A Novel Four-DOF Parallel Manipulator Mechanism and Its Kinematics

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***Abstract*—A novel 4-UPU parallel manipulator mechanism that can perform three-dimensional translations and rotation about Z axis is presented. The principle that the mechanism can perform the above motions is analyzed based on the screw theory, the mobility of the mechanism is calculated, and the rationality of the chosen input joints is discussed. The forward and inverse position kinematics solutions of the mechanism and corresponding numerical examples are given, the workspace and the singularity of the parallel mechanism are discussed. The mechanism having the advantages of simple symmetric structure and large stiffness can be applied to the developments of NC positioning platforms, parallel machine tools, four-dimensional force sensors and micro-positional parallel manipulators.**

***Keywords*—parallel mechanism, kinematics, singularity, workspace**

I. INTRODUCTION

Spatial parallel mechanisms have received much attention because they can be used as industrial robots and novel virtual-axes machine tools, and a great many of papers have been published. Some spatial three-DOF parallel robots have interested many researchers, for the type parallel robots are simpler and cheaper than that of six-DOF ones. There are many special instances. In this field, Hunt[1] proposed various kinematic structures for parallel robots. An important one being widely noticed is the three-DOF three-RPS parallel robot. Lee[2] used the mechanism as a micromanipulator. Waldron[3] adopted this mechanism as a micromanipulator placed between the wrist and hand of a 10-DOF serial-parallel manipulator. Lee[4] reported their researches on this mechanism. Many other novel 3-DOF parallel manipulators were also proposed. Song[5] studied the reactional force compensation based on three-degree-of freedom parallel platforms. Tsai[6] studied a simpler three-diminution translation manipulator. Huang[6] introduced several new 3-DOF-pyramid mechanisms. Gosselin[8] also published his paper on the spherical manipulator. In 2000, Zhao[9] proposed a novel synthesis method of parallel mechanisms with fewer than six DOF based on screw theory, and obtained many parallel mechanisms by using this method[10,11]. Zhao and Huang[12] presented the kinematics of a spatial translational parallel mechanism.

This paper presents a novel 4-UPU spatial parallel mechanism that can perform one rotation about Z-axis and three-dimensional translation, gives the principle that the mechanism can perform the above motions based on the screw theory, calculates the mobility of the mechanism, and discusses the rationality of the chosen input joints. Then the forward and inverse position kinematics solutions of the mechanism and corresponding numerical examples are given, the workspace and the singularity of the parallel mechanism are discussed.

II. STRUCTURAL CHARACTERISTICS AND MOBILITY OF 4-UPU PLATFORM MECHANISM

A. Structural Characteristics of 4-UPU Platform Mechanism

The presented 4-UPU platform mechanism is shown in Fig.1. The coordinates systems O-XYZ and p-xyz are fixed to the base platform system and the upper platform, respectively. The Z-axis and z-axis are vertical upwards. This mechanism has four limbs each of which consists of four revolute joints and one prismatic joint. The axes of the first and the fifth joints are parallel to the Z-axis. The axes of the second and the fourth joints are parallel to each other also and perpendicular to Z-axis. The axes of the fourth revolute joints in all limbs are coplanar.

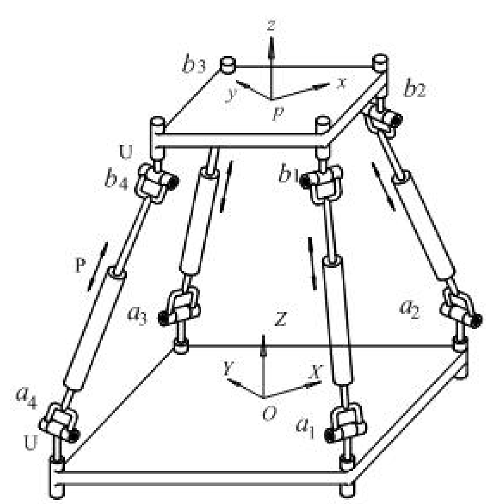

Figure 1. 4-UPU platform mechanism.

Let $b_i(i=1,2,3,4)$ represent the intersection point of the axes of the fourth to the fifth pair in the ith limb, all $b_i(i=1,2,3,4)$ are in a common plane of the upper platform, and form a square with 2d side length. $a_i(i=1,2,3,4)$ is the axis intersection point of the first revolute joint and the second revolute joint. All $a_i(i=1,2,3,4)$ are in a common plane of the base platform and form a rectangle. One side length is $2R_1$ and another is $2R_2$. The origin of O-XYZ is the center of rectangle $a_1a_2a_3a_4$.

B. *Mobility of 4-UPU Platform Mechanism*

In frame O-XYZ, the motion screws of joint axes can be expressed as

$$\begin{aligned}
\$_{i1} &= (0,\quad 0,\quad 1;\quad 0,\quad 0,\quad 0) \\
\$_{i2} &= (l_{i2},\quad m_{i2},\quad 0;\quad 0,\quad 0,\quad 0) \\
\$_{i3} &= (0,\quad 0,\quad 0;\quad P_{i3},\quad Q_{i3},\quad R_{i3}) \\
\$_{i4} &= (l_{i4},\quad m_{i4},\quad 0;\quad P_{i4},\quad Q_{i4},\quad R_{i4}) \\
\$_{i5} &= (0,\quad 0,\quad 1;\quad P_{i5},\quad Q_{i5},\quad 0)
\end{aligned} \tag{1}$$

where l_{i2} and m_{i2} are the direction cosines of the axes of the second and the fourth joints, respectively. P_{i3}, Q_{i3} and R_{i3} are the parameters determined by the position of the third joint. $P_{i4}, Q_{i4}, R_{i4}, P_{i5}, Q_{i5}$ are the parameters determined by the corresponding positional vectors. The five motion screws in every limb are linearly independent, the reciprocal screw of the ith limb can be obtained

$$\$_{i1}{}^r = (0,\quad 0,\quad 0;\quad l_{i2}{}^r,\quad m_{i2}{}^r,\quad 0)(i=1,\ 2,\ 3,\ 4) \tag{2}$$

where $l_{i2}l_{i2}{}^r + m_{i2}m_{i2}{}^r = 0$. It represents a pure couple and restricts the rotation of the upper platform about the line vector parallel to plane XOY. So the constraining couples of the upper platform acted by the four limbs can be expressed as $\$_{11}{}^r, \$_{21}{}^r, \$_{31}{}^r, \$_{41}{}^r$. The four couples are linear dependent. They can be obtained by the combination of the two couples:

$$\$_1{}^r = (0,\quad 0,\quad 0;\quad 1,\quad 0,\quad 0) \tag{3}$$

$$\$_2{}^r = (0,\quad 0,\quad 0;\quad 0,\quad 1,\quad 0) \tag{4}$$

where $\$_1{}^r$, $\$_2{}^r$ are two couples. They restrict the rotation about the X-axis and the Y-axis, respectively. That is the upper platform loses two degrees of freedom. Since the four reciprocal screws $\$_{11}{}^r, \$_{21}{}^r, \$_{31}{}^r, \$_{41}{}^r$ restrict the two rotations of the upper platform, so there are two extra idle constraints in the mechanism. Therefore, this mechanism is an overconstrained mechanism. With Kutzbach-Grubler formula,

$$M = d(n-g-1) + \sum_{i=1}^{g} f_i + \varsigma \tag{5}$$

the degrees of freedom of the mechanism can be calculated, where M expresses the degrees of freedom of the mechanism, d is the order of the mechanism, $d = 6 - \lambda$, λ is the amount of common constrictions of the mechanism, n is the number of the components in the mechanism, g is the number of joints in the mechanism, f_i is the degrees of freedom of the ith joint, ς is an amending item determined by the local freedoms and negative freedoms as well as idle constraints in the mechanism. Since there are two idle constraints in the mechanism, the degrees of freedom of the mechanism can be computed as

$$M = 6 \times (10 - 12 - 1) + (2 \times 8 + 4 \times 1) + 2 = 4 \tag{6}$$

III. Identification of Input Rationality

Generally, there is an input in each limb in spatial parallel mechanism. Since the 4-UPU parallel mechanism is an overconstrained multi-loop mechanism, it is difficult to identify the rationality of a set of inputs. This paper identifies the input rationality of the 4-UPU mechanism based on linear dependency of screw system. When the four prismatic joints are chosen to be the four inputs, after rigidizing the prismatic joint, the screw system of each limb can be expressed as

$$\begin{aligned}
\$_{i1} &= (0,\quad 0,\quad 1;\quad 0,\quad 0,\quad 0) \\
\$_{i2} &= (l_{i2},\quad m_{i2},\quad 0;\quad 0,\quad 0,\quad 0) \\
\$_{i4} &= (l_{i4},\quad m_{i4},\quad 0;\quad P_{i4},\quad Q_{i4},\quad R_{i4}) \\
\$_{i5} &= (0,\quad 0,\quad 1;\quad P_{i5},\quad Q_{i5},\quad 0)
\end{aligned} \tag{7}$$

Accordingly, the constraining screws acted on the upper platform by each limb are

$$\begin{aligned}
\$_{i1}{}^r &= (0,\quad 0,\quad 0;\quad l_{i2}{}^r,\quad m_{i2}{}^r,\quad 0) \\
\$_{i2}{}^r &= (p_i,\quad q_i,\quad r_i;\quad u_i,\quad v_i,\quad w_i)
\end{aligned} \tag{8}$$

where $(p_i,\quad q_i,\quad r_i)$ represents the direction vector of line l_{iab} from point ai to point bi, $(u_i,\quad v_i,\quad w_i)$ the line moment of line l_{iab} relative to the coordinate system O-XYZ. It can be expressed as $l_{iab} \times l_{Oai}$, where l_{Oai} denotes the line from the origin O to point ai. From (8), we can see that every limb increases a constraint expressed as reciprocal screw $\$_{i2}{}^r$ to the upper platform. $\$_{i2}{}^r$ represents a force line vector passing point ai and along line l_{iab}. Consequently, the upper platform is acted by the other four constraining forces after rigidizing the four inputs. The four constraining forces are not linear dependent if only they are not intersecting at a common point in the space, and not linear dependent to the two constraining couples expressed in (3). Therefore, the order of the eight constraining forces in (8) is 4+2＝6, there are 6 non-linear dependent reciprocal screws acting on the upper platform, is loses all of the degrees of freedom, so the set of inputs is rational.

IV. Position Kinematics of 4-UPU Parallel Mechanism

In Fig. 1, points a_1, a_2, a_3, and a_4 form a rectangle which length is $2R_1$ and width is $2R_2$, points b_1, b_2, b_3 and b_4 form a square which single side length is 2d. Figure 2 is the platform view of 4-UPU Parallel Mechanism at the initial position.

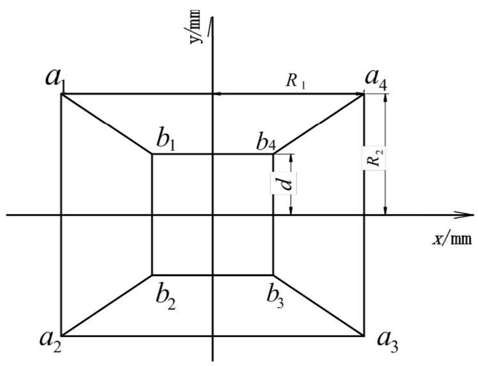

Figure 2. The platform view of 4-UPU Parallel Mechanism.

Let $(X_P,\ Y_P,\ Z_P,\ \alpha,\ \beta,\ \gamma)$ denote the displacements of the origin p and angle displacements of the upper platform, owing to the upper platform can only rotate about Z-axis, so $\alpha = \beta = 0$. Let $\{X_{bi},\ Y_{bi},\ Z_{bi}\}$ and $(x_{bi},\ y_{bi},\ z_{bi})$ be the coordinates of point b_i in frame O-XYZ and in frame p-xyz, respectively, $\{X_{a_1},\ Y_{a_2},\ Z_{a_1}\}$ the coordinates of point a_i in frame O-XYZ. So the following equation can be get

$$\{X_{bi},\ Y_{bi},\ Z_{bi},\ 1\}^T = \boldsymbol{T}_{Op}\{x_{bi},\ y_{bi},\ z_{bi},\ 1\}^T . \tag{9}$$

$$\boldsymbol{T}_{Op} = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & X_p \\ \sin\gamma & \cos\gamma & 0 & Y_p \\ 0 & 0 & 1 & Z_p \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{10}$$

$\boldsymbol{T}_{Op}$ is the transformation matrix from frame p-xyz to O-XYZ. When the structure parameters are known, the coordinates of point b_i in frame O-XYZ can be obtained from (9). If the orientation parameters of the upper platform are given, the constraining equations can be obtained as follows

$$L_i = \sqrt{(X_{bi}-X_{ai})^2 + (Y_{bi}-Y_{ai})^2 + (Z_{bi}-Z_{ai})^2}\,(i=1,2,3,4) . \tag{11}$$

where L_i represents the length from point ai to point bi, also the input of the ith limb. Substituting $\{X_{bi},\ Y_{bi},\ Z_{bi}\}$ and $(x_{bi},\ y_{bi},\ z_{bi})$ into (9) and (11) , and squaring the both sides of (11), transposing and packing up (11) as an equation of X_p, Y_p, Z_p, γ, the equations about Xp, Yp and γ can be get

$$X_p = \frac{(L_1^2-L_3^2)(d\sin\gamma+d\cos\gamma-R_2)-(L_2^2-L_4^2)(d\sin\gamma-d\cos\gamma+R_2)}{8(d^2-R_1d\cos\gamma-R_2d\cos\gamma+R_1R_2)} \tag{12}$$

$$Y_p = \frac{(L_1^2-L_3^2)(d\cos\gamma-d\sin\gamma-R_1)-(L_2^2-L_4^2)(d\sin\gamma+d\cos\gamma-R_1)}{8(d^2-R_1d\cos\gamma-R_2d\cos\gamma+R_1R_2)} \tag{13}$$

$$\gamma = -\arcsin\left[\frac{L_1^2 - L_2^2 + L_3^2 - L_4^2}{8d(R_2 - R_1)}\right] . \tag{14}$$

Substituting the value of γ from (14) in (12) and (13), X_p and Y_p can be solved out, moreover Z_p can be obtained.

Numerical Examples

V. Numerical Examples

Let $d = 200$ mm， $R_1 = 500$ mm and $R_2 = 400$ mm. Table I shows the results of the inverse displacement analysis of this mechanism. Table II shows the results of the positive displacement analysis of this mechanism.

VI. Singularity of 4-UPU Parallel Mechanism

The following equation can be get from above equations about X_p, Y_p, Z_p, γ

$$\dot{\boldsymbol{D}} = \{\dot{\boldsymbol{L}}_1\quad \dot{\boldsymbol{L}}_2\quad \dot{\boldsymbol{L}}_3\quad \dot{\boldsymbol{L}}_4\}^{\mathrm{T}} = \boldsymbol{J}_p \begin{Bmatrix} \boldsymbol{V_p} \\ \boldsymbol{\omega_Z} \end{Bmatrix} \tag{15}$$

Where $\boldsymbol{V_p} = (v_{pX}\quad v_{pY}\quad v_{pZ})^T$, it is a three-dimensional velocity vector of point p.

$$\boldsymbol{J_p} = \begin{bmatrix} \dfrac{-ds\gamma-dc\gamma+X_p+R_1}{L_1} & \dfrac{-ds\gamma+dc\gamma+Y_p-R_2}{L_1} & \dfrac{Z_p}{L_1} & \dfrac{D_1}{L_1} \\ \dfrac{ds\gamma-dc\gamma+X_p+R_1}{L_2} & \dfrac{-ds\gamma-dc\gamma+Y_p+R_2}{L_2} & \dfrac{Z_p}{L_2} & \dfrac{D_2}{L_2} \\ \dfrac{ds\gamma+dc\gamma+X_p-R_1}{L_3} & \dfrac{ds\gamma-dc\gamma+Y_p+R_2}{L_3} & \dfrac{Z_p}{L_3} & \dfrac{D_3}{L_3} \\ \dfrac{-ds\gamma+dc\gamma+X_p-R_1}{L_4} & \dfrac{ds\gamma+dc\gamma+Y_p-R_2}{L_4} & \dfrac{Z_p}{L_4} & \dfrac{D_4}{L_4} \end{bmatrix} \tag{16}$$

$$D_1 = X_p ds\gamma + R_1 ds\gamma - X_p dc\gamma - R_1 dc\gamma - Y_p dc\gamma + R_2 dc\gamma - Y_p ds\gamma + R_2 ds\gamma$$
$$D_2 = X_p ds\gamma + R_1 ds\gamma + X_p dc\gamma + R_1 dc\gamma - Y_p dc\gamma - R_2 dc\gamma + Y_p ds\gamma + R_2 ds\gamma$$
$$D_3 = -X_p ds\gamma + R_1 ds\gamma + X_p dc\gamma - R_1 dc\gamma + Y_p dc\gamma + R_2 dc\gamma + Y_p ds\gamma + R_2 ds\gamma$$
$$D_4 = -X_p ds\gamma + R_1 ds\gamma - X_p dc\gamma + R_1 dc\gamma + Y_p dc\gamma - R_2 dc\gamma - Y_p ds\gamma + R_2 ds\gamma$$

where, $s\gamma = \sin\gamma, c\gamma = \cos\gamma$.

Equation (15) gives a mapping, 4×4 matrix $\boldsymbol{J}_p$ is a mapping matrix. If $\boldsymbol{J}_p$ are not singular, (15) can be rewritten as the following form

$$\begin{Bmatrix} \boldsymbol{V}_p \\ \boldsymbol{\omega}_Z \end{Bmatrix} = \boldsymbol{J}_p^{-1} \dot{\boldsymbol{D}} \tag{17}$$

where $\boldsymbol{J}_p^{-1}$ is the Jocabian matrix of the 4-UPU parallel mechanism. If $\boldsymbol{J}_p$ is singular, this mechanism is singular. The following is to study the singularity of the 4-UPU parallel mechanism by the use of analyzing the singularity conditions of matrix $\boldsymbol{J}_p$.

Li (*i*=1, 2, 3, 4) is the length of the ith pole among the elements of matrix $\boldsymbol{J}_p$ from (16), the singularity conditions of matrix $\boldsymbol{J}_p$, namely the singularity of the 4-UPU parallel mechanism can be acquired by analyzing (16), the conclusions are as follows.

1). If Zp=0，the upper platform has no displacement along Z-axis, the value of the determinant of $\boldsymbol{J}_p$ is zero, matrix $\boldsymbol{J}_p$ is singular, this configuration of the 4-UPU parallel mechanism is singular. At the configuration, the 4-UPU parallel mechanism can not only translate along Z-axis no matter what the inputs are. The cause is that the action lines of the four driving forces from the four translating pairs are in common plane parallel to the upper platform, the translating along Z-axis of the upper platform is impossible.

2). When $R_1 = R_2$, the base platform is a square also, the value of the determinant of $\boldsymbol{J}_p$ is zero, matrix $\boldsymbol{J}_p$ is singular. This configuration of the mechanism is questionably, it is a special configuration with non-general content, under this condition, no matter what configuration the mechanism is at, the determinant of $\boldsymbol{J}_p$ is zero without exception, this kind of singularity is named as structure singularity or geometry singularity. Thus, it ought to avoid the case that the two quadrangles formed by the four joint centers on the upper platform and the four joint centers on the base platform are squares simultaneously or are resembling quadrangles symmetrical about X-axis and Y-axis at the same time while designing robots adopting the 4-UPU parallel mechanism.

3).When $\gamma = \pm \pi/2$，the value of the determinant of $\boldsymbol{J}_p$ is zero, this configuration of the mechanism is singularity, it is a position singularity.

VII. WORKSPACE OF 4-UPU PARALLEL MECHANISM

The workspace can be classified as single orientation workspace and agility workspace. The single orientation workspace is a concourse of all the points arrived by the reference point of the upper platform at some orientation. The agility workspace is a concourse of all the points arrived by the reference point of the upper platform at any an orientation. Considering the structure restricting of the 4-UPU parallel mechanism, the upper platform can not rotate 360° around any a line parallel to Z-axis, therefore the 4-UPU parallel mechanism has no agility workspace. The following only researches the single orientation workspace of the parallel mechanism. Under the condition of $-1000\,\text{mm} \le x \le 1000\,\text{mm}$, $-1000\,\text{mm} \le y \le 1000\,\text{mm}$, $400\,\text{mm} \le z \le 1600\,\text{mm}$, the section drawing and the solid drawing protracted using search method of the workspace at some orientations are shown in Fig.3, Fig.4 and Fig.5. Fig.3 and Fig.4 are the section of the workspace at the orientation of $\gamma = 0°$, Fig.5 is the solid drawing of the workspace at the orientation of $\gamma = 0°$. Here the structure parameters of the mechanism are $L_{i\min} = 600\,\text{mm}$, $L_{i\max} = 1500\,\text{mm}$, $d = 200\,\text{mm}$, $R_1 = 500\,\text{mm}$, and $R_2 = 400\,\text{mm}$.

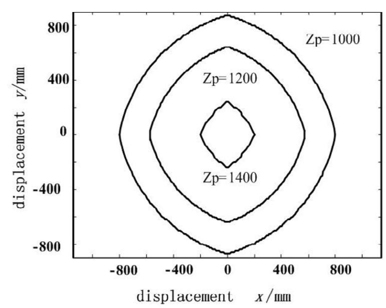

Figure 3. Workspace boundaries of sections perpendicular to Z-axis.

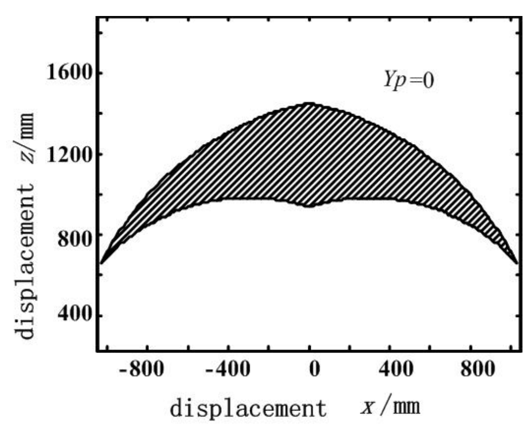

Figure 4. Workspace boundary of section perpendicular to Y-axis.

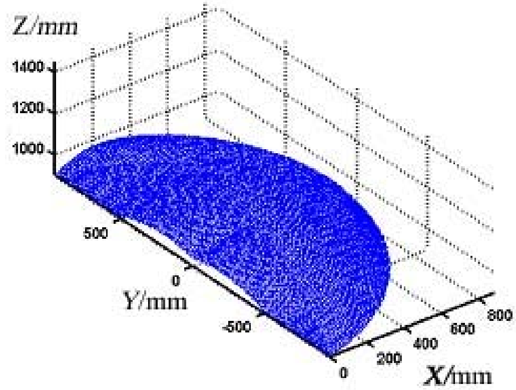

Figure 5. The cutaway view of the solid drawing of the workspace at $\gamma = 0°$ and $X > 0$.

VIII. Conclusions

This paper presents a 4-UPU parallel manipulator mechanism, and indicates that the 4-UPU parallel mechanism is an overconstrained mechanism. It has four degrees of freedom, and can perform three-dimensional translations and rotation about the Z axis. The four prismatic joints on the four limbs can serve as inputs. The closure equations of the forward position solution are deduced. The singularity conditions of the mechanism are given, and the workspace is described.

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