

SA-CCR Allocation under consideration of margining



Andreas Höcherl

Christ Church College

University of Oxford

A thesis submitted for the degree of

Master of Science

Trinity 2020

Abstract

This will be the abstract.

Contents

1	Introduction	1
1.1	Motivation	1
1.2	Literature review	2
1.3	Structure of the thesis	2
2	Applied models and methods	4
2.1	Instruments, pricing and market data	4
2.1.1	European Equity Option	6
2.1.2	Overnight indexed swap	6
2.1.3	Interest rate swap	6
2.1.4	Swaption	6
2.2	Margining	6
2.2.1	Market structure and associated margining approaches	8
2.3	ISDA SIMM	9
2.3.1	Regulatory requirements for an internal initial margin model	9
2.3.2	The ISDA-SIMM model	10
2.3.3	Calculation of ISDA SIMM compliant sensitivities	10
2.4	SA-CCR	10
2.4.1	Capital requirements for counterparty credit risk	10
2.4.2	Calculation of SA-CCR	11
2.4.3	Trade level adjusted notional	14
2.4.4	Supervisory delta adjustments	15
2.4.5	AddOn for interest rate derivatives	21
2.5	Allocation of Risk Measures	22
2.5.1	Incremental allocation	23
2.5.2	Marginal allocation	24
3	Results	25
3.1	Allocation of initial margin	25
3.2	Allocation of SA-CCR	25
3.2.1	Allocation without margining	25
3.2.2	Allocation under VM collateralization	25
3.2.3	Allocation under VM and IM collateralization	25
4	Outlook and conclusion	26

4.1 Outlook	26
4.2 Conclusion	26
Appendix A Architectural blueprint for SA-CCR allocation	27
Glossary	28
Bibliography	28

List of Figures

2.1	13
2.2	17
2.3	18

List of Tables

Chapter 1

Introduction

1.1 Motivation

With increasing sophistication of risk models, own capital models and margining models the need for equally sophisticated tools for allocating these measures rises as well. For any risk metric that considers portfolio effects calculating the contribution to the risk measure of individual trades is a challenge. As part of the Basel 3 reform, regulators have updated the standardized models for market risk own capital requirement and credit risk own capital requirements. The new standardized model for market risk own capital requirements is the FRTB-SA and the new standardized model for credit risk own capital requirements is the SA-CCR model . Both of these models are portfolio based risk models. Gregory [3, Chapter 10.7] states that three allocation approaches are used in practice:

- Incremental allocation for pre trade risk checks and for front office incentivization
- Marginal allocation for risk analytics of existing portfolios
- Pro rata allocation if trade contributions must not be negative, if risk sensitivity is not required or if the allocated risk measure does not take portfolio effects into account

These allocation approaches and their advantages and disadvantages are analyzed in further detail in section 2.5. Calculation of pro rata and incremental allocation is fairly straightforward and can generally be performed under any circumstances. Marginal allocation on the other hand is more challenging. This thesis will also include results for incremental and pro rata allocations for the sake of completeness but the main focus will be the analysis of marginal allocation.

Schulze [8] has analytically calculated the marginal allocation for the FRTB-SA. However, an approach to marginal allocation of SA-CCR has not been published yet. This thesis intends to close this gap by showcasing a numerical marginal allocation

Add appropriate citation to Basel 3 or CRD 2

Citation to FRTB-SA needed

Enter Citation of SA-CCR

approach for SA-CCR. A particular challenge in the allocation of SA-CCR is its consideration of margining. This makes the allocation of SA-CCR dependent on margin models which can themselves be significantly more complex than the SA-CCR model itself.

The goal of this thesis is to find an approach for allocating SA-CCR while considering margining. Particular focus is put on the allocation of SA-CCR under consideration of variation margin and an internal initial margin model.

1.2 Literature review

1.3 Structure of the thesis

Throughout this document a small but diverse selection of derivatives is used for exemplary calculations and to explore edge cases of the different models. Section 2.1 does briefly introduce these instruments and the financial models and market data that is used to price them.

Since margining is an integral part of the SA-CCR model, section 2.2 will afterwards introduce different margin types and typical counterparty relations and margin models observed in the market. Section 2.2 will also establish why out of the different margin models, the ISDA SIMM model needs to be investigated the closest for the purpose of this thesis.

Section 2.3 introduces the ISDA SIMM which is the most commonly used model for initial margin calculation of uncleared derivatives. As the ISDA SIMM is based on first order sensitivities, section 2.3 also describes how to calculate ISDA SIMM compliant sensitivities for the financial instruments introduced in section 2.1.

After the different margin components have been introduced, section 2.4 presents the SA-CCR model that is used for the calculation of the EAD of derivatives. Special emphasis will be put on the inclusion of margining.

Section 2.5 presents literature results regarding the allocation of risk measures. Additionally, the theoretical foundation for the Euler allocation is laid out as the investigation if and when an Euler allocation is possible is the main subject for the analysis section of the thesis.

At this point all relevant concepts, models and financial instruments have been introduced to conduct the analysis chapter of this thesis. The main goal is to establish a numerical allocation approach for SA-CCR.

As the received initial margin is a subcomponent of SA-CCR it might be necessary to also allocate initial margin. Due to the complexity of initial margin models allocation of this subcomponent is investigated separately. Section 3.1 establishes numerical approaches for marginal and incremental allocation of initial margin figures.

Finally, section 3.2 uses the results of all previous sections as the basis to analyze the marginal and incremental allocation of SA-CCR. The prerequisites for marginal allocation are higher than those for incremental allocation. Therefore, the section investigates in detail under which circumstances marginal allocation of SA-CCR is possible and examines portfolios that represent edge cases.

Chapter 4 provides an outlook to adjacent topics and other use cases of SA-CCR allocation which might yield different challenges. The chapter also contains the conclusion of the thesis.

For the interested reader, Appendix A showcases an architecture blueprint for the implementation of SA-CCR and initial margin allocation as presented in this thesis.

Chapter 2

Applied models and methods

2.1 Instruments, pricing and market data

For the analysis presented in chapter 3 a small but diverse set of financial instruments is required. Due to the structure of the ISDA SIMM and the SA-CCR model the set of financial instruments should meet the following criteria:

1. The instruments should range across multiple asset classes
2. Non-linear instruments should be included
3. The instruments should range across multiple currencies
4. The instruments should be commonly traded as bilateral, uncleared derivatives to be relevant for ISDA SIMM
5. Pricing and sensitivity calculation should be possible without implementation of simulation approaches
6. Inferring market data objects required for pricing from market quotes of traded instruments must be simple

Items 4 and 5 of the above list are slightly conflicting. Bilaterally traded derivatives are usually more complex than cleared derivatives. Due to this increased complexity many of them have to be priced with a Monte Carlo simulation since an analytical solution is not possible.

Item 6 rises from the requirement of the ISDA SIMM model to calculate all sensitivities against market quotes. This means for example, that interest rate sensitivities mustn't be calculated with regard to a movement of the interest rate curve used as a pricing input but with regard to the price of the traded instrument that is used to build the interest rate curve in the first place. In the case of interest rate curves the process to build an interest rate curve is commonly referred to as *bootstrapping* and has to be performed again whenever a sensitivity is calculated to be compliant with

ISDA SIMM. Designing a pricing framework that can handle this required interdependence of market quotes, market data objects such as curves and priced instruments is a steep task even for deceptively simple instruments such as plain vanilla interest rate swaps. For this reason the implementation is based on QuantLib which offers an excellent and proven framework to monitor these interdependencies with ease. Calculation of ISDA SIMM compliant sensitivities of the instruments introduced in this section is the topic of section 2.3.3.

Careful consideration of the criteria listed above and the available market data lead to the following set of financial instruments that will be used for analysis:

- European equity options
- Overnight indexed swaps
- Interest rate swaps
- Swaptions

2.1.1 European Equity Option

2.1.1.1 Market data

2.1.1.2 Pricing

2.1.2 Overnight indexed swap

2.1.2.1 Market data

2.1.2.2 Pricing

2.1.3 Interest rate swap

2.1.3.1 Market data

2.1.3.2 Pricing

2.1.4 Swaption

2.1.4.1 Market data

2.1.4.2 Pricing

2.2 Margining

In the context of derivatives, margining refers to the process of posting and receiving collateral in the form of cash or securities to reduce the counterparty credit risk associated with the derivatives.

Counterparty credit risk refers to the risk of a default of the counterparty of a derivative. Derivatives are contracts between two institutions which give rise to future cash flows dependent of the performance of its underlying. These future cash flows may be at risk if the counterparty defaults during the lifetime of the derivative.

Over the past decades several measures were established in the derivatives markets to mitigate counterparty credit risk.

The most impactful measure is close-out netting. Close-out netting is a contractual agreement of two counterparties how their bilateral derivative contracts have to be settled if one of the counterparties defaults. With close-out netting, in case one of the two counterparties defaults all derivatives which are yet to mature are immediately closed out based on the current market value. The market values of the individual derivatives are summed up and the netted amount needs to be paid by whichever party

is trailing across the portfolio. In case of a default this close-out netting procedure takes priority over all other claims of creditors against the defaulted counterparty. Close-out netting has two major advantages. First, the non-defaulting counterparty only suffers a counterparty credit loss, if it is ahead across the entire portfolio of currently running derivatives with the defaulted party. Just having a positive market value on a few derivatives does not necessarily result in a counterparty credit loss. Secondly, the immediate close-out of the open derivatives of the defaulted counterparty greatly facilitates unwinding its portfolio. A disadvantage of close-out netting is, that it may prove difficult to find an objective market value of the derivatives that have to be closed out - especially in a stressed market environment, which is likely to be present if e.g. a large investment bank defaults. The contractual obligation to perform close-out netting is agreed upon in a master agreement, which was introduced to the derivatives market by ISDA in 1985. Currently, close out netting is in effect for almost all traded derivatives and it will implicitly always be assumed to be in effect throughout this thesis. More details on close-out netting may be found in [3, Chapter 5].

The second most effective measure in mitigating counterparty credit risk is the exchange of variation margin. If the obligation to post variation margin is agreed as part of a master agreement the accrued mark-to-market of the derivative portfolio has to be collateralized by the trailing counterparty. This measure effectively resets counterparty credit risk to zero for both parties every time a variation margin payment is made or the exchanged variation margin is adjusted to the current market value of the portfolio. The exchange of variation margin was common but not a given in the inter-bank market before the financial crisis of 2008. After the crisis it has become commonplace in the interbank market and recently has even been mandated by regulators¹. Non-financial counterparties oftentimes do not collateralize their derivatives since they are not mandated to do so, shy away from the operational burden and have a harder time funding the significant amount of cash necessary to cover the current mark-to-market value of their entire derivatives portfolio. Collateralizing a derivatives portfolio not only significantly reduces CCR but also significantly alters how the remaining CCR behaves. The CCR of a collateralized portfolio may rather be driven by the terms of the CSA or residual phenomenon such as collateral spikes than by the underlying instruments.

As a final measure, initial margin can be exchanged. Historically, initial margin was a collateral amount that was calculated and exchanged once at the inception of a new derivative and held until maturity - hence the name *initial* margin. One common formulation used w.r.t. initial margin, which has also found its way into regulatory documents is that initial margin is collateral, that - in contrast to variation margin - is not based on the MtM of a portfolio or derivative. The idea behind initial margin is that it secures the counterparties against losses that can incur between the last time variation margin has been exchanged prior to a default until the original position has

¹In the European Union the exchange of Variation Margin for inter bank bilateral OTC derivatives is compulsory since September of 2016 for large banks or March of 2017 for smaller banks.

Explain what a CSA is

Quote something regarding collateral Spikes

cite location in regulatory document that does this

been restored. This time period is referred to as the margin period of risk and this time period results as the sum of

1. The contractually agreed regular frequency of variation margin exchanges
2. The time it takes from a counterparty not complying with a margin call to ascertain that the counterparty has indeed defaulted
3. The necessary time to reopen the defaulted derivatives with new counterparties to re hedge the bank and thereby restoring the risk profile held prior to the default of the counterparty

Initial margin should cover the gap between the MtM of the derivatives with the defaulted counterparty when variation margin was last exchanged to the price for which the derivatives are reopened in step 3 under the assumption that the market has moved unfavorably during the MPoR. In the context of CCR, *unfavorable* means that the banks position would have increased in value throughout the margin period of risk and therefore the market price for which the bank repurchases the derivatives is higher than the value at which variation margin was exchanged last.

A more comprehensive introduction to counterparty credit risk and its reduction through netting and margining may be found in chapters four through six of [3].

2.2.1 Market structure and associated margining approaches

The derivative market is divided into exchange traded derivatives, cleared OTC derivatives and uncleared bilateral derivatives. Uncleared bilateral derivatives can either be uncollateralized, collateralized with VM or collateralized with IM and VM. Below, these five counterparty relations are briefly introduced. They are ordered w.r.t. their associated counterparty credit risk.

Uncollateralized bilateral derivatives Derivatives are arranged between two counterparties without involvement of a third party. No collateral is exchanged at any point, only the contractual cashflows of the derivatives are exchanged. The CCR is very high as the entire MtM of the portfolio is at risk. Since no margin is posted at all, the MPoR is the maturity of the traded derivatives and can therefore easily eclipse multiple years. IM posted, IM received and VM are zero at all times.

Bilateral derivatives collateralized with VM The CCR is still significant. When the counterparty defaults the bank can suffer unmitigated losses for a couple of days until it can rebuild its position. The MtM of the portfolio is collateralized with VM. VM exchange is subject to contract parameters such as the threshold, minimum transfer amount or the exchange frequency. Values of these parameters impact how well CCR is mitigated.

Bilateral derivatives collateralized with VM and IM Counterparty credit risk is low. Only in edge cases is it possible that the counterparties credit losses

surpass the available collateral. An MPoR of at least 10 days is mandated by the regulator. The IM is calculated with an internal or standardized bilateral initial margin model. Posted and received IM are recalculated daily. VM exchange obeys the same mechanics as for Bilateral derivatives collateralized with VM.

Cleared OTC derivatives Derivatives are initially arranged bilaterally between two counterparties and then cleared by a CCP. The CCP takes over positions in case of a default of either party mitigating any CCR in the traditional sense. The bank has no direct counterparty credit risk. It may however suffer losses to its clearing fund contribution if another Member of the CCP defaults. The MPoR is mandated by the regulator to be five days. The initial margin that is posted by the bank to the CCP is calculated by the CCP with his proprietary internal initial margin model. In line with the assumption that the CCP can not default, the CCP does not post IM to its clearing members. Therefore, the IM received from the perspective of the bank is always zero. The daily PnL of the portfolio is exchanged as VM between the CCP and the Bank.

Exchange traded derivatives Banks enter positions in exchange traded derivatives listed by a CCP. Positions are matched by the CCP and the counterparties of a transaction remain anonymous to each other. Associated CCR and margining is largely the same as for Cleared OTC derivatives but the MPoR is generally below five days since it is assumed that positions in exchange traded derivatives can be closed faster than in cleared OTC derivatives. The used internal initial margin model may differ e.g. since the regulator requires coverage of a 99% quantile instead of the 99.5% mandated for cleared OTC derivatives.

According to [3, Figure 3.2] based on notional 9% of derivatives are exchange traded, 55% are cleared OTC derivatives and 36% are uncleared OTC derivatives. It has to be noted that these figures are from 2014 and it can be assumed, that the fraction of cleared OTC derivatives has increased since then at the expense of the fraction of uncleared OTC derivatives. The reason for this is, that regulators have incentivized or even mandated the clearing of simpler OTC derivatives over the course of the last years. In connection with this development the large CCPs such as Eurex or the LCH have extended the product range for which they offer OTC clearing in recent years.

2.3 ISDA SIMM

2.3.1 Regulatory requirements for an internal initial margin model

After the 2008 financial crisis the G20 agreed to reduce systematic, counterparty and operational risk and as a result of this commitment has been put into effect by regulators worldwide. In Europe the European Market Infrastructure Regulation

(EMIR) came into force in August 2012 and focused on promoting or mandating central clearing as the primary measure to reduce counterparty risk.

2.3.2 The ISDA-SIMM model

In December of 2013 the International Swap and Derivatives Association (ISDA) published a motivation and basic methodological outline of a common internal initial margin model called Standard Initial Margin Model (SIMMTM)[11]. The goal of the model is to meet the model requirements to an internal model of all regulators, while being among others easy to replicate, quick to calculate and relatively cheap to operate, implement and validate.

SIMM is a Delta-Gamma VaR model using Delta and Vega sensitivities calculated by the banks themselves and risk weights and correlations provided and recalibrated annually by ISDA. ISDA provides member with a methodological paper [10] and a paper describing the input format of sensitivities [9].

2.3.3 Calculation of ISDA SIMM compliant sensitivities

2.4 SA-CCR

2.4.1 Capital requirements for counterparty credit risk

Counterparty credit risk is considered to be a part of credit risk by the regulator. Risk weighted assets have to be calculated and need to be backed by own capital. The three main inputs for calculating credit risk are the probability of default (PD) the loss given default (LGD) and the exposure at default (EAD). Assuming the default of a counterparty over the course of the next year, the EAD is the current estimation of money indebted by the counterparty to the bank at the time of default. Estimating EAD for traditional credit instruments s.a. loans, credit cards, mortgages or bonds is relatively simple. Such instruments do often times have deterministic payment schedules making it easy to predict the exposure in one years time. Credit lines or credit cards behave less deterministic but it is still simple to determine an upper bound to the future exposure by assuming that the entire credit line is exhausted. The counterparty credit risk incurred by derivatives has first been regarded in regulatory capital calculation in Basel II [6]. Due to the stochastic nature of derivatives EAD calculation for counterparty credit risk has always been regulated separately ever since To calculate the counterparty credit risk associated with derivatives a different approach to calculating

2.4.2 Calculation of SA-CCR

For analysis we create an SA-CCR object that implements SA-CCR as specified in [7]

When using *SA-CCR* the exposure at default (EAD) has to be calculated as:

$$EAD = \alpha * (RC + PFE)$$

where $\alpha = 1.4$

RC : Replacement Cost

PFE : Potential Future Exposure

```
[In]: SA_CCR.calculate_sa_ccr_ead(rc = 10, pfe = 20)
```

```
[Out]: 42.0
```

Relation of RC and PFE The purpose of the RC is to assess the immediate loss suffered by the default of a counterparty. It is based on the current MtM of the derivative less the accessible collateral. If a bank has posted collateral to non-segregated accounts of a counterparty this collateral is also assumed to be lost in case of a default which increases the replacement cost.

The potential future exposure (PFE) on the other hand assesses how the RC might develop in the future. The future being defined as during the next year. If the RC today is 0 but is likely to be larger than 0 in the near future the estimated EAD should take this expected increase in RC into account.

See also Paragraph 130 and 131 of [7]

Paragraph 130 - case without margining:

For unmargined transactions, the RC intends to capture the loss that would occur if a counterparty were to default and were closed out of its transactions immediately. The PFE add-on represents a potential conservative increase in exposure over a one-year time horizon from the present date (i.e. the calculation date).

Paragraph 131 - case with margining:

For margined trades, the RC intends to capture the loss that would occur if a counterparty were to default at the present or at a future time, assuming that the closeout and replacement of transactions occur instantaneously. However, there may be a period (the margin period of risk)

between the last exchange of collateral before default and replacement of the trades in the market. The *PFE* add-on represents the potential change in value of the trades during this time period.

Definition of Potential Future Exposure (PFE)

$$PFE = \text{multiplier} * AddOn^{\text{aggregate}}$$

where $AddOn^{\text{aggregate}}$: aggregate add-on component
multiplier : $f(V, C, AddOn^{\text{aggregate}})$

AddOn is calculated differently for each asset *a* class. Since no netting is allowed between asset classes the aggregate is calculated as:

$$AddOn^{\text{aggregate}} = \sum_a AddOn^a$$

Collateralization is taken into account of the PFE calculation through the multiplier that uses the collateral held as an input. As overcollateralization e.g. through IM increases, the multiplier decreases. However, the multiplier is floored at 5%.

$$\text{multiplier} = \min \left\{ 1; Floor + (1 - Floor) \exp \left(\frac{V - C}{2(1 - Floor) AddOn^{\text{aggregate}}} \right) \right\}$$

where $Floor = 5\%$

It is important to note that the multiplier can only be below 1 if $C > V$, i.e. if the portfolio is overcollateralized. If the portfolio is overcollateralized, the *AddOn* comes into play. The idea behind the *AddOn* is related to the idea of value at risk. The higher the *AddOn* the faster the SA-CCR model expects the positions to lose in value. Therefore, the higher the value, the higher the multiplier.

In the example below the current NPV of the portfolio is 30 and the received collateral ($IM + VM$) is 37. The portfolio is overcollateralized as it should be when initial margin is used. On the other hand, as collateralization decreases e.g. $V=C$ since only VM is exchanged or $C=0$ in the case of an uncollateralized portfolio the multiplier increases.

2.4.2.1 AddOn calculation

Most of the SA-CCR logic is hidden inside the AddOn calculation. At first it is important to define the following four data parameters:

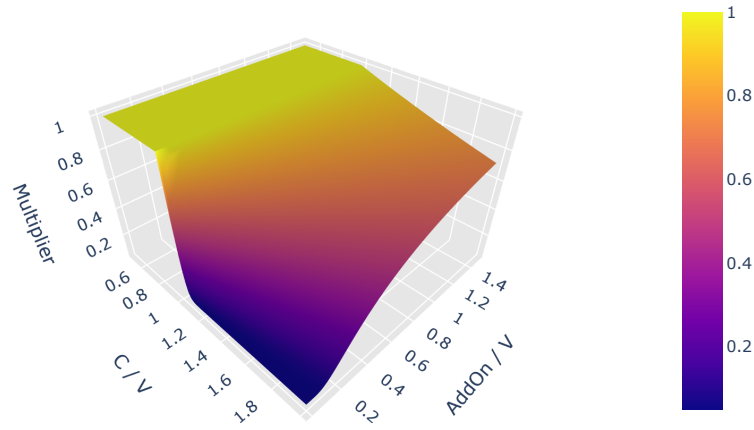


Figure 2.1:

M_i

Maturity of the derivative contract. If the underlying of a derivative is another derivative - e.g. in the case of a swaption the maturity date of the underlying needs to be chosen.

S_i

For interest rate and credit derivatives the start date of the time period referenced by an interest rate or credit contract. If the derivatives underlying is another interest rate or credit instrument (eg swaption or bond option) S_i is the start date of the underlying instead.

E_i

Defined as S_i but referencing the end date instead of the start date.

T_i

For options across all asset classes this is the latest contractual exercise date.

2.4.3 Trade level adjusted notional

Each trade i has a trade level adjusted notional d_i^a assigned to it. This is calculated differently for the different asset classes.

Interest rate and credit derivatives

The notional of the trade is usually a well defined value in domestic currency for interest rate and credit derivatives. It is multiplied by a supervisory duration factor. The basic idea is, that the value of the derivative can change more the longer the remaining

$$d_i = \text{Notional}_i * SD_i$$

$$\text{where } SD_i = \frac{\exp(-0.05 * S_i) - \exp(-0.05 * E_i)}{0.05}$$

FX derivatives

While the wording in the BCBS paper is a bit more specific we will just assume that every FX traded derivative has a USD leg and set the notional equal to the USD notional.

Equity and commodity derivatives

The notional is defined as the price of the underlying. Therefore, it fluctuates over time.

Notional of exotic derivatives

For more exotic derivatives which do have adjustable notionals, resetting notionals etc. detailed handling of the notional is defined in paragraph 158.

Within this thesis we investigate only equity and interest rate derivatives. For these we can make a few exemplary calculations of the trade level adjusted notional.

For equity trades determining the trade level adjusted notional is trivial as it always is the spot price of the underlying. As an example consider the two trades defined below:

```
[In]: #When the strike is not set explicitly an at the money option is_
      ↪ created with  $K = S(t_0)$ 
      eqOption1 = EquityOption(maturity = ql.Period(1, ql.Years),
```

```

tradeType= TradeType.CALL,
tradeDirection= TradeDirection.LONG,
underlying= Stock.ADS)

eqOption2 = EquityOption(maturity = ql.Period(1,    ql.Years),
tradeType= TradeType.PUT,
tradeDirection= TradeDirection.SHORT,
underlying= Stock.ADS,
strike = 60)

```

Let the spot price of Adidas stock be 42. Then, the adjusted notional of `eqOption1`, an at the money call on Adidas, is 42 and the adjusted notional of `eqOption2`, a short in the money put on Adidas, is also 42.

For interest rate derivatives such as interest rate swaps or swaptions on the other hand, the notional is adjusted by the supervisory duration factor. As the supervisory duration depends on S and E it is important to understand how these are determined for the different interest rate derivatives.

Trade Type	S	E
Interest Rate Swap	Current date	Maturity date
Forward starting IRS	Start date of the underlying swap	Maturity date of the underlying swap
Swaption	Start date of the underlying swap	Maturity data of the underlying swap

2.4.4 Supervisory delta adjustments

For linear derivatives δ is 1 for long derivatives and -1 for short derivatives.

For options δ is defined as under Black-Scholes:

$$\delta_{\text{long Call}} = +\Phi \left(\frac{\ln(P_i/K_i) + 0.5 * \sigma_i^2 * T_i}{\sigma_i * \sqrt{T_i}} \right)$$

where Φ : standard normal cdf

σ_i : supervisory volatility as defined in Table 2 in paragraph 183

This delta is multiplied by -1 in case of a long Put option or a short Call option. This formula is used for both, equity options and swaptions.

No detail is given at this point on the delta calculation of CDO tranches as these are not in the scope of this thesis.

In the case of an european equity option the parametrization is quite straight forward.

σ_i : 1.2 is the supervisory volatility for a single stock option

K_i : The strike of the option

P_i : The spot price of the underlying stock

T_i : The maturity of the option

A swaption on the other hand is parametrized as follows for calculation of its supervisory delta:

σ_i : 0.5 is the supervisory volatility for any interest rate option.

K_i : The strike of the option is the fixed rate of the underlying swap

P_i : Is the current par rate of the underlying (forward starting) swap

T_i : The maturity of the option. Please note the difference to E_i used for calculation of the adjusted notional, which is the maturity of the underlying swap.

SA-CCR uses the same Black-Scholes based formula for Swaps as it uses for Equities. It differentiates options in two dimensions. Whether they are *bought* or *sold* and whether they are *Call* or *Put* options (Compare paragraph 159).

SA-CCR defines an option as a call option if it rises in value as the underlying rises in value. A fixed payer swap rises in value as the underlying interest rate rises in value. Therefore, an option to buy a fixed payer swap at a predetermined strike also rises in value as the underlying interest rate rises in value. Therefore, a swaption on a payer swap is considered a *Call* under SA-CCR, while a swaption on a receiver swap is considered a *Put*.

For the at the money option `eqOption1` that was set up above we yield a supervisory delta adjustment of 0.7257.

For an exemplary short european swaption that has a par swap as underlying (i.e. the NPV of the swap is 0) that is set up as follows:

```
[In]: swap = IRS(notional=100,
                timeToSwapStart=ql.Period(1, ql.Years),
                timeToSwapEnd=ql.Period(3, ql.Years),
                swapDirection=SwapDirection.PAYER,
                index=InterestRateIndex.EURIBOR6M
                )

swaption = Swaption(underlyingSwap=swap,
                    optionMaturity=ql.Period(1, ql.Years),
```

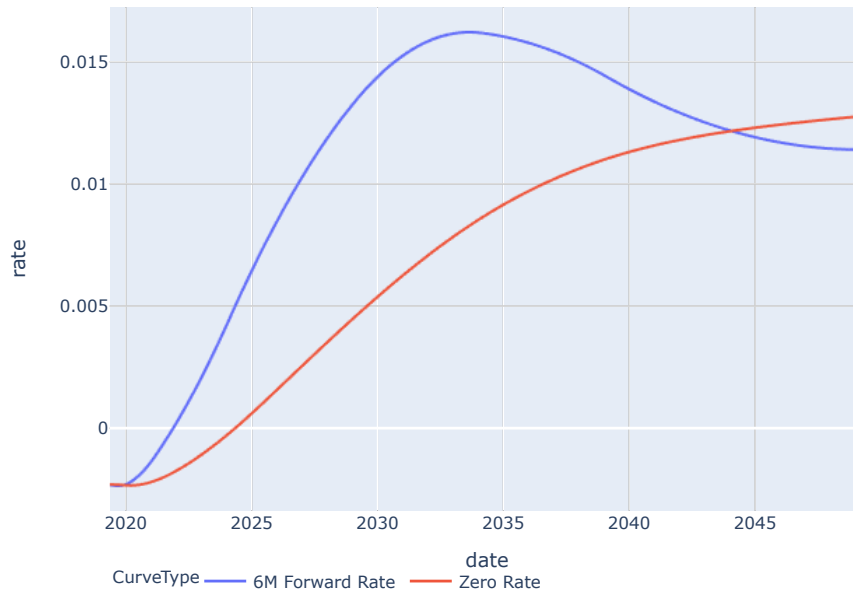


Figure 2.2:

```
tradeDirection=TradeDirection.SHORT)

SA_CCR.calculate_sa_ccr_delta(swaption)
```

[Out]: -0.5987063256829928

we yield a regulatory delta of -0.5987.

However, the approach of SA-CCR to use the Black-Scholes for interest rate derivatives does have a significant flaw.

Obviously, $\ln(P_i/K_i)$ is not defined, if P_i/K_i is negative. This has, however been very commonplace in recent years, especially for Euro swaptions. One can always set the fixed rate of the underlying swap and therefore K_i as a negative value. But based on the market data of the 10th of May 2019 it is even possible to yield negative par rates for the underlying swap.

As an example we are plotting the zero and the 6M forward curve for the EURIBOR 6M index below. When zooming further in, we can see that that the 6M forward curve crosses 0 on the 15th of November 2021.

To highlight the relationship between the 6M forward curve and the par rate P_i of forward starting swap we can set up the following two swaps depicted below. Since the $EndDate - StartDate$ for these two swaps is six months their par rate or fair price can directly be read from the 6M Forward curve.

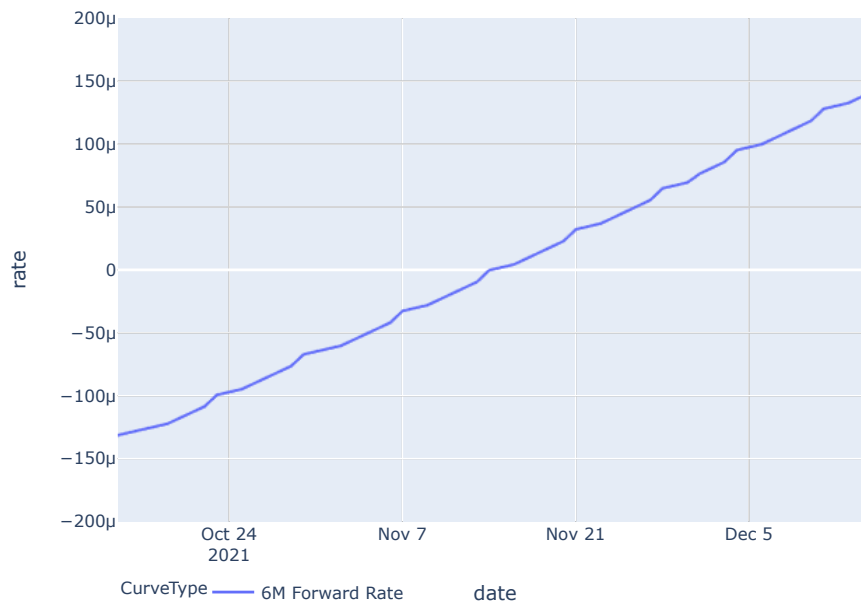


Figure 2.3:

```
[In]: print('StartDate: ' + str(today+ql.Period(30,ql.Months)))
swap1 = IRS(notional=100,
            timeToSwapStart=ql.Period(30,ql.Months),
            timeToSwapEnd=ql.Period(36,ql.Months),
            swapDirection=SwapDirection.PAYER,
            index=InterestRateIndex.EURIBOR6M,
            fixed_rate=0.01
        )
print('ParRate: %.6f%%' % (swap1.get_par_rate()*100))
```

StartDate: November 10th, 2021

ParRate: -0.002346%

```
[In]: print('StartDate: ' + str(today+ql.Period(31,ql.Months)))
swap2 = IRS(notional=100,
            timeToSwapStart=ql.Period(31,ql.Months),
            timeToSwapEnd=ql.Period(37, ql.Months),
            swapDirection=SwapDirection.PAYER,
            index=InterestRateIndex.EURIBOR6M,
            fixed_rate=0.01
        )
print('ParRate: %.6f%%' % (swap2.get_par_rate()*100))
```


StartDate: December 10th, 2021
ParRate: 0.011976%

```
[In]: swap3 = IRS(notional=100,
                  timeToSwapStart=ql.Period(30, ql.Months),
                  timeToSwapEnd=ql.Period(36, ql.Months),
                  swapDirection=SwapDirection.PAYER,
                  index=InterestRateIndex.EURIBOR6M,
                  fixed_rate=-0.01)
```

```
[In]: #calculatin SA_CCR delta for swaptions that convert into these
      ↪ swaps
```

```
swaption1 = Swaption(underlyingSwap=swap1,
                     optionMaturity=ql.Period(30, ql.Months),
                     tradeDirection=TradeDirection.LONG)

SA_CCR.calculate_sa_ccr_delta(swaption1)
```

[Out]: 0.004138768807253323

```
[In]: swaption2 = Swaption(underlyingSwap=swap2,
                           optionMaturity=ql.Period(31, ql.Months),
                           tradeDirection=TradeDirection.LONG)

SA_CCR.calculate_sa_ccr_delta(swaption2)
```

[Out]: 0.005109036476851662

The following is a very deep in the money swaption with negative strike and price. The delta should be close to 1. It is however close to zero without offsetting.

```
[In]: swaption3 = Swaption(underlyingSwap=swap3,
                           optionMaturity=ql.Period(30, ql.Months),
                           tradeDirection=TradeDirection.LONG)

SA_CCR.calculate_sa_ccr_delta(swaption3)
```

[Out]: 0.9996934007461944

```
[In]: print(swaption1.get_delta())
      print(swaption2.get_delta())
```

4.875082678775719
5.248490507272288

2.4.4.1 Risk Horizon

For unmargined transaction the margining factor is

$$MF_i^{\text{unmargined}} = \sqrt{\frac{\min(M_i; 1 \text{ year})}{1 \text{ year}}}$$

This factor can be used to scale down a risk weight calibrated for a 1 year horizon to a shorter period.

With margining the margin period of risk (MPOR) is:

- 10 business days for small, uncleared OTC portfolios
- 5 business days for cleared derivatives
- 20 business days for netting sets with more than 5000 transactions that are not with a central counterparty
- and doubling this period for portfolios with outstanding disputes

The margining factor is then

$$MF_i^{\text{margined}} = \frac{3}{2} \sqrt{\frac{MPOR_i}{1 \text{ year}}}$$

At this point we need to introduce a collateral agreement object. For simplicities sake we will not differentiate between collateral and netting sets in this thesis. All trades that are covered by the same collateral agreement are also admissible for netting with each other. (Also refer to the introduction of close out netting above). To take into account the different parameters determining the risk horizon a couple of parameters are required to create a collateral agreement. As an example, below we are setting up a collateral agreement for uncleared derivatives without exchange of variation margin or initial margin.

```
[In]: ca = CollateralAgreement(  
    margining=Margining.UNMARGINED,  
    clearing=Clearing.UNCLEARED,  
    initialMargining=InitialMargining.NO_IM,  
    tradecount=Tradecount.UNDER_FIVE_THOUSAND,  
    dispute=Dispute.NO_OUTSTANDING_DISPUTES,  
    threshold=0.0,           #Threshold to trigger a margin call  
    mta=0.0,                 #received initial margin  
)
```

With this collateral set object we can define a function for calculation the margining factor:

For trades of differing maturity let's compare the margining factor for the three most common scenarios:

1. No margining
2. Bilateral margining
3. Centrally cleared

[Out]:

	Three days	Two weeks	Six months	One year	Ten years
No margining	0.2000	0.2000	0.7071	1.0000	1.0000
Bilateral margining	0.3000	0.3000	0.3000	0.3000	0.3000
Centrally cleared	0.2121	0.2121	0.2121	0.2121	0.2121

2.4.5 AddOn for interest rate derivatives

Step 1 - calculation of effective notional D_{jk}^{IR}

$$D_{jk}^{IR} = \sum_{i \in \{Ccy_j, MB_k\}} \delta_i * d_i^{IR} * MF_i$$

Here, the notation $i \in \{Ccy_j, MB_k\}$ refers to trades whose underlying is the interest rate of a common currency j and which mature in a common maturity bucket k

We can test our implementation of the effective notional against a small exemplary portfolio in Annex 4a Example 1 of the SA_CCR paper.

It consists of the following trades:

Trade #	Nature	ResidualBase MaturityCcy	Notional (tsd)	Pay Leg	Receive Leg	Market value (tsd)
1	Interest rate swap	10 years USD	10000	Fixed	Floating	30
2	Interest rate swap	4 years USD	10000	Floating	Fixed	-20
3	European Swaption	1 into 10 years EUR	5000	Floating	Fixed	50

To set up this exemplary portfolio we need to find fixed rates for the swaps and underlying swaps to match the desired market values.

Through optimization and using the market data of the 10th of May 2019 the fixed rates to match the market values in Example 1 were identified.

For trade 1 the matching fixed rate is 2.3754%, for trade 1 it is 2.2108% and for the underlying swap of trade 3 it is 0.1610%

```
[In]: ca=CollateralAgreement()

print(SA_CCR.calculate_sa_ccr_delta(trade_1))
print(SA_CCR.trade_level_adjusted_notional(trade_1))
print(SA_CCR.calculate_sa_ccr_delta(trade_2))
print(SA_CCR.trade_level_adjusted_notional(trade_2))
print(SA_CCR.calculate_sa_ccr_delta(trade_3))
print(SA_CCR.trade_level_adjusted_notional(trade_3))

SA_CCR.interest_rate_addOn(test_portfolio,ca)
```

```
1
78638320.21725275
-1
36198301.54418307
-0.00343382383244269
31712286.360503413
```

```
[Out]: 89019.81894092409
```

```
[In]: export("SA_CCR_ird_addon.ipynb")
```

2.5 Allocation of Risk Measures

With increasing sophistication of risk, own capital and margining models the need for equally sophisticated tools for attributing these measures rises as well. Allocating the variation margin or the current exposure method (CEM) to individual trades is trivial as these measures may just be calculated for an individual trade and then added up across all trades to obtain the correct aggregate value. For measures which take portfolio effects into account such as a state of the art VaR model, ISDA-SIMM or SA-CCR however, this approach is not possible. The advent of portfolio based models for internal risk measurement in the late 1990s and for regulatory risk measurement in the late 2000s sparked research into how such measures should be reallocated. Gregory [3, Chapter 10.7] states that three approaches are used in practice:

- Incremental allocation
- Marginal allocation

- Pro rata allocation

Additionally [4] Allocation properties

- Native additivity
- Risk sensitivity
- Independent of portfolio constellation
- Stable through time

2.5.1 Incremental allocation

Incremental allocation can only be applied when observing the development of a portfolio through time. Given a pre-existing portfolio P consisting of n trades t_1 through t_n and a portfolio-based measure M the incremental contribution of the first and second additional trade may be calculated as:

$$\begin{aligned} M_{\text{inc}, t_{n+1}} &= M(t_1 \dots t_{n+1}) - M(t_1 \dots t_n) \\ M_{\text{inc}, t_{n+2}} &= M(t_1 \dots t_{n+2}) - M(t_1 \dots t_{n+1}) \end{aligned}$$

It can be easily seen that this approach yields a natively additive allocation since it forms a telescoping sum² :

$$\begin{aligned} M_{\text{inc}, t_1} &= M(t_1) \\ M_{\text{inc}, t_i} &= M(t_i) - M(t_{i-1}) \\ M_{\text{inc}, t_n} &= M(t_n) - M(t_{n-1}) \\ \sum_{i=1}^n M_{\text{inc}, i} &= M(t_1) - M(t_1) + \dots + M(t_{n-1}) - M(t_{n-1}) + M(t_n) = M(t_n) \end{aligned}$$

The incremental allocation can be calculated as or before a new trade is added to the portfolio. It is a risk sensitive value when it is calculated as it accurately reflect how the additional trade changes the risk measure. If the trade is mitigating risk at the time of its inception according to M its incremental allocation M_{inc} is negative. If it increases the risk its M_{inc} is positive. However, M_{inc} does not adapt over time and is likely to loose its accurate risk depiction as additional trade are added to the portfolio. As a portfolio develops it may well be possible, that a trade for which a negative M_{inc} was calculated at its inception may loose its risk mitigation. Due to this property M_{inc} of a given trade should ideally only be used at or before trade inception. One such use case is the PnL calculation of a new trade to determine the performance of the trading desk or trader which initiated the trade. Another would be to use it prior to an investment decision [12]. It can however not be used to analyze an existing portfolio to e.g. identify trades which drive risk or determine how increases or decreases in a given position would impact the portfolio measure. It

²For brevity in Notation let $M(t_i)$ be equivalent to $M(t_1 \dots t_i)$

also cant be calculated deterministically a posteriori for a portfolio without knowing its composition through time.

In the past, some academic work focused on approximating the incremental VaR as it requires a recalculation of the VaR for the entire portfolio. An overview of these works and their potential pitfalls may be found in [12]. Since this work will rather focus on marginal than incremental allocation further details may be found in the referred paper. In the empirical analysis the incremental allocation will be calculated exactly.

2.5.2 Marginal allocation

Chapter 3

Results

3.1 Allocation of initial margin

3.2 Allocation of SA-CCR

3.2.1 Allocation without margining

3.2.2 Allocation under VM collateralization

3.2.3 Allocation under VM and IM collateralization

Chapter 4

Outlook and conclusion

4.1 Outlook

4.2 Conclusion

Remove 2cm
Margin from
preamble

Appendix A

Architectural blueprint for SA-CCR allocation

Glossary

CCP	Central counterparty
CCR	Counterparty credit risk
CRD2	Capital requirements directives two of the european union
CSA	Credit support annex
EAD	Exposure at default
IM	Initial margin
ISDA	International Swaps and Derivatives Association
ISDA SIMM	Internal initial margin model for uncleared derivatives developed by ISDA and used by most market participants
MPoR	Margin period of risk
MtM	Mark to market - current market value of a portfolio or a financial instrument
OTC	Over the counter derivatives as opposed to exchange trades derivatives
PnL	Profit and loss
SA-CCR	Standard approach for counterparty credit risk EAD calculation under CRD2
VM	Variation Margin

Bibliography

- [1] Eurex. Eurex Default Management Process. <http://www.eurexclearing.com/clearing-en/risk-management/default-management-process>. Accessed: 2019-01-24.
- [2] Eurex. Eurex Monthly Statistics December 2018. http://www.eurexexchange.com/blob/3571518/1aa0157c4b96709caa97b254a4075aaf/data/monthlystat_201812.pdf. Accessed: 2019-01-22.
- [3] Jon Gregory. *The XVA Challenge: Counterparty Credit Risk, Funding, Collateral and Capital*. John Wiley & Sons, 2015.
- [4] Ugur Koyluoglu and James Stoker. Risk decomposition honour your contribution. *RISK-LONDON-RISK MAGAZINE LIMITED-*, 15(4):90–94, 2002.
- [5] Bank of International Settlements (BIS). OTC derivatives outstanding. <https://www.bis.org/statistics/derstats.htm>. Accessed: 2019-01-24.
- [6] Basel Committee on Banking Supervision (BCBS). International convergence of capital measurement and capital standards, a revised framework - comprehensive version. <https://www.bis.org/publ/bcbs128.htm>, 2009. Accessed: 2019-04-29.
- [7] Basel Committee on Banking Supervision (BCBS). The standardised approach for measuring counterparty credit risk exposures. <https://www.bis.org/publ/bcbs279.htm>, 2014. Accessed: 2020-04-25.
- [8] Robert Schulze. Capital allocation under the frtb regime via marginal measures. *Available at SSRN 3265320*, 2018.
- [9] International Swaps and Derivatives Association (ISDA). ISDA SIMM™ Methodology: Risk Data Standards, version 1.36. <https://www.isda.org/a/owEDE/risk-data-standards-v1-36-public.pdf>. Accessed: 2019-04-29.
- [10] International Swaps and Derivatives Association (ISDA). ISDA SIMM™ Methodology, version 2.1. <https://www.isda.org/a/zSpEE/ISDA-SIMM-v2.1-PUBLIC.pdf>. Accessed: 2019-04-29.
- [11] International Swaps and Derivatives Association (ISDA). ISDA Standard Initial Margin Model (SIMM™) for Non-Cleared Derivatives. <https://www.isda.org>.

[org/a/gwEDE/march-26-simm-for-non-cleared-paper.pdf](#). Accessed: 2019-04-29.

- [12] Luisa Tibiletti. Incremental value-at-risk: traps and misinter-pretations. In *Mathematical Finance*, pages 355–364. Springer, 2001.