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Type Synthesis of 4-DOF Parallel Manipulators

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Abstract **– The number of 4-DOF (degrees of freedom) parallel manipulators with full-cycle mobility has been small for a long time. In this paper, the type synthesis of 4-DOF parallel manipulators is performed systematically by the constraint-synthesis method. Novel 4-DOF symmetrical and asymmetrical parallel manipulators with full-cycle mobility are identified and enumerated. Moreover, structural characteristics of the 4-DOF parallel manipulators are revealed.**

I. INTRODUCTION

In recent years, the use of lower-mobility parallel manipulators has drawn a lot of interest. A lower-mobility parallel mechanism has fewer than six DOF, which has the advantages of simple mechanical architecture, lower cost of design and manufacturing. Thus lower-mobility parallel mechanisms are suitable for many tasks requiring fewer than six DOF.

The 3-DOF parallel manipulator has been studied extensively, including the 3-DOF translational parallel manipulator [1,2], the 3-DOF spherical rotational parallel manipulator [3,4] and the 3-RPS parallel manipulator [5], which has two rotational DOF and one translational DOF.

The number of the 4-DOF parallel manipulators is relatively small compared with its 3-DOF counterpart. In 1999, Rolland proposed two 4-DOF parallel mechanisms, the Manta and the Kanuk[6], for material handling. The design of the Manta and the Kanuk robot is not far from the Delta robot [1] since they also use parallelograms to eliminate rotations. In 2000, Zhao and Huang proposed a 4-DOF 4-URU parallel mechanism [7], which has three translational DOF and one rotational DOF about the normal of the base. In 2001, Zlatanov and Gosselin came up with a 4-DOF 4-(RRR)$^{b}\underline{\text{RR}}$ parallel mechanism[8], where $^{b}\underline{\text{RR}}$ denotes two successive revolute pairs whose axes are parallel to the base plane and (RRR) denotes a 3R spherical subchain. The mechanism has three rotational DOF and one translational DOF. The translational DOF is along the normal of the moving platform plane. In 2002, Huang and Li invented two 4-DOF parallel mechanisms[9], $4-{}^{b}\underline{\text{RRR}}(\text{RR})$ and $4-{}^{b}\underline{\text{R}}\text{P}\underline{\text{R}}(\text{RR})$, where the underline denotes that the two revolute axes at the two end of the prismatic pair are parallel and (RR) denotes a 2R spherical subchain. These two mechanisms have three rotational DOF and one translational DOF along the normal of the base plane.

In this paper, we perform a systematic type synthesis of the 4-DOF parallel manipulators by the constraint-synthesis method [10]. Novel 4-DOF parallel manipulators with full-cycle mobility are identified and enumerated. Moreover, structural characteristics of the 4-DOF parallel manipulators are revealed.

II. SOME BASIC CONCEPTS

In screw theory [11, 12], the unit screw associated with a revolute pair or representing a force is given by $\$ = (\boldsymbol{s}; \boldsymbol{r} \times \boldsymbol{s}) = (l\ m\ n; a\ b\ c)$, where $\boldsymbol{s}$ is a unit vector along the screw axis, $\boldsymbol{r}$ is the position vector of any point on the screw axis, l, m, n denote three direction cosines. The unit screw associated with a prismatic pair or representing a couple is given by $\$ = (0; \boldsymbol{s}) = (0\ 0\ 0; l\ m\ n)$.

For a lower-mobility parallel mechanism, each limb exerts some structural constraints on the moving platform. The combined effect of all the limb structural constraints determines the mobility of the mechanism. We use the limb constraint system to describe the structural constraints of a single limb, which is defined as a screw system formed by all screws reciprocal to the unit twist associated with all kinematic pairs in a limb. We use the mechanism constraint system to describe the combined effect of all limb constraints.

Based on linear dependency of the limb structural constraints under different geometrical conditions, we can obtain the mechanism constraint system as well as the constrained DOF of the moving platform [10]. Reference [10] also proposed two tables describing the linear dependency of the limb constraints and the constrained DOF of the moving platforms.

Because twists and wrenches are instantaneous, it is necessary to identify whether the mechanism is instantaneous. This can be obtained by verifying the mechanism constraint system after any arbitrary finite displacement. If the mechanism constraint system remains unchanged, the mechanism is non-instantaneous. Such a verification can be done by simple analysis and inspection of structural or geometrical conditions among the kinemaic pairs in each limb and the mechanism. It is not necessary to know the analytical expressions of the mechanism constraints including the analytical expressions of the finite displacements in all joints of each limb. We will show this in the following examples.

For convenience, $\mathbf{\$}_{ij}$ is used to represent the unit twist associated with the j^{th} kinematic pair in the i^{th} limb; $\mathbf{\$}_{ij}^{r}$ is used to represent the j^{th} unit wrench exerted by the i^{th} limb; $\mathbf{\$}_{mj}$ is used to represent the j^{th} unit twist in the mechanism twist system; $\mathbf{\$}_{mj}^{r}$ is used to represent the j^{th} unit wrench in the mechanism constraint system.

III. STRUCTURAL CONSTRAINT CHARACTERISTICS OF 4-DOF PARALLEL MANIPULATORS

Because most of the lower-mobility are over-constrained, the common constraint and the redundant constraint should be taken into consideration. Thus, we rewrite the general Grübler-Kutzbach criterion as

$$M = d(n-g-1)+\sum_{i=1}^{g} f_i + v\,, \qquad (1)$$

where M denotes the mobility of the mechanism, d represents the order of the mechanism and $d=6-\lambda$, λ is the number of common constraints; n is the number of links; g the number of kinematic pairs; f_i the freedom of the i^{th} pair; v the redundant constraints.

The 4-DOF parallel mechanism falls into three categories according to their mobility. The first category has three rotational DOF and one translational DOF, denoted by 3R1T. The second category has three translational DOF and one rotational DOF, denoted by 3T1R. The last category has two rotational DOF and two translational DOF, denoted by 2R2T.

The four DOF of the mechanism can be represented by four linearly independent twists, which form a 4-system. Clearly, there exist only two screws reciprocal to the 4-system, which forms the mechanism constraint system and means the moving platform loses only two freedoms. Thus, the maximum linearly independent number of all the limb constraints must be two, i.e., they form a 2-system. Obviously, the two constraint screws must be coplanar in terms of geometry.

IV. TYPE SYNTHESIS OF 4-DOF (3R1T) PARALLEL MANIPULATORS

Without loss of genearality, we focus on the 4-DOF parallel mechanism with three rotational DOF and one translational DOF along the Z axis. The standard base of the mechanism twist system is given by

$$\begin{matrix} \mathbf{\$}_{m1}=(1\,0\,0\,;0\,0\,0) & \mathbf{\$}_{m2}=(0\,1\,0\,;0\,0\,0) \\ \mathbf{\$}_{m3}=(0\,0\,1\,;0\,0\,0) & \mathbf{\$}_{m4}=(0\,0\,0\,;0\,0\,1) \end{matrix}\,. \qquad (2)$$

The standard base of the mechanism constraint system is given by

$$\begin{matrix} \mathbf{\$}_{m1}^{r}=(1\,0\,0\,;0\,0\,0) \\ \mathbf{\$}_{m2}^{r}=(0\,1\,0\,;0\,0\,0) \end{matrix}\,. \qquad (3)$$

To form such a mechanism constraint system, the limb constraint system can be divided into two classes.

Case 1 *The limb kinematic chain consists of four kinematic pairs and exists two constraints.*

In this case, the standard base of the limb twist system is the same as (2). The linear combination of $\mathbf{\$}_{m1}$, $\mathbf{\$}_{m2}$ and $\mathbf{\$}_{m3}$ can only produce a 3R spherical subchain. A 2R spherical subchain leads to an instantaneous mechanism. Thus, the parallel mechanisms in this case can only be $4-{}^{z}\mathrm{P(RRR)}$ or $4-{}^{b}\mathrm{R(RRR)}$, where the superscript z denotes the prismatic pair is perpendicular to the base plane and the superscript b denotes that the revolute axis is parallel to the base plane. We call the rotational center of the 3R spherical subchain as limb central point. Note that the four limb central points must coincide with one another.

Case 2 *The limb kinematic chain consists of five kinematic pairs and exerts one constraint.*

In this case, since any twist representing a rotational DOF is linearly dependent with $\mathbf{\$}_{m1}$, $\mathbf{\$}_{m2}$ and $\mathbf{\$}_{m3}$, we have to add a twist representing a translational DOF to the limb twist system. Adding a twist $\mathbf{\$}_{i5}=(0\,0\,0\,;0\,1\,0)$ to (2) yields the new limb twist system

$$\begin{matrix} \mathbf{\$}_{i1}=(1\,0\,0\,;0\,0\,0) \\ \mathbf{\$}_{i2}=(0\,1\,0\,;0\,0\,0) \\ \mathbf{\$}_{i3}=(0\,0\,1\,;0\,0\,0) \\ \mathbf{\$}_{i4}=(0\,0\,0\,;0\,0\,1) \\ \mathbf{\$}_{i5}=(0\,0\,0\,;0\,1\,0) \end{matrix}\,. \qquad (4)$$

Now the limb constraint system reciprocal to equation (4) only contains one constraint

$$\mathbf{\$}_{i1}^{r}=(1\,0\,0\,;0\,0\,0), \qquad (5)$$

which is a constraint force in the x_i direction and passes through the origin of the limb frame.

Using (4) as the standard base of the limb twist system, we can obtain the limb kinematic chain by linear combination of the five twists in (4).

In (4), $\mathbf{\$}_{i2}$ and $\mathbf{\$}_{i3}$ can only be combined with $\mathbf{\$}_{i1}$ to avoid the undesired presence of helical pairs, the general form of $\mathbf{\$}_{i2}$ and $\mathbf{\$}_{i3}$ after linear combination is

$$\begin{matrix} \mathbf{\$}_{i2}=(l_2\ m_2\ n_2\,;0\,0\,0) \\ \mathbf{\$}_{i3}=(l_3\ m_3\ n_3\,;0\,0\,0) \end{matrix}, \qquad (6)$$

which forms a 2R spherical chain, and the rotation center of the chain is called the limb central point, which is set as the origin of the limb frame and the reference frame.

The linear combination of $\mathbf{\$}_{i1}$ with $\mathbf{\$}_{i2}$ and $\mathbf{\$}_{i3}$ leads to

$$\mathbf{\$}_{i1}=(l_1\ m_1\ n_1\,;0\,0\,0), \qquad (7)$$

which represents a revolute pair whose axis passes the limb central point. It is noted that $\mathbf{\$}_{i2}$, $\mathbf{\$}_{i3}$ in (6) and $\mathbf{\$}_{i1}$ in (7) forms a 3R spherical subchain. Thus, the limb kinematic chain must include a 2R or 3R spherical subchain.

The linear combination of $\boldsymbol{\$}_{i1}$ with $\boldsymbol{\$}_{i4}$ and $\boldsymbol{\$}_{i5}$ leads to $\boldsymbol{\$}_{i1}=\left(1\,0\,0\,;0\,b_1\,c_1\right)$, which is a revolute pair in the x_i – axis direction. Note that $\boldsymbol{\$}_{i4}$ and $\boldsymbol{\$}_{i5}$ represent a z_i – axis and a y_i – axis prismatic pair, respectively. And the linear combination of $\boldsymbol{\$}_{i4}$ and $\boldsymbol{\$}_{i5}$ with themselves only leads to a prismatic pair in yz plane, namely, $\boldsymbol{\$}_{i4}=\left(0\,0\,0\,;0\,m_4\,n_4\right)$ or $\boldsymbol{\$}_{m5}=\left(0\,0\,0\,;0\,m_5\,n_5\right)$. Thus, in such a 4-DOF parallel mechanism, the prismatic pair must not be parallel to the base plane.

To avoid the presence of helical pairs, only $\boldsymbol{\$}_{i1}$ can be linearly combined with $\boldsymbol{\$}_{i4}$ and $\boldsymbol{\$}_{i5}$ to transform them into revolute pairs. Obviously, the linear combination of $\boldsymbol{\$}_{i4}$ and $\boldsymbol{\$}_{i5}$ with $\boldsymbol{\$}_{i1}$ only leads to revolute pairs whose axes are in the x_i – axis direction, that is, $\boldsymbol{\$}_{i4}=\left(1\,0\,0\,;0\,b_4\,c_4\right)$ and $\boldsymbol{\$}_{i5}=\left(1\,0\,0\,;0\,b_5\,c_5\right)$. Hence, the revolute axes except those in the 2R or 3R subchain must be parallel to the base plane.

Because force is not free vector in space, a point on the force axis is necessary to determine a force vector in space. Note that all the limb constraint forces are actually parallel to the base plane and passing through their own limb central point. We set all the limb central point coincident with one another and form a mechanism central point so that all the limb constraint forces are coplanar and intersecting at the mechanism central point. Under this structural condition, the standard base of the mechanism constraint system is given by equation (3), constraining two translational DOF in the base plane.

Though $\boldsymbol{\$}_{i1}$, $\boldsymbol{\$}_{i2}$ and $\boldsymbol{\$}_{i3}$ can form a spherical pair, the center of all the spherical pairs in all limbs cannot coincide with each other. Thus, the limb kinematic chain contains no spherical pairs.

For example, first, we transform $\boldsymbol{\$}_{i1}$ by linear combination with $\boldsymbol{\$}_{i4}$ and $\boldsymbol{\$}_{i5}$ into a revolute pair in the x_i – axis direction, namely, $\boldsymbol{\$}_{i1}=\left(1\,0\,0\,;0\,b_1\,c_1\right)$; then, we transform $\boldsymbol{\$}_{i5}$ by linear combination of with $\boldsymbol{\$}_{i4}$ into a prismatic pair in the yz plane and perpendicular to $\boldsymbol{\$}_{i1}$, i. e., $\boldsymbol{\$}_{i5}=\left(0\,0\,0\,;0\,m_5\,n_5\right)$; we transform $\boldsymbol{\$}_{i4}$ by linear combination with $\boldsymbol{\$}_{i1}$ and $\boldsymbol{\$}_{i5}$ into a revolute prismatic pair in the x_i – axis direction, i. e., $\boldsymbol{\$}_{i4}=\left(1\,0\,0\,;0\,b_4\,c_4\right)$; we keep $\boldsymbol{\$}_{i2}$ and $\boldsymbol{\$}_{i3}$ as a 2R spherical subchain. Finally, by rearranging the sequence as $\boldsymbol{\$}_{i1}-\boldsymbol{\$}_{i5}-\boldsymbol{\$}_{i4}-\boldsymbol{\$}_{i2}-\boldsymbol{\$}_{i3}$, we have a limb kinematic chain denoted by ${}^{b}\underline{\mathrm{R}}\,\underset{\leftrightarrow}{\mathrm{P}}\,\underline{\mathrm{R}}(\mathrm{RR})$, where the $\leftrightarrow$ under P denotes that the prismatic pair is perpendicular to the adjacent two revolute pairs pointed by the arrow; the underline denotes that the two revolute axes are parallel to one another;

It is necessary to state that these structural conditions for the limb kinematic chain are essentially determined by the standard base of the limb twist system. Once the structural conditions are maintained in any arbitrary finite motion, the standard base of the limb twist system is always the same as equation (4). Consequently, the limb constraint system remains unchanged. This is important in identifying the instantaneous mechanism.

Based on the above analysis, we can obtain the usable limb kinematic chain by linear combination of the five twists in equation (4). The enumeration of the symmetrical parallel mechanisms with full-cycle mobility is given as follows.

- The limb chain include five kinematic pairs

1. No prismatic pairs in the limb

$4-{}^{b}\underline{\mathrm{RRR}}(\mathrm{RR})$ $\quad 4-{}^{b}\underline{\mathrm{RR}}\,{}_{1}\mathrm{U}^{s}\mathrm{R}$ $\quad 4-{}^{b}\underline{\mathrm{RR}}(\mathrm{RRR})$

$4-{}^{b}\underline{\mathrm{R}}\,{}_{1}\mathrm{U}^{s}(\mathrm{RR})$

2. One prismatic pair in the limb

$4-\underset{\rightarrow}{\mathrm{P}}\,{}^{b}\underline{\mathrm{RR}}(\mathrm{RR})$ $\quad 4-\underset{\rightarrow}{\mathrm{P}}\,{}^{b}\underline{\mathrm{R}}\,{}_{1}\mathrm{U}^{s}\mathrm{R}$ $\quad 4-{}^{b}\underline{\mathrm{R}}\,\underset{\leftrightarrow}{\mathrm{P}}\,\underline{\mathrm{R}}(\mathrm{RR})$

$4-{}^{b}\overline{\mathrm{R}}\,\underset{\leftrightarrow}{\mathrm{P}}\,{}^{1}\mathrm{U}_{s}\mathrm{R}$ $\quad 4-{}^{b}\underline{\mathrm{RR}}\,\underset{\leftarrow}{\mathrm{P}}(\mathrm{RR})$ $\quad 4-\underset{\rightarrow}{\mathrm{P}}\,{}^{b}\mathrm{R}(\mathrm{RRR})$

$4-\underset{\rightarrow}{\mathrm{P}}\,{}^{b}\mathrm{U}_{s}(\mathrm{RR})$ $\quad 4-{}^{b}\mathrm{R}\,\underset{\leftarrow}{\mathrm{P}}(\mathrm{RRR})$ $\quad 4-{}^{b}\mathrm{R}\,\underset{\leftarrow}{\mathrm{C}}\,{}^{s}(\mathrm{RR})$

Actually, $4-{}^{b}\underline{\mathrm{RR}}(\mathrm{RRR})$ are first proposed by Zaltnov and Gosselin in 2001 [8].

Fig. 1 shows a $4-{}^{b}\overline{\mathrm{R}}\,\underset{\leftrightarrow}{\mathrm{P}}\,{}^{1}\mathrm{U}_{s}\mathrm{R}$ parallel mechanism. In each limb, counting from the base, the first revolute axis $\boldsymbol{\$}_{i1}$ and the first revolute axis of the universal pair, $\boldsymbol{\$}_{i3}$, are parallel to the base plane. The second prismatic pair $\boldsymbol{\$}_{i2}$ is perpendicular to the adjacent two revolute axes. The second revolute axis of the universal pair $\boldsymbol{\$}_{i4}$ and the last revolute axis $\boldsymbol{\$}_{i5}$ form a 2R spherical subchain. The four limb central

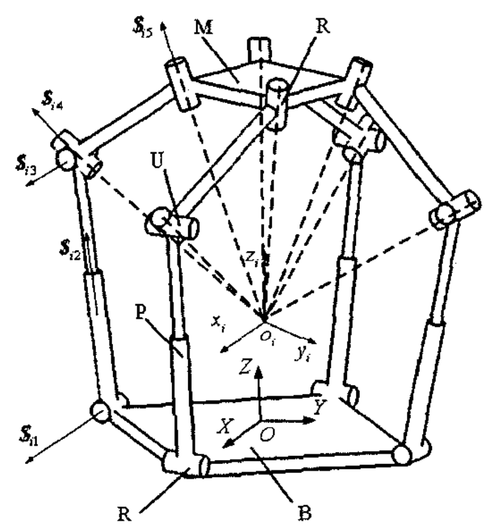

Fig. 1 4-DOF $4-{}^{b}\overline{\mathrm{R}}\,\underset{\leftrightarrow}{\mathrm{P}}\,{}^{1}\mathrm{U}_{s}\mathrm{R}$ parallel mechanism points coincide with each other and form a mechanism central point.

Set the central point as the origin of the limb frame, the z_i axis parallel to the normal of the base plane, the x_i axis parallel to the first revolute axis. The limb twist system in non-singular configuration is

$$\begin{aligned} &\boldsymbol{\$}_{i1} = (1\,0\,0\,;0\,b_1\,c_1) \quad \boldsymbol{\$}_{i2} = (0\,0\,0\,;0\,m_2\,n_2) \\ &\boldsymbol{\$}_{i3} = (1\,0\,0\,;0\,b_3\,c_3) \quad \boldsymbol{\$}_{i4} = (l_4\,m_4\,n_4\,;0\,0\,0) \\ &\boldsymbol{\$}_{i5} = (l_5\,m_5\,n_5\,;0\,0\,0) \end{aligned} \tag{8}$$

By calculating the screws reciprocal to the limb twist system in (8), we have

$$\boldsymbol{\$}^r_{i1} = (1\,0\,0\,;0\,0\,0) \tag{9}$$

The constraint force, $\boldsymbol{\$}^r_{i1}$, passes through the mechanism central point and is parallel to the first revolute axis, $\boldsymbol{\$}_{i1}$. The four limbs exert four constraint forces on the moving platform. The forces are coplanar in a constraint plane parallel to the base plane, and intersecting at the mechanism central point. Because the mechanism central point is fixed relative to the moving platform and the first revolute axis is set parallel to the base plane, the constraint plane is always parallel to the base plane. Thus, the standard base of the mechanism constraint system is given by (3) and restricts the two translational DOF of the moving platform in the base plane.

It should be noted that after the moving platform undergoes any feasible arbitrary finite motion, the first revolute axis and the third revolute axis are always parallel to the base plane, the second prismatic pair is always perpendicular to the two adjacent revolute axes; and the 2R subchain remains itself. Thus, the limb twist system of such a ${}^{b}\overline{\mathrm{R}}\underset{\leftrightarrow}{\mathrm{P}}\,{}^{1}\mathrm{U}_s\mathrm{R}$ leg in a general configuration always takes the form of that in (8). Consequently, the limb constraint system remains the one in (9). Moreover, the existence of the mechanism central point guarantees that the three limb constraint forces are always passing through a common point. Hence, the mechanism constraint system is unchanged and mechanism is not instantaneous and has finite mobility.

Because these constraints are not coaxial, $\lambda = 0$. Note that only two of the four limb constraint forces are linearly independent, two redundant constraints exist, and $v = 2$. Using (1), we have

$$M = 6(18-20-1)+20+2 = 4\,. \tag{10}$$

Note that unsymmetrical parallel mechanism can also be constructed by using the limb kinematic chains in above enumeration. For example, we can construct a 4-DOF $\underset{\rightarrow}{\mathrm{P}}\,{}^{b}\underline{\mathrm{RR}}(\mathrm{RR})\text{-}{}^{b}\underline{\mathrm{RRR}}(\mathrm{RR})\text{-}{}^{b}\overline{\mathrm{R}}\underset{\leftrightarrow}{\mathrm{P}}\,{}^{1}\mathrm{U}_s\mathrm{R}$ mechanism.

V. TYPE SYNTHESIS OF 4-DOF (3T1R) PARALLEL MANIPULATORS

Without loss of generality, we focus on the 4-DOF parallel mechanism with three translational DOF and one rotational DOF about the Z axis. The standard base of the mechanism twist system is given by

$$\begin{aligned} \boldsymbol{\$}_{m1} &= (0\,0\,1\,;0\,0\,0) \\ \boldsymbol{\$}_{m2} &= (0\,0\,0\,;1\,0\,0) \\ \boldsymbol{\$}_{m3} &= (0\,0\,0\,;0\,1\,0) \\ \boldsymbol{\$}_{m4} &= (0\,0\,0\,;0\,0\,1) \end{aligned}\,. \tag{11}$$

The standard base of the mechanism constraint system is given by

$$\begin{aligned} \boldsymbol{\$}^r_{m1} &= (0\,0\,0\,;1\,0\,0) \\ \boldsymbol{\$}^r_{m2} &= (0\,0\,0\,;0\,1\,0) \end{aligned}\,. \tag{12}$$

To form such a mechanism constraint system, the limb constraint system can be divided into two cases.

Case 1 *The limb kinematic chain consists of four kinematic pairs and exists two constraints.*

In this case, the standard base of the limb twist system is the same as (11). We can obtain the limb kinematic chain by linear combination of the four twists in (11).

The linear combination of $\boldsymbol{\$}_{m2}$, $\boldsymbol{\$}_{m3}$ and $\boldsymbol{\$}_{m4}$ yields a prismatic pair in arbitrary space direction. Only the two prismatic pair $\boldsymbol{\$}_{m2}$ and $\boldsymbol{\$}_{m3}$ can be transformed into revolute pairs by linear combination with $\boldsymbol{\$}_{m1}$, thereby producing two revolute pairs in the z_i – axis direction. The prismatic pair $\boldsymbol{\$}_{m4}$ cannot be transformed into revolute pair by linear combination with $\boldsymbol{\$}_{m1}$.

Based on the above analysis, we can obtain the usable *limb kinematic chain by linear combination of the five twists* in equation (11). The enumeration of the parallel mechanisms with full-cycle mobility is given as follows.

$4-{}^{Z}\mathrm{RPPP}$ $4-\mathrm{P}{}^{Z}\mathrm{RPP}$ $4-\mathrm{PP}{}^{Z}\mathrm{RP}$ $4-{}^{Z}\underline{\mathrm{RR}}\mathrm{PP}$ $4-{}^{Z}\mathrm{RPRP}$

$4\text{-}\mathrm{RPPR}$ $4-\mathrm{P}{}^{Z}\underline{\mathrm{R}}\mathrm{P}{}^{Z}\underline{\mathrm{R}}$ $4-\mathrm{P}{}^{Z}\underline{\mathrm{RR}}\mathrm{P}$ $4-{}^{Z}\underline{\mathrm{RRR}}\mathrm{P}$ $4-{}^{Z}\underline{\mathrm{RRPR}}$

$4-{}^{Z}\underline{\mathrm{R}}\mathrm{P}{}^{Z}\underline{\mathrm{RR}}$ $4-\mathrm{P}{}^{Z}\underline{\mathrm{RRR}}$ $4-{}^{Z}\mathrm{CPP}$ $4-\mathrm{P}{}^{Z}\mathrm{CP}$ $4-{}^{Z}\mathrm{C}{}^{Z}\underline{\mathrm{RR}}$

$4-{}^{Z}\mathrm{R}\,{}^{Z}\underline{\mathrm{C}}{}^{Z}R$ $4-{}^{Z}\mathrm{C}{}^{Z}\underline{\mathrm{R}}\mathrm{P}$ $4-\mathrm{P}{}^{Z}\mathrm{C}{}^{Z}\underline{\mathrm{R}}$ $4-\mathrm{P}{}^{Z}\underline{\mathrm{R}}{}^{Z}\mathrm{C}$

$4-{}^{Z}\underline{\mathrm{R}}{}^{Z}\mathrm{CP}$ $4-{}^{Z}\underline{\mathrm{R}}\mathrm{P}{}^{Z}\mathrm{C}$ $4-{}^{Z}\mathrm{C}{}^{Z}\underline{\mathrm{R}}\mathrm{P}$

Case 2 *The limb kinematic chain consists of five kinematic pairs and exerts one constraint.*

In this case, the limb kinematic chain only exerts one constraint on the moving platform. We need to add one twist to (11) to eliminate one corresponding constraint, that is, we add one kinematic pair to the limb kinematic chain represented by (11). Adding a twist $\boldsymbol{\$}_{i5} = (1\,0\,0;0\,0\,0)$ to equation (11) yields the new limb twist system

$$\begin{aligned} &\boldsymbol{\$}_{i1} = (0\,0\,1\,;0\,0\,0) \quad \boldsymbol{\$}_{i2} = (0\,0\,0\,;1\,0\,0) \\ &\boldsymbol{\$}_{i3} = (0\,0\,0\,;0\,1\,0) \quad \boldsymbol{\$}_{i4} = (0\,0\,0\,;0\,0\,1) \\ &\$_{i5} = (1\,0\,0\,;0\,0\,0) \end{aligned} \tag{13}$$

The limb constraint system reciprocal to (13) is

$$\$_{i1}^{r} = (0\,0\,0\,;0\,1\,0), \qquad (14)$$

which is a constraint couple in the y_i direction. Thus, all the limb constraint couples must be coplanar so that the standard base of the mechanism constraint system is the same as that in (12).

Using (13) as the standard base of the limb twist system, we can obtain the limb kinematic chain by linear combination of the five twists in (13).

By linear combination with $\$_{i5}$, the prismatic pairs $\$_{i3}$ and $\$_{i4}$ can also be transformed into revolute pairs whose axes is parallel to $\$_{i5}$, thereby they forming a 3R parallel subchain whose axes are parallel to the base plane. Note that by linear combination with $\$_{i1}$, $\$_{i2}$ can be transformed into a revolute pair whose axis is parallel to $\$_{i1}$, thereby they forming a 2R parallel subchain, whose axes are perpendicular to the base plane. Note that $\$_{i3}$ also can be transformed into a revolute pair by linear combination with $\$_{i1}$. Hence, $\$_{i1}$, $\$_{i2}$ and $\$_{i3}$ also can form a 3R parallel subchain while $\$_{i5}$ and $\$_{i4}$ form a 2R parallel subchain. Obviously, the limb kinematic chain contains no spherical pairs.

The universal pairs and the prismatic pairs can also be obtained by linear combination. For example, $\$_{i1}$ and $\$_{i5}$ can form a universal pair, in which one revolute axis $\$_{i1}$ is perpendicular to the base plane and the other revolute axis $\$_{i5}$ is parallel to the base plane. By linear combination with $\$_{i3}$ and $\$_{i4}$, $\$_{i2}$ can be transformed into a prismatic pair in arbitrary direction in space, namely, $\$_{i2} = (0\,0\,0\,;l_2\;m_2\;n_2)$. Moreover, by linear combination with $\$_{i1}$ and $\$_{i2}$, $\$_{i3}$ can be transformed into a revolute pair whose axis is parallel to $\$_{i1}$, namely, $\$_{i3} = (0\,0\,1\,;a_3\;b_3\;0)$; by linear combination with $\$_{i5}$ and $\$_{i3}$, $\$_{i4}$ can be transformed into a revolute pair whose axis is parallel to $\$_{i5}$, namely, $\$_{i4} = (1\,0\,0\,;0\;b_4\;c_4)$. $\$_{i3}$ and $\$_{i4}$ can also form a universal pair. By rearranging the sequence as $\$_{i1} - \$_{i5} - \$_{i2} - \$_{i4} - \$_{i3}$, a ${}^{z}\mathrm{U}_1\underset{\leftrightarrow}{\mathrm{P}}{}_1\mathrm{U}^{z}$ limb kinematic chain can be obtained. Obviously, all the universal joint planes in such a limb are perpendicular to the base plane.

The enumeration of the parallel mechanisms with full-cycle mobility is listed as follows.

- The limb chain include five kinematic pairs

1. No prismatic pairs in the limb

$4-{}^{z}\underline{\mathrm{RRR}}\,{}^{b}\overline{\mathrm{RR}}$ $\quad 4-{}^{b}\overline{\mathrm{RR}}\,{}^{z}\underline{\mathrm{RRR}}$ $\quad 4-{}^{z}\underline{\mathrm{RR}}_1\mathrm{U}^1\,{}^{b}\overline{\mathrm{R}}$

$4-{}^{b}\underline{\mathrm{R}}_1\mathrm{U}^1\,{}^{z}\overline{\mathrm{RR}}$ $\quad 4-{}^{z}\mathrm{U}_1{}^{b}\underline{\mathrm{R}}_1\mathrm{U}^{z}$

2. One prismatic pair in the limb

$4-\mathrm{P}\,{}^{z}\underline{\mathrm{RR}}\,{}^{b}\overline{\mathrm{RR}}$ $\quad 4-{}^{z}\underline{\mathrm{RR}}\mathrm{P}\,{}^{b}\overline{\mathrm{RR}}$ $\quad 4-\mathrm{P}\,{}^{z}\underline{\mathrm{R}}_1\mathrm{U}^1\,{}^{b}\overline{\mathrm{R}}$

$4-{}^{z}\underline{\mathrm{R}}\mathrm{P}_1\mathrm{U}^1\,{}^{b}\overline{\mathrm{R}}$ $\quad 4-{}^{z}\underline{\mathrm{R}}_1\mathrm{U}^1\mathrm{P}\,{}^{b}\overline{\mathrm{R}}$ $\quad 4-\mathrm{P}\,{}^{b}\underline{\mathrm{RR}}\,{}^{z}\overline{\mathrm{RR}}$

$4-{}^{b}\underline{\mathrm{RR}}\mathrm{P}\,{}^{z}\overline{\mathrm{RR}}$ $\quad 4-\mathrm{P}\,{}^{b}\underline{\mathrm{R}}_1\mathrm{U}^1\,{}^{z}\overline{\mathrm{R}}$ $\quad 4-{}^{b}\underline{\mathrm{R}}\mathrm{P}_1\mathrm{U}^1\,{}^{z}\overline{\mathrm{R}}$

$4-{}^{b}\underline{\mathrm{R}}_1\mathrm{U}^1\mathrm{P}\,{}^{z}\overline{\mathrm{R}}$ $\quad 4-\mathrm{P}\,{}^{z}\mathrm{R}\,{}^{b}\overline{\mathrm{RRR}}$ $\quad 4-{}^{z}\mathrm{R}\mathrm{P}\,{}^{b}\overline{\mathrm{RRR}}$

$4-\mathrm{P}\,{}^{z}\mathrm{U}^1\,{}^{b}\overline{\mathrm{RR}}$ $\quad 4-{}^{z}\mathrm{U}^1\mathrm{P}\,{}^{b}\overline{\mathrm{RR}}$ $\quad 4-\mathrm{P}\,{}^{b}\mathrm{R}\,{}^{z}\overline{\mathrm{RRR}}$

$4-{}^{b}\mathrm{R}\mathrm{P}\,{}^{z}\overline{\mathrm{RRR}}$ $\quad 4-\mathrm{P}\,{}^{b}\mathrm{U}^1\,{}^{z}\overline{\mathrm{RR}}$ $\quad 4-{}^{b}\mathrm{U}^1\mathrm{P}\,{}^{z}\overline{\mathrm{RR}}$

$4-\mathrm{P}\,{}^{b}\overline{\mathrm{RRR}}\,{}^{z}\mathrm{R}$ $\quad 4-{}^{b}\overline{\mathrm{RRR}}\mathrm{P}\,{}^{z}\mathrm{R}$ $\quad 4-\mathrm{P}\,{}^{b}\overline{\mathrm{RR}}\,{}^{z}\mathrm{U}^{b}$

$4-{}^{b}\overline{\mathrm{RR}}\mathrm{P}\,{}^{z}\mathrm{U}^{b}$ $\quad 4-\mathrm{P}\,{}^{z}\overline{\mathrm{RRR}}\,{}^{b}\mathrm{R}$ $\quad 4-{}^{z}\overline{\mathrm{RRR}}\mathrm{P}\,{}^{b}\mathrm{R}$

$4-\mathrm{P}\,{}^{z}\overline{\mathrm{RR}}\,{}^{b}\mathrm{U}^{z}$ $\quad 4-{}^{z}\overline{\mathrm{RR}}\mathrm{P}\,{}^{b}\mathrm{U}^{z}$ $\quad 4-{}^{z}\underline{\mathrm{R}}\mathrm{P}\underline{\mathrm{R}}\,{}^{b}\overline{\mathrm{RR}}$

$4-{}^{b}\underline{\mathrm{R}}\mathrm{P}\underline{\mathrm{R}}\,{}^{z}\overline{\mathrm{RR}}$ $\quad 4-{}^{z}\underline{\mathrm{R}}\mathrm{P}_1\mathrm{U}^1\,{}^{b}\overline{\mathrm{R}}$ $\quad 4-{}^{b}\underline{\mathrm{R}}\mathrm{P}_1\mathrm{U}^1\,{}^{z}\overline{\mathrm{R}}$

$4-{}^{z}\mathrm{U}_1\underset{\leftrightarrow}{\mathrm{P}}{}_1\mathrm{U}^{z}$

Fig. 2 shows such a $4-{}^{z}\mathrm{U}_1\underset{\leftrightarrow}{\mathrm{P}}{}_1\mathrm{U}^{z}$ parallel mechanism. Counting from the base, the first and the fifth revolute axes, $\$_{i1}$ and $\$_{i5}$, are perpendicular to the base plane and the moving platform plane, respectively. The second and the fourth revolute axes, $\$_{i2}$ and $\$_{i4}$, are parallel to the base plane. The prismatic pair axis, $\$_{i3}$, is perpendicular to the two adjacent revolute axes. Thus all the universal joint planes are perpendicular to the base plane.

Note that after any feasible finite motion, the universal joint planes in each branch keep being perpendicular to the base plane since the moving platform can only translate in space and rotate about the normal of itself. Thus, the mechanism constraint system remains unchanged and the mechanism is non-instantaneous and has finite mobility.

VI. TYPE SYNTHESIS OF 4-DOF (2T2R) PARALLEL MANIPULATORS

Similarly, we can perform the type synthesis of 4-DOF(2T2T) parallel mechanisms. Here, for simplicity, we

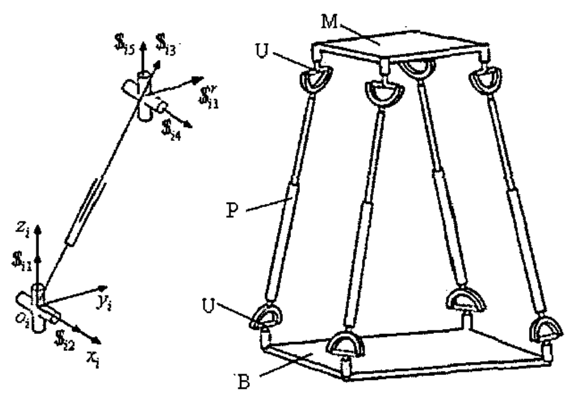

Fig. 2 4-DOF $4-{}^{z}\mathrm{U}_1\underset{\leftrightarrow}{\mathrm{P}}{}_1\mathrm{U}^{z}$ parallel mechanism only consider the situation when each limb consists of five 1-DOF pairs. Obviously, each limb must exert one constraint force on the moving platform and all these constraint forces must be form a 2-system while the combined effect of them must equal one force and one couple. Fig. 3 shows such a $2-{}^{b}\underline{\text{RRR}}(\text{RR})$ parallel mechanism, in which the two limb centers do not coincide. Each limb exerts a constraint force, which is passing the limb center and perpendicular on the base, on the plat-form. Thus, the two constraint forces are parallel in space and form a 2-system. The translation normal to the base and the rotation about the normal of the plane formed by the two forces are constrained.

In the mechanism shown in Fig. 3, each limb has two actuators. Generally, we expect only one actuator in each limb. Thus, we can add another two ${}^{b}\underline{\text{RRR}}(\text{RR})$ limbs to the mechanism. Note that the limb center of one new ${}^{b}\underline{\text{RRR}}(\text{RR})$ limb must coincide with point A and the limb center of the other new ${}^{b}\underline{\text{RRR}}(\text{RR})$ limb must coincide with point B.

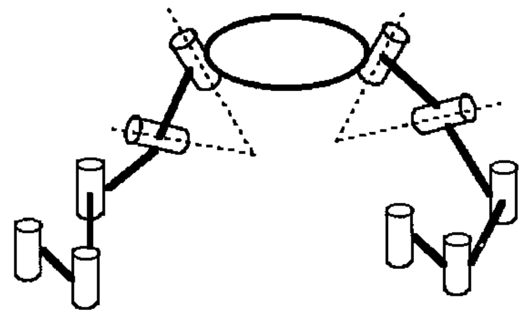

Fig. 3 4-DOF $2-{}^{b}\underline{\text{RRR}}(\text{RR})$ parallel mechanism

VII. CONCLUSION

The type synthesis of 4-DOF parallel manipulators with full-cycle mobility is performed systematically by constraint-synthesis method. Novel 4-DOF symmetrical and asymmetrical parallel manipulators are enumerated. In addition, some conclusions on the structural characteristics are obtained.

(1). Spherical pairs cannot be used in the limb kinematic chain of 4-DOF symmetrical parallel manipulators.

(2). In the parallel manipulator with three rotational DOF and one translational DOF along the Z axis, the prismatic pair must not be set parallel to the base plane and the revolute pair except those in the 2R or 3R spherical subchain must be parallel to the base plane.

(3). In the parallel manipulator with three translational DOF and one rotational DOF about the Z axis, the prismatic pair can be in any arbitrary direction in space. In each limb kinematic chain, the revolute pairs must be divided into two groups. The axes of one group must be perpendicular to the base plane and the axes of the other group must be parallel to the base plane.

VIII. ACKNOWLEDGMENTS

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IX. REFERENCES

[1] R. Clavel, "Delta, a fast robot with parallel geometry," in *Proceedings of the Int. Symp. On Industrial Robot*, Switzerland, 1988, pp. 91-100.

[2] R. E. Stamper, L. W. Tsai, and G. C. Walsh, "Optimization of a three DOF translational platform for well-conditioned workspace," in *Proceedings of IEEE International Conference on Robotics and Automation*, Albuquerque, New Mexico, 1997, pp. 3250-3255.

[3] C. M. Gosselin, "On the kinematic design of spherical 3-dof parallel manipulators," *International Journal of Robotics Research*, vol. 12(4), 1993, pp. 394-402.

[4] C. M. Gosselin, and E. St-Pierre, "Development and experimentation of a fast 3-dof camara-orienting device," *International Journal of Robotics Research*, vol.16(5), 1997, pp. 619-630.

[5] K. H. Hunt, "Structural kinematic of In-Parallel-Actuated robot arms," *Journal of Mechanisms, Transmissions and Automation in Design*, vol. 105, 1983, pp. 705-712.

[6] L. Rolland, "The Manta and the Kanuk: Novel 4-DOF Parallel Mechanisms for Industrial Handling," in *Proc. ASME Dynamic Systems and Control Division, IMECE'99 Conference*, Nashville, USA. Vol. 67, 1999, pp. 831-844.

[7] T. S. Zhao and Z. Huang, "A novel spatial four-dof parallel mechanism and its position analysis," *Mechanical Science and Technology*, vol. 19(6), 2000, pp. 927-929.

[8] D. Zlatanov and C. M. Gosselin. "A new parallel architecture with four degrees of freedom," in *Proceedings of the 2nd workshop on Computational Kinematics*, May 19-22, Seoul, Korea, 2001, pp. 57-66.

[9] Z. Huang and Q. C. Li, "Some novel lower-mobility parallel mechanisms," *in Proc. of ASME 2002 Design Engineering Technical Conferences*, Montreal, Canada, MECH-34299.

[10] Z. Huang and Q. C. Li, "General Methodology for Type Synthesis of Lower-Mobility Symmetrical Parallel Manipulators and Several Novel manipulators," *International Journal of Robotics Research*, vol. 21(2), 2002, pp. 131-145.

[11] R. S. Ball, *The Theory of Screws*, Cambridge University Press, 1900.

[12] K. H. Hunt, *Kinematic Geoemtry of Mechanisms*. Oxford University Press, 1978.