out in [2] or [10]. However, for this thesis higher accuracy in curve construction was not required.

payments and discounting generally does not coincide.

The presence of multiple interest rate curves per currency, i.e. a multi-curve universe has become widely accepted since the financial crisis of 2008 raised awareness for the credit risks associated with interbank lending. This has complicated interest rate curve construction and made a bootstrapping approach in which the interest rate curves of a currency are built up incrementally starting with the OIS curve necessary. The bootstrapping of a multi curve environment is a rather intricate process and is described in detail for example in [2] and [10]. However, for the purpose of this thesis, it is sufficient to know that under bootstrapping, the zero rate curve of a reference Libor curve such as the 6M EURIBOR curve is dependent on both, the par rates quoted for 6M EURIBOR IRS[5] as well as the par rates quoted for OIS-curve of the same currency.

To calculate an ISDA SIMM compliant sensitivity against a specific quoted par rate that was used to construct the OIS or tenor curve is shifted, the bootstrapping is repeated to yield the OIS curve and the forward curve of the underlying interest rate curve and the IRS is repriced.

2.5.3 European Equity Options

For the purpose of this thesis, European equity options are priced analytically with a Black Scholes model as it is for example introduced in [20, Chapter 14]. The model was originally published in [8].

To do so, QuantLib offers the *BlackScholesProcess* class which can be constructed from the current spot value of an equity, a discount curve of the relevant currency and a volatility surface of implied volatilities with option maturities in one dimension and option strikes in the other.

As a discount curve, the OIS curve introduced in section 2.5.1 is used. On the other hand, equity spot and volatility surface dummy data was fabricated for two equities with the volatility surfaces being flat. This fabricated data is sufficient for the purpose of this thesis as its structure allows for an easy calculation of ISDA SIMM compliant sensitivities[6] and for the analysis conducted for section 3 it is irrelevant whether the used market data has actually been observed or has been fabricated.

The relationship between the market data from which a volatility surface is built and the volatility surface itself is straight forward. Implied volatilities of options are directly quoted by the market and can be put on a grid based on the option maturity and strike of the option for which they are quoted. Afterwards, the volatility surface

[5]To be exact, not only interest rate swaps but also forward rate agreements on the 3M USD LIBOR and 6M EURIBOR were used. Again market data as observed on the 10th of May 2019 is used for all calculations presented in section 3.

[6]The used constructed volatility surface has for example exactly the maturity vertices that ISDA SIMM expects without further interpolation being necessary.

is created by simply interpolating between these points. For this thesis a bilinear interpolation and extrapolation was chosen.

When pricing a European equity option the relevant risk free interest rate can be identified by retrieving the zero rate for the option maturity from the OIS curve of the currency in which the equity is traded. Afterwards, an equity volatility can be retrieved from the volatility surface by interpolating the implied volatility for the strike and option maturity of the option that should be priced. These two values can then be used as the risk free rate and equity volatility in the call or put Black Scholes formula to price the option.

According to ISDA SIMM, an equity option has equity delta, equity vega, interest rate delta and FX delta risk. The FX delta risk is calculated in the same way as described for an OIS in section 2.5.1. An interest rate delta sensitivity is calculated by shifting an OIS par rate, rebuilding the OIS curve as pointed out in section 2.5.1 and afterwards inferring a new risk free rate from this rebuilt OIS curve for a new valuation under the Black Scholes model.

An ISDA SIMM compliant equity delta is calculated by increasing the spot price of the underlying equity by one percent and repricing the option under the Black Scholes model. Finally, ISDA SIMM compliant equity volatility sensitivities, i.e. vegas, are calculated by increasing all quoted implied volatilities that share a common option maturity by 0.01, interpolating again to retrieve a new volatility surface and proceed with repricing the option as described above.

Rules on how equity sensitivities and equity volatility sensitivities have to be calculated for the purpose of ISDA SIMM are defined under [34, Section 2.5 and 2.8] and [35, Points 21, 26 and section C.3].

2.5.4 Swaptions

For the purpose of this thesis, the primary approach to pricing swaptions is use of the Black model [7]. This model was originally developed for European options on commodity or equity future contracts and is very similar to the Black-Scholes model but models the development of a forward price instead of spot prices. Both, the Black-Scholes model and Black model assume that returns follow a log-normal distribution. If forward rates of interest rates become negative, the Black model can therefore not be used since log return of negative values can't be calculated. In these cases, an alternative model by Bachelier [5] which assumes the forward rate to follow a normal process can be used. The application of the Bachelier model for swaption pricing is e.g. presented in [16].

Both, the Black model and the Bachelier model require the volatility of the underlying as in input. Similar to what was described for the Black-Scholes model in section 2.5.3 these can be captured in a volatility surface or, depending of the granularity of used input instruments, a structure of even higher dimension. It is important to

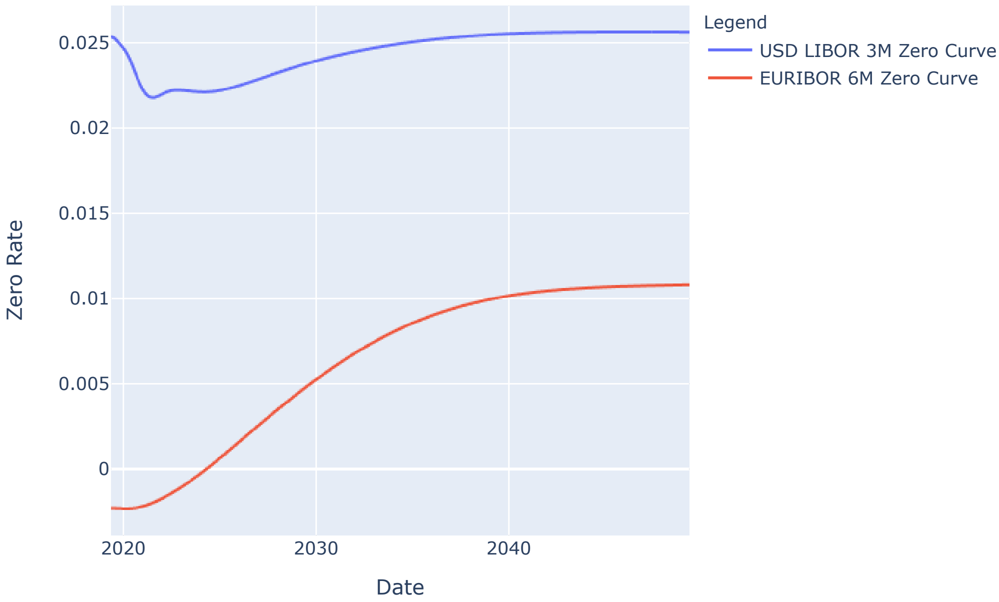


Figure 2.2: EURIBOR 6M and USD LIBOR 3M zero curve on the 10th of May 2019. It can be seen that the USD LIBOR zero rate is constantly well above zero percent while the zero rate of the EURIBOR remains negative for the first couple of years.

note that the implied volatilities used for either the Bachelier or the Black model need to be inferred from option prices using the respective model. One cannot, for example, calculate the implied volatility of a swaption with a Black model and use the resulting number afterwards in the Bachelier formula. The Chicago Mercantile Exchange publishes both the implied volatilities under the normal and under the log-normal model of swaptions on their website [7].

For this thesis swaptions reference swaps on either the six month EURIBOR or the three month USD LIBOR. Market data of the 10th of May 2019 has been used, a time at which the interest rates between the two currencies were quite different, with the EURIBOR 6M zero curve being partially negative as can be seen in figure 2.2[8].

For this reason, swaptions on the EURIBOR 6M index are priced with a Bachelier model while swaptions on the USD LIBOR 3M index are priced with a Black model, both of which offer analytical solutions to price a swaption.

In similar fashion to Black-Scholes pricing of equity options the swaption is priced by first interpolating a volatility that matches the option maturity and swap maturity of the priced swaption[9] and inferring a the forward rate from the interest rate curve

[7] ftp://ftp.cmegroup.com/irs/

[8] This figure is produced in Appendix A.1.

[9] We are working with a two dimensional swaption volatility surface which only contains the

that matches the start and end-point of the underlying swap. This forward rate and volatility are then plugged into the respective Black or Bachelier formula for a European Call if the underlying is a receiver swap or of a European Put if the underlying is a payer swap.

Calculation of ISDA SIMM compliant sensitivities happens in same way as for the previous products. To yield interest rate delta sensitivities par quotes used to build either the OIS or LIBOR or EURIBOR curve are shifted, and bootstrapping is re-executed to yield a new forward interest rate curve. From this forward curve the relevant forward rate of the underlying swap is inferred, plugged into the Bachelier of Black pricing formula and the swaption is repriced to yield the delta sensitivity against the initially shifted par quote.

Interest rate vega sensitivities against the volatility are calculated in the same way as those of an European equity option. They are calculated by increasing all quoted implied volatilities that share a common option maturity by 0.01, interpolating again to retrieve a new volatility surface and proceed with repricing the swaption as described above.

Finally, FX sensitivities are calculated again as one percent of the NPV of the swaption as was also the case for the other instruments.

implied volatilities of at-the-money swaptions and does not differentiate based on moneyness of the option. This is in line with the grid for interest rate vegas that the ISDA SIMM model defines in [34, Section 2.8] and [35, Point 10]

Chapter 3

Results

In this chapter the results produced for the thesis are presented. We will first show with a small sample portfolio that numerical Euler allocation of SA-CCR is possible in section 3.1. On these exemplary portfolios we will also point out a couple of observations highlighting how an Euler allocation can offer great insight due to its risk sensitive nature.

Afterwards, section 3.2 lists scenarios, under which Euler allocation prerequisites are violated and suggests approaches on how to mitigate these issues or proposes a workaround.

3.1 Exemplary Euler allocation of SA-CCR under consideration of margining

In this section we assume, that the minimum transfer amount, variation margin threshold and initial margin threshold as defined in section 2.1 are all zero. This means, that the margin calculated by the used variation margin and initial margin model is entirely incorporated in the SA-CCR model for EAD calculation.

The reason for this is, that assumptions other than zero for the thresholds and MTA generally violate the homogeneity prerequisite for Euler allocation. In practice, this is a strong and somewhat unrealistic assumption as for example an initial margin threshold of 50Mn is usual for bilateral portfolios as it is the highest amount allowed by the regulator [28, Requirement 2.2]. Due to the high practical relevance, the impact of thresholds and MTA on Euler allocation is analyzed in detail in section 3.2.

3.1.1 Exemplary allocation of SA-CCR for a small portfolio of equity options

In this section we analyze an Euler allocation of a small portfolio of equity options. The detailed computation steps are demonstrated in appendix A.2. First, we consider a portfolio consisting of two million call options and three million put options on a fabricated ADS stock. All options are struck at the current stock price and long. Obviously, the two positions are in a hedge relation and being long, at the money options, both positions do have a significant, positive present value.

With the ISDA SIMM and SA-CCR portfolio risk measures as introduced in sections 2.2.2 and 2.3 and considering different margining approaches we can calculate portfolio risk measure values displayed in the portfolio risk measure column of table 3.1. It can

	Portfolio Risk Measure	Allocation to 2Mn ADS Call	Allocation to 3Mn ADS Put
SIMM	14,231,564 EUR	-33.75%	133.75%
EAD No margin	37,643,536 EUR	99.21%	0.79%
EAD VM only	3,519,458 EUR	232.47%	-132.47%
EAD VM and IM	345,874 USD	622.10%	-522.10%

Table 3.1: Risk measure results for an exemplary equity portfolio

be seen that incorporation of the variation margin significantly drops the portfolio EAD. The main reason for this is that the RC in formula 2.3.1 drops from 18.5Mn which is equal to the portfolio PV to zero when VM is incorporated. The additional overcollateralization of 14.2Mn USD through IM is then the reason for the EAD to again drop by 90% to 346k USD when IM is incorporated in addition to VM.

We can now calculate the Euler allocation by applying the forward difference formula 2.4.22 with a bump size of $\epsilon = 0.0001$ as this is the first time this is explicitly done we are writing this down step by step for EAD under consideration of VM and IM.

1. We increase the position in Adidas call options by ϵ adding 200 call options to the portfolio.
2. We recalculate the EAD of the updated portfolio as $\rho_{\text{Inc Call}}$ and remove the 200 Mn call options again.
3. We repeat this with a position increase of 100 Adidas put options yielding result $\rho_{\text{Inc Put}}$.
4. We now use the *bumped* risk measure results and the original ρ of 345,874 USD as displayed in table 3.1 to yield the allocation.
5. The allocated value for the call position is $\text{alloc}_{\text{call}} = \frac{\rho_{\text{Inc Call}} - \rho}{0.0001} =$ 2.152 Mn USD.

6. The allocated value for the put position is $\text{alloc}_{\text{call}} = \frac{\rho\text{Inc Put} - \rho}{0.0001} = -1.806$ Mn USD.
7. This yield the relative allocated values of 622% and -522% displayed in table 3.1.

Comparing the allocations of the different risk measures in table 3.1 with each other uncovers a couple of interesting observations.

First, under consideration of no margin, the contribution of the put position is close to zero. The reason for this is that a marginal increase in the put position impacts the PFE and RC component of equation 2.3.1 differently. A marginal notional increase increases the RC due to associated increasing positive market value. The potential future exposure (PFE) on the other hand decreases, since the put hedges some of the portfolio risk incurred by the larger call position.

Secondly, for the ISDA SIMM risk measure the call position is considered as a hedge position, i.e. has a negative allocated value, while for the VM only EAD model the put position is considered a hedge. A marginal increase in the call position decreases the charged IM while it increases the calculated EAD under consideration of VM only. These two effects reinforce each other when allocating the EAD under consideration of VM and IM. Here, a marginal increase in the call position simultaneously increases the portfolio risk under the SA-CCR EAD risk measure while also resulting in a decrease in received IM and therefore reducing overcollateralization which further raises the calculated EAD.

On the other hand, a marginal increase in the put position decreases the EAD risk measure while simultaneously increasing overcollateralization. This is the reason for the stark increase of the relative allocated risk from the VM only case to the VM+IM case.

Finally, it is worth discussing why the ISDA SIMM risk measure and the SA-CCR risk measure evaluate the portfolio so differently with one considering the put a hedge position while the other one considers the call as the hedge position. Due to the large differences between the two models and the dependency of the ISDA SIMM model on market data is is difficult to pinpoint a single driving factor for this phenomenon. However, the different holding periods of ten days for the ISDA SIMM model and one year for the SA-CCR model appears to be a likely candidate. Indeed, if we reduce the maturity of the options from one year to ten days and thereby effectively reduce the holding period of the SA-CCR model to ten days we can see from the results in table 3.2 that the SA-CCR model then considers the smaller call position to be the hedge trade.

	Allocation to 2Mn ADS Call 10D	Allocation to 3Mn ADS Put 10D	Portfolio Risk Measure
EAD VM only	-358.06%	458.06%	1,701,707 EUR

Table 3.2: Allocation amount under ten day maturity

A reliable test, whether the Euler allocation was successful and therefore if the allocated function exhibits homogeneity is to calculate if equation 2.4.18 holds.

When doing so the value allocated to a certain trade alloc_t is $\frac{\partial f}{\partial x_i}$ and the portfolio risk measure ρ is the $f(x)$. The degree k is one. This results in formula 3.1.1 which is just an alternative notation of Eulers theorem of homogeneous functions. If this equation is met, the risk measure ρ does locally exhibit homogeneity and the results alloc_t can be considered a risk sensitive allocation and used for risk analysis or portfolio optimization purposes.

$$\sum_t \text{alloc}_t = \rho \tag{3.1.1}$$

For the allocation of EAD under VM and IM shown in table 3.1 equation 3.1.1 does not hold as we yield a residual of 1068 EUR or about 0.3% of the portfolio risk measure. Since alloc_t has just been approximated with a finite difference some deviation may be expected. This deviation appears to be within the expected order of magnitude of a forward difference approximation of the two derivatives.

The approximated derivate for the call position is $6.221 * 345,874 = 2.151\text{Mn}$ while the approximated derivative of the put position is -1.806Mn. In absolute terms this results in a sum of about 4Mn. Considering the $\epsilon = 0.0001$ this indicates, that the sum of the error of the two derivatives should be in the order of $\mathcal{O}\,(4\text{Mn}/0.0001 = 400\text{USD})$ which is in line with the observed deviation.

Theoretically, application of a central difference approach should bring the order of magnitude of the error down to $\mathcal{O}\,(4Mn/0.0001^2 = 0.04\text{USD})$. This behavior can also be observed. If the partial derivatives are instead calculated as $(\rho_{Incr} - \rho_{Decr})/0.0001$ the sum of the allocations deviates from the portfolio risk measures by only 0.01 EUR. This indicates that for proper native additivity of the Euler allocation of EAD, the computationally more expensive central difference approach should be used. Another case for application of a central difference approach will be made in section 3.2.3.

3.1.2 Exemplary allocation of SA-CCR for a small portfolio of interest rate derivatives

In this section we investigate a small rates portfolio. This section will further highlight how the interaction of the two EAD and initial margin risk measures can yield surprising results stressing the need for a risk sensitive allocation methodology for analysis purposes. The detailed computation steps are demonstrated in appendix A.3.

Initially, we consider a 1Bn USD Receiver IRS and a 180Mn EUR Payer IRS and create three portfolios. The first only contains the USD IRS, the second only contains the EUR IRS and the third contains both trades. For these portfolios we calculate three risk measures: the ISDA SIMM initial margin, the EAD under consideration of VM and the EAD under consideration of VM and IM. The results are displayed in table 3.3.

	EAD VM only	ISDA SIMM	EAD VM + IM
EUR Swap	1,957,315 USD	6,079,460 USD	286,420 USD
USD Swap	10,873,970 USD	28,762,683 USD	2,014,873 USD
Both Swaps	12,831,284 USD	28,059,093 USD	3,074,959 USD

Table 3.3: Risk measures for exemplary rates portfolio

Interest rate risks across different currencies are handled differently between the SA-CCR and the ISDA SIMM model. In the SA-CCR model each interest rate currency forms a separate so called *hedging set*. SA-CCR does not allow for any hedge effects across the borders of a hedging set. We can observe this in the 'EAD VM only' column of table 3.3, since the EAD of the portfolio containing both trades is simply the sum of the two standalone portfolios. The ISDA SIMM risk measure on the other hand does allow hedge effects across currencies within the interest rate asset class. When aggregating sensitivities across multiple currency buckets, ISDA SIMM assumes a correlation of 22% [35, Section D.2]. This does show in the ISDA SIMM column of table 3.3 with the ISDA SIMM charged for the portfolio containing both trades being smaller than the sum of the two standalone portfolios.

This difference in models leads to the phenomenon that can be observed in the EAD under VM and IM column. The calculated EAD for the portfolio of both IRS is significantly higher than the sum of the EAD of the two standalone portfolios. We have found a counterexample showing that SA-CCR under consideration of VM and IM is not a sub-additive risk measure. Subadditivity is one of the properties of a coherent risk measure and counterexamples showing that a risk measure does not exhibit subadditivity can for example be constructed for all VaR-based risk measures. However, for EAD under IM and VM it appears to be especially simple to construct a counterexample.

The SA-CCR model considers the portfolio of both trades to be just as risky as the two trades independently. However, the available overcollateralization of the portfolio with both trades is relatively lower than the overcollateralization of the two standalone portfolios since the ISDA SIMM model does recognize hedge effect trades. This constellation leads to the observed effect that the EAD of the joint portfolio is higher than the sum of the standalone EAD of the trades.

When performing an Euler allocation of the different risk measures of the portfolio containing both IRS we yield the allocation as depicted in table 3.4.

The results are related to those observed in table 3.1. As $15.25\% * 12.83Mn = 1.96Mn$ the Euler allocation results coincide with the standalone results when allocating the EAD under consideration of VM only. For the allocation of ISDA SIMM on the other

	Allocated EAD VM only	Allocated ISDA SIMM	Allocated EAD VM + IM
180Mn EUR Swap	15.25%	-0.19%	34.95%
1000Mn USD Swap	84.75%	100.19%	65.05%

Table 3.4: Risk measure allocation of an exemplary rates portfolio

hand the EUR trade is considered a hedge trade and almost none of the risk measure is allocated to it. The fact that the EUR trade reduces overcollateralization then also leads to the overproportionate fraction of the risk measure that is allocated to it under consideration of VM and IM.

Considering the larger USD swap as the baseline trade the EUR swap contribution to the portfolio EAD under IM and VM is overproportionate since it is considered to be a risk mitigating trade by the ISDA SIMM model while the SA-CCR model considers it to increase the risk. The opposite phenomenon can be observed when we add a USD receiver swaption as a third trade to the portfolio.

For this we add a one year swaption on a five year 500Mn USD receiver swap to the portfolio. The risk measures of the resulting portfolio are allocated and the result displayed in table 3.5.

	Allocated EAD VM only	Allocated ISDA SIMM	Allocated EAD VM + IM
180Mn EUR Swap	17.70%	0.50%	43.69%
1000Mn USD Swap	98.33%	80.07%	125.93%
500Mn USD Swaption	-16.04%	19.44%	-69.62%

Table 3.5: Allocation results after addition of a swaption

As can be seen, the swaption is considered to be marginally risk decreasing by the EAD risk measure with only VM while it is considered to be a risk increasing trade by the ISDA SIMM risk measure. Consequently, the swaption reduces the EAD under consideration of VM and IM by an overproportionate amount as it decreases risk while increasing overcollateralization which is indicated by the large negative Euler allocation.

In line with the results of the Euler allocation we can see in table 3.6 that inclusion of the swaption increases portfolio ISDA SIMM by 24%, while decreasing portfolio EAD under VM by -14% and portfolio EAD und VM and IM by -48%. This is exactly the opposite effect as observed for the EUR swap beforehand.

	EAD VM only	ISDA SIMM	EAD VM + IM
Portfolio	11,058,114 USD	34,796,088 USD	1,586,748 USD

Table 3.6: Impact of the swaption on the portfolio risk measure

For the swaption it is more difficult to pinpoint where the difference between the ISDA SIMM and SA-CCR model that results in the very different risk assessment of the swaption in relation to the rest of the portfolio is originating. Sensitivities of the swaps and swaption are calculated very differently between the two models with the ISDA SIMM model being based on current market data whilst the SA-CCR makes much more simplifying assumptions.

3.1.3 Exemplary allocation of SA-CCR on subportfolios

One of the advantages of the Euler allocation is, that once allocated, values on trade level can be aggregated to also produce risk sensitive results for subportfolios. It is also possible to directly allocate risk metrics on subportfolios if no further granularity is required and thereby saving computation time. The detailed computation steps for the results of this section are demonstrated in appendix A.4.

For this we put the five exemplary derivatives of the two previous sections in a joint portfolio, i.e. the EUR IRS, the USD IRS, the USD swaption and the position in ADS put and call options. The resulting portfolio risk measures are displayed in the last row of table 3.7 with the portfolio results of the standalone portfolios from the previous section displayed as reference.

	EAD VM only	ISDA SIMM	EAD VM + IM
Equity Portfolio	3,519,458 USD	14,231,564 USD	345,874 USD
Rates Portfolio	11,058,114 USD	34,796,088 USD	1,586,748 USD
Equity & Rates Portfolio	14,577,571 USD	49,027,652 USD	1,890,742 USD

Table 3.7: Risk measure results of a mixed equity and rates portfolio

As we can see, for EAD under VM and ISDA SIMM the multi asset class portfolio is simply the sum of the risk measures of the portfolios containing only trades of the individual asset class. The reason for this is, that for both models, SA-CCR and ISDA SIMM no hedge effects are recognized between different asset classes. Nevertheless, for EAD under VM and IM we can observe in table 3.7 that the EAD of the multi asset class portfolio is slightly lower than the sum of the equity and the rates portfolio. The reason for this is that initial margin pledged for interest rates trades may also be used to mitigate losses caused by equity trades and vice versa. This slightly increases the overcollateralization of the joint portfolio leading to a lower EAD under VM and IM.

When allocating the joint portfolio risk measure to the individual trades we yield the trade allocations displayed in table 3.8 and can also aggregate the results on subportfolio level as shown in the table for an equity and rates subportfolio.

If an allocation on trade level is more granular than necessary it is also possible to directly allocate the portfolio risk measure on subportfolios. Therefore, each u_i from

	Allocated EAD VM only	Allocated ISDA SIMM	Allocated EAD VM + IM
2Mn ADS Call	8,181,543 USD	-4,802,460 USD	2,959,180 USD
3Mn ADS Put	-4,662,085 USD	19,034,023 USD	-2,643,342 USD
Equity subportfolio	3,519,458 USD	14,231,564 USD	315,837 USD
180Mn EUR Swap	1,957,315 USD	172,265 USD	630,350 USD
1000Mn USD Swap	10,873,970 USD	27,859,887 USD	1,921,998 USD
500Mn USD Swaption	-1,773,170 USD	6,763,936 USD	-977,443 USD
Rates subportfolio	11,058,114 USD	34,796,088 USD	1,574,905 USD

Table 3.8: Allocation result of the mixed portfolio

equation 2.4.2 needs to be interpreted not as a single trade but rather as a selection of trades. When calculating the Euler allocation with a central difference approach this means that the notional of all trades within the subportfolio is bumped simultaneously by a relative amount $\epsilon/2$. This does save computation time as in the example shown in table 3.8 only four new portfolio risk measure aggregations are necessary to allocate to the equity and rates subportfolio directly while ten aggregations are necessary to allocate a risk measure on all of the five trades. An allocation of the subportfolios is calculated with this approach in appendix A.4 and yields exactly the same result as the summed up subportfolio results in table 3.8.

This might also be a valid approach for a risk analysis architecture that calculates allocations only when requested by a user in real time on the required aggregation level.

3.2 Consideration of edge cases

While numerical Euler allocation of SA-CCR and ISDA SIMM is generally possible as shown in section 3.1, a couple of cases can be identified, in which Euler allocation fails since its prerequisites of homogeneity and differentiability are violated. Both, ISDA SIMM and SA-CCR are complex, convoluted formulas, making general observations on differentiability and homogeneity difficult. In fact, we will see that depending on the portfolio and parametrization of the collateral agreement, both ISDA SIMM and SA-CCR are not homogeneous risk measures, and that they are both occasionally not partially differentiable w.r.t. position size.

In this section, all identified cases under which prerequisites for Euler allocation are violated are presented and for some a workaround is presented to allow nevertheless for risk sensitive and naturally additive allocation.

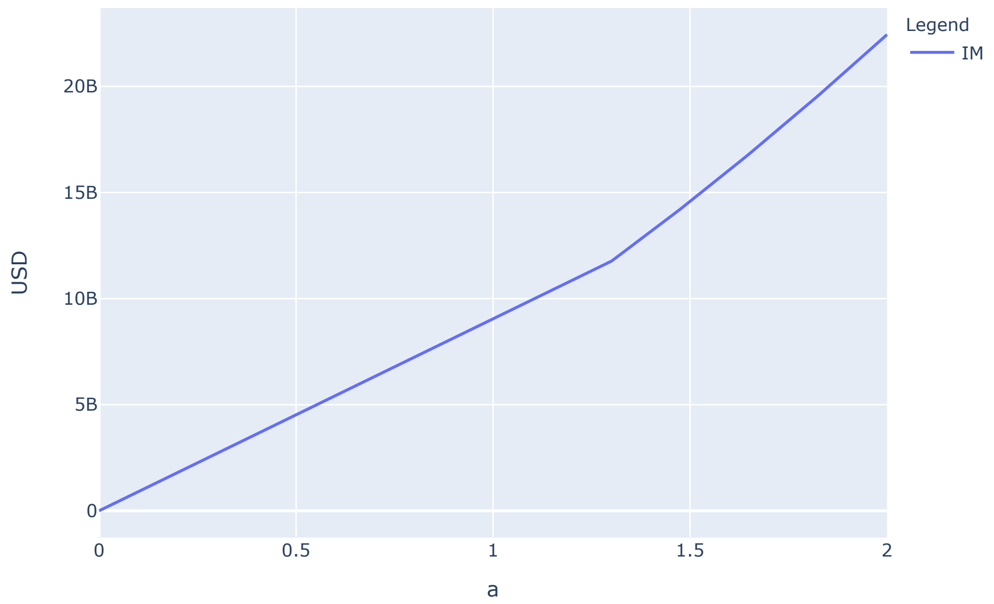


Figure 3.1: ISDA SIMM calculated for different notionals invested in the trade with $a = 1$ being an investment of 200 billion USD

3.2.1 Allocation when an ISDA SIMM liquidity threshold is exceeded

As pointed out in section 2.4.2 a risk measure needs to exhibit positive homogeneity of degree 1 to be able to perform an Euler allocation. This precondition is violated if a liquidity threshold of the ISDA SIMM model is exceeded.

We can show this by exploring whether ISDA SIMM does exhibit positive homogeneity for a minimal example.

For this we set up an USD Libor IRS with ten years time to maturity and a notional of 200 billion USD. This is our initial portfolio $\mathbf{u}$. ISDA SIMM would fulfill the required positive homogeneity condition if $a\rho(\mathbf{u}) = \rho(a\mathbf{u})$ for $a > 0$. In figure 3.1 $\rho(a\mathbf{u})$ is plotted for $0 < a \leq 2$ in blue. The function exhibits homogeneity for $0 < a < 1.3$ but not for higher a. The reason for this is, that at this point the concentration risk charge of ISDA SIMM does kick in. The concentration risk for interest rate risks for our minimal example is defined as [35, Article 7.b]

$$CR = \max\left(1, \left(\frac{|\sum s|}{T}\right)^{1/2}\right)$$

with s being the sensitivities against USD interest rate risk and T being 230Mn USD as specified in [35, Article 74]. Due to subsequent variance-covariance aggregation the concentration risk impacts the calculated IM as

$$IM_{\text{with conc. risk}} = CR^2 \cdot IM_{\text{without conc. risk}}$$

This causes the change in slope and implied loss of homogeneity visible in figure 3.1. If the portfolio would consist of a more diverse set of risk factors than the minimal example displayed in figure 3.1 the associated concentration risk would kick in at different levels of a. The slope of the function would increase with each additional concentration risk not being floored at one any more.

It is important to note that as soon as the sensitivity against a single risk factor in the portfolio is above the concentration threshold the ISDA SIMM risk measure does not exhibit homogeneity anymore.

Even a trivial example with just one trade is sufficient to show that Euler allocation does not work in the inhomogeneous part of the ISDA SIMM equation. For this, we compare two sample portfolios one consisting of one USD IRS with 200 bn notional and one consisting of one USD IRS with 400 bn notional. Critically, the second portfolio is penalized by the model since its USD IRS risk is too large. We calculate the Euler calculation with a forward finite difference approach as displayed in equation 2.4.22.

Assuming that we calculate the finite difference with an $\epsilon = 0.0001$ this means that we calculate the ISDA SIMM of an IRS with 200Bn notional ($SIMM_{200Bn}$) and the ISDA SIMM of an equivalent IRS with 200.02 Bn notional ($SIMM_{200.02Bn}$) and this yields an Euler allocation to this trade as

$$\frac{SIMM_{200.02Bn} - SIMM_{200Bn}}{0.0001} = 9.04Bn$$

We can see in figure 3.1 that this value is both, the slope and the IM value at $a = 1$. The portfolios IM was correctly fully allocated to the single trade of which it consists.

However, performing the same calculation for an equivalent IRS with 400Bn notional yields

$$\frac{SIMM_{400.04Bn} - SIMM_{400Bn}}{0.0001} = 33.67Bn$$

again, we can refer to figure 3.1 to check if this is a reasonable allocation result. As $a = 1$ represents the IM charge for investing 200Bn of notional in the IRS, $a = 2$ represents an investment of 400Bn notional. The associated IM is just 22.44Bn - allocating 33.67Bn of the risk measure to the only trade in the portfolio is therefore clearly wrong. The Euler allocation of 33.67Bn can also be read off figure 3.1 - it is the slope at $a = 2$ times two.

This result extends to the Euler allocation of the SA-CCR. When allocation of the IM is not possible any more, allocation of the SA-CCR is neither if initial margin is taken into consideration during SA-CCR calculation.

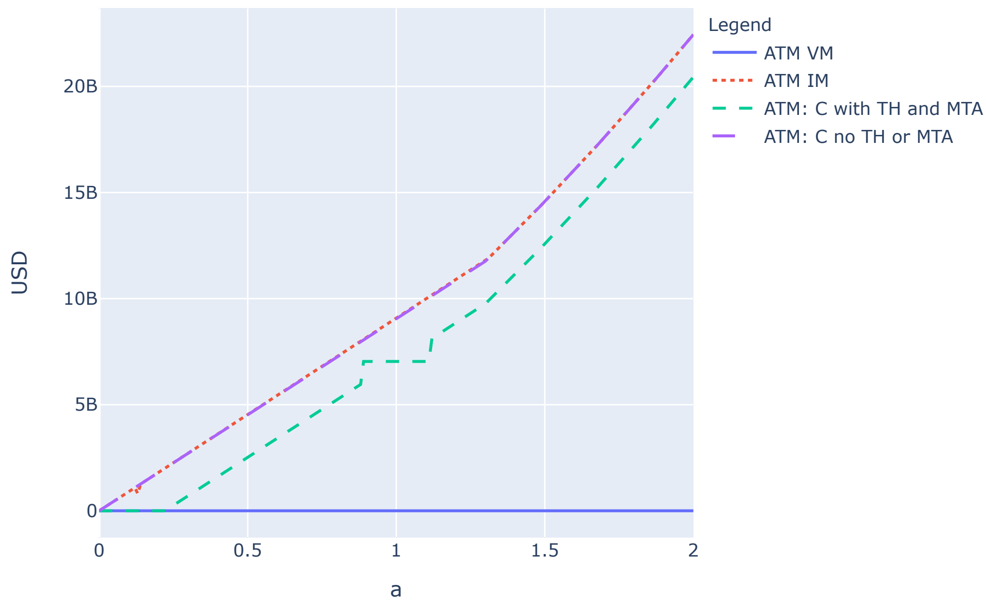


Figure 3.2: VM, IM, C and C under consideration of TH and MTA for a portfolio consisting of a single at the money interest rate swap. Values are calculated based on different notionals invested in the IRS with $a = 1$ referring to a notional of 200Bn USD. More details on creation can be found in Appendix A.7.

3.2.2 Incorporation of a minimum transfer amount and threshold

To allocate SA-CCR under consideration of margining, the available collateral C is of special interest. As pointed out in table 2.1 depending on the margining approach, C can be calculated as $C = \text{VM}$ or $C = \text{VM} + \text{IM}_{\text{received}}$.

The actually exchanged collateral, however has to be calculated under consideration of the threshold and minimum transfer amount as displayed in equation 2.3.6. With this consideration of threshold and minimum transfer amount C is not a homogeneous function.

This can exemplary be seen in figures 3.2 and 3.3. These figures display C for an at the money 10Y USD interest swap and the same swap with a lower fixed rate making it an in the money swap. Again, a very high notional of 200 Bn is chosen to showcase the concentration risk charge of ISDA SIMM and the initial margin threshold and minimum transfer amount have also been chosen to be very high at 2Bn and 1Bn USD. The variation margin threshold in the two figures is assumed to be zero.

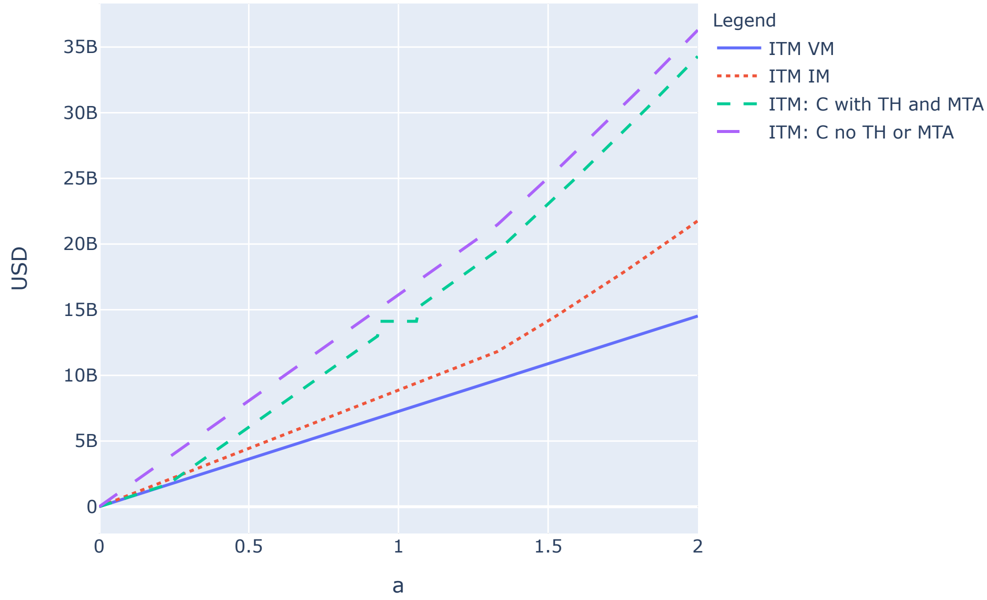


Figure 3.3: Same as figure 3.2 but for an in the money trade with an payer interest rate at 2% which is below par.

3.2.2.1 Incorporation of a VM threshold

Generally, a variation margin threshold $\mathrm{TH}_{\mathrm{VM}}$ can not be in place when bilateral initial margin is being exchanged. The reason for this is, that the regulator poses a VM threshold of zero as a prerequisite for bilateral initial margining [28, Requirement 2.1].

No exchange of IM implies that no overcollateralization can be available and therefore margining has no impact on the PFE as defined in formula 2.3.2. Instead the VM threshold solely impacts the RC which, based on formula 2.3.4, under VM but in absence of IM simplifies to $RC = \mathrm{TH}_{\mathrm{VM}}$ independent of the portfolio constellation. Consequently, a VM threshold will always increase the portfolio EAD by $1.4 * \mathrm{TH}_{\mathrm{VM}}$. Even if the portfolio is empty, the EAD will be $1.4 * \mathrm{TH}_{\mathrm{VM}}$.

When performing an Euler allocation of a portfolio with a VM threshold, the sum of the allocations will be $EAD - 1.4 * \mathrm{TH}_{\mathrm{VM}}$. This is arguably a correct result, as this remainder is independent of the trades and even remains when all trades are removed from the portfolio. A bank allocating its EAD can decide freely weather the VM threshold impact on the EAD of $1.4 * \mathrm{TH}_{\mathrm{VM}}$ shall remain unallocated or to allocate it to the trades of the portfolio according to some pro-rata approach.

Some exemplary calculations exploring the effects of a VM threshold can be found in appendix A.9.

3.2.2.2 Incorporation of a minimum transfer amount

Again, similar to the exemplary calculation in section 3.2.1 it can be shown with a trivial example, that Euler allocation of SA-CCR is not possible under recognition of a MTA. Detailed documentation of this exemplary calculation may be found in appendix A.8.

We consider the same 200Bn IRS as in section 3.2.1. We assume that the currently posted margin is 9.04Bn which is the calculated ISDA SIMM margin. Setting C at 9.04Bn in the SA-CCR equation 2.3.3 and then calculating the SA-CCR EAD for this single IRS yields an regulatory EAD of 582.8Mn USD. Any natively additive allocation should allocate this full amount to the IRS. The VM is zero as the IRS is struck at par. In table 3.9 we assume that the initially received collateral is the currently calculated collateral. When calculating the Euler allocation with a forward difference in line with equation 2.4.22, the received collateral when rising the notional to 200.02Bn increases when no MTA is assumed while it remains unchanged with consideration of an MTA. Ultimately, this difference leads to a correct allocation of the entire EAD to the single IRS with the *No MTA* approach while the *MTA* approach obviously yields a wrong result by allocating 244% of the portfolios EAD to its only trade.

	$\text{SA-CCR}_{\text{MTA}}$	$\text{SA-CCR}_{\text{No MTA}}$
Initial C	9038.2Mn	9038.2Mn
$\text{EAD}_{\text{200Bn IRS}}$	582.8Mn	582.8Mn
$\text{C}_{\text{200Bn IRS}}$	9038.2Mn	9038.2Mn
$\text{C}_{\text{200.02Bn IRS}}$	9038.2Mn	9039.1Mn
$\text{EAD}_{\text{200.02Bn IRS}}$	583.0Mn	582.9Mn
$\frac{\text{EAD}_{\text{200.02Bn IRS}}-\text{EAD}_{\text{200Bn IRS}}}{0.0001}$	1425.3Mn	582.8Mn

Table 3.9: Numerical Euler allocation of SA-CCR with and without consideration of a minimum transfer amount for an example of a portfolio with a single 200Bn notional IRS. Euler allocation is only successful if the MTA is not considered for the recalculation of the received margin C. A threshold of 0 is assumed.

In the above example any adverse impact that the MTA may have on the Euler allocation can be mitigated by simply assuming an MTA of zero during the calculation of the Euler allocation. However, a scenario can be constructed in which this approach still leads to an allocation, whose sum deviates from the portfolio risk measure.

As established in table 2.1 the MTA impacts the RC of the SA-CCR calculation with $RC = \text{TH}_{\text{IM}} + \text{MTA}$ under VM collateralization and $RC = \max(0, \text{MTA} - \text{IM}_{\text{rec}})$

under VM and IM collateralization. If only VM is exchanged this does not need to be recognized in an Euler allocation. Following the same line of argument that has already been used in section 3.2.2.1 the MTA increases the EAD under all circumstances by a flat amount of $1.4 * EAD$ which does not necessarily have to be allocated to any trade.

On the other hand, if VM and IM are exchanged a MTA does not increase the RC by a flat amount but the impact of the MTA on the RC depends on the received initial margin. Accordingly, if $\text{IM}_{\text{rec}} < MTA$ the impact of a trade on the exchanged initial margin not only impacts the PFE of equation 2.3.1 but also the RC. This effect is lost if the Euler allocation is calculated under an assumption of $MTA = 0$. An example for this phenomenon is presented in Appendix A.8.1.

However, the window for this phenomenon to occur in practice is rather small. If bilateral initial margin is exchanged, the regulator mandates that a minimum transfer amount may not exceed half a million EUR [28, Requirement 2.3]. In comparison to the calculated bilateral initial margin of common portfolios or the mandated maximum IM threshold of 50Mn EUR [28, Requirement 2.2] this is a rather small amount. Accordingly unlikely is it that the received initial margin satisfies $0 < \text{IM}_{\text{rec}} < \text{MTA}$. Considering this it appears to be feasible in all cases to just assume a MTA of zero when calculating an Euler allocation and to just ignore the fact that a fraction of the SA-CCR EAD of up to $1.4 * MTA$ may remain unallocated after doing so.

3.2.2.3 Incorporation of an IM threshold

In comparison with recognizing a minimum transfer amount or a variation margin threshold adjusting for the inclusion of IM threshold appears to be a much bigger issue. As we can see in figures 3.2 and 3.3, inclusion of an initial margin threshold does reduce the received IM to zero until the threshold is reached and afterwards reduces the exchanged IM amount by the IM threshold. Theoretically, this behavior is determined by the bilateral collateral agreement and the two parties could also agree to exchange the full calculated IM amount once the threshold is exceeded. However, subsequent deduction of the IM threshold once the threshold itself is exceeded is explicitly allowed by the regulator [28, Background discussion 2(h)] and allows the counterparties to exchange as little IM as possible while still being regulatory compliant. This setup should therefore be the most common in practice which is why it is the only one considered in this section.

We investigate the impact of an initial margin threshold by means of an exemplary portfolio consisting of an equity option and an interest rate swap. Detailed documentation of this exemplary calculation may be found in appendix A.10.

The notional of the two trades is chosen such that they have the same EAD under VM but different IM and consequently different EAD under VM and IM. The different risk measures of the two standalone portfolios are displayed in 3.10 all figures do not assume any threshold. As we can see the IRS has a lower EAD under VM and IM

	EAD under VM	IM	EAD under VM and IM
Equity option	1,651,405	2,806,744	530,996
Interest rate swap	1,651,405	4,519,079	291,441
Portfolio of both	3,302,809	7,325,823	777,229

Table 3.10: Risk measures for exemplary standalone trades to subportfolio - all values are in USD

than the equity option which is due to the fact that the ISDA SIMM model charges considerably more IM for the IRS which is then recognized as overcollateralization for the subsequent EAD calculation.

When we now assume an IM threshold of zero, five million and ten million USD and perform a numerical Euler allocation with a central difference approach we yield the results displayed in 3.11. As we can see, the results for a 10Mn threshold are exactly

	No IM threshold	5Mn IM threshold	10Mn IM threshold
Allocated to Equity option	505,528 (65%)	331,448 (16%)	1,651,405 (50%)
Allocated to Interest rate swap	271,701 (35%)	-381,962 (-19%)	1,651,405 (50%)
Portfolio risk measure	777,229	2,032,638	3,302,809

Table 3.11: EAD allocation under different IM thresholds - all values are in USD

the same as those assuming no IM. Since the calculated IM is lower than the IM threshold no IM is exchanged and the allocation works fine since it is the same as an allocation under VM only. On the other hand, the allocation assuming no IM threshold is also working fine. As we can see, the Euler allocation correctly allocates a lower fraction of 35% instead of 50% of the EAD to the IRS in this scenario since it has a higher contribution towards the overcollateralization of the portfolio.

However, the Euler allocation fails when we assume a threshold of 5Mn USD. This could be expected as the EAD locally does not exhibit homogeneity since as we were able to infer from figures 3.2 and 3.3. The calculated figures do not represent any form of a risk sensitive allocation - neither in absolute nor in relative terms.

While we can't use the results of the Euler allocation under the assumption of a 5Mn IM threshold we can use the allocation results under no IM threshold and under VM only to draw conclusion on a risk sensitive allocation for the case with a 5Mn IM threshold. Considering the IRS we do know, that allocating 50% of the EAD to the IRS would be too much since some IM is available which is lowering the EAD from 3.3Mn to 2Mn USD and the contribution of the IRS to this IM is overproportionate. On the other hand, and allocation of 35% of the EAD to the IRS would be too little since this would be the relative amount allocated if the entire IM calculated would be exchange.

Based on these considerations we can establish that the relative fraction of EAD under an IM threshold should be within a bounded range. This range is bound by the relative result of an Euler allocation under assumption of no IM threshold and the relative result of an Euler allocation under the assumption of no IM being exchanged.

3.2.3 Allocation of hedged portfolios

As pointed out in section 2.4, Euler allocation is a risk sensitive allocation and as such does generally attribute negative contributions to trades that are decreasing the risk of the portfolio. If we consider a portfolio of a 200Mn payer IRS, an equivalent 100Mn receiver IRS and 1Mn long stock call options we yield the result depicted in table 3.12 when calculating the Euler allocation numerically with a forward difference for the SA-CCR under IM and VM and ISDA SIMM risk measures. The 100Mn receiver IRS

	SA-CCR	ISDA SIMM
$IRS_{100Mn\ Rec}$	-246k	-4.52Mn
$IRS_{200Mn\ Pay}$	493k	9.04Mn
Equity Option	1.30Mn	6.95Mn
Sum of allocations	1.54Mn	11.47Mn
Portfolio risk measure	1.54Mn	11.47Mn

Table 3.12: Allocation of a hedged portfolio without a perfect hedge

partially hedges the risk induced by the 200Mn payer IRS. Both, the ISDA SIMM and the SA-CCR model do not recognize any hedge effects across asset classes and therefore the risk associated with the equity option is completely independent from the two IRS trades. Appropriately, a negative IM and EAD is allocated to the smaller IRS trade and the allocation exhibits native additivity as the sum of the allocation of the three trades coincides with the risk measures of the portfolio.

	Forward	Central	Backward
$IRS_{200Mn\ Rec}$	188k	0k	-188k
$IRS_{200Mn\ Pay}$	188k	0k	-188k
Equity Option	1.32Mn	1.32Mn	1.32Mn
Sum of allocations	1.69Mn	1.32Mn	944k
Portfolio risk measure		1.32Mn	

Table 3.13: Allocation of SA-CCR for a perfectly hedged portfolio with different finite difference approaches

However, if we decrease the notional of the payer swap to 100Mn as well and allocate the SA-CCR EAD again with a forward difference approach, we yield the result depicted in the leftmost column of table 3.13. This clearly isn't a viable allocation as it is far off the risk measure.

As we can see in table 3.13 the chosen finite difference approach completely changes the resulting allocation with only the central difference approach exhibiting subadditivity. The same observation can also be made for the allocation of the ISDA SIMM IM depicted in table 3.14.

	Forward	Central	Backward
$\text{IRS}_{\text{200Mn Rec}}$	4.52Mn	0.00Mn	-4.52Mn
$\text{IRS}_{\text{200Mn Pay}}$	4.52Mn	0.00Mn	-4.52Mn
Equity Option	6.95Mn	6.95Mn	6.95Mn
Sum of allocations	15.99Mn	6.95Mn	-2.09Mn
Portfolio risk measure	6.95Mn		

Table 3.14: Allocation of ISDA SIMM for a perfectly hedged portfolio with different finite difference approaches

The reason for this observation is that the portfolios risk measures happen to not be differentiable w.r.t. position size for the two swap trades. This can be seen in figure 3.4 for the EAD and in figure 3.5 for ISDA SIMM IM. As we can see, both an increase and decrease of position size in either IRS leads to an increase in initial margin and the slope of this increase is constant.

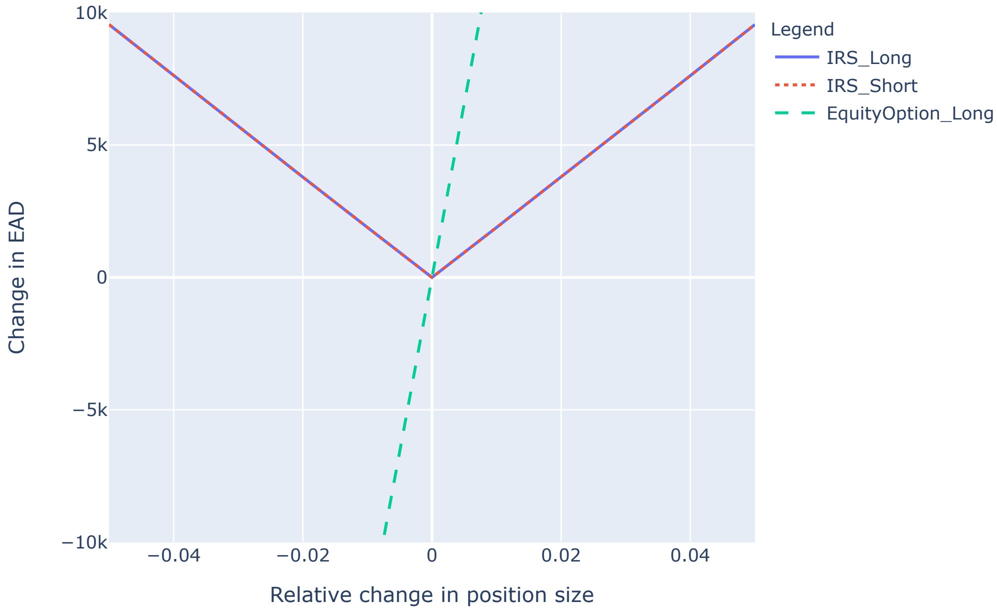


Figure 3.4: When increasing or decreasing the notional of one of the two IRS we can see that SA-CCR EAD is not differentiable with regard to trade notional due to the perfect hedge between the two IRS.

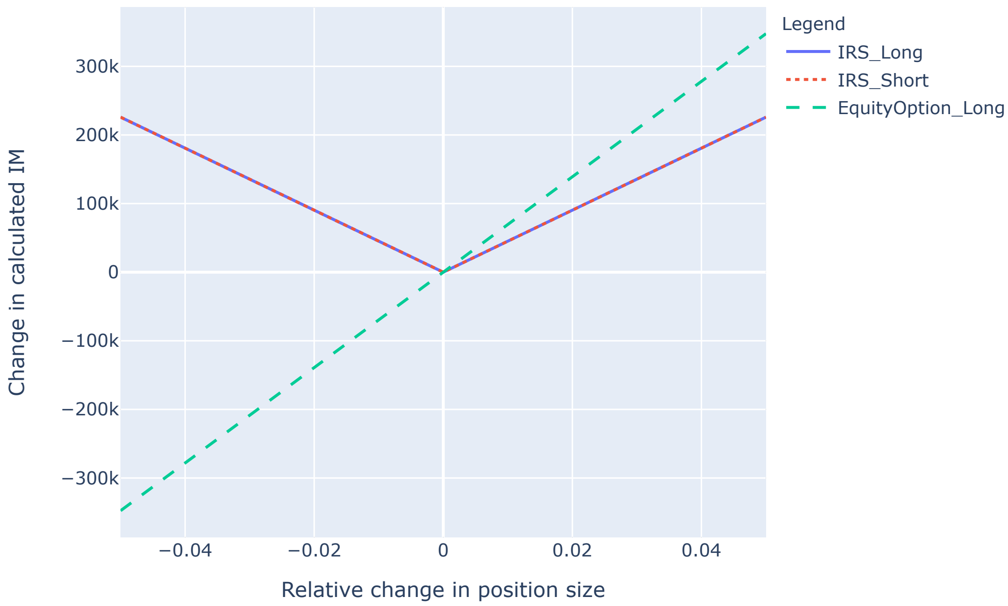


Figure 3.5: Same as figure 3.4 but for ISDA SIMM initial margin

Chapter 4

Discussion of results

In most cases an Euler allocation of the SA-CCR EAD model is possible and yields risk sensitive results that can support risk analysis or portfolio optimization tasks.

The SA-CCR EAD model is a rather unconventional risk measure as it embeds another risk measure, namely the initial margin model whose result mitigates risk of the EAD model. This risk mitigation is a major part of the model and must not be dismissed when an allocation of the risk measure is required. For the exemplary equity portfolio in 3.1.1 we can see that inclusion of risk mitigation through IM reduces the calculated EAD by a factor of ten.

The exemplary portfolios also showed that a risk sensitive allocation can differ tremendously between the different margining approaches.

When allocating the SA-CCR EAD without variation margining the risk characteristics of the trades can be overshadowed by its present value. This means that a significant fraction of the EAD may stem from its RC which unlike the PFE does not take future hedge characteristics of the portfolio into account but is rather based on the current PV of the trades in the portfolio.

When allocating the SA-CCR under consideration of VM but not IM the RC should, assuming a MTA and VM threshold of zero, be zero and the portfolio EAD consists solely of the PFE component. No overcollateralization is in place and therefore the risk sensitive allocation is entirely based on the PFE mechanics of the SA-CCR model.

Finally, with initial margining in play, the allocation depends a lot on the interplay between the SA-CCR PFE risk measure and the risk measure used to calculate the initial margin. In this thesis, we performed exemplary allocations assuming the ISDA SIMM model as the initial margin model. Depending on the portfolio all constellations between the two risk measures could be observed. Examples have been found in which a trade locally reduces the SA-CCR risk measure without overcollateralization while simultaneously increasing the received initial margin. These two effects amplify each other leading to a very high, but explicable negative amount of the EAD being allocated to the trade. The opposite phenomenon of a trade simultaneously increasing the SA-CCR risk measure without overcollateralization while simultaneously decreasing the received initial margin could also be observed and justified.

These results indicate that risk sensitive allocation of SA-CCR is only possible when fully recognizing margining during the allocation. Simpler workarounds such as e.g. allocating SA-CCR without margining and then scaling the results can yield deceptive results that the user might assume to be risk sensitive although they may be very far from capturing the actual portfolio mechanics.

The interaction of the two risk measures as part of SA-CCR EAD recognizing initial margin also increases the value that a risk sensitive allocation approach such as the Euler allocation may add to the risk analytics function of a bank. The exemplary portfolios analyzed in section 3.1 were quite simple but their Euler allocation still unveiled some surprising portfolio mechanics especially for SA-CCR recognizing IM. Once unveiled by the allocation the underlying mechanics could be understood upon further analysis. This demonstrates that a risk sensitive allocation is a powerful tool for SA-CCR risk EAD analysis and portfolio optimization purposes.

Although not explicitly shown, a numerical allocation of SA-CCR under consideration of initial margin is also possible if the received IM is calculated under the standard approach discussed in section 2.2.1 instead of the internal ISDA SIMM model.

4.1 Evaluation of the numerical allocation approach

To calculate the Euler allocation a numerical, finite difference approach was chosen in this thesis. Alternatively, one could have tried to establish an analytical solution similar to the approach that Schulze has chosen in [32] for allocation of the FRTB standard approach. The ISDA SIMM model's structure is very related to the FRTB standard approach but it lends itself less to calculating analytical derivatives as its convoluted formulation e.g. nests more Min and Max expressions that need to be handled on a case by case basis when trying to establish partial derivatives analytically. When nesting the ISDA SIMM risk measure within the SA-CCR model this complexity increases further as the SA-CCR model too e.g. has maximum conditions in its calculation of the RC component. Considering this complexity, a manual analytic calculation of the partial derivatives appears to be prohibitively complex, error prone and difficult to implement and maintain in practice. A more practical approach to establish an analytic solution would probably be an adjoint algorithmic differentiation, which was outside the scope of this thesis but is briefly broached in section 5.2.

Overall, the numerical, finite difference approach for calculation of both the ISDA SIMM and SA-CCR allocation worked well. Without extensive analysis of the aggregation formulas the user can always check whether the risk measure exhibits the prerequisite homogeneity and differentiability by just checking if equation 2.4.18 holds after performing the allocation. Within an appropriate architecture it can be very simple and require very little additional code to implement classes to perform a numerical Euler allocation.

Despite the increased computational cost a central difference approach for numerical finite difference calculation is generally advisable. The increased accuracy of the central difference approach makes it easier to evaluate whether equation 2.4.18 holds and the allocated results are trustworthy or whether the theorem of homogeneous functions is violated inferring, that the prerequisites for Euler allocation are violated which should generally trigger an investigation into the processed portfolio. Additionally, the case of a perfectly hedged portfolio that is investigated in 3.2.3 showcases a scenario that could have sufficient practical relevance in which the central difference approach unlike a forward difference, backward difference or even an analytic partial derivative yields a result that does not break the native additivity of the Euler allocation.

The numerical approach also lends itself to direct allocation on subportfolios as illustrated in section 3.1.3. If subportfolio results rather then trade results are required, simultaneously bumping multiple trades may significantly improve runtimes.

4.2 Limitations of an Euler allocation of SA-CCR

In practice, the incorporation of parameters of the collateral agreement such as thresholds or the minimum transfer amount is a challenge.

The issues caused by the minimal transfer amount are relatively minor. If the received initial margin is high enough to cap the RC at zero, the MTA can be ignored for the allocation calculation and still yield risk sensitive results that exhibit native additivity. If the initial margin is too low or when the MTA is taken into account for margin recalculations on the other hand, the SA-CCR risk measure does not exhibit homogeneity anymore and an Euler allocation is not possible.

The observations made for the MTA do also indicate that treating margin as an immutable portfolio-independent external constant will render an Euler allocation impossible as the SA-CCR risk measure does not exhibit homogeneity under this assumption.

Euler allocation under the presence of a variation margin threshold is not a problem. A variation margin threshold increases the EAD of a portfolio by a flat amount under all circumstances and this amount is not really attributable to the trades.

Out of the collateral agreements parameters the initial margin threshold is the hardest to handle. As long as the threshold is not exceeded, i.e. no initial margin is exchanged the allocation works fine but when the threshold is exceeded and is subtracted from the calculated initial margin the SA-CCR model does not exhibit homogeneity anymore.

It is possible to identify relative bounds how much should allocated to each trade within these bounds *some* metric can be use to find an allocation that adds up to the portfolio risk measure but an undisputable risk sensitive way of doing so could not be identified in this thesis. In practice, allocating the impact on EAD of the IM threshold could turn out to be the biggest issue when performing and Euler allocation of SA-CCR EAD.

Finally, whether the inability to perform an Euler allocation when an ISDA SIMM threshold is exceeded as described in section 3.2.1 prohibits a bank from using Euler allocation entirely or is rather just a theoretical observation depends entirely on the structure and size of the portfolios that should be allocated. While the USD interest rate concentration threshold chosen as an example in section3.2.1 is extremely high due to the high liquidity, other threshold might be easier to exceed. If a threshold is exceeded the calculated allocation will not fulfill equation 2.4.15 anymore and it should therefore be relatively easy for a risk department to identify such a case. However, there is no simple workaround to adjust for such a case and Euler allocation appears to be generally not possible anymore once the portfolio exceeds any ISDA SIMM thresholds.

Chapter 5

Conclusion and Outlook

5.1 Conclusion

The goal of this thesis was to identify a suitable allocation approach for the SA-CCR own capital requirements for bilateral portfolios while accurately recognizing collateralization.

Through literature review, a combination of an incremental and Euler allocation was identified as the most promising as both approaches are risk sensitive and exhibit native additivity. The incremental approach is applicable to any risk measure and as such is also applicable to the SA-CCR model with proper recognition of collateralization. Especially in the front office questions regarding to pre-trade analysis, calculating the regulatory capital consumption or the incremental margin requirement of a newly arranged derivative are commonplace. If such or similar tasks rise in the context of SA-CCR or ISDA SIMM the incremental allocation approach can and should be used.

On the other hand, tasks such as identifying risk drivers within a portfolio or identifying possibilities to reduce the regulatory capital or initial margin consumed by a portfolio can not be tackled with an incremental allocation. For such tasks which tend to rather take rise in the middle office of a bank the incremental allocation is not suited and the Euler allocation should be used instead. Unfortunately, the calculation of the Euler allocation and the prerequisites for it are much more demanding than those for the incremental allocation.

In this thesis we have chosen a numerical approach to calculate the Euler allocation as it is easier to implement in practice and less prone to error due to the complexity of the SA-CCR model with a nested initial margin model such as ISDA SIMM. The analysis conducted for this thesis has established that a numerical Euler allocation of SA-CCR under recognition of bilateral margining is generally possible and can provide valuable insight. There are however a couple of circumstances under which the prerequisites to perform an Euler allocation are violated. The most important of these limitations is probably the recognition of an initial margin threshold. If an initial margin threshold is in place and exceeded, the prerequisites for an Euler allocation are violated. For this case upper and lower bounds for the margin allocated to each trade could be established but no unambiguous risk sensitive overall allocation could be identified.

Disregarding the issue with initial margin thresholds, Euler allocation could still be used in practice despite not working in some edge cases. Conveniently, it is very easy to identify if the numerical Euler allocation is not working and, consequently, the portfolio in question is violating the prerequisites for an Euler allocation. In such cases the result looses its native additivity, i.e. the sum of the trade allocations will be far off the portfolio risk measure which can be easily identified and trigger an investigation of the portfolio or the use of a fall-back allocation approach.

If any margin is exchanged, an Euler allocation of SA-CCR is not possible when the margin is assumed to be an external constant value that does not have partial derivatives against the trade notionals in the portfolio. If an Euler allocation is calculated under such an assumption the prerequisites for the allocation are violated. Additionally, exemplary portfolios analyzed for this thesis have also shown that a relative, risk sensitive allocation of SA-CCR without margining can differ tremendously from a risk sensitive allocation of SA-CCR with margining. Therefore, no shortcut exists when trying to calculate a risk sensitive allocation of the SA-CCR of a bilateral margined portfolio. Both margin components, variation margin and received initial margin have to be treated as a function of the trade notionals in the portfolio. Their partial derivatives against the notional of a trade are required to calculate the respective partial derivative and consequently the Euler allocation of the SA-CCR own capital requirements.

Finally, when numerically calculating the Euler allocation of SA-CCR one should use a central difference approach. The increased precision of the central difference approach makes it easier to identify cases in which the Euler allocation is not working due to violated prerequisites. Additionally, the central difference approach is also more robust against undifferentiability w.r.t the trade notional that appears if the analyzed portfolio exhibits certain hedge constellations.

5.2 Outlook

When extending on the results of this thesis three areas come to mind.

1. Exploring the allocation of alternative models for the calculation of regulatory capital requirements for counterparty credit risk and alternative models for initial margin requirements.

2. Exploring alternative approaches to calculate the partial derivatives required for an Euler allocation.

3. Establishing a risk sensitive allocation for the cases identified in section 3.2 for which no satisfactory Euler allocation could be established.

When considering alternative CCR models for RWA and IM models of higher or lower complexity than those analyzed in this thesis may be explored. Less complex alternatives are the simplified SA-CCR model [31, Article 281 and following] and the standardized initial margin model briefly discussed in section 2.2.1. More complex alternatives are internal CCR models [26, CRE 53] and internal initial margin models of CCPs.

If additional models were to be explored it is likely that similar restrictions are uncovered as those listed in section 3.2 for the interplay of the SA-CCR and ISDA SIMM model. Just as the ISDA SIMM model, internal initial margin models such as the PRISMA model of Eurex Clearing contain a liquidity risk component that increases disproportionately with the position size [11, Section 3.5] which results in a risk measure that may not exhibit homogeneity. Similarly, observations made in section 3.2 considering CSA parameters such as the initial margin threshold or MTA should be transferable to internal CCR RWA models as these CSA parameters are also taken into account in these internal models. It should be noted that CCP IM models should be less interwoven with CCR RWA models than it is the case for bilateral initial margin models such as the ISDA SIMM. The reason for this is that for cleared derivatives initial margin is only deposited by the bank at the CCP while the bank does not receive any initial margin from the CCP. As no initial margin is received it can consequently also not mitigate any risk in subsequent CCR RWA calculations.

Although the chosen numerical approach chosen worked well for the analysis conducted for this thesis it comes with the usual high computational requirements of any finite difference approach - especially when a central difference approach is used due to its advantages discussed in section 4.1. Alternatively, one might consider to establish an analytical solution for the Euler allocation of SA-CCR and ISDA SIMM similar to what Schulze has established for the FRTB-SA [32]. However, as already discussed due to the high complexity of the SA-CCR model with a nested ISDA SIMM model this approach might not be practical.

A more promising approach might be to perform the Euler allocation using adjoint algorithmic differentiation (AAD). While AAD is somewhat established in finance for the calculation of sensitivities, i.e. partial derivatives of the price w.r.t. market data [17] it should generally also be applicable to calculate a partial derivative of a risk measure w.r.t. a trade notional as required for the Euler allocation. The performance gain over the numerical approach should be large for any portfolio containing at least a double-digit trade count. Allocating a portfolio of n trades with a numerical central difference allocation takes $2n$ times as long as just calculating the portfolio risk measure[1]. The computation time of an allocation through AAD on the other hand should be independent of the trade count and take less then ten times as long

[1]Not taking into consideration any parallelization that could generally be applied without any restrictions for this use case.

as the calculation of the portfolio risk measure [15].

As pointed out in the conclusion, especially the inability to perform an Euler allocation when an initial margin threshold exists and is exceeded may prohibit numerical Euler allocation of SA-CCR from being applied in practice. Accordingly, it would be important to establish an unambiguous risk sensitive allocation for this case. One possible approach might be to revisit the Shapley allocation and explore whether it could yield a risk sensitive and natively additive result in those edge cases in which the prerequisites for an Euler allocation are violated.

Appendix A

Detailed documentation of results

The purpose of this section is to document how the results presented primarily in section 3 were computed. For the purpose of this thesis a library has been implemented in Python and Java that can be found at [19]. Using this library, results for this thesis were computed within Jupyter Notebooks. These Jupyter notebooks are presented in this section. Some, more technical notebook cells are not displayed for brevity.

A.1 USD LIBOR and EURIBOR zero curves

The purpose of the notebook is to plot the zero rates of the USD LIBOR and EURIBOR zero curve for the effective date 10th of May 2019 which is used for pricing throughout this thesis.

[In]:
```
today = ql.Date(10, ql.May, 2019)
ql.Settings.instance().evaluationDate = today
```

We import the market data object for the two curves which are set up in the market data package of the library developed for this thesis.

[In]:
```
from marketdata.interestRateCurves import LiborCurve
euribor6m = LiborCurve.EURIBOR6M.value
usdlibor3m = LiborCurve.USDLIBOR3M.value

usdlibor3m.__class__
```

[Out]:
```
QuantLib.QuantLib.YieldTermStructureHandle
```

The curves are built as core QuantLib objects which offers a functionality to infer zero rates.

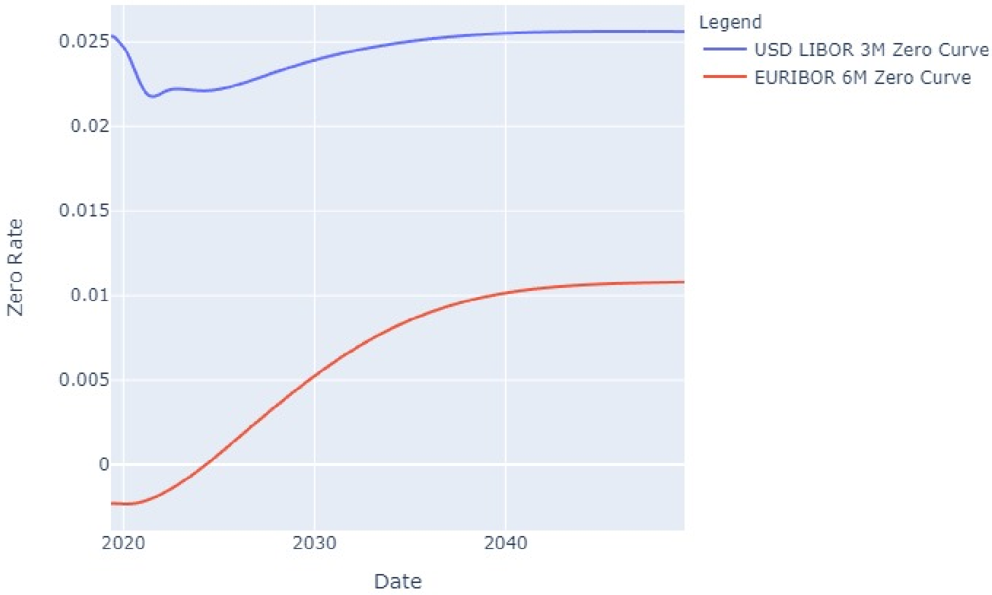
[In]:
```python
end = today + ql.Period(30, ql.Years)
# dates is just a list of all days until 30 years in the future
dates = [ql.Date(serial) for serial in range((today+ql.Period(4, ql.
 ↪Days)).serialNumber(), end.serialNumber()+1)]
zero_rates_euribor = [euribor6m.zeroRate(d, ql.Actual360(), ql.
 ↪Continuous).rate() for d in dates]
zero_rates_usd_libor = [usdlibor3m.zeroRate(d, ql.Actual360(), ql.
 ↪Continuous).rate() for d in dates]
```

When plotting these zero rates we do yield

[In]:
```python
def plotdf(y, x, legend):
    df = pd.DataFrame()
    df['Zero Rate'] = y
    df['Date'] = [d.to_date() for d in x]
    df['Legend'] = legend
    return df

plot_data = plotdf(zero_rates_usd_libor, dates, 'USD LIBOR 3M Zero␣
 ↪Curve')\
    .append(plotdf(zero_rates_euribor, dates, 'EURIBOR 6M Zero␣
 ↪Curve'))
```

[In]:
```python
fig = px.line(plot_data, x = 'Date', y = 'Zero Rate', color =␣
 ↪'Legend')
```

[Out]:



A.2 SA-CCR Euler allocation of an exemplary equity portfolio

We set up a collateral agreement with no thresholds or MTA and add two equity options to it. A big put option on an imagainary stock ADS and a smaller call option on the same stock. Both options are struck at the current market price $S(0)$.

We consider three cases of margining

1. No margining
2. VM only
3. VM and bilateral IM

[In]:
```
ca = CollateralAgreement(threshold=0,
                         threshold_vm=0,
                         mta=0,
                         margining=Margining.UNMARGINED,
                         initialMargining=InitialMargining.NO_IM)
ca.link_sa_ccr_instance(SA_CCR(ca))

ca_vm = CollateralAgreement(threshold=0,
```

```
                            threshold_vm=0,
                            mta=0,
                            margining=Margining.MARGINED,
                            initialMargining=InitialMargining.NO_IM)
ca_vm.link_sa_ccr_instance(SA_CCR(ca_vm))

ca_im = CollateralAgreement(threshold=0,
                            threshold_vm=0,
                            mta=0,
                            margining=Margining.MARGINED,
                            initialMargining=InitialMargining.SIMM)
ca_im.link_sa_ccr_instance(SA_CCR(ca_im))
```

[In]:
```
eq_opt_ads_call = EquityOption(underlying=Stock.ADS,
                               maturity=ql.Period(1, ql.Years),
                               notional=2000000,
                               tradeType=TradeType.CALL,
                               tradeDirection=TradeDirection.LONG)

eq_opt_ads_put = EquityOption(underlying=Stock.ADS,
                              maturity=ql.Period(1, ql.Years),
                              notional=3000000,
                              tradeType=TradeType.PUT,
                              tradeDirection=TradeDirection.LONG)
```

The current value of these trades is 6,601,467 EUR for the call option and 10,378,881 EUR for the put option.

When putting these trades in the three portfolios we observe a high difference between the unmargined and VM margined EAD. This difference is primarily driven through the RC and therefore through the high positive PV.

[In]:
```
ca.add_trades([eq_opt_ads_call, eq_opt_ads_put])
ca_vm.add_trades([eq_opt_ads_call, eq_opt_ads_put])
ca_im.add_trades([eq_opt_ads_call,eq_opt_ads_put])
```

[In]:
```
print('EAD with no margining: {:,.2f} USD'.format(ca.
 ↪get_sa_ccr_model().get_ead()))
print('EAD with VM margining: {:,.2f} USD'.format(ca_vm.
 ↪get_sa_ccr_model().get_ead()))
print('RC with no margining: {:,.2f} USD'.format(ca.
 ↪get_sa_ccr_model().get_rc()))
print('RC with VM margining: {:,.2f} USD'.format(ca_vm.
 ↪get_sa_ccr_model().get_rc()))
print('Portfolio PV: {:,.2f} USD'.format(ca.get_V()))
```

```
EAD with no margining: 37,643,536.02 USD
EAD with VM margining: 3,519,457.62 USD
RC with no margining: 18,508,579.01 USD
RC with VM margining: 0.00 USD
Portfolio PV: 18,508,579.01 USD
```

Next, we perform an Euler allocation:

[In]:
```
eulerAllocator1 = EulerAllocator(ca)
eulerAllocator2 = EulerAllocator(ca_vm)
eulerAllocator3 = EulerAllocator(ca_im)
allocation_no_margin = eulerAllocator1.allocate_ead()
allocation_with_vm = eulerAllocator2.allocate_ead()
allocation_with_im_and_im = eulerAllocator3.allocate_ead()
allocation_im = eulerAllocator3.allocate_im()
```

we can calculate how far the sum of the allocated values deviates from the risk measure:

```
Diff EAD no margin:    0.00 EUR
Diff EAD only VM:      0.00 EUR
Diff EAD VM + IM:   1068.64 EUR
Diff calculated IM:    6.74 EUR
```

In relation to the EAD of 345,874 EUR the deviation of the allocated EAD under VM and IM of 1,068.64 EUR is not large but can be improved nevertheless.

By, default the implemented Euler allocation class uses a forward difference approach. If we switch over to a central difference approach the deviation shrinks significantly.

[In]:
```
eulerAllocator3.fdApproach2 = FdApproach2.Central
allocation_im = eulerAllocator3.allocate_im()
allocation_with_im_and_im = eulerAllocator3.allocate_ead()
```

```
Diff EAD VM + IM:   0.01 EUR
Diff calculated IM: 0.00 EUR
```

Displaying the allocation results

[Out]:

	2Mn ADS Call	3Mn ADS Put	Portfolio Risk Measure
SIMM	-33.75%	133.75%	14,231,564 USD
No margin	99.21%	0.79%	37,643,536 USD
VM only	232.47%	-132.47%	3,519,458 USD
VM+IM	622.10%	-522.10%	345,874 USD

For SIMM the Put has the higher risk and the Call is considered a hedge trade while for SA-CCR with only VM, the Call has the higher risk and the Put is considered a hedge trade.

The reason for this are the different holding periods between the two models. If we lower the maturity of the trades to 10 days instead, we can see that for SA-CCR with only VM the call is considered the hedge trade.

[In]:
```
eq_opt_ads_call_10d = EquityOption(underlying=Stock.ADS,
                                   maturity=ql.Period(10, ql.Days),
                                   notional=2000000,
                                   tradeType=TradeType.CALL,
                                   tradeDirection=TradeDirection.LONG)

eq_opt_ads_put_10d = EquityOption(underlying=Stock.ADS,
                                  maturity=ql.Period(10, ql.Days),
                                  notional=3000000,
                                  tradeType=TradeType.PUT,
                                  tradeDirection=TradeDirection.LONG)
```

[In]:
```
ca_vm.remove_all_trades()
ca_vm.add_trades([eq_opt_ads_call_10d,eq_opt_ads_put_10d])
```

[In]:
```
allocation_with_vm = eulerAllocator2.allocate_ead()
```

[Out]:
```
         2Mn ADS Call 1W3D 3Mn ADS Put 1W3D Portfolio Risk Measure
VM only          -358.06%          458.06%           1,701,707 USD
```

Going back to 1Y maturity equity options we can see that the allocations of the preexisting trades can change significantly, when we add another equity option to the portfolio. We choose a position of 10Mn call options on an imaginary DBK stock.

[In]:
```
eq_opt_dbk_call = EquityOption(underlying=Stock.DBK,
                               maturity=ql.Period(1, ql.Years),
                               notional = 10000000,
                               tradeType=TradeType.CALL,
                               tradeDirection=TradeDirection.LONG)

ca.remove_all_trades()
ca.add_trades([eq_opt_ads_call,eq_opt_ads_put,eq_opt_dbk_call])
ca_vm.remove_all_trades()
ca_vm.add_trades([eq_opt_ads_call,eq_opt_ads_put,eq_opt_dbk_call])
ca_im.remove_all_trades()
ca_im.add_trades([eq_opt_ads_call,eq_opt_ads_put,eq_opt_dbk_call])
```

[In]:
```
allocation_no_margin = eulerAllocator1.allocate_ead()
allocation_with_vm = eulerAllocator2.allocate_ead()
allocation_with_im_and_im = eulerAllocator3.allocate_ead()
allocation_im = eulerAllocator3.allocate_im()
```

[Out]:

	10Mn DBK Call	2Mn ADS Call	3Mn ADS Put	PF Risk Measure
SIMM	63.10%	15.23%	21.67%	27,551,513 USD
No margin	57.45%	33.16%	9.39%	76,295,560 USD
VM only	80.79%	44.65%	-25.44%	10,230,051 USD
VM+IM	106.19%	86.85%	-93.04%	1,847,365 USD

Further analysis of the results shown above may be found in section 3.1

A.3 Euler allocation of an exemplary rates portfolio

We create a portfolio consisting of a big USD payer swap and a smaller EUR receiver swap of same maturity.

[In]:
```
payer_usd_6Y = IRS(notional = 1000000000,
                   swapDirection=SwapDirection.PAYER,
                   timeToSwapStart=ql.Period(2, ql.Days),
                   timeToSwapEnd=ql.Period(6, ql.Years),
                   index = InterestRateIndex.USDLIBOR3M)

receiver_eur_6Y = IRS(notional=180000000,
                      swapDirection=SwapDirection.RECEIVER,
                      timeToSwapStart=ql.Period(2,ql.Days),
                      timeToSwapEnd=ql.Period(6,ql.Years),
                      index=InterestRateIndex.EURIBOR6M)
```

We create three collateral agreements for which only VM but not IM is exchanged. The portfolios associated with the first two collateral agreements only consist of one of the two trades, while the portfolio associated with the third collateral agreement contains both trades.

[In]:
```
ca_together_vm_only = CollateralAgreement(initialMargining =␣
 ↪InitialMargining.NO_IM)
ca_together_vm_only.
 ↪link_sa_ccr_instance(SA_CCR(ca_together_vm_only))
ca_together_vm_only.add_trades([payer_usd_6Y, receiver_eur_6Y])

ca_usd_vm_only = CollateralAgreement(initialMargining =␣
 ↪InitialMargining.NO_IM)
ca_usd_vm_only.link_sa_ccr_instance(SA_CCR(ca_usd_vm_only))
ca_usd_vm_only.add_trades(payer_usd_6Y)
```

```
ca_eur_vm_only = CollateralAgreement(initialMargining =␣
 ↪InitialMargining.NO_IM)
ca_eur_vm_only.link_sa_ccr_instance(SA_CCR(ca_eur_vm_only))
ca_eur_vm_only.add_trades(receiver_eur_6Y)
```

We allocate the EAD of the portfolio containing both trades to the two individual trades.

[In]:
```
eulerAllocator = EulerAllocator(ca_together_vm_only)
eulerAllocator.fdApproach2 = FdApproach2.Central
alloc = eulerAllocator.allocate_ead()
```

We repeat the same process for collateral agreements for which VM and IM is exchanged. In this case, we also allocate the IM in addition to the EAD.

[In]:
```
ca_together_vm_im = CollateralAgreement(initialMargining =␣
 ↪InitialMargining.SIMM)
ca_together_vm_im.link_sa_ccr_instance(SA_CCR(ca_together_vm_im))
ca_together_vm_im.add_trades([payer_usd_6Y, receiver_eur_6Y])

ca_usd_vm_im = CollateralAgreement(initialMargining =␣
 ↪InitialMargining.SIMM)
ca_usd_vm_im.link_sa_ccr_instance(SA_CCR(ca_usd_vm_im))
ca_usd_vm_im.add_trades(payer_usd_6Y)

ca_eur_vm_im = CollateralAgreement(initialMargining =␣
 ↪InitialMargining.SIMM)
ca_eur_vm_im.link_sa_ccr_instance(SA_CCR(ca_eur_vm_im))
ca_eur_vm_im.add_trades(receiver_eur_6Y)

eulerAllocator2 = EulerAllocator(ca_together_vm_im)
eulerAllocator2.fdApproach2 = FdApproach2.Central
alloc2 = eulerAllocator2.allocate_ead()

alloc_im = eulerAllocator2.allocate_im()
```

This yields the following results for the calculated EADs and IMs:

[Out]:

	(Standalone) EAD VM only	(Standalone) IM	(Standalone) EAD VM + IM
180Mn EURIBOR6M RECEIVER IRS 6Y	1,957,315 USD	6,079,460 USD	286,420 USD
1,000Mn USDLIBOR3M PAYER IRS 6Y	10,873,970 USD	28,762,683 USD	2,014,873 USD
Portfolio	12,831,284 USD	28,059,093 USD	3,074,959 USD

And the following allocations of the risk measures:

[Out]:

	Allocated EAD VM only	Allocated IM	Allocated EAD VM + IM
180Mn EURIBOR6M RECEIVER IRS 6Y	15.25%	-0.19%	34.95%
1,000Mn USDLIBOR3M PAYER IRS 6Y	84.75%	100.19%	65.05%

As can be seen the EUR Receiver Swap is considered to increase risk for the EAD risk measure while the IM risk measure considers it to be a hedge position. Further discussion of this result in section 3.1.2.

To further showcase the differences between the two risk measures and their potentially unexpected interaction we add another trade to the portfolio.

For this we add a one year Swaption on a five year USD receiver swap to the portfolio and reallocate:

[In]:

```
ul_swap = IRS(notional=500000000,
              swapDirection=SwapDirection.RECEIVER,
              timeToSwapStart=ql.Period(1, ql.Years),
              timeToSwapEnd=ql.Period(6, ql.Years),
              index = InterestRateIndex.USDLIBOR3M)
rec_swaption_1_6 = Swaption(underlyingSwap=ul_swap,
                            optionMaturity=ql.Period(1, ql.Years))

ca_together_vm_im.add_trades(rec_swaption_1_6)
ca_together_vm_only.add_trades(rec_swaption_1_6)
```

```
alloc = eulerAllocator.allocate_ead()
alloc2 = eulerAllocator2.allocate_ead()
alloc_im = eulerAllocator2.allocate_im()
```

This yields the following results:

[Out]:

	Allocated EAD VM only	Allocated IM	Allocated EAD VM + IM
180Mn EURIBOR6M RECEIVER IRS 6Y	17.70%	0.50%	43.69%
1,000Mn USDLIBOR3M PAYER IRS 6Y	98.33%	80.07%	125.93%
500,000,000 USDLIBOR3M RECEIVER Swaption 1Y to 6Y	-16.04%	19.44%	-69.62%

[Out]:

	EAD VM only	IM	EAD VM + IM
Portfolio	11,058,114 USD	34,796,088 USD	1,586,748 USD

We observe that the Swaption is considered a hedge for the SA-CCR EAD risk measure while it increases risk under the ISDA-SIMM IM risk measure. The displayed results are discussed further in 3.1.2.

A.4 SA-CCR Euler allocation of an exemplary multi asset class portfolio

We want to show at an example, how the Euler allocation of trades can be summed up to subportfolios to still accurately represent the risk dynamics of the allocated risk measure.

We combine the exemplary equity portfolio (just the ADS options) with the exemplary rates portfolio set up in the previous sections within a joint portfolio.

[In]:

```
trades = [eq_opt_ads_call,
          eq_opt_ads_put,
          payer_usd_6Y,
```

```
          receiver_eur_6Y,
          rec_swaption_1_6]

joint_ca_vm_only = CollateralAgreement(
    margining=Margining.MARGINED,
    initialMargining=InitialMargining.NO_IM)
joint_ca_vm_only.link_sa_ccr_instance(SA_CCR(joint_ca_vm_only))
joint_ca_vm_only.add_trades(trades)

joint_ca_vm_and_im = CollateralAgreement(
    margining=Margining.MARGINED,
    initialMargining=InitialMargining.SIMM)
joint_ca_vm_and_im.link_sa_ccr_instance(SA_CCR(joint_ca_vm_and_im))
joint_ca_vm_and_im.add_trades(trades)
```

And perform an Euler allocation using a central difference approach.

[In]:
```
eulerAllocator_vm = EulerAllocator(joint_ca_vm_only)
eulerAllocator_vm.fdApproach2 = FdApproach2.Central
eulerAllocator_vm_and_im = EulerAllocator(joint_ca_vm_and_im)
eulerAllocator_vm_and_im.fdApproach2 = FdApproach2.Central

alloc_ead_under_vm = eulerAllocator_vm.allocate_ead()
alloc_ead_under_vm_and_im = eulerAllocator_vm_and_im.allocate_ead()
alloc_im = eulerAllocator_vm_and_im.allocate_im()
```

We then display the results on a trade level:

[Out]:

	Allocated EAD VM only	Allocated IM	Allocated EAD VM + IM
2Mn ADS Call 1Y	8,181,543 USD	-4,802,460 USD	2,959,180 USD
3Mn ADS Put 1Y	-4,662,085 USD	19,034,023 USD	-2,643,342 USD
180Mn EURIBOR6M RECEIVER IRS 6Y	1,957,315 USD	172,265 USD	630,350 USD
1000Mn USDLIBOR3M PAYER IRS 6Y	10,873,970 USD	27,859,887 USD	1,921,998 USD
500Mn USDLIBOR3M RECEIVER Swaption 1Y to 6Y	-1,773,170 USD	6,763,936 USD	-977,443 USD
Overall Portfolio	14,577,571 USD	49,027,652 USD	1,890,742 USD

We can aggregate the results across the asset classes.

[Out]:

	Allocated EAD VM only	Allocated IM	Allocated EAD VM + IM
Equity Subportfolio	3,519,458 USD	14,231,564 USD	315,837 USD
Rates Subportfolio	11,058,114 USD	34,796,088 USD	1,574,905 USD

This result is discussed further in section 3.1.3.

Subportfolio allocations can also be calculated directly by calculating partial derivatives against an entire subportfolio instead of a single trade which saves computational effort.

Using a central difference approach we first calculate the partial derivative of the EAD under VM and IM of the equity subportfolio:

[In]:

```
bump = 0.00005

joint_ca_vm_and_im.replace_trade(
    [eq_opt_ads_put,
```

```
     eq_opt_ads_call],
    [eq_opt_ads_put.get_bumped_copy(rel_bump_size=-bump),
     eq_opt_ads_call.get_bumped_copy(rel_bump_size=-bump),])

bump_down_ead = joint_ca_vm_and_im.get_sa_ccr_model().get_ead()

joint_ca_vm_and_im.remove_all_trades()
joint_ca_vm_and_im.add_trades(trades)
joint_ca_vm_and_im.replace_trade(
    [eq_opt_ads_put,
     eq_opt_ads_call],
    [eq_opt_ads_put.get_bumped_copy(rel_bump_size=bump),
     eq_opt_ads_call.get_bumped_copy(rel_bump_size=bump),])

bump_up_ead = joint_ca_vm_and_im.get_sa_ccr_model().get_ead()

eq_pf_central_difference = (bump_up_ead-bump_down_ead)/(2*bump)
```

[Out]: '315,837 USD'

[In]:
```
joint_ca_vm_and_im.remove_all_trades()
joint_ca_vm_and_im.add_trades(trades)
joint_ca_vm_and_im.replace_trade(
    [receiver_eur_6Y,
     payer_usd_6Y,
     rec_swaption_1_6],
    [receiver_eur_6Y.get_bumped_copy(rel_bump_size=-bump),
     payer_usd_6Y.get_bumped_copy(rel_bump_size=-bump),
     rec_swaption_1_6.get_bumped_copy(rel_bump_size=-bump),])

bump_down_ead = joint_ca_vm_and_im.get_sa_ccr_model().get_ead()

joint_ca_vm_and_im.remove_all_trades()
joint_ca_vm_and_im.add_trades(trades)
joint_ca_vm_and_im.replace_trade(
    [receiver_eur_6Y,
     payer_usd_6Y,
     rec_swaption_1_6],
    [receiver_eur_6Y.get_bumped_copy(rel_bump_size=bump),
     payer_usd_6Y.get_bumped_copy(rel_bump_size=bump),
     rec_swaption_1_6.get_bumped_copy(rel_bump_size=bump),])

bump_up_ead = joint_ca_vm_and_im.get_sa_ccr_model().get_ead()
```

```
ir_pf_central_difference = (bump_up_ead-bump_down_ead)/(2*bump)
```

[Out]: '1,574,904 USD'

When bumping all trades of a subportfolio simultaneously we yield the same results of 315,837 USD for the equity subportfolio and 1,574,904 USD for the rates subportfolio whilst performing less recalculations of the portfolio.

A.5 ISDA SIMM homogeneity for portfolio of a single trade

We want to showcase that the concentration risk addOn breaks homogeneity of the ISDA SIMM risk measure. The concentration threshold for USD interest rate risk is 230Mn USD per Basis Point change. Considering that IRS trades roughly have a delta of one against the interest rate this means that a trade with a notional of $\frac{230\text{Mn}}{0.0001} = 2300\text{Bn}$ and a maturity of 1 year would incur a risk above the threshold. If the maturity increases to 10 years a notional of roughly 230Bn should be enough to exceed the concentration threshold.

[In]:
```
notional = 200000000000

irs = IRS(notional = notional,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          swapDirection=SwapDirection.PAYER,
          index = InterestRateIndex.USDLIBOR3M)

simm_sensis = irs.get_simm_sensis()
ir_delta = sum([float(entry['amountUSD'])
                for entry
                in simm_sensis
                if entry['riskType'] == 'Risk_IRCurve'])
ir_delta
```

[Out]: 177293785.56303406

The trade has an aggregated delta sensitivity against a 1BP move of the USD interest rate of 177,293,786 USD. With the available market data the delta of the IRS appears to be slightly lower than one.

We create a collateral agreement with associated ISDA SIMM and SA CCR model and load the IRS in the portfolio. The collateral agreement uses ISDA SIMM for IM calculation.

[In]:
```
ca = CollateralAgreement(initialMargining=InitialMargining.SIMM,
                         margin_currency=Currency.USD)
ca.link_sa_ccr_instance(SA_CCR(ca))
ca.add_trades(irs)
```

We now want to investigate if the ISDA SIMM exhibits homogeneity for this single trade portfolio. To do so we test if

$$f(\alpha \mathbf{x}) = \alpha^k f(\mathbf{x})$$
$$\text{for } \alpha > 0$$

holds. We test for $0 < \alpha \leq 2$ with an increment size of 0.01.

[In]:
```
bumps = arange(0,2.01,0.01)
resultDataframe = pd.DataFrame(columns = ['USD','k','Legend'])
```

We create a utility function that supports IM, VM and Collateral although we are just exploring IM right now.

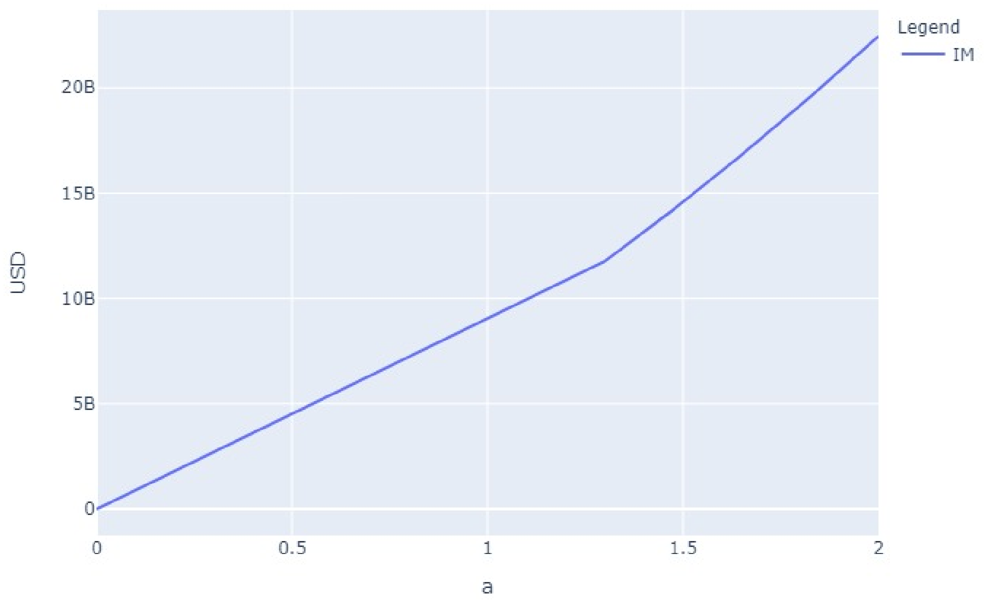
[In]:
```
def bump_and_get_results(bumpsize, trade, collateralagreement):
    record = {}
    record['Bumpsize']=bumpsize
    bumped_copy = trade.get_bumped_copy(rel_bump_size=bumpsize-1)
    collateralagreement.remove_all_trades()
    collateralagreement.add_trades(bumped_copy)
    record['IM'] = collateralagreement.get_im_model().
 ↪get_risk_measure()
    record['VM'] = collateralagreement.get_vm_model().
 ↪get_risk_measure()
    record['Collateral'] = collateralagreement.get_C()
    collateralagreement.remove_all_trades()
    collateralagreement.add_trades(trade)
    return record
```

[In]:
```
for bump in bumps:
    result = bump_and_get_results(bump, irs, ca)
    im_record = {'X': result['Bumpsize'],
                 'Y': result['IM'],
                 'Legend': 'IM'}
    resultDataframe = resultDataframe.append(im_record,␣
 ↪ignore_index=True)
```

[In]:
```
fig = px.line(resultDataframe[resultDataframe.Legend == 'IM'],␣
 ↪x='X', y='Y', color='Legend', line_dash='Legend')
```

```
fig.update_layout(xaxis_title = 'a', yaxis_title = 'USD')
img_bytes = fig.to_image(format='jpeg')
Image(img_bytes)
```

[Out]:



As can be seen, ISDA SIMM does not exhibit homogeneity. We can find the notional at which the IR delta of the trade exceeds 230Mn USD.

[In]:
```
def aggregate_usd_ir_by_notional(notional):
    irs2 = IRS(notional = notional,
            timeToSwapStart=ql.Period(2, ql.Days),
            timeToSwapEnd=ql.Period(10, ql.Years),
            swapDirection=SwapDirection.PAYER,
            index = InterestRateIndex.USDLIBOR3M)
    irs2.get_simm_sensis_ircurve()
    delta = sum([float(k['amount']) for k in irs2.
 ↪get_simm_sensis_ircurve()])
    return (delta-230000000)**2

res = scipy.optimize.minimize_scalar(
    aggregate_usd_ir_by_notional,
    bounds = (1, 10e12),
    method = 'bounded')
f'Notional: {res.x:,.0f} USD'
```

```
[Out]: 'Notional: 259,456,358,913 USD'
```

```
[In]: 'a: %.2f'%(res.x / irs.notional)
```

```
[Out]: 'a: 1.30'
```

Further discussion of this in section 3.2.1.

A.6 SA-CCR and ISDA SIMM Euler allocation under a perfect hedge

The goal is to create a portfolio with a perfect hedge and see if and under which circumstances EAD allocation is still possible. We load two perfectly offsetting IRS (one payer, one receiver). To avoid the unrealistic case of a zero IM and EAD portfolio we add an unrelated equity option into the portfolio.

```
[In]: IRS_pay = IRS(notional=100000000,
                     timeToSwapStart=ql.Period(2, ql.Days),
                     timeToSwapEnd=ql.Period(10, ql.Years),
                     swapDirection=SwapDirection.PAYER,
                     index=InterestRateIndex.USDLIBOR3M)

      IRS_rec = IRS(notional=100000000,
                     timeToSwapStart=ql.Period(2, ql.Days),
                     timeToSwapEnd=ql.Period(10, ql.Years),
                     swapDirection=SwapDirection.RECEIVER,
                     index=InterestRateIndex.USDLIBOR3M)

      eqOpt = EquityOption(notional = 1000000,
                           underlying=Stock.ADS,
                           tradeType = TradeType.CALL,
                           tradeDirection = TradeDirection.LONG,
                           maturity = ql.Period(1, ql.Years))

      ca = CollateralAgreement()
      ca.link_sa_ccr_instance(SA_CCR(ca))
```

```
[In]: ca.add_trades([IRS_pay, IRS_rec, eqOpt])
      print(ca.get_sa_ccr_model().get_risk_measure())
      print(ca.get_im_model().get_risk_measure())
```

```
1315355.6739375147
6952717.387110085
```

We now create an Euler allocator which can be used to perform a numerical Euler allocation of the ISDA-SIMM IM or the SA-CCR EAD risk measure. The allocator can be set to use forward, backward or central differentiation. We will see that the differentiation approach makes a big difference for this perfectly hedged portfolio.

[In]:
```
eulerAllocator = EulerAllocator(ca)
im_alloc_forward = eulerAllocator.allocate_im()
saccr_alloc_forward = eulerAllocator.allocate_ead()

eulerAllocator.fdApproach2=FdApproach2.Central
im_alloc_central = eulerAllocator.allocate_im()
saccr_alloc_central = eulerAllocator.allocate_ead()

eulerAllocator.fdApproach2=FdApproach2.Backward
im_alloc_backward = eulerAllocator.allocate_im()
saccr_alloc_backward = eulerAllocator.allocate_ead()
```

Below the resulting allocation for the IM is displayed. The allocation only exhibits nativ additivity when using the central difference approach since then the allocated values sum up to the IM value of 6952717.39 USD.

[In]:
```
display_table(im_alloc_forward, im_alloc_central, im_alloc_backward)
```

[Out]:

	Backward	Central	Forward
IRS_Long	-4.518969e+06	-2.421718e-01	4.518969e+06
IRS_Short	-4.519001e+06	-6.323463e+01	4.518937e+06
EquityOption_Long	6.952717e+06	6.952717e+06	6.952717e+06
Sum	-2.085253e+06	6.952654e+06	1.599062e+07

Below the resulting allocation for the EAD is displayed. The allocation only exhibits nativ additivity when using the central difference approach since then the allocated values sum up to the IM value of 1315355.67 USD.

[In]:
```
display_table(saccr_alloc_forward, saccr_alloc_central,␣
↪saccr_alloc_backward)
```

[Out]:

	Backward	Central	Forward
IRS_Long	-1.884133e+05	4.845671e-02	1.884134e+05
IRS_Short	-1.884070e+05	1.265202e+01	1.884196e+05
EquityOption_Long	1.315356e+06	1.315356e+06	1.315356e+06
Sum	9.385354e+05	1.315368e+06	1.692189e+06

The reason for the Euler allocation not working is that the SA-CCR is not differentiable in case of a perfect hedge. This can be shown by plotting the function SA-CCR w.r.t. the position size in the three trades.

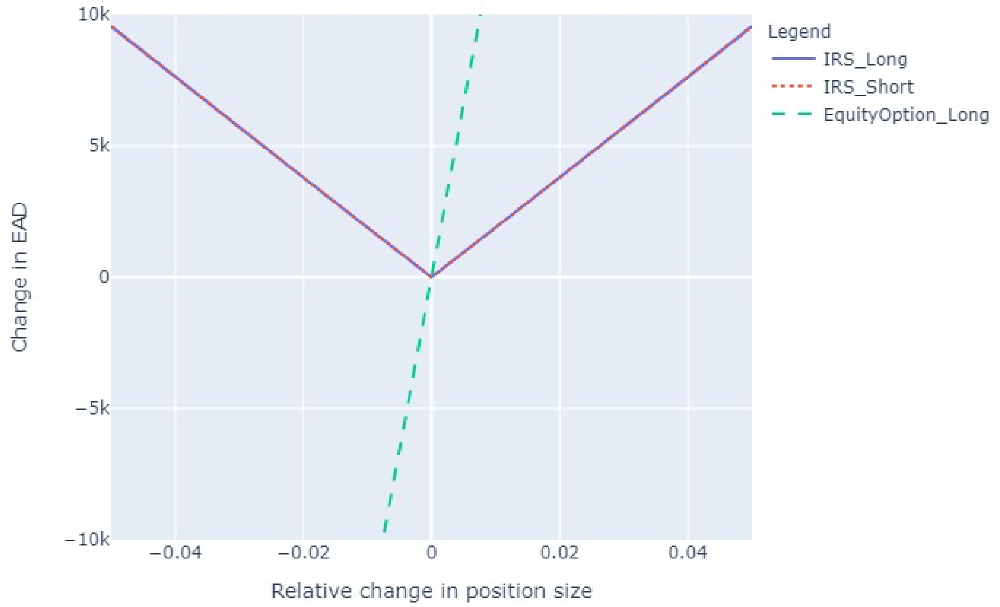
[In]:
```
bumps = arange(-0.05, 0.06, 0.01)
```

[In]:
```
def bump_one_trade_and_return_diff(bump, trade: Trade, ca:␣
↪CollateralAgreement, method):
    base = method()
    ca.remove_trades(trade)
    bumped_trade = trade.get_bumped_copy(rel_bump_size=bump)
    ca.add_trades(bumped_trade)
    result = method()
    ca.remove_trades(bumped_trade)
    ca.add_trades(trade)
    return result-base
```

[In]:
```
result_df = pd.DataFrame()

for t in ca.trades:
    for bump in bumps:
        record = \
            {'Relative change in position size': bump,
             'Change in EAD':
                 bump_one_trade_and_return_diff(
                     bump, t, ca,
                     ca.get_sa_ccr_model().get_risk_measure),
             'Legend':
                 ast.literal_eval(str(t))['Instrument'] + '_'
                 + ast.literal_eval(str(t))['TradeDirection']
             }

        result_df = result_df.append(record, ignore_index=True)
```

Displaying `result_df` yields:

[Out]:

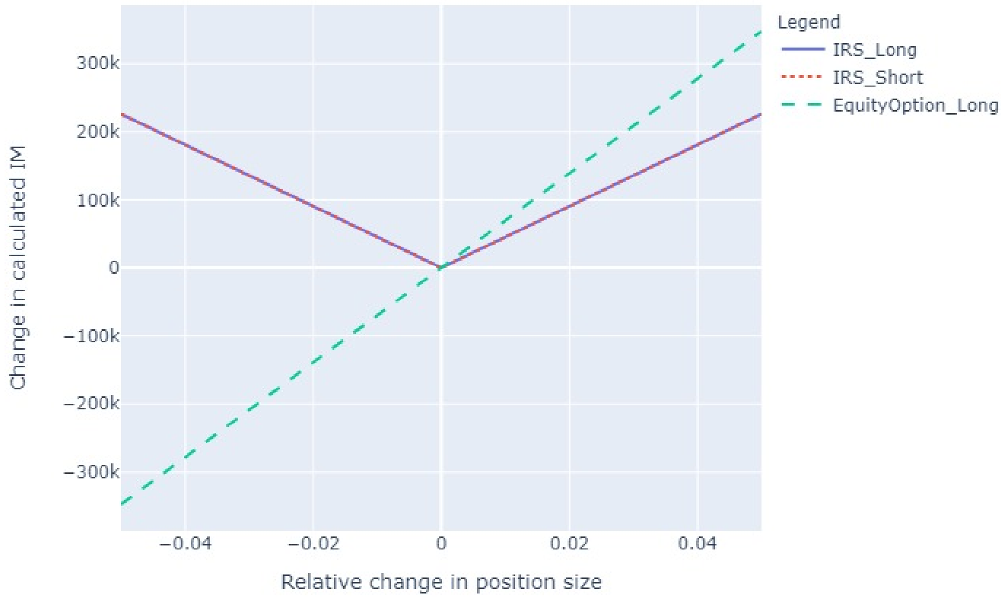


Do the same for the IM:

[In]:
```
result_df = pd.DataFrame()

for t in ca.trades:
    for bump in bumps:
        record = \
            {'Relative change in position size': bump,
             'Change in calculated IM':␣
 ↪bump_one_trade_and_return_diff(
                 bump, t, ca,
                 ca.get_im_model().get_risk_measure),
             'Legend':
                 ast.literal_eval(str(t))['Instrument'] + '_'
                 + ast.literal_eval(str(t))['TradeDirection']
             }

        result_df = result_df.append(record, ignore_index=True)
```

Displaying `result_df` yields:

[Out]:



The same phenomenon does not appear for if a hedge is not perfect i.e. if the hedge size can be increased to further diminish the risk metric.

[In]:
```
IRS_pay = IRS(notional=200000000,
              timeToSwapStart=ql.Period(2, ql.Days),
              timeToSwapEnd=ql.Period(10, ql.Years),
              swapDirection=SwapDirection.PAYER,
              index=InterestRateIndex.USDLIBOR3M)

IRS_rec = IRS(notional=100000000,
              timeToSwapStart=ql.Period(2, ql.Days),
              timeToSwapEnd=ql.Period(10, ql.Years),
              swapDirection=SwapDirection.RECEIVER,
              index=InterestRateIndex.USDLIBOR3M)

eqOpt = EquityOption(notional = 1000000)
```

The IM for this portfolio is 11471795.91 USD and the EAD is 1538770.53 USD.

For the IM the allocation yields:

[Out]:

	Backward	Central	Forward
IRS_Short	-4.519079e+06	-4.519078e+06	-4.519079e+06

```
IRS_Long              9.038157e+06  9.038157e+06  9.038157e+06
EquityOption_Long     6.952717e+06  6.952717e+06  6.952717e+06
Sum                   1.147180e+07  1.147180e+07  1.147180e+07
```

and for the EAD the different differentiation approaches yield:

[Out]:
```
                          Backward       Central       Forward
IRS_Short            -2.466093e+05 -2.466078e+05 -2.466063e+05
IRS_Long              4.932094e+05  4.932156e+05  4.932217e+05
EquityOption_Long     1.292161e+06  1.292163e+06  1.292164e+06
Sum                   1.538761e+06  1.538771e+06  1.538780e+06
```

A.7 Homogeneity of C for a single trade portfolio

We want to investigate under which circumstances the C representing the received collateral in the calculation of SA-CCR exhibits homogeneity. For this we consider a portfolio consisting of a single IRS as follows

[In]:
```
notional = 200000000000
irs = IRS(notional = notional,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          swapDirection=SwapDirection.PAYER,
          index = InterestRateIndex.USDLIBOR3M)
```

Since we have not explicitly set a fixed rate, the IRS is struck at par and should have a present value of close to 0.

[In]:
```
print('Fixed rate: %f' %irs.get_fixed_rate())
print('Present value: %.2f' %irs.get_price())
```

```
Fixed rate: 0.024093
Present value: 0.00
```

We also set up a fixed payer IRS that is in the money as it has a fixed rate of 2%.

[In]:
```
irs_itm = IRS(notional = notional,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          swapDirection=SwapDirection.PAYER,
          index = InterestRateIndex.USDLIBOR3M,
          fixed_rate=0.02)

print('Fixed rate: %f' %irs_itm.get_fixed_rate())
```

```
print('Present value: %.2f' %irs_itm.get_price())
```

```
Fixed rate: 0.020000
Present value: 7258788031.38
```

Additionally, we set up a collateral agreement exchanging IM in accordance with ISDA-SIMM with a minimum transfer amount of 1Bn and a threshold of 2Bn. For technical reasons we need to first create the collateral agreement and afterwards link it to an instance of the SA-CCR model. We put each of the two trades created above in a separate portfolio and collateral agreement.

[In]:
```
ca = CollateralAgreement(mta = 1000000000,
                          threshold= 2000000000,
                          initialMargining=InitialMargining.SIMM,
                          margin_currency=Currency.USD)
ca.link_sa_ccr_instance(SA_CCR(ca))
```

Again we explore homogeneity at this example by exploring whether

$$f(\alpha \mathbf{x}) = \alpha^k f(\mathbf{x})$$
$$\text{for } \alpha > 0$$

holds based on our initial portfolio for a range $0 < \alpha \leq 2$

Next, we calculate IM, VM, and C for the two IRS. C is once calculated with consideration of MTA and threshold and once without.

To use the MTA we need to set a current margin amount. We will set this as the currently calculated C. With the MTA in place C will afterwards only be updated if the sum of VM and IM differ from the current margin amount by more than the MTA.

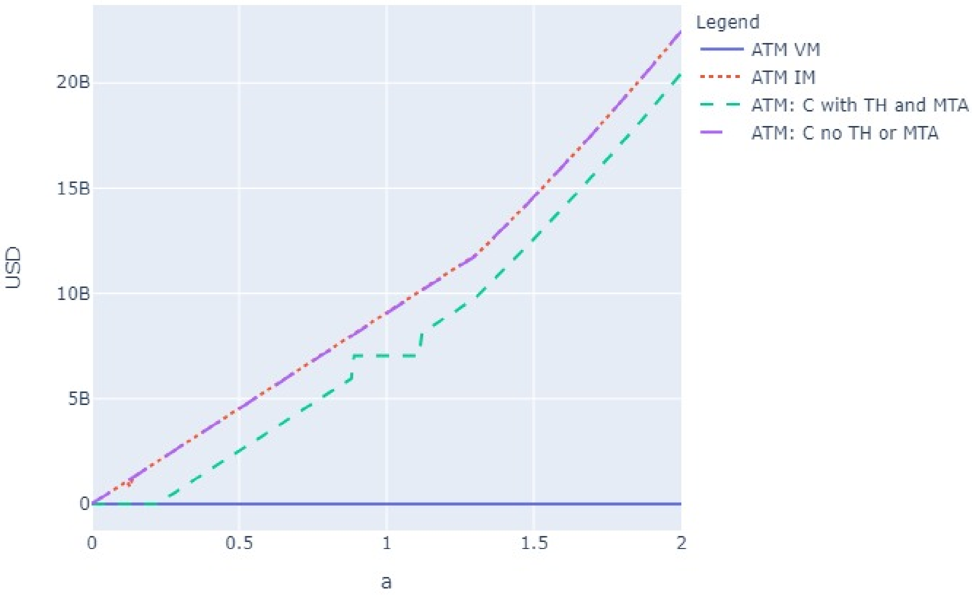
[In]:
```
bumps = arange(0,2.01,0.01)
resultDataframe = pd.DataFrame(columns = ['X','Y','Legend'])
```

[In]:
```
# At the money IRS, with threshold and mta
ca.add_trades(irs)
ca.set_start_collateral_amount(ca.get_C())
for bump in bumps:
    result = bump_and_get_results(bump, irs, ca)
    result_to_record('ATM VM', 'VM', result)
    result_to_record('ATM IM', 'IM', result)
    result_to_record('ATM: C with TH and MTA', 'Collateral', result)

# In the money IRS, with threshold and mta
```

```
ca.remove_all_trades()
ca.add_trades(irs_itm)
ca.set_start_collateral_amount(ca.get_C())
for bump in bumps:
    result = bump_and_get_results(bump, irs_itm, ca)
    result_to_record('ITM VM', 'VM', result)
    result_to_record('ITM IM', 'IM', result)
    result_to_record('ITM: C with TH and MTA', 'Collateral', result)

# In the money IRS without threshold or mta
ca.threshold = 0
ca.mta = 0

for bump in bumps:
    result = bump_and_get_results(bump, irs_itm, ca)
    result_to_record('ITM: C no TH or MTA', 'Collateral', result)

# At the money IRS without threshold or mta
ca.remove_all_trades()
ca.add_trades(irs)
ca.set_start_collateral_amount(ca.get_C())
for bump in bumps:
    result = bump_and_get_results(bump, irs, ca)
    result_to_record('ATM: C no TH or MTA', 'Collateral', result)
```

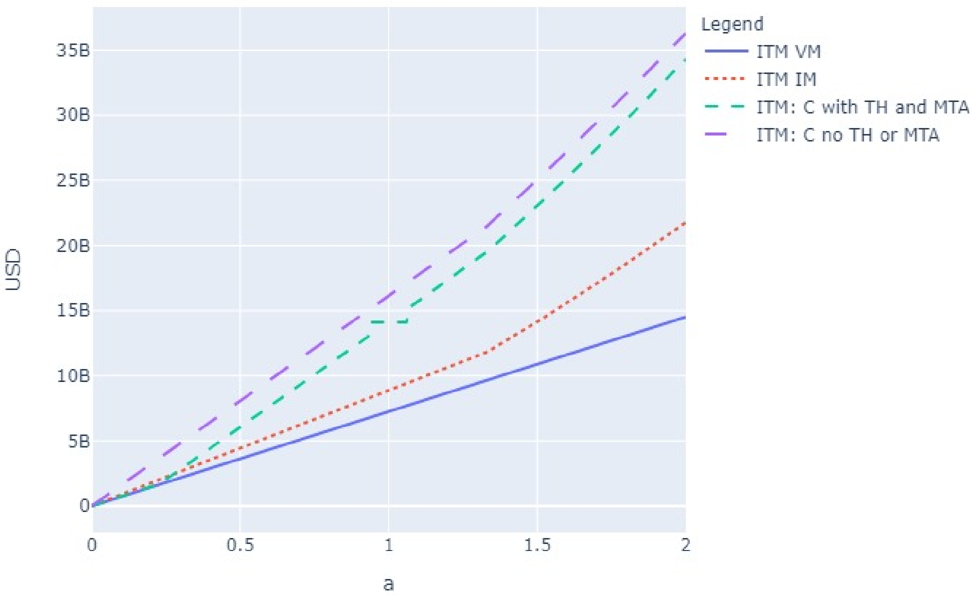
Displaying the result for the at the money IRS it can be seen that the VM is flat at zero since the IRS is at par and therefore has a PV of 0. The IM shows the behaviour described in appendix A.5. C with threshold and minimum transfer amount is not homogeneous at all, while C without threshold and MTA is partially. This is further discussed in section 3.2.2.

[Out]:



Results are the same for the in the money IRS with the exception, that the VM is not 0 but VM is still a homogeneous function.

[Out]:



A.8 Exemplary SA-CCR allocation under consideration of a minimum transfer amount.

The result of Appendix A.7 shows, that inclusion of the MTA results in a local plateau of C.

In this section we want to investigate if inclusion of the MTA breaks homogeneity of the SA-CCR EAD function and what can be done to mitigate this effect.

We initialize a 200Bn IRS, a collateral agreement with a 0 threshold and a 1Bn MTA.

[In]:
```
irs = IRS(notional = 200000000000,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          index = InterestRateIndex.USDLIBOR3M)
ca = CollateralAgreement(threshold=0,
                         mta=1000000000)
ca.link_sa_ccr_instance(SA_CCR(ca))
ca.add_trades(irs)

ca.set_start_collateral_amount(ca.get_C())
```

The starting collateral C_{t-1} is set to 9038157077 USD which is also the calculated IM since the VM of this par IRS is 0.

The EAD is:

[In]:
```
original_ead = ca.get_sa_ccr_model().get_risk_measure()
original_ead
```

[Out]:
```
582881953.4866074
```

When bumping the notional of the irs by 0.01%, we can see that the collateral of the portfolio does not change due to the MTA.

[In]:
```
ca.remove_all_trades()
ca.add_trades(irs.get_bumped_copy(rel_bump_size=0.0001))
ead_bumped_mta = ca.get_sa_ccr_model().get_risk_measure()
print('EAD:  %d' %ead_bumped_mta)
print('C:    %d' %ca.get_C())
```

```
EAD:  583024482
C:    9038157077
```

[In]:
```
ead_bumped_mta = ca.get_sa_ccr_model().get_risk_measure()
```

When temporarily disabling the MTA the resulting EAD and C differ.

[In]:
```
ca.mta = 0
ead_bumped_no_mta = ca.get_sa_ccr_model().get_risk_measure()
print('EAD:  %d' %ead_bumped_no_mta)
print('C:   %d' %ca.get_C())
```

```
EAD:  582940242
C:   9039060887
```

Calculating the forward difference with and without consideration of the MTA yields

[In]:
```
print('With MTA:    %d' %((ead_bumped_mta-original_ead)/0.0001))
print('Without MTA:  %d' %((ead_bumped_no_mta-original_ead)/0.0001))
```

```
With MTA:    1425289375
Without MTA:  582887602
```

As can be seen, only when the allocation is performed without recognition of the MTA the allocated amount corresponds to the actual EAD of this single trade portfolio.

A.8.1 Impact of the minimum transfer amount on RC

the MTA also impacts RC as displayed in table 2.1. Since IM reduces the RC the most relevant case is when the calculated IM is below the threshold.

In an example we try to allocate the EAD of a portfolio consisting of a single 100Mn IRS. The associated collateral agreement has a threshold of 50Mn and a minimum transfer amount of 2Mn.

[In]:
```
irs = IRS(notional=100000000,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          index=InterestRateIndex.USDLIBOR3M)

ca = CollateralAgreement(threshold=50000000,
                         mta = 2000000)
ca.link_sa_ccr_instance(SA_CCR(ca))
ca.add_trades(irs)

print('RC:      %d USD' %ca.get_sa_ccr_model().get_rc())
print('RC*1.4:  %d USD' %(ca.get_sa_ccr_model().get_rc()*1.4))
print('PFE:     %d USD' %(ca.get_sa_ccr_model().get_pfe()))
print('PFE*1.4: %d USD' %(ca.get_sa_ccr_model().get_pfe()*1.4))
```

```
print('EAD:     %d USD' %ca.get_sa_ccr_model().get_risk_measure())
```

```
RC:       2000000 USD
RC*1.4:   2800000 USD
PFE:      1179574 USD
PFE*1.4: 1651404 USD
EAD:      4451404 USD
```

The EAD is the sum of the RC and the PFE component time the α factor of 1.4. In all previous examples, the RC has always been floored at 0 since the received IM was higher than the MTA or since the MTA was 0.

Again, Euler allocation is not possible, because the EAD is a sum of the PFE, which is a function of the portfolio notional and the RC which, at least locally, is a constant. Similar to the issue with threshold described in A.10 one can only allocate without MTA and then allocate the remainder $EAD_{\text{MTA}} - EAD_{\text{no MTA}}$ according to some rule.

Below, we allocate the ead assuming a mta of 0.

[In]:
```
ca.mta = 0
eulerAllocator = EulerAllocator(ca)
allocation = eulerAllocator.allocate_ead()
ca.mta = 2000000
allocation[irs]
```

[Out]:
```
1651404.724563472
```

As we can see the result equals $PFE * 1.4$ of the entire portfolio but the RC has not been allocated.

The RC is also > 0, if $TH_{IM} < IM_{calc} < MTA$. Based on the available marketdata, chosen MTA and chosen threshold this is e.g. the case for an IRS with a notional of 1130Mn USD.

[In]:
```
irs2 = IRS(notional=1130000000,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          index=InterestRateIndex.USDLIBOR3M)

ca.remove_all_trades()
ca.add_trades(irs2)

print('RC:        %d USD' %ca.get_sa_ccr_model().get_rc())
print('PFE:     %d USD' %(ca.get_sa_ccr_model().get_pfe()))
print('EAD/1.4: %d USD' %(ca.get_sa_ccr_model().get_risk_measure()/
 ↪1.4))
print('EAD:     %d USD' %ca.get_sa_ccr_model().get_risk_measure())
```

```
RC:        934412 USD
PFE:     13329195 USD
EAD/1.4: 14263607 USD
EAD:     19969050 USD
```

These observations are discussed further in 3.2.2.2.

A.9 EAD impact of a variation margin threshold

We set up a single trade portfolio in a vm margined collateral agreement with a VM threshold:

[In]:
```
irs = IRS(notional=2000000000,
          index=InterestRateIndex.USDLIBOR3M,
          timeToSwapStart=ql.Period(2, ql.Days),
          timeToSwapEnd=ql.Period(10, ql.Years),
          fixed_rate=0.024)

irs.get_price()
```

[Out]: 1643257.9608547091

[In]:
```
ca_10mn = CollateralAgreement(
    margining=Margining.MARGINED,
    initialMargining=InitialMargining.NO_IM,
    threshold_vm=10000000)
ca_10mn.link_sa_ccr_instance(SA_CCR(ca_10mn))
ca_10mn.add_trades(irs)

ca_1mn = CollateralAgreement(
    margining=Margining.MARGINED,
    initialMargining=InitialMargining.NO_IM,
    threshold_vm=1000000)
ca_1mn.link_sa_ccr_instance(SA_CCR(ca_1mn))
ca_1mn.add_trades(irs)

ca_0 = CollateralAgreement(
    margining=Margining.MARGINED,
    initialMargining=InitialMargining.NO_IM,
    threshold_vm=0)
ca_0.link_sa_ccr_instance(SA_CCR(ca_0))
ca_0.add_trades(irs)
```

When calculating the EAD, PFE and RC of the three portfolios we can see that a

VM threshold only impacts the RC and that inclusion of a variation margin threshold simply increases the EAD by $1.4 * \text{VM threshold}$.

[In]:
```
print(ca_10mn.get_sa_ccr_model().get_ead())
print(ca_1mn.get_sa_ccr_model().get_ead())
print(ca_0.get_sa_ccr_model().get_ead())
```

```
47028094.49124615
34428094.49124615
33028094.491246153
```

[In]:
```
print(ca_10mn.get_sa_ccr_model().get_rc())
print(ca_1mn.get_sa_ccr_model().get_rc())
print(ca_0.get_sa_ccr_model().get_rc())
```

```
10000000.0
1000000.0
0.0
```

[In]:
```
print(ca_10mn.get_sa_ccr_model().get_pfe())
print(ca_1mn.get_sa_ccr_model().get_pfe())
print(ca_0.get_sa_ccr_model().get_pfe())
```

```
23591496.065175824
23591496.065175824
23591496.065175824
```

This impact of the EAD even persists if the portfolio is empty.

[In]:
```
ca_10mn.remove_all_trades()
ca_10mn.get_sa_ccr_model().get_ead()
```

[Out]:
```
14000000.0
```

This result is discussed in section 3.2.2.1.

A.10 Exemplary SA-CCR allocation under consideration of an initial margin threshold

We consider a portfolio of two trades, an IRS and an equity option.

[In]:
```
irs = IRS(notional=100000000,
          index = InterestRateIndex.USDLIBOR3M,
          timeToSwapStart=ql.Period(2, ql.Days),
```

```
            timeToSwapEnd=ql.Period(10, ql.Years))

ca_irs = CollateralAgreement(initialMargining=InitialMargining.
 ↪NO_IM)
ca_irs.link_sa_ccr_instance(SA_CCR(ca_irs))
ca_irs.add_trades(irs)
```

[In]:
```
eqOpt = EquityOption(notional = 403690.28,
                     maturity = ql.Period(1, ql.Years),
                     underlying = Stock.ADS,
                     tradeType = TradeType.CALL,
                     tradeDirection = TradeDirection.LONG)
```

[In]:
```
ca_eqOpt = CollateralAgreement(initialMargining=InitialMargining.
 ↪NO_IM)
ca_eqOpt.link_sa_ccr_instance(SA_CCR(ca_eqOpt))
ca_eqOpt.add_trades(eqOpt)
```

These two trades happen to have the same EAD under VM only.

[Out]: 'IRS EAD under VM: 1,651,405 USD'

[Out]: 'EQ EAD under VM: 1,651,405 USD'

If they are put into a common portfolio under VM only, this common portfolio just has the sum as EAD as no hedge effects take place.

[In]:
```
ca_common = CollateralAgreement(
    initialMargining=InitialMargining.NO_IM)
ca_common.link_sa_ccr_instance(SA_CCR(ca_common))
ca_common.add_trades([eqOpt,irs])
```

[Out]: 'Portfolio EAD under VM: 3,302,809 USD'

However the IM and consequently the EAD under IM differ between the two trades.

[In]:
```
ca_eqOpt.initialMargining = InitialMargining.SIMM
ca_irs.initialMargining = InitialMargining.SIMM
```

[Out]: 'EQ IM: 2,806,744 USD'

[Out]: 'IRS IM: 4,519,079 USD'

[Out]: 'EQ EAD under VM and IM: 530,996 USD'

[Out]: 'IRS EAD under VM and IM: 291,441 USD'

The IM of the common portfolio is the sum of the individual portfolios while the EAD of the common portfolio is a bit smaller than the respective sum.

[In]:
```
ca_common.initialMargining = InitialMargining.SIMM
```

[Out]: 'Portfolio IM: 7,325,823 USD'

[Out]: 'Portfolio EAD under VM and IM: 777,229 USD'

We can compare the allocation of EAD with and without IM to see that with IM, less is attributed to the IRS as it commands a higher margin.

Without initial margin:

[In]:
```
eulerAllocator = EulerAllocator(ca_common)
eulerAllocator.fdApproach2=FdApproach2.Central
ca_common.initialMargining = InitialMargining.NO_IM
{k.__class__.__name__: f'{v:,.0f} USD'
 for k, v in eulerAllocator.allocate_ead().items()}
```

[Out]: {'EquityOption': '1,651,405 USD', 'IRS': '1,651,405 USD'}

With initial margin:

[ ]:
```
ca_common.initialMargining = InitialMargining.SIMM
{k.__class__.__name__: f'{v:,.0f} USD'
 for k, v in eulerAllocator.allocate_ead().items()}
```

We introduce an initial margin threshold that is higher than the calculated IM and see that the resulting allocation is subadditive and the same as in the case without initial margin.

[In]:
```
ca_common.threshold = 10000000
{k.__class__.__name__: f'{v:,.0f} USD'
 for k, v in eulerAllocator.allocate_ead().items()}
```

[Out]: {'EquityOption': '1,651,405 USD', 'IRS': '1,651,405 USD'}

[Out]: 'Portfolio EAD with 10Mn IM threshold: 3,302,809 USD'

When we lower the threshold below the exchanged IM the allocation fails

[In]:
```
ca_common.threshold = 5000000
{k.__class__.__name__: f'{v:,.0f} USD'
 for k, v in eulerAllocator.allocate_ead().items()}
```

```
[Out]: {'EquityOption': '331,448 USD', 'IRS': '-381,962 USD'}
```

```
[Out]: 'Portfolio EAD with 10Mn IM threshold: 2,032,638 USD'
```

Further discussion of these results in section 3.2.2.3.

Glossary

AAD	Adjoint Algorithmic Differentiation
CCP	Central counterparty
CCR	Counterparty credit risk
CEM	Current exposure method
CRIF	Common Risk Interchange Format - standardized file format for inputs for the ISDA SIMM model
CRR2	Capital requirements regulation two of the european union [31]
CSA	Credit support annex
EAD	Exposure at default
EURIBOR	Euro Interbank Offered Rate
FRTB	Fundamental review of the trading book regulation
FRTB-SA	Standard approach for market risk RWA calculation under FRTB
IM	Initial margin
IMM	Internal model method
IRS	Interest Rate Swap
ISDA	International Swaps and Derivatives Association
ISDA SIMM	Internal initial margin model for uncleared derivatives developed by ISDA and used by most market participants
LIBOR	London Inter-bank Offered Rate - Interest rate index for a range of currencies, e.g. the USD
MPoR	Margin period of risk
MTA	Minimum transfer amount of a collateral agreement
MtM	Mark to market - current market value of a portfolio or a financial instrument

NPV	Net present value
OIS	Overnight Indexed Swap
OTC	Over the counter derivatives as opposed to exchange trades derivatives
PnL	Profit and loss
PV	Present Value
RWA	Risk weighted assets
SA-CCR	Standard approach for counterparty credit risk EAD calculation under CRR2
TH	Threshold of a collateral agreement under which no margin is exchanged
VaR	Value at risk
VM	Variation Margin

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