Spline-Based Time-Optimal Control for Smooth Trajectory Generation of CNC Machines with Geometric Constraints

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***Abstract*—High demands in a modern industry require computerized numerical control machines to accurately track the desired workpiece contour in a minimum motion time for productivity. Generally, minimum motion time and accuracy are contradictory; therefore, the machine slows down tracking in the workpieces with strict geometric constraints. This study presents time-optimal trajectory generation of the workpieces, focusing on straight-line, circle, and spline contour segments by considering fitting errors as geometric constraints. For machining purposes, the given workpiece is divided into several G-code segments represented by the piecewise continuous functions for trajectory generation. In this study, the cubic B-spline function represents each segment trajectory, and the optimal control problem is formulated. The effectiveness is investigated considering different geometric constraints, and the motion time of the resulting trajectories are compared and discussed.**

***Index Terms*—B-Spline, Computerized Numerical Control (CNC) Machines, Geometric Constraints, Time-Optimization.**

I. Introduction

Computerized numerical control (CNC) machines play an important role in the modern manufacturing industry for the productivity and precision of the workpieces [1], [2]. Due to the high demands of complicated workpieces, the geometric path is created in the computer-aided design software and converted into several G-code segments such as straight lines (G01), circles (G02/G03), and spline contours (G05). These G-code segments are usually represented by the smooth piecewise continuous functions, and the trajectory planning is considered to track the required workpiece contour considering kinematic and dynamic limitations of the machines and generate reference trajectories for CNC controllers [3], [4].

Methods for minimum time trajectory generation have been investigated in the literature to increase the productivity. Generally, there are two approaches for motion planning: decoupled, where the optimal control problem (OCP) is separated into the geometric path planning [5], [6] and feedrate generation [7]–[9], and the coupled approach, where the OCP is solved in a single step. Since decoupled approaches require the exact contour for trajectories, coupled OCPs are formulated considering the accuracy of the workpiece as a constraint or as an objective function. Nshama *et al.* [10] proposed bi-objective trajectory generation for the linear segments considering the tolerance at cornering motions. Lu *et al.* [11] divided a given curve into several linear segments, which were represented by the S-curve trajectories with axial constraints. In [12], the high-speed cornering motion of CNC machines was presented along with the tool path contour error under axial acceleration limits.

Splines are the piecewise continuous functions that are widely used in geometric modeling of curves and surfaces [13]. Due to the continuity, flexibility, and exact representation of complicated profiles, splines are extensively applied to parameterize the primitive motion trajectory of an OCP [14]. Gasparetto and Zanotto [15], [16] proposed a time-jerk trajectory generation of robot manipulators, satisfying kinematic limits by the convex hull property of B-splines. In [17], a smooth cornering approach for two linear segments with a required tolerance is implemented by NURBS parameterization. Mercy *et al.* [18] proposed a method for time-optimal trajectory generation for the workpiece consisting of straight-line and circle segments. However, spline contour segments were not addressed in the formulation of an OCP.

This study presents a simple and effective approach for time-optimal trajectory generation of CNC machines, considering not only for straight-line (G01) and circle (G02/G03) segments but also for spline contour (G05) segments for a more complicated profile. The cubic B-spline parameterizes the motion trajectory, and the fitting error is proposed as the closest distance between the discretized via-points and the initialized curve of each G-code segment. Moreover, initial and final states, the velocity and acceleration continuity between segment trajectories, and the machine kinematic limits satisfaction are considered as equality and inequality constraints of an OCP. Thereafter, time-optimal solutions are formulated using the sequential quadratic programming (SQP) [19].

The rest of the paper is organized as follows: Section II describes the general problem formulation of time-optimal trajectory for multiple segments. Section III shows the cubic B-spline parameterization, followed by the determination of fitting errors and the kinematic constraints along the trajectory. The effectiveness of the proposed method is validated by the simulation with a complicated geometric profile considering different limits on fitting errors in Section IV. Conclusion and future work of this paper are presented in Section V.

II. Problem Formulation

The geometric path of CNC machines consists of the three-dimensional position of the workpiece $\boldsymbol{s}(t) = [X(t), Y(t), Z(t)]^T$, where t is the motion time. The geometric path is generally divided into several G-code segments, such as straight-line segments (G01), circle segments (G02/G03), and spline contour segments (G05), and are fitted by the piecewise continuous functions to have the continuity in acceleration or jerk of the trajectory. Moreover, the velocity, acceleration, and jerk values must satisfy the upper and lower kinematic limits of CNC machines as follows:

$$\begin{aligned} \boldsymbol{v}_{\text{min}} &\leqslant \dot{\boldsymbol{s}}(t) \leqslant \boldsymbol{v}_{\text{max}}, \\ \boldsymbol{a}_{\text{min}} &\leqslant \ddot{\boldsymbol{s}}(t) \leqslant \boldsymbol{a}_{\text{max}}, \\ \boldsymbol{j}_{\text{min}} &\leqslant \dddot{\boldsymbol{s}}(t) \leqslant \boldsymbol{j}_{\text{max}}. \end{aligned} \tag{1}$$

where, $\dot{\boldsymbol{s}}(t)$, $\ddot{\boldsymbol{s}}(t)$, and $\dddot{\boldsymbol{s}}(t)$ are the velocity, acceleration, and jerk of the trajectory, respectively. Moreover, the velocity and acceleration must be zero at the start and end of the trajectory, satisfying the initial and final positions as follows:

$$\begin{aligned} &\boldsymbol{s}(0) = \boldsymbol{q}_0,\ \boldsymbol{s}(T_{\text{tot}}) = \boldsymbol{q}_f, \\ &\dot{\boldsymbol{s}}(0) = \boldsymbol{0},\ \ \dot{\boldsymbol{s}}(T_{\text{tot}}) = \boldsymbol{0}, \\ &\ddot{\boldsymbol{s}}(0) = \boldsymbol{0},\ \ \ddot{\boldsymbol{s}}(T_{\text{tot}}) = \boldsymbol{0}. \end{aligned} \tag{2}$$

where, $T_{\text{tot}} = \sum_{i=0}^{N} t_i$ represents the total motion time of the $N+1$ segment trajectories, $\boldsymbol{q}_0$ and $\boldsymbol{q}_f$ are the initial and final positions, respectively. For smooth transitions between segment trajectories, the equality constraints for position, velocity, and acceleration are determined as follows:

$$\begin{aligned} \boldsymbol{s}_i(t_i) &= \boldsymbol{s}_{i+1}(0),\ i = 0,1,2,...,N-1, \\ \dot{\boldsymbol{s}}_i(t_i) &= \dot{\boldsymbol{s}}_{i+1}(0),\ i = 0,1,2,...,N-1, \\ \ddot{\boldsymbol{s}}_i(t_i) &= \ddot{\boldsymbol{s}}_{i+1}(0),\ i = 0,1,2,...,N-1. \end{aligned} \tag{3}$$

For the accuracy of the generated workpiece, each G-code segment is discretized into a number of $m+1$ points so-called via-points, and the positions should satisfy the following geometric constraint at the specific via-points of the workpiece as follows:

$$\begin{aligned} &\boldsymbol{g}_{\text{min}} \leqslant \boldsymbol{s}_i(t_{k,i}) \leqslant \boldsymbol{g}_{\text{max}},\ k = \{0,1,2,...,m\}, \\ &i = \{0,1,2,...,N\}. \end{aligned} \tag{4}$$

where, $\boldsymbol{g}_{\text{min}}$ and $\boldsymbol{g}_{\text{max}}$ are the minimum and maximum geometric limits, respectively. If the upper and lower kinematic limits are considered symmetric for all axes, the original OCP for minimizing the total motion time is formulated as follows:

$$\min_{\boldsymbol{s}(.),\, t_0,...,t_N} \sum_{i=0}^{N} t_i, \tag{5}$$

subject to (2)-(4), and

$$|\dot{\boldsymbol{s}}_i(t)| \leqslant \boldsymbol{v}_{\text{lim}},\ \forall\, t \in [0, t_i],\ i = 0,1,2,...,N. \tag{6}$$

$$|\ddot{\boldsymbol{s}}_i(t)| \leqslant \boldsymbol{a}_{\text{lim}},\ \forall\, t \in [0, t_i],\ i = 0,1,2,...,N. \tag{7}$$

$$|\dddot{\boldsymbol{s}}_i(t)| \leqslant \boldsymbol{j}_{\text{lim}},\ \forall\, t \in [0, t_i],\ i = 0,1,2,...,N. \tag{8}$$

where, $\boldsymbol{v}_{\text{lim}}$, $\boldsymbol{a}_{\text{lim}}$, and $\boldsymbol{j}_{\text{lim}}$ are the absolute symmetric velocity, acceleration, and jerk limits of the CNC machine, respectively.

III. Smooth Trajectory Generation

A. Parameterization by Cubic B-splines

This paper focuses on the time-optimal trajectory generation of CNC machines, whose contours are considered as straight line, circle, and spline segments in the x- and y-directions. In order to fit the discretized via-points for each segment $\boldsymbol{D}_i = [\boldsymbol{d}_{0,i}, \boldsymbol{d}_{1,i}, \boldsymbol{d}_{2,i}, ..., \boldsymbol{d}_{m,i}]^T$, and satisfy the velocity and acceleration constraints in (2) and (3), the cubic B-spline function with the order ($q = 4$) is used to represent the trajectory for each segment as follows:

$$\boldsymbol{s}_i(u) = \sum_{j=0}^{m+4} B_{j,q}(u)\, \boldsymbol{c}_{j,i}^{\text{pos}},\ \forall u \in [0,1],\ i = 0,1,2,...,N. \tag{9}$$

with

$$B_{j,1}(u) = \begin{cases} 1, & \text{for } \ u_j \leqslant u < u_{j+1}. \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

$$B_{j,q}(u) = \frac{(u-u_j)\, B_{j,q-1}(u)}{(u_{j+q-1}-u_j)} + \frac{(u_{j+q}-u)\, B_{j+1,q-1}(u)}{(u_{j+q}-u_{j+1})}. \tag{11}$$

where, $\boldsymbol{s}_i(u)$ is the position with the dimensionless parameter u, $B_{j,q}(u)$ is the basis function, and $\boldsymbol{c}_{j,i}^{\text{pos}}$ represents the vector position control points of the trajectory. Thereafter, the knot vector for (9) are constructed as follows:

$$\boldsymbol{u} = [\,\underbrace{0,\ ...,\ 0}_{q\text{-times}},\ u_1, u_2, ..., u_{m-1},\ \underbrace{1,\ ...,\ 1}_{q\text{-times}}\,]. \tag{12}$$

At the start and end of the knot vector in (12), the knots are q-times clamped; therefore, the first and last control points $\boldsymbol{c}_{0,i}^{\text{pos}}$ and $\boldsymbol{c}_{m+4,i}^{\text{pos}}$ are equal to the position values at $u = 0$ and $u = 1$, respectively. The inner knots are considered as uniformly distributed between the interval $[0,1]$.

The r^{th} derivatives of B-spline are determined as follows:

$$\boldsymbol{s}_i^r(u) = \sum_{j=0}^{m+4} B_{j,q}^r(u)\ \ \boldsymbol{c}_{j,i}^{\text{pos}},\ \ r = 1,2,3. \tag{13}$$

with their basis functions given by

$$B_{j,q}^r(u) = (q-1)\left[\frac{B_{j,q-1}^{r-1}(u)}{u_{j+q-1}-u_j} - \frac{B_{j+1,q-1}^{r-1}(u)}{u_{j+q}-u_{j+1}}\right]. \tag{14}$$

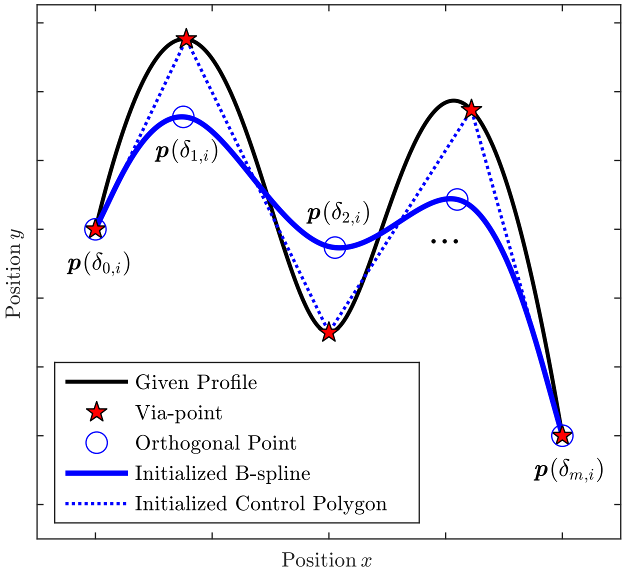

Fig. 1. Demonstration of orthogonal points from the initialized B-spline to the discretized via-points.

B. Initialization and Determination of the Fitting Error

In order to fit the G-code segments described in Section III-A by the cubic B-spline, we have to determine the error between the fitted curve and the discretized via-points for each segment. In this study, the fitting error is considered as the closest distance, where the initialized curve is orthogonal to each via-point. These fitting errors will be considered as constraints of an OCP in Section III-C.

Firstly, discretized via-points are assumed as control points of B-splines, repeating $(q-1)$ times at both ends, and the initialization of the curve is implemented. After that, the orthogonal distance to each via-point is computed (See Fig. 1) by solving the following nonlinear equation as in [5]:

$$\begin{aligned}&(\boldsymbol{d}_{k,i}-\boldsymbol{p}_i(u))\cdot\boldsymbol{p}_i^{(1)}(u)=0,\\&k=\{0,1,2,...,m\},\ i=\{0,1,2,...,N\}.\end{aligned} \tag{15}$$

where, $\boldsymbol{p}_i(u)$ and $\boldsymbol{p}_i^{(1)}(u)$ are the position and first derivative of the initialized curve, respectively. Equation (15) is solved numerically by the Newton's iteration method and the curve parameters that satisfy the solutions are determined as $\boldsymbol{\delta}_i = [\delta_{0,i}, \delta_{1,i}, \delta_{2,i}, ..., \delta_{m,i}]^T$.

Therefore, the k^{th} fitting error for each i^{th} segment for the OCP is defined as follows:

$$\epsilon_{k,i}=||\boldsymbol{d}_{k,i}-\boldsymbol{s}(\delta_{k,i})||_2. \tag{16}$$

where, $||.||_2$ is the Euclidean norm.

C. Spline-Based Time-Optimal Trajectory Generation

The generation of time-optimal trajectories must satisfy the upper and lower kinematic limits of CNC machines for all horizons. Here, the constraints on velocity, acceleration, and jerk of the trajectory are determined using the convex hull property of B-splines [13]. A B-spline curve and its derivatives are contained within the convex hulls, determined by its control points; therefore, we determine the respective control points for each i^{th} segment trajectory as follows:

$$\boldsymbol{c}_{j,i}^{\text{vel}}=\frac{(q-1)}{(u_{j+q}-u_{j+1})}(\boldsymbol{c}_{j+1,i}^{\text{pos}}-\boldsymbol{c}_{j,i}^{\text{pos}}),\ j=0,1,...,m+3. \tag{17}$$

$$\boldsymbol{c}_{j,i}^{\text{acc}}=\frac{(q-2)}{(u_{j+q}-u_{j+2})}(\boldsymbol{c}_{j+1,i}^{\text{vel}}-\boldsymbol{c}_{j,i}^{\text{vel}}),\ j=0,1,...,m+2. \tag{18}$$

$$\boldsymbol{c}_{j,i}^{\text{jerk}}=\frac{(q-3)}{(u_{j+q}-u_{j+3})}(\boldsymbol{c}_{j+1,i}^{\text{acc}}-\boldsymbol{c}_{j,i}^{\text{acc}}),\ j=0,1,...,m+1. \tag{19}$$

where, $\boldsymbol{c}_{j,i}^{\text{vel}}$, $\boldsymbol{c}_{j,i}^{\text{acc}}$, and $\boldsymbol{c}_{j,i}^{\text{jerk}}$ denote the control points for velocity, acceleration, and jerk of each segment, respectively. For fluent transitions between segment trajectories, the following continuity constraints are adopted as in [18]:

$$\begin{aligned}\boldsymbol{s}_i(1)&=\boldsymbol{s}_{i+1}(0),\\\dot{\boldsymbol{s}}_i(1)\cdot t_{i+1}&=\dot{\boldsymbol{s}}_{i+1}(0)\cdot t_i,\\\ddot{\boldsymbol{s}}_i(1)\cdot t_{i+1}^2&=\ddot{\boldsymbol{s}}_{i+1}(0)\cdot t_i^2.\end{aligned} \tag{20}$$

Therefore, the OCP for smooth and time-optimal trajectory generation considering fitting errors as geometric constraints is formulated as follows:

$$\min_{\boldsymbol{\phi}}\sum_{i=0}^{N}t_i, \tag{21}$$

with

$$\boldsymbol{\phi}=\{\boldsymbol{c}_{0,i}^{\text{pos}},...,\boldsymbol{c}_{m+4,i}^{\text{pos}},t_0,...,t_N\}. \tag{22}$$

subject to

$$\begin{aligned}&\boldsymbol{s}_0(0)=\boldsymbol{q}_0,\ \boldsymbol{s}_N(1)=\boldsymbol{q}_f,\\&\dot{\boldsymbol{s}}_0(0)=\mathbf{0},\ \ \dot{\boldsymbol{s}}_N(1)=\mathbf{0},\\&\ddot{\boldsymbol{s}}_0(0)=\mathbf{0},\ \ \ddot{\boldsymbol{s}}_N(1)=\mathbf{0},\\&|\boldsymbol{c}_{j,i}^{\text{vel}}|\leqslant\boldsymbol{v}_{\text{lim}}\cdot t_i,\ j=\{0,1,...,m+3\},i=\{0,1,...,N\},\\&|\boldsymbol{c}_{j,i}^{\text{acc}}|\leqslant\boldsymbol{a}_{\text{lim}}\cdot t_i^2,\ j=\{0,1,...,m+2\},i=\{0,1,...,N\},\\&|\boldsymbol{c}_{j,i}^{\text{jerk}}|\leqslant\boldsymbol{j}_{\text{lim}}\cdot t_i^3,\ j=\{0,1,...,m+1\},i=\{0,1,...,N\},\\&\epsilon_{k,i}\leqslant\epsilon_{\text{lim}},k=\{0,1,...,m\},i=\{0,1,...,N\},\\&\boldsymbol{s}_i(1)=\boldsymbol{s}_{i+1}(0),i=\{0,1,...,N\},\\&\dot{\boldsymbol{s}}_i(1)\cdot t_{i+1}=\dot{\boldsymbol{s}}_{i+1}(0)\cdot t_i,i=\{0,1,...,N\},\\&\ddot{\boldsymbol{s}}_i(1)\cdot t_{i+1}^2=\ddot{\boldsymbol{s}}_{i+1}(0)\cdot t_i^2,i=\{0,1,...,N\}.\end{aligned} \tag{23}$$

where, ϵ_{lim} is the absolute limit of fitting error of the geometric path.

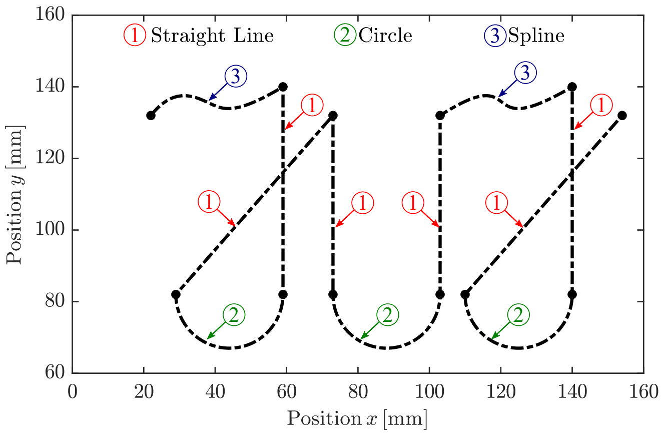

Fig. 2. TUT geometric profile consisting of straight-line (G01), circle (G02/G03), spline contour (G05) segments.

IV. Optimization Results

A. Calculation Conditions

In order to investigate the performance of the proposed method, simulation is conducted in this Section. For solving the OCP described in Section III-C, the rolling horizon approach [20] is used, where the solutions for the first n segments are computed, and the horizon is shifted to $n+1$ segments. The time-optimal solutions are calculated using the SQP, ("fmincon" function) in MATLAB® environment of a laptop computer with Windows 10 64-bit Intel(R) Core(TM) i7-7500U CPU @2.70 GHz and memory of 8 GB. The proposed method considers the fitting error constraints only at the discretized via-points of the given G-code segments.

For the application of the algorithm, we choose the letter profile TUT, representing the acronym of the Toyohashi University of Technology, which consists of 6 straight-line segments, 3 circle segments, and 2 spline contour segments as shown in Fig 2. Each G-code segment is discretized into 10 via-points, and the cubic B-spline with 14 control points is used to represent each segment trajectory of the TUT profile. The absolute symmetric velocity, acceleration, and jerk limits are set as $\boldsymbol{v}_{\text{lim}} = [80, 80]\,\text{mm/s}$, $\boldsymbol{a}_{\text{lim}} = [500, 500]\,\text{mm/s}^2$, and $\boldsymbol{j}_{\text{lim}} = [10000, 10000]\,\text{mm/s}^3$, respectively. Three segments $(n = 3)$ are used for simultaneous optimization by the rolling horizon approach. The initial guess for solving the OCP is the same as the control points of initialized curve in Section III-B. In this study, various fitting error constraints $\epsilon_{\text{lim}} = 0.2\,\text{mm}$ and $\epsilon_{\text{lim}} = 1\mu\text{m}$ are chosen for formulating the trajectory to investigate the relation between the total motion time and the accuracy of the workpiece.

B. Application Results

Fig. 3 illustrates the x- and y-axial positions of the optimal paths with the various fitting errors. It is observed that both optimal paths can approximate the contour of the given TUT profile, satisfying initial and final boundary positions, including the fitting error constraint of the path for each via-point.

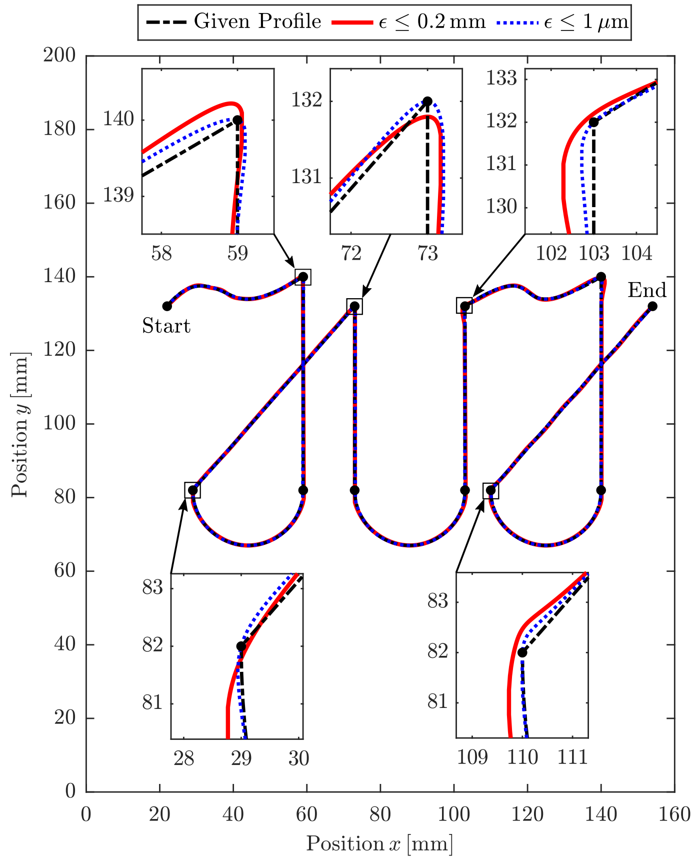

Fig. 3. Comparison of optimal paths and the cornering positions with different fitting errors.

The given geometric profile has no fluent connection between G-code segments, resulting in the discontinuity in velocity and acceleration of the trajectory. On the other hand, the proposed method satisfies the smooth transitions of the path between segment trajectories by the modification of position control points which considers not only fitting error limits but also kinematic limits of the machines. The optimal solution with the error constraint $1\mu\text{m}$ is more accurate than the solution with $0.2\,\text{mm}$. The resulting path passes through the corner points and all via-points within the designated accuracy, and is closer to the given G-code segments.

Fig. 4 shows that the proposed method can fit the straight-line (G01), circle (G02/G03), and spline contour (G05) segments compared to the via-points of the given profile while assuring the time-optimality of the generated trajectory. The time-optimal solution usually finds the shorter distance of the path with a small curvature; therefore, the algorithm can accurately fit the G01 segments. For G02/G03 and G05 segments, the optimal paths are slightly deviated from the via-points and in-between the via-points, depending on the limitation of the constraints of the problem. Although increasing via-points in an OCP can preserve the finer approximation of the given path, more optimization variables and constraints may increase the computational complexity of the problem.

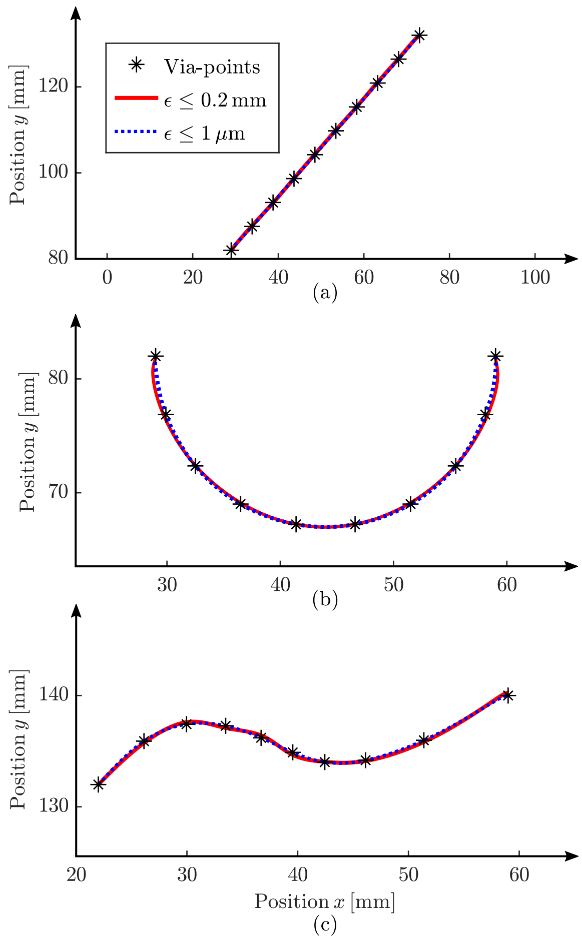

Fig. 4. Fitting for (a) straight-line (G01) (b) circle (G02/G03) and (c) spline contour (G05) segments of the TUT profile with different fitting errors of $0.2\,\mathrm{mm}$ and $1\,\mu\mathrm{m}$ at the via-points.

TABLE I
COMPARISON OF MOTION TIMES FOR SEVERAL G-CODE SEGMENTS (FIG. 4) AND THE TOTAL MOTION TIME (T_{tot}) ACCORDING TO DIFFERENT FITTING ERROR CONSTRAINTS (ϵ_{lim}).

ϵ_{lim}	Fig. 4a	Fig. 4b	Fig. 4c	T_{tot}
$0.2\,\mathrm{mm}$	$0.802\,\mathrm{s}$	$0.674\,\mathrm{s}$	$0.953\,\mathrm{s}$	$8.933\,\mathrm{s}$
$1\,\mu\mathrm{m}$	$0.836\,\mathrm{s}$	$0.747\,\mathrm{s}$	$1.008\,\mathrm{s}$	$9.581\,\mathrm{s}$

In order to investigate the relationship between the motion time and the accuracy of the workpiece, the motion times according to different fitting error constraints are compared in Table I. The optimal path with the accuracy of $0.2\,\mathrm{mm}$ has more flexibility dealing with the control points of B-splines than the path with $1\,\mu\mathrm{m}$ accuracy; therefore, the respective motion times of the straight-line, circle, and spline contour segments are approximately 4%, 9.77%, and 5.45% faster, respectively. Therefore, as a result, the total motion time of $0.2\,\mathrm{mm}$ accuracy is 6.76% faster than the path with $1\,\mu\mathrm{m}$ accuracy under the same kinematic limits.

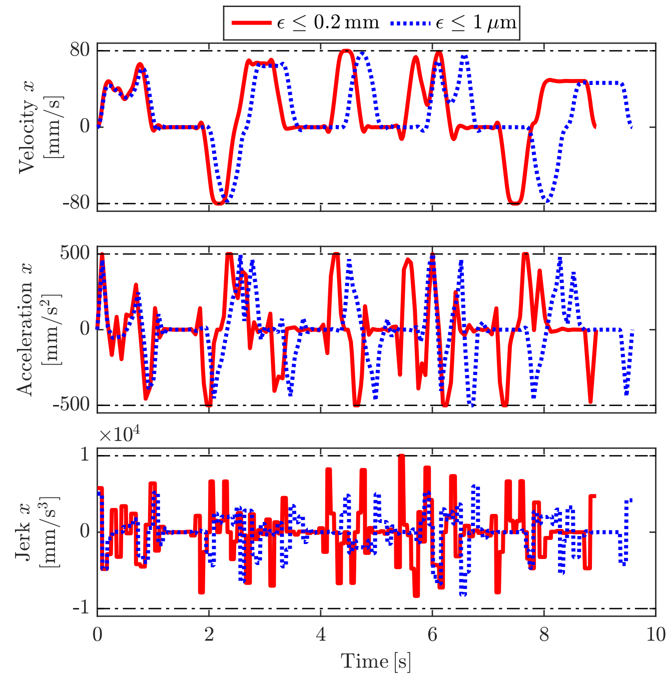

Fig. 5. Satisfaction of velocity, acceleration, and jerk limits on x-axis of the TUT profile with the fitting errors of 0.2mm and 1μm.

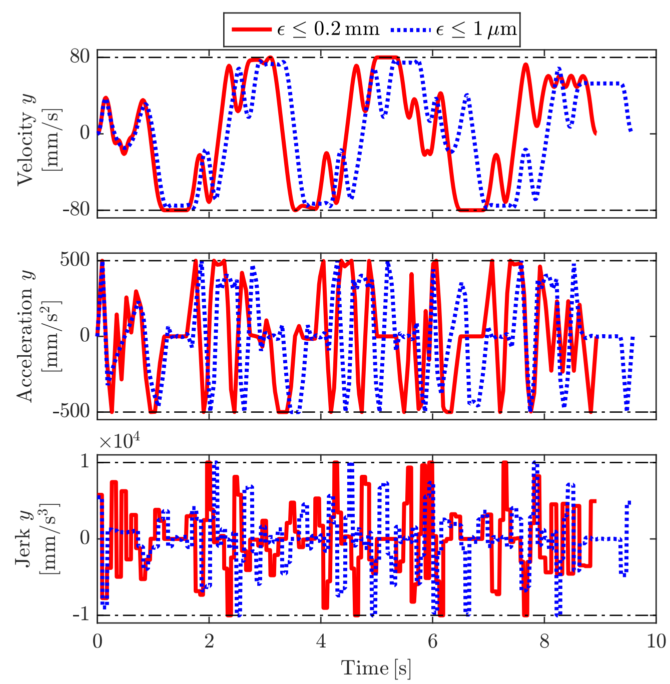

Fig. 6. Satisfaction of velocity, acceleration, and jerk limits on y-axis of the TUT profile with the fitting errors of 0.2mm and 1μm.

Figs. 5 and 6 illustrate the x- and y-axial velocity, acceleration, and jerk of the optimal paths with the fitting error of $0.2\,\mathrm{mm}$ and $1\,\mu\mathrm{m}$, respectively. It is observed that the velocity, acceleration, and jerk values of the optimal trajectory with $0.2\,\mathrm{mm}$ accuracy are closer to machine kinematic limits than the trajectory with $1\,\mu\mathrm{m}$; therefore, resulting in a faster trajectory. Both trajectories satisfy zero velocity and acceleration at the start and end positions, and the kinematic limits are satisfied for all horizons due to the convex hull property of B-splines. The continuity in axial velocities and accelerations are also satisfied along the trajectory.

V. Conclusion

This paper presents a technique for time-optimal trajectory generation of CNC machines while considering discrete fitting errors as geometric constraints of an OCP. Therefore, the proposed method enables to fit not only straight-line (G01) and circle (G02/G03) segments but also spline contour (G05) segments. Since the fitting error constraints are considered at the via-points of the G-code segments, the resulting geometric path can be slightly deviated in-between the via-points depending on the limitations of constraints, and this problem will be considered as future work.

The effectiveness of the proposed method is investigated by simulations with a complicated geometric profile considering different limits on fitting errors. Smooth transitions between segment trajectories are achieved and machine kinematic limits are satisfied for all horizons. The simulation results demonstrated the trade-off between the time-optimality and accuracy of the workpiece by the cubic B-spline parameterization. Therefore, the proposed trajectory generation method is effective for improving the productivity with the required accuracy of workpieces for CNC machines.

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