SA-CCR Allocation under consideration of margining

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Abstract

This will be the abstract.

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Chapter 1

Introduction

1.1 Motivation

With increasing sophistication of risk models, own capital models and margining models the need for equally sophisticated tools for allocating these measures rises as well. For any risk metric that considers portfolio effects calculating the contribution to the risk measure of individual trades is a challenge. As part of the Basel 3 reform, regulators have updated the standardized models for market risk own capital requirement and credit risk own capital requirements. The new standardized model for market risk own capital requirements is the FRTB-SA and the new standardized model for credit risk own capital requirements is the SA-CCR model . Both of these models are portfolio based risk models. Gregory [4, Chapter 10.7] states that three allocation approaches are used in practice:

Add appropriate citation to Basel 3 or CRD 2

Citation to FRTB-SA needed

Enter Citation of SA-CCR

- Incremental allocation for pre trade risk checks and for front office incentivization
- Marginal allocation for risk analytics of existing portfolios
- Pro rata allocation if trade contributions must not be negative, if risk sensitivity is not required or if the allocated risk measure does not take portfolio effects into account

These allocation approaches and their advantages and disadvantages are analyzed in further detail in section 2.5. Calculation of pro rata and incremental allocation is fairly straightforward and can generally be performed under any circumstances. Marginal allocation on the other hand is more challenging. This thesis will also include results for incremental and pro rata allocations for the sake of completeness but the main focus will be the analysis of marginal allocation.

Schulze [10] has analytically calculated the marginal allocation for the FRTB-SA. However, an approach to marginal allocation of SA-CCR has not been published yet. This thesis intends to close this gap by showcasing a numerical marginal allocation approach for SA-CCR. A particular challenge in the allocation of SA-CCR is its consideration of margining. This makes the allocation of SA-CCR dependent on margin models which can themselves be significantly more complex than the SA-CCR model itself.

The goal of this thesis is to find an approach for allocating SA-CCR while considering margining. Particular focus is put on the allocation of SA-CCR under consideration of variation margin an an internal initial margin model.

1.2 Literature review

1.3 Structure of the thesis

Throughout this document a small but diverse selection of derivatives is used for exemplary calculations and to explore edge cases of the different models. Section 2.1 does briefly introduce these instruments and the financial models and market data that is used to price them.

Since margining is an integral part of the SA-CCR model, section 2.2 will afterwards introduce different margin types and typical counterparty relations and margin models observed in the market. Section 2.2 will also establish why out of the different margin models, the ISDA SIMM model needs to be investigated the closest for the purpose of this thesis.

Section 2.3 introduces the ISDA SIMM which is the most commonly used model for initial margin calculation of uncleared derivatives. As the ISDA SIMM is based on first order sensitivities, section 2.3 also describes how to calculate ISDA SIMM compliant sensitivities for the financial instruments introduced in section 2.1.

After the different margin components have been introduced, section 2.4 presents the SA-CCR model that is used for the calculation of the EAD of derivatives. Special emphasis will be put on the inclusion of margining.

Section 2.5 presents literature results regarding the allocation of risk measures. Additionally, the theoretical foundation for the Euler allocation is laid out as the investigation if an when an Euler allocation is possible is the main subject for the analysis section of the thesis.

At this point all relevant concepts, models and financial instruments have been introduced to conduct the analysis chapter of this thesis. The main goal is to establish a numerical allocation approach for SA-CCR.

As the received initial margin is a subcomponent of SA-CCR it might be necessary to also allocate initial margin. Due to the complexity of initial margin models allocation of this subcomponent is investigated separately. Section 3.1 establishes numerical approaches for marginal and incremental allocation of initial margin figures.

Finally, section 3.2 uses the results of all previous sections as the basis to analyze the marginal and incremental allocation of SA-CCR. The prerequisites for marginal allocation are higher than those for incremental allocation. Therefore, the section investigates in detail under which circumstances marginal allocation of SA-CCR is possible and examines portfolios that represent edge cases.

Chapter 4 provides an outlook to adjacent topics and other use cases of SA-CCR allocation which might yield different challenges. The chapter also contains the conclusion of the thesis.

For the interested reader, Appendix A showcases an architecture blueprint for the implementation of SA-CCR and initial margin allocation as presented in this thesis.

Chapter 2

Applied models and methods

2.1 Instruments, pricing and market data

For the analysis presented in chapter 3 a small but diverse set of financial instruments is required. Due to the structure of the ISDA SIMM and the SA-CCR model the set of financial instruments should meet the following criteria:

1. The instruments should range across multiple asset classes
2. Non-linear instruments should be included
3. The instruments should range across multiple currencies
4. The instruments should be commonly traded as bilateral, uncleared derivatives to be relevant for ISDA SIMM
5. Pricing and sensitivity calculation should be possible without implementation of simulation approaches
6. Inferring market data objects required for pricing from market quotes of traded instruments must be simple

Items 4 and 5 of the above list are slightly conflicting. Bilaterally traded derivatives are usually more complex than cleared derivatives. Due to this increased complexity many of them have to be priced with a Monte Carlo simulation since an analytical solution is not possible.

Item 6 rises from the requirement of the ISDA SIMM model to calculate all sensitivities against market quotes. This means for example, that interest rate sensitivities mustn't be calculated with regard to a movement of the interest rate curve used as a pricing input but with regard to the price of the traded instrument that is used to build the interest rate curve in the first place. In the case of interest rate curves the process to build an interest rate curve is commonly referred to as *bootstrapping* and has to be performed again whenever a sensitivity is calculated to be compliant with

ISDA SIMM. Designing a pricing framework that can handle this required interdependence of market quotes, market data objects such as curves and priced instruments is a steep task even for deceptively simple instruments such as plain vanilla interest rate swaps. For this reason the implementation is based on QuantLib which offers an excellent and proven framework to monitor these interdepencies with ease. Calculation of ISDA SIMM compliant sensitivities of the instruments introduced in this section is the topic of section 2.3.2.2.

Careful consideration of the criteria listed above and the available market data lead to the following set of financial instruments that will be used for analysis:

- European equity options
- Overnight indexed swaps
- Interest rate swaps
- Swaptions

2.1.1 European Equity Option

2.1.1.1 Market data

2.1.1.2 Pricing

2.1.2 Overnight indexed swap

2.1.2.1 Market data

2.1.2.2 Pricing

2.1.3 Interest rate swap

2.1.3.1 Market data

2.1.3.2 Pricing

2.1.4 Swaption

2.1.4.1 Market data

2.1.4.2 Pricing

2.2 Margining

In the context of derivatives, margining refers to the process of posting and receiving collateral in the form of cash or securities to reduce the counterparty credit risk associated with the derivatives.

Counterparty credit risk refers to the risk of a default of the counterparty of a derivative. Derivatives are contracts between two institutions which give raise to future cash flows dependent of the performance of its underlying. These future cash flows may be at risk if the counterparty defaults during the lifetime of the derivative.

Over the past decades several measures were established in the derivatives markets to mitigate counterparty credit risk.

The most impactful measure is close-out netting. Close-out netting is a contractual agreement of two counterparties how their bilateral derivative contracts have to be settled if one of the counterparties defaults. With close-out netting, in case one of the two counterparties defaults all derivatives which are yet to mature are immediately closed out based on the current market value. The market values of the individual derivatives are summed up and the netted amount needs to be paid by whichever party is trailing across the portfolio. In case of a default this close-out netting procedure takes priority over all other claims of creditors against the defaulted counterparty. Close-out netting has two major advantages. First, the non-defaulting counterparty only suffers a counterparty credit loss, if it is ahead across the entire portfolio of currently running derivatives with the defaulted party. Just having a positive market value on a few derivatives does not necessarily result in a counterparty credit loss. Secondly, the immediate close-out of the open derivatives of the defaulted counterparty greatly facilitates unwinding its portfolio. A disadvantage of close-out netting is, that it may proof difficult to find an objective market value of the derivatives that have to be closed out - especially in a stressed market environment, which is likely to be present if e.g. a large investment bank defaults. The contractual obligation to perform close-out netting is agreed upon in a master agreement, which was introduced to the derivatives market by ISDA in 1985. Currently, close out netting is in effect for almost all traded derivatives and it will implicitly always be assumed to be in effect throughout this thesis. More details on close-out netting may be found in [4, Chapter 5].

The second most effective measure in mitigating counterparty credit risk is the exchange of variation margin. If the obligation to post variation margin is agreed as part of a master agreement the accrued mark-to-market of the derivative portfolio has to be collateralized by the trailing counterparty. This measure effectively resets counterparty credit risk to zero for both parties every time a variation margin payment is made or the exchanged variation margin is adjusted to the current market value of the portfolio. The exchange of variation margin was common but not a given in the inter-bank market before the financial crisis of 2008. After the crisis it has become commonplace in the interbank market and recently has even been mandated by regulators[1]. Non-financial counterparties oftentimes do not collateralize their derivatives since they are not mandated to do so, shy away from the operational burden and have a harder time funding the significant amount of cash necessary to cover the current mark-to-market value of their entire derivatives portfolio. Collateralizing a derivatives portfolio not only significantly reduces CCR but also significantly alters how the remaining CCR behaves. The CCR of a collateralized portfolio may rather be driven by the terms of the CSA or residual phenomenon such as collateral spikes than by the underlying instruments.

Explain what a CSA is

Quote something regarding collateral Spikes

As a final measure, initial margin can be exchanged. Historically, initial margin was a collateral amount that was calculated and exchanged once at the inception of a new derivative and held until maturity - hence the name *initial* margin. One common formulation used w.r.t. initial margin, which has also found its way into regulatory documents is that initial margin is collateral, that - in contrast to variation margin - is not based on the MtM of a portfolio or derivative. The idea behind initial margin is that it secures the counterparties against losses that can incur between the last time variation margin has been exchanged prior to a default until the original position has

cite location in regulatory document that does this

[1] In the European Union the exchange of Variation Margin for inter bank bilateral OTC derivatives is compulsory since September of 2016 for large banks or March of 2017 for smaller banks.

been restored. This time period is referred to as the margin period of risk and this time period results as the sum of

1. The contractually agreed regular frequency of variation margin exchanges
2. The time it takes from a counterparty not complying with a margin call to ascertain that the counterparty has indeed defaulted
3. The necessary time to reopen the defaulted derivatives with new counterparties to rehedge the bank and thereby restoring the risk profile held prior to the default of the counterparty

Initial margin should cover the gap between the MtM of the derivatives with the defaulted counterparty when variation margin was last exchanged to the price for which the derivatives are reopened in step 3 under the assumption that the market has moved unfavorably during the MPoR. In the context of CCR, *unfavorable* means that the banks position would have increased in value throughout the margin period of risk and therefore the market price for which the bank repurchases the derivatives is higher than the value at which variation margin was exchanged last.

A more comprehensive introduction to counterparty credit risk and its reduction through netting and margining may be found in chapters four through six of [4].

2.2.1 Market structure and associated margining approaches

The derivative market is divided into exchange traded derivatives, cleared OTC derivatives and uncleared bilateral derivatives. Uncleared bilateral derivatives can either be uncollateralized, collateralized with VM or collateralized with IM and VM. Below, these five counterparty relations are briefly introduced. They are ordered w.r.t. their associated counterparty credit risk.

Uncollateralized bilateral derivatives Derivatives are arranged between two counterparties without involvement of a third party. No collateral is exchanged at any point, only the contractual cashflows of the derivatives are exchanged. The CCR is very high as the entire MtM of the portfolio is at risk. Since no margin is posted at all, the MPoR is the maturity of the traded derivatives and can therefore easily eclipse multiple years. IM posted, IM received and VM are zero at all times.

Bilateral derivatives collateralized with VM The CCR is still significant. When the counterparty defaults the bank can suffer unmitigated losses for a couple of days until it can rebuild its position. The MtM of the portfolio is collateralized with VM. VM exchange is subject to contract parameters such as the threshold, minimum transfer amount or the exchange frequency. Values of these parameters impact how well CCR is mitigated.

Bilateral derivatives collateralized with VM and IM Counterparty credit risk is low. Only in edge cases is it possible that the counterparties credit losses surpass the available collateral. An MPoR of at least 10 days is mandated by the regulator. The IM is calculated with an internal or standardized bilateral initial margin model. Posted and received IM are recalculated daily. VM exchange obeys the same mechanics as for Bilateral derivatives collateralized with VM.

Cleared OTC derivatives Derivatives are initially arranged bilaterally between two counterparties and then cleared by a CCP. The CCP takes over positions in case of a default of either party mitigating any CCR in the traditional sense. The bank has no direct counterparty credit risk. It may however suffer losses to its clearing fund contribution if another Member of the CCP defaults. The MPoR is mandated by the regulator to be five days. The initial margin that is posted by the bank to the CCP is calculated by the CCP with his proprietary internal initial margin model. In line with the assumption that the CCP can not default, the CCP does not post IM to its clearing members. Therefore, the IM received from the perspective of the bank is always zero. The daily PnL of the portfolio is exchanged as VM between the CCP and the Bank.

Exchange traded derivatives Banks enter positions in exchange traded derivatives listed by a CCP. Positions are matched by the CCP and the counterparties of a transaction remain anonymous to each other. Associated CCR and margining is largely the same as for Cleared OTC derivatives but the MPoR is generally below five days since it is assumed that positions in exchange traded derivatives can be closed faster than in cleared OTC derivatives. The used internal initial margin model may differ e.g. since the regulator requires coverage of a 99% quantile instead of the 99.5% mandated for cleared OTC derivatives.

According to [4, Figure 3.2] based on notional 9% of derivatives are exchange traded, 55% are cleared OTC derivatives and 36% are uncleared OTC derivatives. It has to be noted that these figures are from 2014 and it can be assumed, that the fraction of cleared OTC derivatives has increased since then at the expense of the fraction of uncleared OTC derivatives. The reason for this is, that regulators have incentivized or even mandated the clearing of simpler OTC derivatives over the course of the last years. In connection with this development the large CCPs such as Eurex or the LCH have extended the product range for which they offer OTC clearing in recent years.

2.3 Bilateral initial margin

After the 2008 financial crisis the G20 agreed to reduce systematic, counterparty and operational risk and as a result of this commitment has been put into effect by regulators worldwide. In Europe the European Market Infrastructure Regulation (EMIR) came into force in August 2012 and focused on promoting or mandating central clearing as the primary measure to reduce counterparty risk.

2.3.1 The standard approach

Is it called standard approach or standardized approach?

Insert paragraph and paper of the IM standard approach

Section in CRD II that mandates the standard approach

Check if negative is correct

insert again BIS and CRD definition of standardized approach

The standard approach to calculate bilateral initial margin has been proposed by the bank for international settlement in and has been implemented in European law in . It is a schedule based approach that calculates an IM contribution on a trade by trade basis multiplying the trades notional with a regulatory factor based on the asset class and term to maturity of the trade. The resulting sum may be reduced by up to 60% through the so called net-gross ratio, if the portfolio has a negative present value from the perspective of the calculating bank . For a detailed specification of the aggregation the reader may refer to . The implementation of this approach is addressed in 2.3.2.1

2.3.2 The ISDA-SIMM model

In December of 2013 the International Swap and Derivatives Association (ISDA) published a motivation and basic methodological outline of a common internal initial margin model called Standard Initial Margin Model (SIMM™)[14]. The goal of the model is to meet the model requirements to an internal model of all regulators, while being among others easy to replicate, quick to calculate and relatively cheap to operate, implement and validate.

SIMM is a Delta-Gamma VaR model using Delta and Vega sensitivities calculated by the banks themselves and risk weights and correlations provided and recalibrated annually by ISDA. ISDA provides member with a methodological paper [13] and a paper describing the input format of sensitivities [12]. Additionally, the authors of ISDA SIMM have provided a technical paper [15] that lays out the mathematical foundation of the model. The core idea of the model is to multiply sensitivities with risk weights and aggregate them with nested variance-covariance computations.

2.3.2.1 Implementation

As already pointed out, ISDA-SIMM is standardized despite being an internal model. Therefore, all market participants using an internal model for bilateral initial margin calculation are forced to calculate ISDA-SIMM compliant sensitivities, convert them into the CRIF format and to aggregate them to an initial margin figure using the ISDA-SIMM aggregation. The process to create ISDA-SIMM compliant sensitivities is individual to each bank. Many vendor solutions for trading and risk have incorporated the creation of ISDA-SIMM compliant sensitivities and a CRIF into their products but the most suitable way to produce a CRIF still needs to be established on a bank to bank basis.

Aggregation on the other hand is absolutely standardized. It uses a single file, the CRIF, as input does not need any auxiliary market data and returns a single value, the

IM. Considering this, Acadiasoft[2] decided to provide an open source implementation of the ISDA-SIMM aggregation [1]. Acadiasoft is an ISDA-affiliated company who also offers a dispute resolution platform for bilateral initial margin that has become the market standard. For the analysis shown in chapter 3, this open source library was used for aggregation. Therefore, only the ISDA SIMM compliant sensitivities needed to be calculated and parsed into a CRIF entry minimizing potential sources of error and necessary testing effort. The open source library by Acadiasoft also offers functionality to calculate bilateral initial margin according to the standard approach presented in section 2.3.1 using an extended CRIF standard.

2.3.2.2 Calculation of compliant sensitivities

2.4 SA-CCR

Counterparty credit risk is considered to be a part of credit risk by the regulator. Risk weighted assets have to be calculated and need to be backed by own capital. The three main inputs for calculating credit risk are the probability of default (PD) the loss given default (LGD) and the exposure at default (EAD). Assuming the default of a counterparty over the course of the next year, the EAD is the current estimation of money indebted by the counterparty to the bank at the time of default. Estimating EAD for traditional credit instruments s.a. loans, credit cards, mortgages or bonds is relatively simple. Such instruments do often times have deterministic payment schedules making it easy to predict the exposure in one years time. Credit lines or credit cards behave less deterministic but it is still simple to determine an upper bound to the future exposure by assuming that the entire credit line is exhausted. The counterparty credit risk incurred by derivatives has first been regarded in regulatory capital calculation in Basel II [8]. Due to the stochastic nature of derivatives EAD calculation for counterparty credit risk has always been regulated separately ever since. Consideration of CCR in regulatory capital was overhauled as part of Basel III . The regulation for the internal margin model (IMM) approach was adjusted and the current exposure model (CEM) that was introduced with Basel II as the standard approach for CCR EAD calculation was replaced with the SA-CCR model and the simplified SA-CCR model. .

proper citation needed

Include timeline of SA-CCR regulatory documents

The SA-CCR model was implemented for the analysis section of this thesis but will not be presented in a comprehensive fashion here. Instead the reader may refer to the latest regulatory documents and or the library developed for this thesis [5]. This section will highlight the aspects of the SA-CCR model and the simplified SA-CCR model that are of special interest within the scope of this thesis such as the consideration of margin.

CRD2 document

latest EBA update

[2]https://acadiasoft.com/

2.4.1 Consideration of margining in SA-CCR

When using SA-CCR the exposure at default has to be calculated as:

$$
\begin{aligned}
& EAD = \alpha * (RC + PFE) \\
\text{where} \quad & \alpha = 1.4 \\
& RC : \text{Replacement Cost} \\
& PFE : \text{Potential Future Exposure}
\end{aligned}
\tag{2.4.1}
$$

The purpose of the RC is to assess the immediate loss suffered by the default of a counterparty. It is based on the current MtM of the derivative less the accessible collateral. If a bank has posted collateral to non-segregated accounts of a counterparty this collateral is also assumed to be lost in case of a default which increases the replacement cost. The potential future exposure (PFE) on the other hand assesses how the RC might develop in the future. The future being defined as during the next year. If the RC today is 0 but is likely to be larger than 0 in the near future the estimated EAD should take this expected increase in RC into account.

See also Paragraph 130 and 131 of [9]

Paragraph 130 - case without margining:

> For unmargined transactions, the *RC* intends to capture the loss that would occur if a counterparty were to default and were closed out of its transactions immediately. The *PFE* add-on represents a potential conservative increase in exposure over a one-year time horizon from the present date (i.e. the calculation date).

Paragraph 131 - case with margining:

> For margined trades, the *RC* intends to capture the loss that would occur if a counterparty were to default at the present or at a future time, assuming that the closeout and replacement of transactions occur instantaneously. However, there may be a period (the margin period of risk) between the last exchange of collateral before default and replacement of the trades in the market. The *PFE* add-on represents the potential change in value of the trades during this time period.

The PFE is defined as

$$
\begin{aligned}
& PFE = \text{multiplier} * AddOn^{\text{agg}} \\
\text{where} \quad & AddOn^{\text{agg}} : \text{aggregate add-on component} \\
& \text{multiplier} : f(V, C, AddOn^{\text{agg}})
\end{aligned}
\tag{2.4.2}
$$

AddOn is calculated differently for each asset a class. Since no netting is allowed between asset classes the aggregate is calculated as:

$$AddOn^{\text{agg}} = \sum_a AddOn^a$$

Collateralization is taken into account of the PFE calculation through the multiplier that uses the collateral held as an input. As overcollateralization increases, the multiplier decreases. The most important source of overcollateralization is initial margin. However, the multiplier is floored at 5%.

$$\begin{aligned} \text{multiplier} &= \min\left\{1; Floor + (1 - Floor)\exp\left(\frac{V - C}{2(1 - Floor)AddOn^{\text{agg}}}\right)\right\} \\ \text{where } Floor &= 5\% \end{aligned} \tag{2.4.3}$$

The RC is defined as

$$RC = \max\{V - C; TH + MTA - NICA; 0\}$$

$$\begin{aligned} \text{where} \qquad V &: \text{Current portfolio value} \\ C &: \text{Net collateral held} \\ TH &: \text{Threshold} \\ MTA &: \text{Minimum Transfer Amount} \\ NICA &: \text{Net Independent Collateral Amount} \end{aligned} \tag{2.4.4}$$

C is defined according to the $NICA$ definition, which in accordance with paragraph 143 of [9]. By making assumption 1

Could add reasoning to make this assumption

Assumption 1. *Variation margin is posted in unsegregated accounts, initial margin is posted in segregated accounts and initial margin is the only form of overcollateralization.*

the calculation of $NICA$ and C simplifies to:

$$\begin{aligned} C &= \text{Variation Margin balance} + NICA \\ NICA &= \text{Received initial margin} \end{aligned} \tag{2.4.5}$$

Assuming also

Assumption 2. $IM_{received} > TH + MTA \mid IM_{received} > 0$

	NICA	C_{calc}	RC
Uncollateralized bilateral derivatives	0	0	V
Bilateral derivatives collateralized with VM	0	VM	TH+MTA
Bilateral derivatives collateralized with VM and IM	IM_received	VM+IM_received	0
Cleared OTC derivatives	0	VM	Unclear
Exchange traded derivatives	0	VM	Unclear

Table 2.1: Calculation of NICA, C and RC under different margining approaches

we do yield the results for the five counterparty relation introduced in 2.2 that are displayed in table 2.1. Based on the calculated C in this table, the MTA and threshold need to be taken into account when calculating the collateral that is actually received. The collateral that is received at a point in time t can be calculated as

Clarify RC under Clearing, need to tidy up subscripts

$$C_t = \begin{cases} 0 & \text{if } |C_{calc}| < \text{TH} \\ C_{t-1} & \text{if } |C_{calc}| > \text{TH and } |C_{calc} - C_{t-1}| < \text{MTA} \\ C_{calc} & \text{oterhwise} \end{cases} \tag{2.4.6}$$

The case that is analyzed the most in this thesis is Bilateral derivatives collateralized with VM and IM. It is important to note that RC is always floored at zero in this case and a change in VM or IM then only impacts the SA-CCR EAD through the use of C in the multiplier calculation of equation 2.4.3. The multiplier is therefore the central point of focus when analyzing the interaction between SA-CCR and margin. The multiplier function is plotted in figure 2.1. The multiplier is ceiled at one if $C > V$, i.e. if the portfolio is overcollateralized which under assumption 1 is the case when the bank receives IM. With increasing overcollateralization the multiplier drops and approaches its floor of 5%. The other factor that drives the multiplier is the portfolios *AddOn*.

The *AddOn* is a portfolio metric that is supposed to represent how quickly the value of the portfolio can rise within the MPoR. The underlying idea is similar to a value at risk and to *AddOn* is designed to be easy to compute while still being portfolio based and taking optionalities into account. Margining does not impact the calculated *AddOn*. Therefore, *AddOn* calculation for SA-CCR is not presented in great detail at this point. The reader is referred to and the library that was implemented for the purpose of this thesis [5].

Cite AddOn Section of SA-CCR

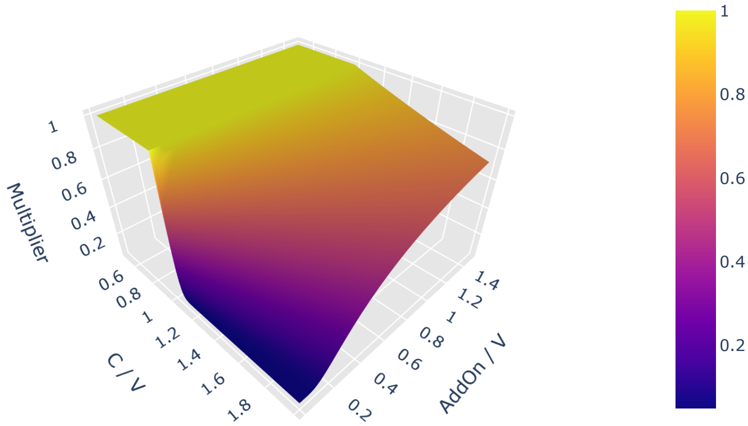

Figure 2.1:

2.5 Allocation of Risk Measures

With increasing sophistication of risk, own capital and margining models the need for equally sophisticated tools for attributing these measures rises as well. Allocating the variation margin or models that disregard portfolio effect entirely such as the current exposure method (CEM) to individual trades is trivial as these measures may just be calculated for an individual trade and then added up across all trades to obtain the correct aggregate value. For measures which take portfolio effects into account such as a VaR model, ISDA SIMM or SA-CCR however, this approach is not possible. The advent of portfolio based models for internal risk measurement in the late 1990s and for regulatory risk measurement in the late 2000s sparked research into how such measures should be reallocated. Gregory [4, Chapter 10.7] states that three approaches are used in practice:

1. Incremental allocation
2. Marginal allocation which will be called Euler allocation in this thesis
3. Pro rata allocation

Based on the paper of Koyluoglu and Stoker [6] the list of approaches can be complemented by:

4. Discrete marginal allocation
5. Shapley value

Unfortunately, naming conventions for the different allocation approaches are not consistent between the different publications. Therefore, a definition of the five approaches is following based on the notation used by Tasche [16]. In the following we will always assume that $X_1, \ldots, X_n$ are real valued random variables that are representing the profits and losses of the trades in a portfolio. $1, \ldots, n$ represents the order in which the trades have been added to the portfolio. X denotes the portfolio-wide PnL, s.t.

$$X = \sum_{i=1}^{n} X_i. \tag{2.5.1}$$

$\rho(X)$ is a risk measure that is supposed to estimate the profit or loss of the portfolio at a certain quantile for a certain time period. Both, the ISDA SIMM model and the SA-CCR model are in their core such risk measures.

The allocation or contribution of trade i to risk measure $\rho(X)$ is denoted as $\rho\left(X_i|X\right)$. Position sizes in the portfolio can be notated through a vector $\mathbf{u} = (u_1, \ldots, u_n)$:

$$X(u) = X(u_1, \ldots, u_n) = \sum_{i=1}^{n} u_i X_i \tag{2.5.2}$$

To make it more convenient to analyze changes to $\mathbf{u}$ we also introduce the function

$$f_{\rho,X}\left(\mathbf{u}\right) \tag{2.5.3}$$

Then, with $\mathbf{1}$ being a vector of ones, $\rho\left(X\left(\mathbf{1}\right)\right) = \rho\left(X\right)$. $\mathbf{u} = \mathbf{1}$ indicates the initial state of the portfolio when calculating an allocation - it does not imply that the notional of each position is 1.

Definition 2.5.4. *Assuming that $\rho\left(X\right)$ is a risk measure, the* ***incremental allocation*** *of trade n can be calculated as*

$$\begin{gathered} \text{with } u_{i \neq n} = 1 \text{ and } u_n = 0 \\ \rho_{inc}\left(X_n|X\right) = \rho\left(X\right) - \rho\left(X\left(u\right)\right) \end{gathered} \tag{2.5.5}$$

The incremental allocation can only be calculated for trade n.

Definition 2.5.6 (). *Assuming that $\rho\left(X\right)$ is a risk measure that is homogeneous of degree one and continuously differentiable, the* ***Euler allocation*** *of an arbitrary trade i can be calculated assume*

$$\rho_{Euler}\left(X_i|X\right) = \frac{d\rho}{dh}\left(X + hX_i\right)|_{h \to 0} = 1\frac{\partial f_\rho}{\partial u_i} \tag{2.5.7}$$

Definition 2.5.8 (). *Assuming that $\rho(X)$ is a risk measure, the* ***pro rata allocation*** *of an arbitrary trade i can be calculated as*

$$\textit{with } u_i = 1 \textit{ and } u_{\neq i} = 0 \tag{2.5.9}$$

$$\rho_{ProRata}(X_i|X) = \frac{\rho(X(u))}{\rho(X)} \tag{2.5.10}$$

Definition 2.5.11. *Assuming that $\rho(X)$ is a risk measure, the* ***discrete marginal allocation*** *of an arbitrary trade i can be calculated as*

$$\begin{gathered} \textit{with } u_i = 0 \textit{ and } u_{\neq i} = 1 \\ \rho_{discrete}(X_n|X) = \rho(X) - \rho(X(u)) \end{gathered} \tag{2.5.12}$$

To calculate the Shapley allocation of a portfolio one needs to iterate through all permutations how the trades in the portfolio could be ordered. For a given trade i the Shapley allocation is the average of the amount by which the risk measure changes when adding trade i to the portfolio in each of the permutations.

Definition 2.5.13.

Need to put Shapley in a formula...

The usefulness of any of the five allocation approaches listed above is dependent on the individual application of the allocation. Criteria by which the allocation approach is judged are also highly dependent of the application. However, the two criteria

1. Native additivity
2. Risk sensitivity

are usually regarded to be the most important one. They are for example the criteria by which Koyluoglu and Stoker [6] compare the different allocation approaches.

A allocation algorithm *alloc* exhibits native additivity if equation 2.5.14 holds.

$$\sum_{i=1}^{n} \rho_{alloc}(X|X_i) = \rho(X) \tag{2.5.14}$$

Risk sensitivity means that $\rho_{alloc}(X|X_i)$ should indicate how the trade i impacts the overall risk $\rho(X)$. No mathematical definition is available for when an allocation is considered to be risk sensitive. A sensible criteria could be that a trade that reduces the risk of the portfolio, i.e. a hedge trade should have a negative contribution to the risk measure.

Depending on the application of the allocation other criteria might be important such as

- Non-negativity of allocations
- The value allocated to a trade must not change through time

- The allocated value needs to be independent from the order in which trades are entered

Generally, such auxiliary requirements raise through operational or technical limitations. Some of the allocation algorithms presented above comply with them, while others do not. Such requirements might be the reason that allocation algorithms that are dismissed as inappropriate in this thesis still find application in the field.

The incremental allocation excels for use at the trading desk. It is well suited as an input when making investment decisions or for calculating the remuneration of traders and trading desks after entering a new trade. Both, Gregory [4] and Koyluoglu and Stoker [6] state that incremental allocation is the best suited allocation for these purposes. It does, however, perform poorly for risk analysis of an existing portfolio. Incremental allocation is further investigated in section 2.5.1.

Euler allocation on the other hand is well suited for analysis of an existing portfolio. It can identify concentration risk within an portfolio or be used for portfolio optimization. In the literature, Euler allocation is generally regarded as the best allocation approach for such purposes as it exhibits native additivity and risk sensitivity and can be calculated for all trades.

Pro rata allocation is generally not risk sensitive for risk measures that take portfolio effects into account. It is, however, very inexpensive to compute, suitable for models that are based on trade contributions such as the CEM or the standardized approach for initial margin (see 2.3.1) and may circumvent some operational or technical issues as trade contributions are always positive. Due to its simplicity and packing risk sensitivity for the models analyzed in this thesis, pro rata allocation will not be analyzed in further detail in this thesis.

While being a very intuitive approach, performance of discrete marginal allocation is relatively poor. The approach does not exhibit native additivity as Tasche [16] shows that

$$\rho_{discrete}\left(X_i|X\right) \leq \rho_{Euler}\left(X_i|X\right)$$

for ρ that are continuously differentiable, sub-additive and homogeneous of degree 1. Koyluoglu and Stoker [6] mention that "it could be argued that discrete marginal allocation is wholly dominated by the Euler allocation".

Check if this is an appropriate way to cite directly

Finally, the Shapley method introduced in [11], like the pro rata model, can not result in negative contributions but exhibits native additivity without the necessity of being normalized by division through $\rho(X)$. With no negative contributions being possible it can be argued, that the Shapley allocation is not risk sensitive. It is certainly considered by authors to be less risk sensitive that the Euler allocation. Since it exhibits natural additivity the Shapley allocation might be considered superior to the pro rata allocation. However, its computation is much more time consuming than any other allocation presented. Therefore it can only be realistically used for very small portfolios or to calculate allocations of subportfolios, e.g. the subportfolios of certain departments. Koyluoglu and Stoker [6] compare Euler and Shapley allocation and find that Shapley allocation is a more robust measure as it does not require differentiability of ρ. The relatively rigorous requirements against ρ to use Euler allocation are introduced in detail in section 2.5.2 and chapter 3 investigates under which circumstances the ISDA SIMM and SA-CCR model comply with these requirements. Overall, Koyluoglu and Stoker suggest to only use Shapley allocation over the Euler allocation for calculating the contribution of few subportfolios if the political cost or confusion caused by negative contributions is considered to be too high.

Non-negativity of Shapley allocation contradicts the results by FIS - investigate

2.5.1 Incremental allocation

Incremental allocation can only be applied when observing the development of a portfolio through time. Given a pre-existing portfolio P consisting of n trades t_1 through t_n and a portfolio-based measure M the incremental contribution of the first and second additional trade may be calculated as:

$$\begin{aligned} M_{\text{inc},t_{n+1}} &= M\left(t_1 \dots t_{n+1}\right) - M\left(t_1 \dots t_n\right) \\ M_{\text{inc},t_{n+2}} &= M\left(t_1 \dots t_{n+2}\right) - M\left(t_1 \dots t_{n+1}\right) \end{aligned}$$

It can be easily seen that this approach yields a natively additive allocation since it forms a telescoping sum[3] :

$$\begin{aligned} M_{\text{inc},t_1} &= M(t_1) \\ M_{\text{inc},t_i} &= M(t_i) - M(t_{i-1}) \\ M_{\text{inc},t_n} &= M(t_n) - M(t_{n-1}) \\ \sum_{i=1}^{n} M_{\text{inc},i} &= M(t_1) - M(t_1) + \cdots + M(t_{n-1}) - M(t_{n-1}) + M(t_n) = M(t_n) \end{aligned}$$

The incremental allocation can be calculated as or before a new trade is added to the portfolio. It is a risk sensitive value when it is calculated as it accurately reflect how the additional trade changes the risk measure. If the trade is mitigating risk at the time of its inception according to M its incremental allocation M_{inc} is negative. If it increases the risk its M_{inc} is positive. However, M_{inc} does not adapt over time and is likely to loose its accurate risk depiction as additional trades are added to the portfolio. As a portfolio develops it may well be possible, that a trade for which a negative M_{inc} was calculated at its inception may loose its risk mitigation. Due to this property M_{inc} of a given trade should ideally only be used at or before trade inception. One such use case is the PnL calculation of a new trade to determine the performance of the trading desk or trader which initiated the trade. Another would be to use it prior to an investment decision [17]. It can however not be used to analyze an existing portfolio to e.g. identify trades which drive risk or determine how increases or decreases in a given position would impact the portfolio measure. It also cant be calculated deterministically a posteriori for a portfolio without knowing its composition through time.

[3]For brevity in Notation let $M(t_i)$ be equivalent to $M(t_1 \dots t_i)$

2.5.2 Euler allocation

The idea of Euler allocation is based on Euler's homogeneous functions theorem.
Definition 2.5.15. *A function f is a positive homogeneous function to a degree of k if*

$$f(\alpha \mathbf{x}) = \alpha^k f(\mathbf{x}) \tag{2.5.16}$$
$$\textit{for } \alpha > 0 \tag{2.5.17}$$

A function would be homogeneous rather than just *positive* homogeneous if equation 2.5.16 would also hold for $\alpha < 0$. Risk measures can only exhibit positive homgenity. Many risk measures do have the property that doubling position size does double the measured risk. However, inverting the postion, e.g. having a short instead of a long position does not result in a negative risk estimate.

Euler's homogeneous functions theorem states
Theorem 2.5.18. *Let $f(\mathbf{x})$ be a homogeneous function of degree k, then*

$$x_i \frac{\partial f}{\partial x_i} = k f(\mathbf{x}) \tag{2.5.19}$$

With if we assume that $k = 1$, use our risk measure $\rho(\mathbf{u})$ as a function of invested position size with $\mathbf{u} = \mathbf{1}$ being the current position size we yield

$$1 \frac{\partial \rho(\mathbf{u})}{\partial u_i} = \rho(\mathbf{u}) \tag{2.5.20}$$

which is what is stated in definition 2.5.6.

While $u = \mathbf{1}$ is defined as the current position size we can also define it as the notional in USD invested in the individual trades, i.e. $\mathbf{n} = (notional_1, \ldots, notional_n)$.

The Euler allocation w.r.t trade i would then be calculated as

$$notional_i \frac{\partial \rho(\mathbf{n})}{\partial notional_i} = \rho(\mathbf{n}) \tag{2.5.21}$$

Need to investigate if investment of 1 is right or investment of notional. Shouldn't one be equivalent to a relative bump and the other be equivalent to a relative bump?

As any partial derivative, $\frac{\partial \rho((u))}{\partial u_i}$ may be approximated as a finite difference.

$$\text{with } \mathbf{h} = (h_0, \ldots, h_n) \text{ and } h_i = \epsilon > 0 \text{ and } h_{\neq i} = 0$$
$$\frac{\partial \rho(\mathbf{u})}{\partial u_i} = \frac{\rho(\mathbf{u}+\mathbf{h}) - \rho(\mathbf{u}-\mathbf{h})}{2\epsilon} + \mathcal{O}(\epsilon^2) \qquad \text{(central difference)} \tag{2.5.22}$$
$$\frac{\partial \rho(\mathbf{u})}{\partial u_i} = \frac{\rho(\mathbf{u}+\mathbf{h}) - \rho(\mathbf{u})}{\epsilon} + \mathcal{O}(\epsilon) \qquad \text{(forward difference)} \tag{2.5.23}$$
$$\frac{\partial \rho(\mathbf{u})}{\partial u_i} = \frac{\rho(\mathbf{u}) - \rho(\mathbf{u}-\mathbf{h})}{\epsilon} + \mathcal{O}(\epsilon) \qquad \text{(backward difference)} \tag{2.5.24}$$

Closed form formulas for contribution derived for standard deviation based models, VaR models, conditional VaR models.

Could show this for a single variance covariance submatrix of ISDA SIMM?!

2.5.3 Shapley allocation

Inclusion of this section optional

Chapter 3

Results

3.1 Allocation of initial margin

The goal of this section is to investigate if a numerical Euler allocation of ISDA SIMM is possible.

As pointed out in section 2.5.2 a risk measure needs to exhibit positive homogeneity of degree 1 to be able to perform an Euler allocation. In a first step we can investigate by calculating ISDA SIMM for a single trade whether ISDA SIMM does exhibit positive homogeneity for a minimal example.

For this we set up an USD Libor IRS with ten years time to maturity and a notional of 200 billion USD. This is our initial portfolio $\mathbf{u}$. ISDA SIMM would fulfill the required positive homogeneity condition if $a\rho(\mathbf{u}) = \rho(a\mathbf{u}$ for $a > 0$. In figure 3.1 $\rho(a\mathbf{u})$ is plotted for $0 < a \leq 2$ in blue. The function exhibits homogeneity for $0 < a < 1.4$ but not for higher a. The reason for this is, that at this point the concentration risk charge of ISDA SIMM does kick in. The concentration risk for interest rate risks for our minimal example is defined as [13, Article 7.b]

find the exact point where homogeneity breaks

$$CR = \max\left(1, \left(\frac{|\sum s|}{T}\right)^{1/2}\right)$$

with s being the sensitivities against USD interest rate risk and T being 230Mn USD as specified in [13, Article 74]. Due to subsequent variance-covariance aggregation the concentration risk impacts the calculated IM as

$$IM_{\text{with conc. risk}} = CR^2 \cdot IM_{\text{without conc. risk}}$$

This causes the change in slope and implied loss of homogeneity visible in figure 3.1. If the portfolio would consist of a more diverse set of risk factors than the minimal example displayed in figure 3.1 the associated concentration risk would kick in at different levels of a. The slope of the function would increase with each additional concentration risk not being floored at one any more.

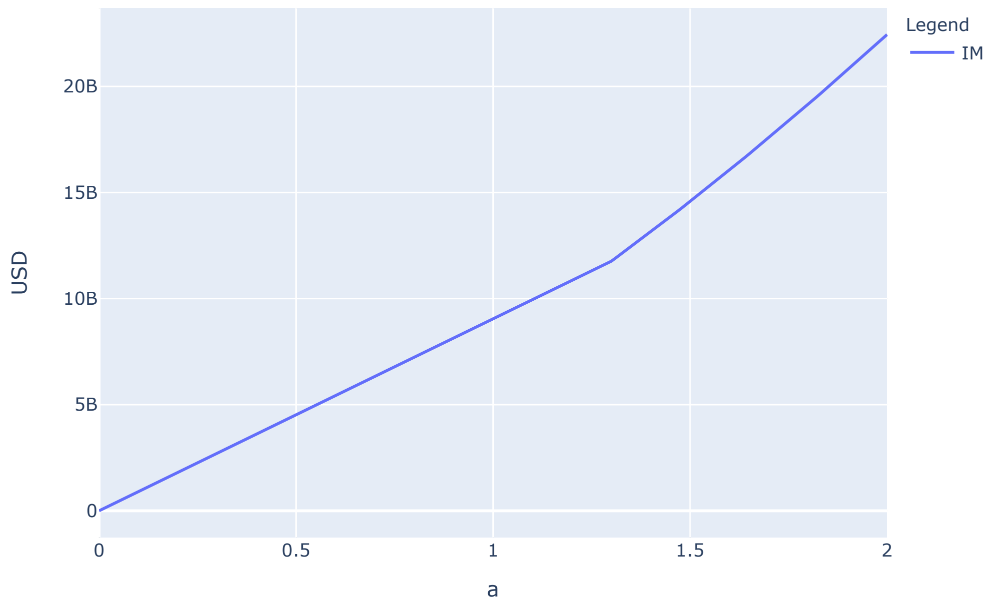

Figure 3.1:

It is important to note that as soon as the sensitivity against a single risk factor in the portfolio is above the concentration threshold the ISDA SIMM risk measure does not exhibit homogeneity anymore.

Even a trivial example with just one trade is sufficient to show that Euler allocation does not work in the inhomogeneous part of the ISDA SIMM equation. For this, we compare two sample portfolios one consisting of one USD IRS with 200 bn notional and one consisting of one USD IRS with 400 bn notional. Critically, the second portfolio is penalized by the model since its USD IRS risk is too large. We calculate the Euler calculation with a forward finite difference approach as displayed in equation 2.5.23.

Assuming that we calculate the finite difference with an $\epsilon = 0.0001$ this means that we calculate the ISDA SIMM of an IRS with 200Bn notional ($SIMM_{200Bn}$) and the ISDA SIMM of an equivalent IRS with 200.02 Bn notional ($SIMM_{200.02Bn}$) and this yields an Euler allocation to this trade as

$$\frac{SIMM_{200.02Bn} - SIMM_{200Bn}}{0.0001} = 9.04Bn$$

We can see in figure 3.1 that this value is both, the slope and the IM value at $a = 1$. The portfolios IM was correctly fully allocated to the single trade of which it consists.

However, performing the same calculation for an equivalent IRS with 400Bn notional

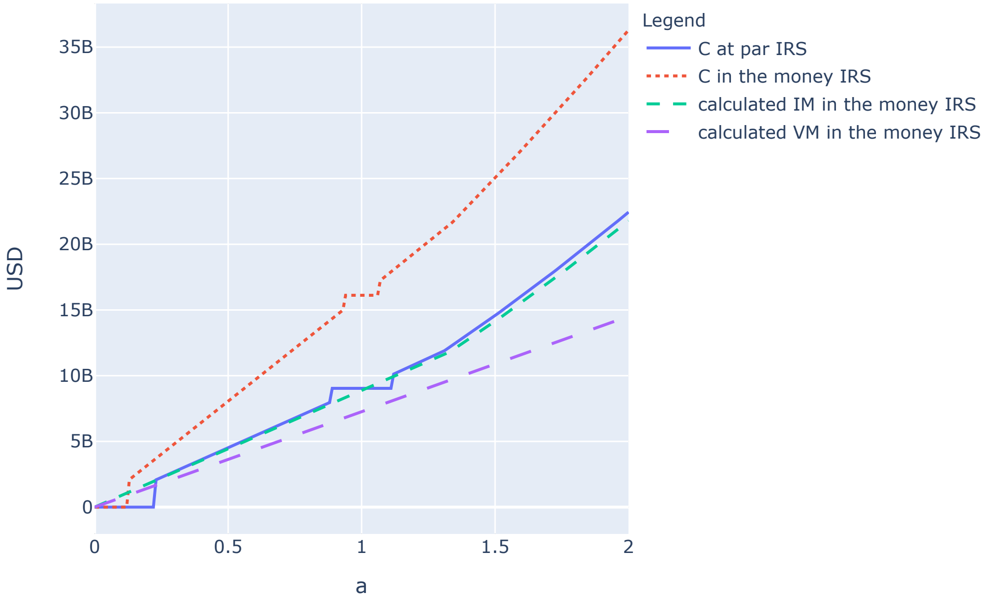

Figure 3.2:

yields

$$\frac{SIMM_{400.04Bn} - SIMM_{400Bn}}{0.0001} = 33.67Bn$$

again, we can refer to figure 3.1 to check if this is a reasonable allocation result. As $a = 1$ represents the IM charge for investing 200Bn of notional in the IRS, $a = 2$ represents an investment of 400Bn notional. The associated IM is just 22.44Bn - allocating 33.67Bn of the risk measure to the only trade in the portfolio is therefore clearly wrong. The Euler allocation of 33.67Bn can also be read off figure 3.1 - it is the slope at $a = 2$ times two.

3.2 Allocation of SA-CCR

3.2.1 Homogeneity of SA-CCR

3.2.1.1 Homogeneity of C

To allocate SA-CCR under consideration of margining, the available collateral C is of special interest. As pointed out in table 2.1 depending on the margining approach C can be calculated as $C = \text{VM}$ or $C = \text{VM} + \text{IM}_{\text{received}}$.

3.2.2 Allocation without margining

3.2.3 Allocation under VM collateralization

3.2.4 Allocation under VM and IM collateralization

Chapter 4

Outlook and conclusion

4.1 Outlook

4.2 Conclusion

Remove 2cm Margin from preamble

Appendix A

Architectural blueprint for SA-CCR allocation

Glossary

CCP	Central counterparty
CCR	Counterparty credit risk
CEM	Current exposure method
CRD2	Capital requirements directives two of the european union
CRIF	Common Risk Interchange Format - standardized file format for inputs for the ISDA SIMM model
CSA	Credit support annex
EAD	Exposure at default
IM	Initial margin
IMM	Internal model method
ISDA	International Swaps and Derivatives Association
ISDA SIMM	Internal initial margin model for uncleared derivatives developed by ISDA and used by most market participants
MPoR	Margin period of risk
MTA	Minimum transfer amount of a collateral agreement
MtM	Mark to market - current market value of a portfolio or a financial instrument
OTC	Over the counter derivatives as opposed to exchange trades derivatives
PnL	Profit and loss
SA-CCR	Standard approach for counterparty credit risk EAD calculation under CRD2
VaR	Value at risk
VM	Variation Margin

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