151-0591-00 **Control Systems I** (Autumn 2018) **Exercise Set 11**

Topic: **PID Control**

Discussion: 7.12.2018

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Learning objectives: The student can

- Design a PID controller.
- Describe the characteristics of the P, I and D part.
- Classify when to use a P, I and D part.
- Use different tuning strategies for PID controllers.

Instructional video: `https://www.youtube.com/watch?v=4Y7zG48uHRo`

Background

Motivation

- The PID controller is the most common controller in industry.
- 95% of industrial systems are controlled by PID controllers.
- PID control easily supports additional control blocks to further enhance performance.

PID Controler Parts

P part:

As the proportional gain increases,

- The closed-loop system becomes more oscillatory.
- The steady-state error decreases.
- The response becomes faster.
- The sensitivity to noise increases.

I part:

As the integral gain increases,

- The closed-loop system becomes more oscillatory (warning!).
- The steady-state error decreases.
- The response becomes faster.
- The sensitivity to noise **does not** change.

D part:

As the differential gain increases,

- The steady-state error is not affected.
- The response becomes less oscillatory, but potentially slower.
- The sensitivity to noise increases!

Ziegler and Nichols design rules:

(roll-off time constant $\tau \approx T_d/10$)
The design rules by Ziegler and Nichols assume a plant model which is approximated by the following transfer function

$$P(s) \approx \tfrac{k}{\tau \cdot s+1} \cdot e^{-T \cdot s}$$

and the ratio $T/(T+\tau)$ should be small (< 0.3).
In practice the design rules by Ziegler/Nichols oftentimes provide a good starting point when coming up with a control design. When a plant model is known, more targeted approaches such as loop shaping will generally yield better results.

type	k_p	T_i	T_d
P	$0.50 \cdot k_p^*$	$\infty \cdot T^*$	$0 \cdot T^*$
PI	$0.45 \cdot k_p^*$	$0.85 \cdot T^*$	$0 \cdot T^*$
PD	$0.55 \cdot k_p^*$	$\infty \cdot T^*$	$0.15 \cdot T^*$
PID	$0.60 \cdot k_p^*$	$0.5 \cdot T^*$	$0.125 \cdot T^*$

Table 1: Zigler/Nichols design rules

The Ziegler/Nichols PID description is equivalent to the PID discussed in the lecture. Instead of k_i, k_d, the parameters T_i, T_d are used as shown below.

$$C(s) = k_p + \frac{k_i}{s} + k_d \cdot s = k_p(1 + \frac{1}{T_i \cdot s} + T_d \cdot s) \tag{1}$$

Additional Readig (for the interested reader)

- Astrom Karl Johan. & Murray, R.M., 2008. Chapter 10. In Feedback systems: an introduction for scientists and engineers. Princeton: Princeton University Press.
- Guzzella, L., 2011. Chapter 11.2. In Analysis and synthesis of single-input single-output control systems. Zürich: vdf Hochschulverlag AG an der ETH Zurich.
- Haager, W., 2016. Chapter 4.3.2. In Regelungstechnik - kompetenzorientiert. Wien: Verlag Hoelder-Pichler-Tempsky. (German)
- Haager, W., 2007. Chapter 2.3.2. In Regelungstechnik. Wien: Verlag Hoelder-Pichler-Tempsky. (German)

Exercise 1 (Ziegler/Nichols I)

Let $P(s)$ be an arbitrary, asymptotically stable plant for which a PI-controller was designed using the design rules of Ziegler and Nichols. Determine the point $L(j\omega^*)$ of the open-loop transfer function (Please note: ω^* is the frequency at which the : $\angle P(j\omega^*) = 180°$. In experiments, the control gain k_p is increased until a steady-state oscillation is achieved. The critical oscillation period $T^* = \frac{2\pi}{\omega^*}$ and the critical gain k_p^* at this point on the stability boundary are then used to determine the parameters using the Ziegler/Nichols design rules). *Hint: Write $C(j\omega)$ and $P(j\omega)$ as a combination of its real part and imaginary part.*

Exercise 2 (Duckietown)

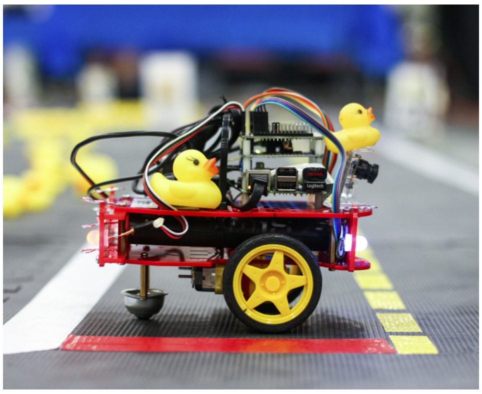

Figure 1: *Duckiebot*

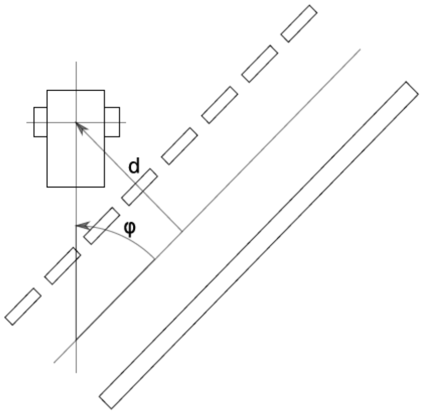

Figure 2: *Lane Pose of the Duckiebot.*

A Duckiebot is a state of the art self driving car development platform. (see figure 1) You and your colleague got the task to design a controller that keeps your fleet of duckies on the road using a PI controller. Your colleague is really good at modeling and already provides you a state space representation of the duckiebot. The continous-time model of a Duckiebot with the states $\vec{x} = [d\ \varphi]^T$, input $u = \omega$ and output y with the states defined as in figure 2 looks like this:

$$\dot{\vec{x}} = A\vec{x} + Bu = \begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad \vec{y} = C\vec{x} = \begin{bmatrix} 6 & 1 \end{bmatrix} \vec{x}$$

a) Calculate the transfer function of the system.

b) We are assuming that $v = 0.22m/s$. Find k_P such that $L(s) = P(s)C(s)$ has a crossover frequency of approximately $4.2rad/s$.

c) Calculate the phase margin of $L(s)$. Use the k_P calculated in part b and $k_I = 1$. (Hint: $\omega_c = 4.2rad/s$)

d) Your boss just popped in and gave you this transfer function of PID controller.

$$C(s) = \frac{s^2 + 5s + 2}{2s}$$

He wants you to tell him what the control parameters (k_p, T_i, T_d) of the PID controller in the loop are.

Exercise 3 (Homework: Ziegler/Nichols II)

a) You are given the Bode plot of a plant $P_1(s)$ without controller (see figure 3).

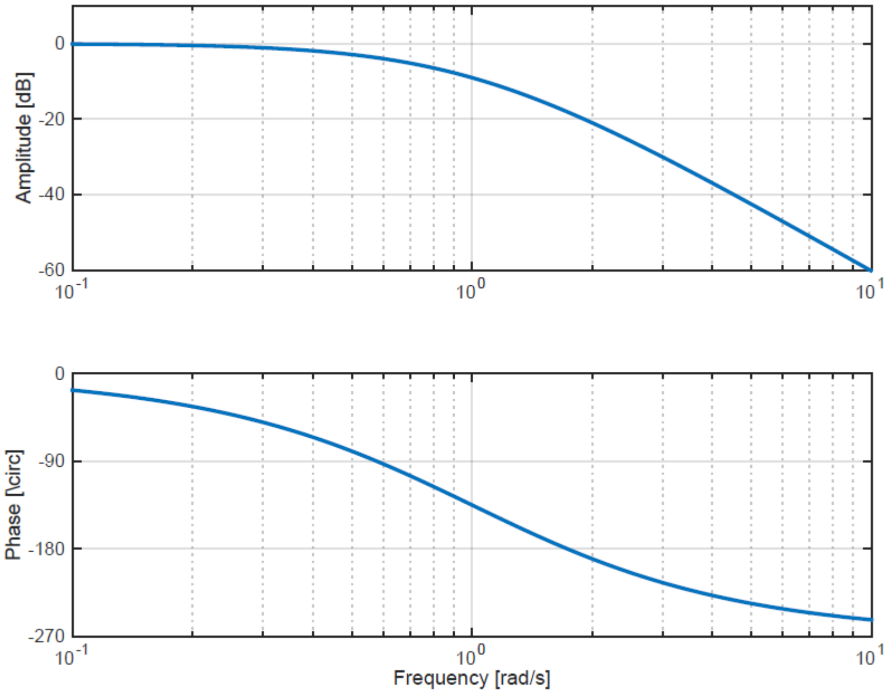

Figure 3: Bode plot of $P_1(s)$

Find the critical frequency ω^*, the critical gain k_p^* and $T*$. With these values, calculate the parameters (k_p, T_i) of a PI-controller following the Ziegler/Nichols design rules.

b) After you determined the parameters of the controller, your colleague realizes he sent you the wrong Bode plot. To be sure that you now have the right model, your colleague modeled the plant and provides you with the following plant transfer function $P_2(s)$:

$$P_2(s) = \frac{100}{(s+10)^2} \cdot e^{-s \cdot \frac{\pi}{20}}$$

Now you are asked to design a PID controller for this plant. Calculate the Ziegler/Nichols PID parameters (k_p, T_i, T_d) analytically for the plant (Hint: Start with the condition in exercise question 2 at the beginning of the exercise set).

c) Your boss gives you access to a machine but did not have the time to derive a model for it. She does however know that the system is asymptotically stable. Please design a PID-controller for the system using the Ziegler/Nichols design rules by empirically determining the critical gain k_p^* and critical oscillation period T^*.
A Python script to simulate the step response of the system is provided to you. By varying the input controller to the system, the critical gain k_p and critical oscillation period T^* can be obtained.

Exercise 4 (Homework: Exam Type Question)

a) The plot below shows the step responses of different variations of PID controllers (P, PI, PD, PID). Which step response in the figure 4 belongs to the PD-controller?

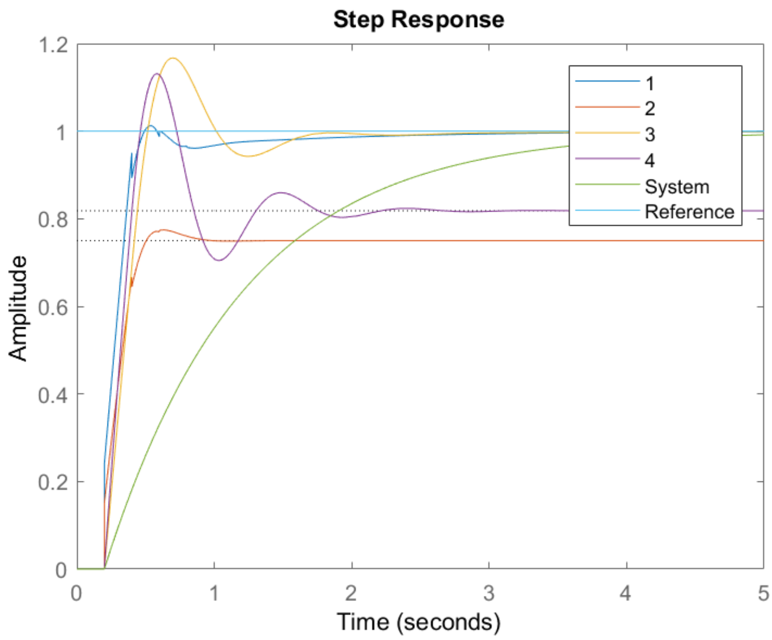

Figure 4: Step responses of PID controllers

b) Your friend created Nyquist plots of a P, a PD and a PI controller for a second-order stable plant with a delay and only real-poles. Which Nyquist plot refers to which controller?

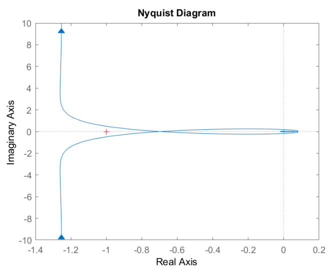

Figure 5: Nyquist 1

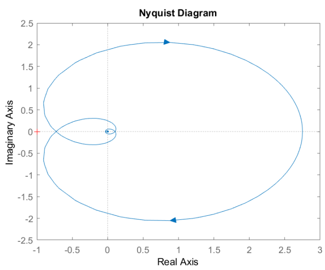

Figure 6: Nyquist 2

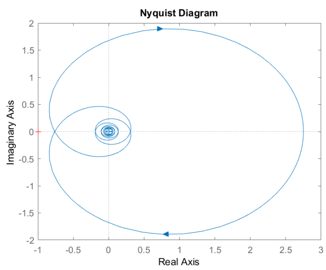

Figure 7: Nyquist 3