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CONSTRUCTION AND KINEMATIC PROPERTIES OF 3-UPU PARALLEL MECHANISMS

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ABSTRACT

The assembly condition of universal pairs in the 3-UPU parallel mechanism determines the mechanism mobility. This paper uses reciprocal screw theory to study mobility analysis of the 3-UPU parallel mechanism under several different assembly conditions. The analysis and the method presented in this paper will be helpful in using the 3-UPU parallel mechanism and introduce new insights into the mobility analysis of parallel mechanisms.

INTRODUCTION

With the development of parallel robot technology, simpler and less expensive spatial parallel robots which have fewer than 6 DOF have interested many researchers. These parallel mechanisms are called as lower-mobility parallel mechanisms.

An important one widely studied is the 3-RPS parallel robot(Hunt 1983, Waldron et al. 1989, Lee and Arjuman 1991, Agrawal 1991, Carretero et al. 1998), which has two rotational degrees of freedom and one translational degree of freedom, where S denotes the spherical joint, R denotes the revolute pair, and P denotes the prismatic pair. Cox and Tesar (1989) studied a 3-DOF spherical mechanism. Gosselin and Angeles (1988, 1989) proposed an optimum kinematic design of a planar and a spherical 3-DOF parallel manipulator. Lee and Shah (1988) studied the design of a spatial 3-DOF parallel manipulator. Clavel (1988) proposed the successful 3-DOF translational parallel mechanism, DELTA robot. Tsai (1997) proposed some 3-DOF translational parallel mechanism in his patent, one of which is the 3-RRC parallel mechanism, where C denotes a cylindrical pair. And Tsai (1996, 1999) also proposed the 3-DOF 3-UPU translational parallel mechanism, where U denotes the universal joint. Huang (1995) presented a 3-DOF cubic type parallel mechanism in an IEEE conference.

The 3-UPU parallel mechanism consists of three UPU kinematic chains. Each UPU chain consists of two universal joints and one prismatic pair in the middle. Different assembly conditions of the U-pairs produce different mobility of the 3-UPU parallel mechanism, which has drawn a lot of interest in robotics community and some results have been well established (Di Gregorio and Parenti-Castelli 1999, Tsai 1999, Frisoli et al. 2000, Carricato and Parenti-Castelli 2001, Bonev and Zlatanov 2001, Zlatanov et al. 2001, Karouia and Hervé 2000).

This paper uses reciprocal screw theory to study the relations between the mobility and the assembly conditions of the U-pairs of the 3-UPU parallel mechanism. Besides identifying the well-established results, we propose one novel assembly condition of the 3-UPU parallel mechanism.

1. BASIC CONCEPTS

In what follows, screw theory is used to aid the mobility analysis of these novel mechanisms. In screw theory (Ball 1900, Hunt 1978), the general form of a unit screw is given by $\boldsymbol{\$}=(\boldsymbol{s};\boldsymbol{r}\times\boldsymbol{s}+h\boldsymbol{s})$, where $\boldsymbol{s}$ is a unit vector along the screw axis, $\boldsymbol{r}$ is the position vector of any point on the screw axis, and h is called the pitch. The unit screw associated with a revolute pair is given by $\boldsymbol{\$}=(\boldsymbol{s};\boldsymbol{r}\times\boldsymbol{s})=(l\ m\ n;a\ b\ c)$, where l,m,n denotes the three direction cosines of the revolute axis. The unit screw associated with a prismatic pair is given by $\boldsymbol{\$}=(0;\boldsymbol{s})=(0\ 0\ 0;l\ m\ n)$.

When all the joints in a mechanism are associated with unit screws and constitute a screw system, we can define a common constraint as a screw reciprocal to every screw of this system.

The number of the linear independent screws reciprocal to this system is referred to the number of common constraints, denoted by λ.

When the rank of a screw system is equal to 6, $r = 6$, there exist no reciprocal screws. When the rank of a screw system is less than six, $r < 6$, there are $6 - r$ common constraints for the mechanism.

The mobility can also be by the general Grübler criterion

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i \,, \tag{1}$$

where M denotes the mobility of the mechanism, d represents the order of the mechanism, $d = 6 - \lambda$, λ is the number of common constraints; n is the number of links; g is the number of kinematic pairs; f_i is the freedom of the ith pair.

The structure constraint acting on the moving platform by limbs can be represented by wrenches, which could be forces, couples or wrenches. We define the wrenches acting on the moving platform by a single limb as the limb structure constraint screws, which constitute the limb constraint system. And all the limb structure constraint screws constitute the mechanism constraint system, which determines the mobility of the mechanism. The maximum linear independent number of the mechanism constraint system equals the constrained DOF of the parallel mechanism. Both the number and property of mobility can be judged according to the mechanism constraint system.

It must be pointed out that the mechanism constraint system is instantaneous. When the mechanism moves, the mechanism constraint system may change, consequently, the mobility will change. When analyzing the mobility, we must identify whether the mechanism constraint system will remain the same one in the mechanism motion. Here the same one does not necessarily mean that the mechanism constraint system must be in the same form as the original, but the mechanism constraint system exerts the same constraint on the moving platform as the original mechanism constraint system This can be detected by examining the standard base or base of the mechanism constraint system, because the specific screws are the linear combination of a group of screws bases. If the mechanism constraint system changes in the mechanism motion, we call this mechanism an instantaneous mechanism.

2. MOBILITY ANALYSIS

2.1 ASSEMBLY CONDITION OF U-PAIRS FOR INSTANTANEOUS 5-DOF 3-UPU MECHANISM

Bonev and Zlatanov (2001) have discussed a similar case. A 3-UPU parallel mechanism and its limb are shown in Fig.1. All the revolute axes are parallel to the base plane. The three revolute axes adjacent to the base from the three different branches are coplanar and intersect at one common point. The three revolute axes adjacent to the moving platform from the three different branches are coplanar and intersect at another common point.

We call the plane determined by the two revolute axes of the universal joint as the universal joint plane. A local frame is set on a branch. The x axis of the local frame passes through the revolute pair fixed on the base. The y axis of the local frame passes through the other revolute pair of the same universal joint. Thus the xy plane of the local frame is always coincident with the universal joint plane and the z axis of the local frame is perpendicular to the universal joint plane. The coordinate of the central point of the universal joint adjacent to the moving platform can be denoted as $(x_1, 0, z_1)$.

According to the geometrical conditions between the two universal joint planes in a UPU branch, we can analyze this mechanism in a straightforward manner.

1. When the two universal joint planes in a UPU branch are parallel to each other.

On the initial assembly configuration, the universal joint plane is parallel to the base plane and the moving platform plane.

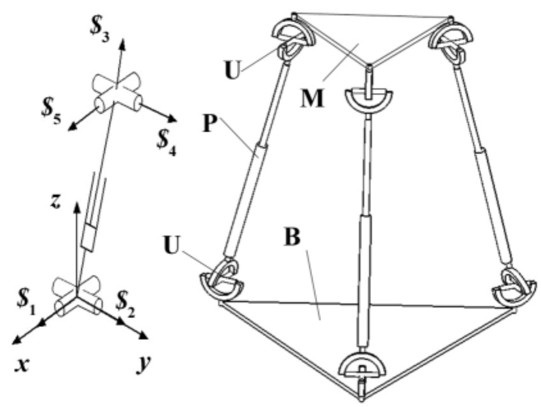

Fig.1 5-DOF 3-UPU instantaneous parallel mechanism

The limb twist system in the local frame at the original configuration is:

$$\begin{aligned}
\$_1 &= (1\ 0\ 0\,;0\ 0\ 0)\\
\$_2 &= (0\ 1\ 0\,;0\ 0\ 0)\\
\$_3 &= (0\,0\,0\,;l_3\ 0\ n_3)\\
\$_4 &= (0\ 1\ 0\,;-z_1\ 0\ x_1)\\
\$_5 &= (1\ 0\ 0\,;0\ z_1\ 0)
\end{aligned} \tag{2}$$

By calculating the reciprocal screws to equation (2), we can get the limb constraint system:

$$\$^r_{II} = (0\,0\,0\,;0\,0\,1) \tag{3}$$

Equation (3) shows that a single UPU limb exerts a constraint couple on the moving platform, which is perpendicular to the universal joint plane.

The mechanism constraint system contains three constraint couples on the moving platform exerted by the three UPU limbs. The three constraint couples are coaxial and linearly dependent, being equivalent to one couple perpendicular to the universal joint plane. Consequently, the standard base of the mechanism constraint system is:

$$\boldsymbol{\$}_{m1}^{r} = \left(0\,0\,0\,;0\,0\,1\right) \tag{4}$$

The constraint couple in equation (4) only restricts the rotation about the normal of the moving platform plane. Thus, this mechanism is a 5-DOF mechanism instead of a 3-DOF translational one.

To use equation (1) to calculate the mobility, we must get the right λ. In the above analysis, it can be seen that each limb constraint couple is reciprocal to all the three limb twist systems. Thus, $\lambda = 1$, and $n = 8$, $g = 9$, we have

$$\begin{aligned} M &= d(n-g-1) + \sum_{i=1}^{g} f_i \\ &= (6-1)(8-9-1) + 15 = 5 \end{aligned} \tag{5}$$

If the moving platform undergoes a pure translation along the normal of the universal joint plane, the mechanism constraint system will not change and the mechanism is still a instantaneous 5-DOF one.

If the moving platform undergoes a pure translation along a direction other than the normal of the universal plane, the two universal joint planes in a single UPU branch are still parallel to each other. However, the three groups of universal joint planes in the three different UPU branches are not parallel to one another. Thus, the three constraint couples in the mechanism constraint system are along three different normal directions and linearly independent. The standard base of the mechanism constraint system is:

$$\begin{aligned} \boldsymbol{\$}_{m1}^{r} &= \left(0\,0\,0\,;1\,0\,0\right) \\ \boldsymbol{\$}_{m2}^{r} &= \left(0\,0\,0\,;0\,1\,0\right) \\ \boldsymbol{\$}_{m3}^{r} &= \left(0\,0\,0\,;0\,0\,1\right) \end{aligned} \tag{6}$$

The constraint couples in equation (6) restrict the three rotational freedoms of the moving platform. The mechanism is a 3-DOF translational mechanism.

2. When the two universal joint planes in a UPU branch are not parallel to each other.

At the initial configuration, the moving platform can rotate about the x axis and y axis. After the moving platform undergoes a rotation, the second universal joint plane adjacent to the moving platform will not be parallel to the first universal joint plane. The limb twist system in the local frame is:

$$\begin{aligned} \boldsymbol{\$}_1 &= \left(1\ \ 0\ \ 0\,;0\ \ 0\ \ 0\right) \\ \boldsymbol{\$}_2 &= \left(0\ \ 1\ \ 0\,;0\ \ 0\ \ 0\right) \\ \boldsymbol{\$}_3 &= \left(0\,0\,0\,;l_3\ \ 0\ \ n_3\right) \\ \boldsymbol{\$}_4 &= \left(0\ \ 1\ \ 0\,;-z_1\ \ 0\ \ x_1\right) \\ \boldsymbol{\$}_5 &= \left(l_5\ \ 0\ \ n_5\,;0\ \ z_1 l_5 - x_1 n_5\ \ 0\right) \end{aligned} \tag{7}$$

By calculating the reciprocal screws to equation (7), we can get the limb constraint system

$$\boldsymbol{\$}_{l1}^{r} = \left(0\ \ n_5\ \ 0\,;0\ \ 0\ \ x_1 n_5 + z_1 l_5\right) \tag{8}$$

Here $\boldsymbol{\$}_{l1}^{r}$ is a constraint force parallel to the second revolute axis counting from the base.

Considering the other two UPU branches, there are two possibilities. One possibility is that all the upper and lower universal joint planes in all the three branches are not parallel to one another. Then the three constraint forces are linearly independent. So the moving platform loses three translational freedoms and can only perform three rotations. However, the rotation axes must intersect all the three constraint forces. The other possibility is that in one or two branches, the upper and lower universal joint planes are still parallel to each other. In that case, the moving platform has three freedoms including both rotational and translational degrees of freedom.

2.2 ASSEMBLY CONDITION OF U-PAIRS FOR A 3-DOF 3-UPU TRANSLATIONAL PARALLEL MECHANISM

The assembly condition of U-pairs for a 3-DOF 3-UPU translational parallel mechanism has been widely studied (Di Gregorio and Parenti-Castelli 1999, Tsai 1999, Frisoli et al. 2000, Carricato and Parenti-Castelli 2001, Bonev and Zlatanov 2001, Zlatanov et al. 2001, Karouia and Hervé 2000). Such a translational 3-UPU mechanism is shown in Fig 2.

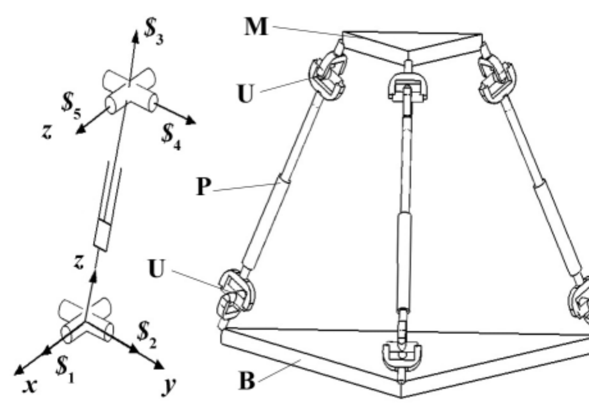

Fig. 2 3-DOF 3-UPU translational parallel mechanism

The x axis of the local frame passes through the revolute pair fixed on the base and the y axis of the local frame passes through the revolute pair adjacent to the prismatic pair. The prismatic pair is perpendicular to the universal joint plane.

The limb twist system in the local frame is:

$$\begin{aligned} \$_1 &= (1\ 0\ 0;0\ 0\ 0) \\ \$_2 &= (0\ 1\ 0;0\ 0\ 0) \\ \$_3 &= (0\,0\,0;0\ 0\ 1) \\ \$_4 &= (0\ 1\ 0;-a_4\ 0\ 0) \\ \$_5 &= (1\ 0\ 0;0\ b_5\ 0) \end{aligned} \quad . \qquad (9)$$

By calculating the reciprocal screws to equation(9), we can get the limb constraint system:

$$\$_{l1}^r = (0\,0\,0;0\,0\,1). \qquad (10)$$

Equation (10) shows that a single UPU limb exerts a constraint couple on the moving platform, which is perpendicular to the universal joint plane. It is easy to visualize that the three constraint couples in mechanism constraint system exerted on the moving platform are not coplanar and linearly independent, thus, the standard base of the mechanism constraint system is the same as the equation (6). Under this assembly configuration, this 3-UPU mechanism is a 3-DOF translational mechanism at this initial assembly configuration.

Notice that the three revolute pairs fixed on the moving platform are always parallel to the moving platform plane. The moving platform is parallel to the base plane at the initial assembly configuration. Since the moving platform can only translate at this moment, the moving platform plane will keep being parallel to the base plane, so are the three revolute axes adjacent to the moving platform.

In the view of this, we check the limb twist system after any translations of the moving platform:

$$\begin{aligned} \$_1 &= (1\ 0\ 0;0\ 0\ 0) \\ \$_2 &= (0\ 1\ 0;0\ 0\ 0) \\ \$_3 &= (0\,0\,0;l_3\ 0\ n_3) \\ \$_4 &= (0\ 1\ 0;-z_1\ 0\ x_1) \\ \$_5 &= (1\ 0\ 0;0\ z_1\ -y_1) \end{aligned} \qquad (11)$$

By calculating the reciprocal screws to equation(11), we can get the limb constraint system:

$$\$_{l1}^r = (0\,0\,0;0\,0\,1) \qquad (12)$$

Equation (12) shows after the moving platform translates, a single UPU limb exerts a constraint couple on the moving platform. The couple is perpendicular to the universal joint plane adjacent to the base. Obviously, considering the practical restriction of the universal joint, as long as the three universal joint planes adjacent to the base are not parallel to each other, the three constraint couples in mechanism constraint system will not be coaxial or coplanar. So the standard base of the mechanism constraint system is the same as equation (6), this redesigned 3-UPU parallel mechanism is a 3-DOF translational mechanism.

There are no common constraints. Thus, $\lambda = 0$, and $n = 8$, $g = 9$, we have

$$M = d(n-g-1) + \sum_{i=1}^{g} f_i = 6(8-9-1)+15 = 3 \qquad (13)$$

2.3 ASSEMBLY CONDITION OF U-PAIRS FOR A 3-DOF 3-UPU SPHERICAL PARALLEL MECHANISM

Karouia and Hervé (2000) first described the 3-DOF 3-UPU spherical parallel mechanism As shown in Fig.3, the moving platform and the base are connected through three identical UPU limbs. The two revolute axes adjacent to the prismatic pair are parallel to the base plane and perpendicular to the prismatic pair. The axes of the three revolute pairs fixed on the base are parallel to the base plane and converge toward a point. The axes of the three revolute pairs fixed on the moving platform also converge toward the same point, which is set as the origin of the local frame. The x axis of the local frame passes through the revolute pair fixed on the base and the y axis of the local frame is parallel to the revolute axis adjacent to the prismatic pair.

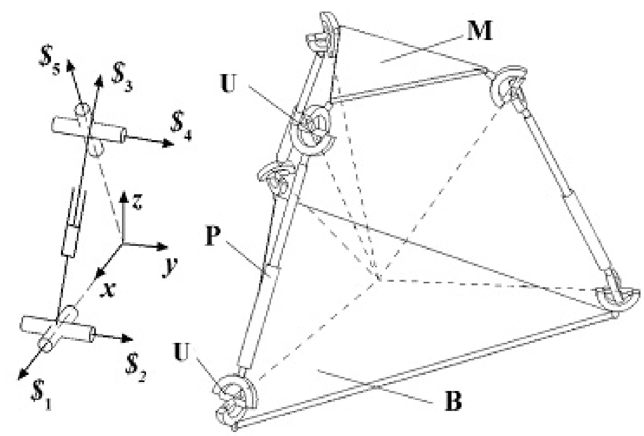

Fig. 3 3-DOF 3-UPU spherical parallel Mechanism

The limb twist system in the local frame is:

$$\begin{aligned} \$_1 &= (1\ 0\ 0;0\ 0\ 0) \\ \$_2 &= (0\ 1\ 0;0\ 0\ c_2) \\ \$_3 &= (0\,0\,0;l_3\ 0\ n_3) \\ \$_4 &= (0\ 1\ 0;a_4\ 0\ c_4) \\ \$_5 &= (l_5\ 0\ n_5;0\ 0\ 0) \end{aligned} \qquad (14)$$

By calculating the reciprocal screws to equation (14), we can get the limb constraint system:

$$\$_{l1}^r = (0\,1\,0;0\,0\,0) \qquad (15)$$

Equation (15) shows that a single UPU limb exerts a constraint force on the moving platform. This force is parallel to the second revolute axis counting from the base and passes through the origin. The three UPU limbs exert three constraint forces in total on the moving platform. These three constraint forces are coplanar and intersect in the same point, they are linearly dependent. Thus, under this geometrical condition, the standard base of the mechanism constraint system is given by:

$$\begin{aligned} \$^r_{m1} &= (1\,0\,0\,;0\,0\,0) \\ \$^r_{m2} &= (0\,1\,0\,;0\,0\,0) \end{aligned} \tag{16}$$

The two constraint forces in equation (16) restrict two translational freedoms of the moving platform in the fixed base plane. Thus, at the initial assembly configuration, this mechanism has three rotational freedoms and one translational freedom along the z axis.

There are no common constraints. Thus, $\lambda = 0$, and one virtual constraint exists, we have

$$\begin{aligned} M &= d(n-g-1) + \sum_{i=1}^{g} f_i \\ &= 6(8-9-1)+15+1 = 4 \end{aligned} \tag{17}$$

Note that if the fifth revolute axis fixed on the moving platform is not parallel to the first universal joint plane, the constraint force produced by this UPU limb is always a force, which is parallel to the second revolute axis.

When the moving platform rotates, the two revolute axes adjacent to the prismatic pair are not parallel to the base plane. Thus the three constraint forces are noncoplanar and intersect at the common point, consequently, the standard base of the mechanism constraint system is given by:

$$\begin{aligned} \$^r_{m1} &= (1\,0\,0\,;0\,0\,0) \\ \$^r_{m2} &= (0\,1\,0\,;0\,0\,0) \\ \$^r_{m3} &= (0\,0\,1\,;0\,0\,0) \end{aligned} \tag{18}$$

The three constraint forces in equation (18) restrict the three translational freedoms of the moving platform. Thus, at this configuration, this 3-UPU mechanism is a 3-DOF rotational parallel mechanism.

There are no common constraints. Thus, $\lambda = 0$, no virtual constraint exists, we have

$$\begin{aligned} M &= d(n-g-1) + \sum_{i=1}^{g} f_i \\ &= 6(8-9-1)+15 = 3 \end{aligned} \tag{19}$$

It may be considered that the initial configuration is singular and the mechanism has an unwanted DOF. When the moving platform rotates to any other configuration, the mechanism has only three rotational degrees of freedom.

2.4 ASSEMBLY CONDITION OF U-PAIRS FOR A NOVEL 4-DOF 3-UPU PARALLEL MECHANISM

A novel 3-UPU parallel mechanism is as shown in Fig. 4. The moving platform and the base are connected through three identical UPU limbs. For a single UPU branch, counting from the base, the first and the fifth revolute axes are perpendicular to the base plane and the moving platform plane. The second and the fourth revolute axes are parallel to the base plane.

The x axis of the local frame passes through the second revolute pair and the z axis of the local frame passes through the first revolute pair fixed on the base. Thus all the universal joint planes are perpendicular to the base plane. The central point of the second universal joint is denoted as $(0,\, y_1,\, z_1)$.

The limb twist system in the local frame is:

$$\begin{aligned} \$_1 &= (0\ \ 0\ \ 1\,;0\ \ 0\ \ 0) \\ \$_2 &= (1\ \ 0\ \ 0\,;0\ \ 0\ \ 0) \\ \$_3 &= (0\,0\,0\,;0\ \ m_3\ \ n_3) \\ \$_4 &= (1\ \ 0\ \ 0\,;0\ \ z_1\ \ -y_1) \\ \$_5 &= (0\ \ 0\ \ 1\,;y_1\ \ 0\ \ 0) \end{aligned} \quad . \tag{20}$$

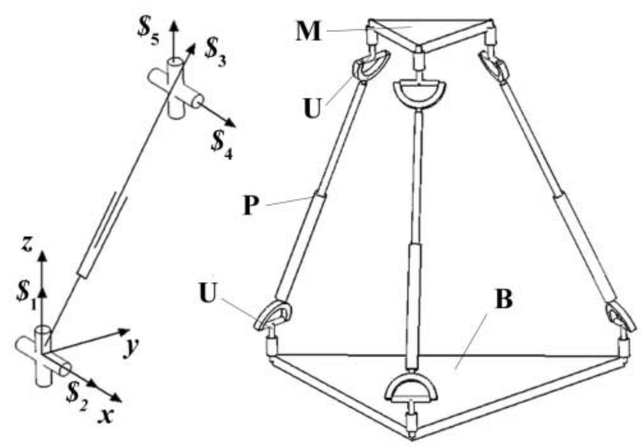

Fig. 4 4-DOF 3-UPU parallel Mechanism

By calculating the reciprocal screws to equation (14), we can get the limb constraint system:

$$\$^r_{l1} = (0\,0\,0\,;0\,1\,0). \tag{21}$$

Equation (21) shows that a single UPU branch exerts a constraint couple on the moving platform. This couple is perpendicular to the first universal joint plane. All the three UPU limbs exert three constraint couples on the moving platform. Because the three constraint couples are coplanar, the standard base of the mechanism constraint system is given by:

$$\begin{aligned} \$^r_{m1} &= (0\,0\,0\,;1\,0\,0) \\ \$^r_{m2} &= (0\,0\,0\,;0\,1\,0) \end{aligned} \tag{22}$$

The two constraint couples in equation (22) restrict two rotational freedoms of the moving platform. The remaining four freedoms of the moving platform include three translational freedoms and one rotational freedom about the normal of the moving platform plane.

There are no common constraints. Thus, $\lambda = 0$, one virtual constraint exists, we have

$$M = d(n-g-1) + \sum_{i=1}^{g} f_i \\ = 6(8-9-1)+15+1 = 4 \tag{23}$$

Note that after any feasible finite motion, the universal joint planes in each branch keep being perpendicular to the base plane, thus, the mechanism constraint system will stay invariable. This mechanism is a 4-DOF one

2.5 DIFFERENT ARRANGEMENT OF U-PAIRS

If the two intermediate revolute axes are perpendicular to each other instead of being parallel, the mobility of the 3-UPU parallel mechanism will change.

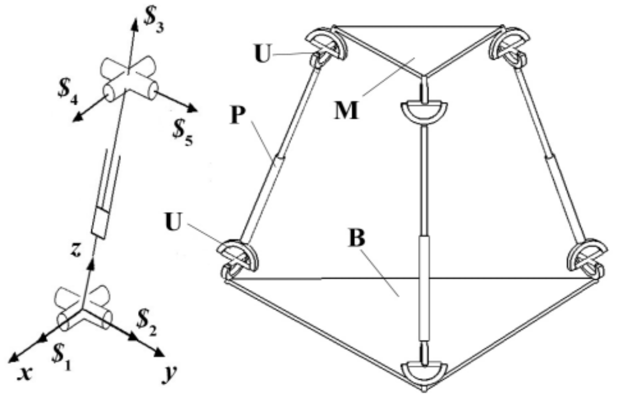

Fig. 5 DIFFERENT ARRANGEMENT OF U-PAIRS

In each branch, as shown in Fig.5, the two intermediate revolute axes adjacent to the prismatic pair are perpendicular to each other. Counting from the base, the second revolute axis and the last revolute axis are not parallel to the base plane. The three revolute axes adjacent to the moving platform from the three different branches are parallel to the base plane and form a triangle.

At this initial configuration, according to the analysis above, the 3-UPU mechanism is a 3-DOF instantaneous translational parallel mechanism. If the moving platform undergoes a pure translation along the normal of the universal joint plane, it is still a 3-DOF instantaneous translational parallel mechanism.

However, after the moving platform translates a finite distance in the base platform plane, the geometrical conditions among these axes will change. As shown in Fig.5, at least in one limb, the prismatic axis will not be perpendicular to the universal joint plane. The fourth revolute axis will not be parallel to the first revolute axis. The fifth revolute axis will not be parallel to the second revolute axis.

And the limb twist system at this moment is given by:

$$\begin{aligned} \$_1 &= (1\ \ 0\ \ 0\,;0\ \ 0\ \ 0) \\ \$_2 &= (0\ \ 1\ \ 0\,;0\ \ 0\ \ 0) \\ \$_3 &= (0\,0\,0\,;l_3\ \ 0\ \ n_3) \\ \$_4 &= (l_4\ \ 0\ \ n_4\,;0\ \ -x_1 n_4\ \ 0) \\ \$_5 &= (l_5\ \ m_5\ \ n_5\,;y_2 n_5\ \ 0\ \ -y_2 l_5) \end{aligned} \tag{24}$$

By calculating the reciprocal screws to equation (24), we can get the limb constraint system:

$$\$_{l1}^r = (L_1\ \ M_1\ \ N_1\,;0\ \ 0\ \ C_1), \tag{25}$$

where $L_1 = -n_3 n_5$, $M_1 = \dfrac{y_2(l_3 l_5 + n_5 n_3)}{x_1}$

$, N_1 = l_3 n_5, C_1 = y_2(l_3 l_5 + n_5 n_3)$.

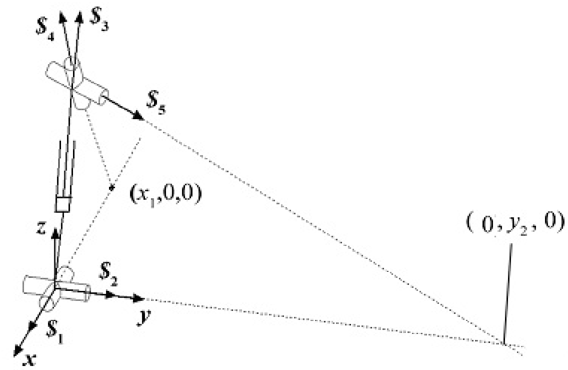

Fig. 6

Under the general configuration, the $\$_{l1}^r$ is a wrench with finite pitch. The three UPU limbs exert three constraint wrenches in total on the moving platform. Because a wrench constrains both the translation along its axis and the rotation about the axis, it takes a complicated analysis for the concrete mobility characteristics of this 3-UPU parallel mechanism. It is not a three-dimension translational mechanism yet. However, we conclude that it has three DOF. Under this geometrical condition, there are no common constraints. Thus, $\lambda = 0$, and $n = 8$, $g = 9$, we have

$$M = d(n-g-1) + \sum_{i=1}^{g} f_i \\ = 6(8-9-1)+15 = 3 \tag{26}$$

3. CONCLUSIONS

The assembly conditions of universal pairs in a 3-UPU parallel mechanism play an essential role in determining the mechanism mobility. Both the DOF and properties may change under different assembly configurations. Under different assembly conditions of the universal pairs, the 3-UPU parallel mechanism can be an instantaneous 5-DOF mechanism, a 3-DOF translational mechanism, a 3-DOF rotational mechanism, or a 4-DOF mechanism, even something not included in this paper.

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