

MODULE *Euclid*

Specification for the algorithm to find the greatest common divisor between two integer numbers greater than zero, aka *Euclid* algorithm.

EXTENDS *Integers*CONSTANTS M, N

VARIABLES x, y

$$PositiveInteger(n) \triangleq n \in Nat \wedge n \neq 0$$
$$\begin{aligned} \textit{TypeInvariant} &\triangleq \textit{PositiveInteger}(x) \\ &\quad \wedge \textit{PositiveInteger}(y) \end{aligned}$$

Initial state

$$Init \stackrel{\Delta}{=} (x = M) \wedge (y = N)$$

Next state of computation

$$\text{Next} \stackrel{\Delta}{=} ((x < y) \wedge (x' = x) \wedge (y' = y - x)) \\ \vee ((y < x) \wedge (y' = y) \wedge (x' = x - y))$$
$$Spec \triangleq Init \wedge \Box[Next]_{\langle x, y \rangle}$$

THEOREM $Spec \Rightarrow TypeInvariant$