BASIC AERODYNAMICS

David Herrera
dherreraa@unal.edu.co

Doctorate in Engineering - Mechanical and Mechatronical Engineering
Universidad Nacional de Colombia

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Objectives

- Understand the fundamentals of flow over airfoils, and drag and lift forces acting on them.
- Have an intuitive understanding of the various physical phenomena such as drag and lift.
- Explain how airplanes fly
- Understand the trimming process.

Contents

Basic Airfoil

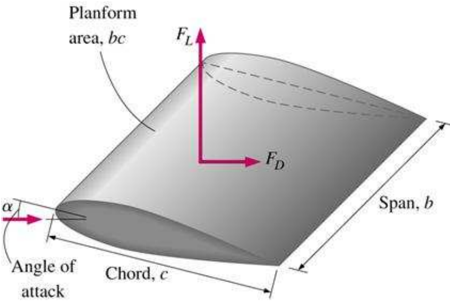


Figure: Airfoil

Fluid dynamic forces are due to pressure and viscous forces acting on the body surface.

- Drag
- Lift
- CP: center of pressure
- AC: aerodynamic center

Definitions

- **Leading edge** is the foremost edge of the airfoil section, where the airflow is separated
- **Trailing edge** is the rear edge, where the airflow is rejoined
- **Chord line** a straight line drawn from the leading edge to the trailing edge
- **Mean (camber) line** is a line joining the leading edge and the trailing edge and having a desired shape. The airfoil is constructed on this camber line by drawing perpendiculars and placing the upper and lower surfaces equal distances above and below the camber line according to a chosen distribution.

Definitions

- **Angle of attack** α is the angle that the chord line makes with the freestream velocity vector
- **Center of pressure (cp)** is the point through which the total force can be tought to be acting
- **Aerodynamic Center (ac)** is a point at wich the aerodynamic moment tends to be invariant with respect to α

Aerodynamic Forces

Two different physical mechanisms contribute to producing an aerodynamic force:

- Each element of surface area multiplied by the pressure at that position, leads to an elemental force normal to the airfoil surface. When this calculation is integrated over the whole surface, the resultant force is in general nonzero
- For each element of area there is a layer of the fluid in contact with the surface, and not moving relative to the surface. If the flow is laminar, we can visualize layers of fluid farther from the surface moving progressively faster, and the molecular forces between layers constitute the *shear stress*

Skin Friction Force

Shear stress multiplied by an element of area leads to an elemental force tangencial to the surface. When the shear forces are integrated over the whole surface, the resultant force is defined to be the *skin friction*. The skin friction force will be proportional to the *wetted area* (area in contact with the fluid)

The combination of the pressure force and the skin friction force is the resultant aerodynamic force on the airfoil.

Lift and Drag

The aerodynamic force is conventionally resolved into two perpendicular components, the *lift* and *drag* components.

Lift

The lift force is to be defined parallel to the freestream velocity vector

Drag

The drag force is parallel to the freestream velocity vector

Lift and drag normally increase as α is increased

Pitching Moment

An aerodynamic moment appears over the airfoil, it is called pitching moment. The pitching moment on an airfoil is the moment (or torque) produced by the aerodynamic force on the airfoil if that aerodynamic force is considered to be applied, not at the center of pressure, but at the aerodynamic center of the airfoil. The pitching moment on the wing of an airplane is part of the total moment that must be balanced using the lift on the horizontal wing on the tail (horizontal stabilizer).

Dynamic Pressure

The forces and moments over an airfoil are proportional to the product of freestream mass density, the square of the freestream velocity and a characteristic area of the body. The product of the first two quantities is known as *dynamic pressure*. The dynamic pressure of the freestream over the airfoil is defined by:

$$\bar{q} = \frac{1}{2}\rho V_T^2 \tag{1}$$

where ρ is the freestream mass density and V_t is the freestream airspeed

Aerodynamic Coefficients

By dividing a measured or calculated aerodynamic force by the dynamic pressure and an arbitrarily chosen reference area, it is possible to determine dimensionless coefficients that represent the ability of the airfoil to produce lift or drag. These *aerodynamic coefficients* depend on:

- Airfoil shape
- Airfoil angle of attack.
- Flow viscosity and compresibility.

In case of geometrically similar airfoils, aerodynamic coefficients may be the same, no matter their size (Similarity Parameters).

Similarity Parameters

The aerodynamic coefficients depends on how the fluid is compressed in the flow around the airfoil, and the fluid's viscosity. If this dependence is expressed in terms of aproppiate parameters, the geometrically similar airfols (same shape, same reference area, not necesarily same size) will have the same aerodynamic coefficient when they are at the same angle of attack in two different flowfields, providing that the two similarity parameters are the same for each.

Similarity Parameters

- Reynolds Number
- Mach Number

Reynolds Number

Reynolds number (Re) is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions. In other words, it's the relationship between the total transferred momentum and the transfered molecular moment. It's defined by:

$$Re = \frac{\rho l V_t}{\mu} \tag{2}$$

where l is a characteristic length (usually the chord) and μ the flow viscosity.

Reynolds Number

The flow in the boundary layer is laminar at low Reynolds numbers, but this situation changes, for example, with increasing the freestream velocity

Critical Reynolds Number

A low Reynolds number implies a laminar flow. In range of few hunddred of thousands, this number rises, and flow becomes turbulent. The Reynolds number also depends on temperature, speed and even humidity.

Mach Number

Mach number (M) is a dimensionless number representing the speed of an object moving through air or other fluid divided by the local speed of sound. It is used as a measure of the relationship between fluid compressibility and its elastic module. Mach number is given by:

$$M = \frac{V_T}{a} \tag{3}$$

where a is the speed of sound. Mach number is classified into:

- Subsonic $M < 1$
- Transonic $0.8 \leq M \leq 1.2$
- Supersonic $1.0 \leq M \leq 5.0$
- Hipersonic $M > 5$

Lift Coefficient

The lift coefficient (c_l) is a dimensionless coefficient that relates the lift generated by a lifting body (airfoil or fixed-wing aircraft), the dynamic pressure of the fluid flow around the body, and a reference area associated with the body. Lift coefficient is defined by:

$$c_l = \frac{F_L}{\frac{1}{2}\rho V_T^2 A} = \frac{F_L}{qA} \tag{4}$$

where F_L is the lift force, $\bar{q}$ is dynamic pressure, and A is the planform area. Lift coefficient is a efectiveness measure of an airfoil for producing lift.

Section lift coefficient

Lift coefficient is also used to refer to the dynamic lift characteristics of a two-dimensional foil section, whereby the reference area is taken as the foil chord. Lift coefficient may be described as the ratio of lift pressure to dynamic pressure where lift pressure is the ratio of lift to reference area.

$$c_l = \frac{l}{\frac{1}{2}\rho V_T^2 c} \tag{5}$$

where c is the chord line and l the span.

Lift coefficient: effects of angle of attack and shape

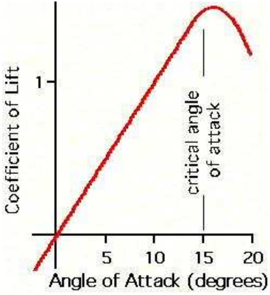


Figure: Lift as a function of the angle of attack

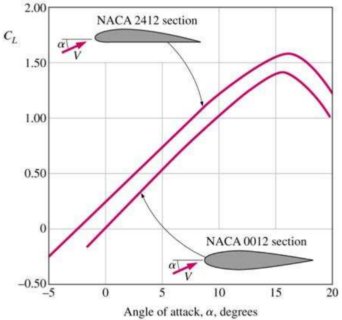


Figure: Shape effect on Lift coefficient

Drag Coefficient

The drag coefficient (c_d) is a dimensionless quantity that is used to quantify the drag or resistance of an object in a fluid environment. It is defined by:

$$c_d = \frac{F_D}{\frac{1}{2}\rho V_T^2 A} = \frac{F_D}{\bar{q}A} \tag{6}$$

where F_D is the lift force, $\bar{q}$ is dynamic pressure, and A is the planform area. The drag coefficient of any object comprises the effects of the two basic contributors to fluid dynamic drag: skin friction and form drag.

Moment Coefficient

The pitching moment coefficient (c_m) is a dimensionless quantity that indicates how much moment is generated by the aerodynamic force on the airfoil. This coefficient is defined by:

$$c_m = \frac{M}{\frac{1}{2}\rho V_T^2 A c} = \frac{M}{\bar{q} A c} \tag{7}$$

where M is the pitching moment, A is the planform area and c is the chord

Aerodynamic Coefficients: Typical plots

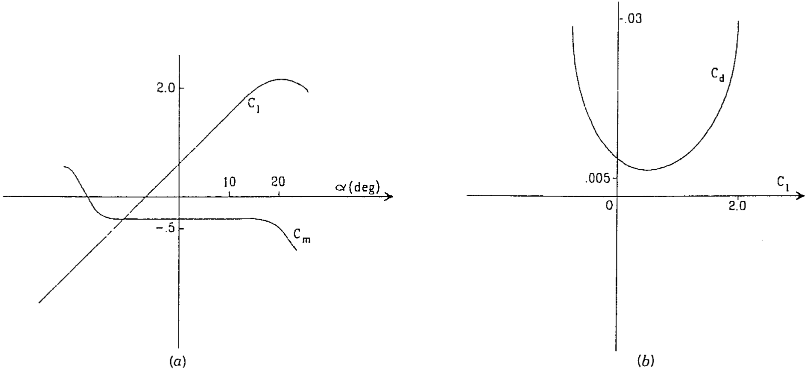


Figure: Typical plots of aerodynamic coefficients

Effect: Angle of Attack

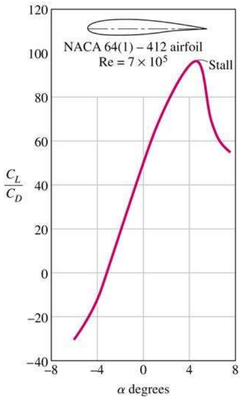


Figure: Angle of attack effect on C_d y C_L

Section Coefficients

- lift per unit span: $\bar{q}cC_l(\alpha, R, R_e)$
- drag per unit spam: $\bar{q}cC_d(\alpha, R, R_e)$
- pitching moment per unit spam: $\bar{q}c^2C_m(\alpha, R, R_e)$

Important Wing-Planform Parameters

- b wing span
- c wing chord
- $\bar{c}$ mean aerodynamic chord
- S wing area
- λ taper ratio (tip chord/root chord)
- Λ leading-edge sweep angle
- $AR = b^2/S$ aspect ratio

What makes Airplanes Fly

A difficult question to answer. There are three ways to explain this phenomenon:

- The Mathematical Aerodynamics Description which is used by aeronautical engineers.
- The Popular Explanation which is based on the Bernoulli principle.
- The Physical Description of lift. This description is based primarily on Newton's laws.

The second one is the most common explanation.

Usual Descriptions

Complex Explanation

This description uses complex mathematics and/or computer simulations to calculate the lift of a wing. These are design tools which are powerful for computing lift but do not lend themselves to an intuitive understanding of flight.

Bernoulli Explanation

The primary advantage of this description is that it is easy to understand and has been taught for many years. Because of its simplicity, it is used to describe lift in most flight training manuals. This explanation relies on the “principle of equal transit times”.

Bernoulli and Lift

Airplanes fly as a result of Bernoulli's principle, which says that if air speeds up the pressure is lowered. Thus a wing generates lift because the air goes faster over the top creating a region of low pressure, and thus lift.

Why air goes faster over the top?

The distance the air must travel is directly related to its speed. When the air separates at the leading edge, the part that goes over the top must converge at the trailing edge with the part that goes under the bottom. This is the so-called "principle of equal transit times".

Problem

Expected Shape?

In order to generate the required lift for a typical small airplane, the distance over the top of the wing must be about 50% longer than under the bottom

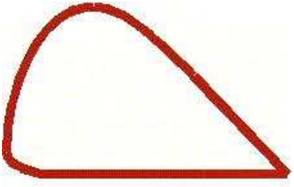


Figure: Expected Airfoil

Principle of equal times Transit

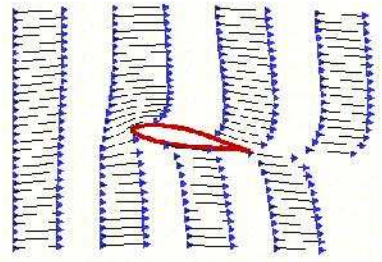


Figure: Air speed

Some simulations shows air that goes over the top of the wing gets to the trailing edge considerably before the air that goes under the wing.

Is Bernoulli principle wrong?

NO, it is just not enough to explain how airplanes fly. It can not explain facts like:

- Inverted flight
- How a wing adjusts for great changes in load
- Power needed for Lifting

Why prevails?

IT IS EASY TO UNDERSTAND. Besides, there is nothing wrong with the Bernoulli principle, or with the statement that the air goes faster over the top of the wing.

Newton's laws and lift

Newton's Laws for lifting:

- First law: if one sees a bend in the flow of air, or if air originally at rest is accelerated into motion, there is a force acting on it.
- Third law: in order to generate lift a wing must do something to the air. What the wing does to the air is the action while lift is the reaction.

Lift needs Power

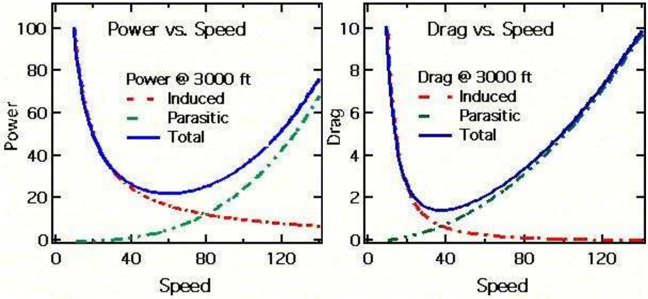


Figure: Effect of Speed on Power and Drag

Definition of axes and Angles

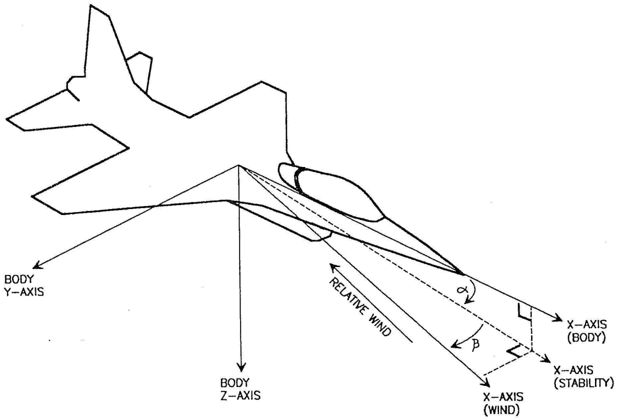


Figure: Aircraft Angles and Axes

Aerodynamic forces and axes system

Definitions

- α_e: "equilibrium" angle of attack, it defines the stability-axes coordinate system
- Forces in the wind-axes system: L lift, D drag and C cross-wind

Rotations from body to stability system, and stability to wind axes:

$$C_{s/b} = \begin{bmatrix} \cos\alpha_e & 0 & \sin\alpha_e \\ 0 & 1 & 0 \\ -\sin\alpha_e & 0 & \cos\alpha_e \end{bmatrix} \qquad C_{w/s} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Combined rotation

The combined rotation from body to wind is:

$$C_{w/b} = \begin{bmatrix} \cos\alpha_e \cos\beta & \sin\beta & \sin\alpha_e \cos\beta \\ -\cos\alpha_e \sin\beta & \cos\beta & -\sin\alpha_e \sin\beta \\ -\sin\alpha_e & 0 & \cos\alpha_e \end{bmatrix}$$

Definition of forces and moments

Coefficients

- C_N normal force coefficient
- C_X axial force coefficient
- C_Z body-axes force coefficient

Aerodynamic forces:

$$F_A^w = \begin{bmatrix} -D \\ -C \\ -L \end{bmatrix} = C_{w/b} \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} = C_{w/b} F_A^b$$

More definitions

Dimensionless force coefficients:

	x	y	z
Wind:	C_D	C_C	C_L
Stability:	$*$	C_Y	C_L
Body:	C_X	C_Y	C_Z

Aerodynamic Moments:

$$M_A^b = \begin{bmatrix} l \\ m \\ n \end{bmatrix} \qquad M_A^s = \begin{bmatrix} l_s \\ m \\ n_s \end{bmatrix} \qquad M_A^w = \begin{bmatrix} l_w \\ m_w \\ n_w \end{bmatrix}$$

More definitions

Dimensionless Moment Coefficients:

$$C_l, \ C_m, \ C_n$$

(same notation in all systems)

Thrust Force and Moment:

$$F_T^b = \begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} \qquad M_T^b = \begin{bmatrix} m_{x,T} \\ m_{y,T} \\ m_{z,T} \end{bmatrix}$$

More definitions

Relative Velocity Components

$$v_{rel}^{b} = C_{b/w} v_{rel}^{w} = C_{b/w} \begin{bmatrix} V_T \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} V_T \cos\alpha \cos\beta \\ V_T \sin\beta \\ V_T \sin\alpha \cos\beta \end{bmatrix}$$

Aerodynamic Angles:

$$\tan(\alpha) = \frac{W'}{V} \qquad \sin(\beta) = \frac{V'}{V_T} \qquad V_T = |V_{rel}|$$

More definitions

Absolute Velocity Components:

$$v^b_{CM/e} = \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

Angular Velocity Components (r denotes any frame):

$$\omega^b_{b/r} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \qquad \omega^s_{b/r} = \begin{bmatrix} P_s \\ Q \\ R_s \end{bmatrix} \qquad \omega^w_{b/r} = \begin{bmatrix} P_w \\ Q_w \\ R_w \end{bmatrix} \quad (R_w = R_s)$$

More Definitions

Control Surface Deflections

- Elevator: δ_e
- Aileron: δ_a
- Rudder: δ_r
- Flap: δ_F
- Throttle Position: δ_t
- Control vector: $\bar{U} = [\delta_t \;\; \delta_e \;\; \delta_a \;\; \delta_r \;\; \delta_F \;\; \ldots]^T$

Control surfaces

A fixed-wing aircraft has several control surfaces attached to the airframe on joints (hinges or tracks) so they may move and thus deflect the air stream passing over them. This redirection of the air stream generates an unbalanced force to rotate the plane about the associated axis.

Main control surfaces

- Aileron
- Rudder
- Elevator

Control Surfaces

Associated Movements

- Ailerons movement causes turn around the roll axis.
- Elevator movement produces rotation around pitch.
- Rudder movement allows rotation around the yaw axis.

Other control surfaces are flaps, slats, spoilers, etc.

Ailerons

The ailerons are horizontal flaps located near the end of an aircraft's wings. These flaps allow one wing to generate more lift than the other, resulting in a rolling motion that allows the plane to bank left or right. Ailerons usually work in opposition.

Figure: Ailerons and their effect

Elevator

The tail of the airplane has two types of small wings, called the horizontal and vertical stabilizers.

Elevators are flaps located on the horizontal tail wing. They enable the plane to go up and down through the air. The elevators change the horizontal stabilizer's angle of attack, and the resulting lift either raises the rear of the aircraft or lowers it.

Figure: Elevator and its effect

Rudder

The rudder is a flap attached to the vertical tail wing (vertical stabilizer). Just like its nautical counterpart on a boat, this part enables the plane to turn left or right and works along the same principle.

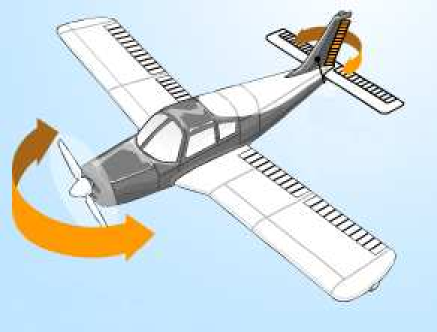


Figure: Rudder and its effect

Force and Moment Coefficients

The forces and moments acting on the complete aircraft are defined in terms of dimensionless aerodynamic coefficients in the same manner as for the airfol section.

- drag: $D = \bar{q}SC_D$
- lift: $L = \bar{q}SC_L$
- crosswind force: $C = \bar{q}SC_C$
- rolling moment: $l_w = \bar{q}SbC_l$
- pitching moment: $m_w = \bar{q}S\bar{c}C_m$
- yawing moment: $n_w = \bar{q}SbC_n$

Aerodynamic Derivatives

The dimensionless forces and moment coefficients are calculated using “aerodynamic derivatives”. The aerodynamic derivatives are used to study the aircraft stability during flight.

Longitudinal Stability Derivatives

- C_{L_α} Lift-curve slope
- C_{m_α} Pitch stiffness
- C_{m_q} Pitch damping
- C_{m_V} Tuck derivative
- $C_{m_{\dot\alpha}}$ Alpha-dot derivative
- C_{D_V} Speed damping
- C_{D_α} Drag versus alpha slope
- C_{L_v} Lift versus speed slope

Longitudinal Stability Derivatives

C_{m_α} Lift-curve slope

The major contribution to pitching stability. It determines the natural frequency of the short period phugoid. It also determines the aircraft response to pilot inputs and gusts. The value should be sufficiently large to give an acceptable response to pilot inputs.

Longitudinal Stability Derivatives

$C_{m_{\dot{\alpha}}}$ Alpha-dot derivative

This derivative also contributes to damping of the short period phugoid.

C_{m_q} Pitch damping

As the aircraft pitches, resistance to the angular velocity is provided by this term. C_{M_q} is commonly referred to a pitch damping as it dampens the short period phugoid. It provides amajor contribution to longitudinal stability and aircraft handling qualities.

Contents

Relationship for Rotation

The orientation of one Cartesian coordinate system with respect to another can always be described by three sucesive rotations, and the angles of rotation are called *Euler Angles*

Standard Aircraft Euler Angles

- Yaw ψ
- Pitch θ
- Roll ϕ

Euler rotations and Angular Velocity

The sequence of rotations is:

1. Right-handed rotation about the *z*-axis (ψ)
2. Right-handed rotation about the *y*-axis (θ)
3. Right-handed rotation about the *x*-axis (ϕ)

The transformation between the Euler derivatives and the roll, pitch and yaw rate components of the aircraft angular-velocity vector is defined by:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta \sin\phi & \tan\theta \cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi / \cos\theta & \cos\phi / \cos\theta \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$$

Contents

Coordinate Systems

- **Earth-Fixed coordinate system** rotates with respect to the earth and is centered in the middle of the earth.
- **Aircraft body fixed coordinate system** is fixed to the aircraft with origin in Center Of Gravity (CoG).
- **Aircraft fixed reference coordinate system** is used to establish a reference for the controller. It is centered in CoG of the aircraft and the orientation is given by the field where the aircraft takeoff and land.
- **Navigational coordinate system** is used to describe the position of the aircraft with respect to starting point of the flight.

Rigid Body Dynamics

Definitions

- F_i an inertial reference frame
- F_b a body-fixed frame in the rigid vehicle
- $v_{CM/i}$ velocity of vehicle cm in F_i
- $\omega_{b/i}$ angular velocity of F_b with respect to F_i
- $M_{A,T}$ sum of aerodynamic and thurst moments about the cm

Angular Motion

The forces are created on the surface of the aircraft. Since the aerodynamic forces are not acting directly on CoG, a moment $M_{A,T}$ is generated, causing an angular acceleration ${}^{i}\dot{\omega}_{b/i}$ of the aircraft. The moment equation is given by:

$$M^{b}_{A,T} = J^{b\,i}\dot{\omega}_{b/i} + \Omega^{b}_{b/i} J^{b} \omega^{b}_{b/i}$$

where $\Omega^{b}_{b/i}$ is a cross-product matrix for $\omega^{b}_{b/i}$ and J^{b} is the inertia matrix:

$$J^{b} = \begin{bmatrix} J_{xx} & -J_{xy} & -J_{xz} \\ -J_{xy} & J_{yy} & -J_{yz} \\ -J_{xz} & -J_{yz} & J_{zz} \end{bmatrix}$$

Euler's equations of motions

If the torque vector has body-axes components given by:

$$M^b_{A,T} = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$

the Euler's equations are:

$$\dot{P} = \frac{(J_y - J_z)QR}{J_x} + \frac{l}{J_x}$$
$$\dot{Q} = \frac{(J_z - J_x)RP}{J_y} + \frac{m}{J_y}$$
$$\dot{R} = \frac{(J_x - J_y)PQ}{J_z} + \frac{n}{J_z}$$

Translational dynamics

Definitions

- $p_{CM/O}$ vehicle cm position relative to ECI origin
- $v_{CM/i} = {}^{i}\dot{p}_{CM/O}$ velocity of cm in i
- $v_{CM/e} = {}^{e}\dot{p}_{CM/O}$ velocity of cm in e
- $\omega_{x/y}$ angular velocity of frame x with respect to frame y
- $F_{A,T}$ vector sum of aerodynamic and thrust forces at cm

Translational motion

By using the equation of Coriolis to relate the derivatives of $p_{CM/O}$ in F_e and F_i, we find that

$$v_{CM/i} =^i \dot{p}_{CM/O} = v_{CM/e} + \omega_{e/i} \times p_{CM/O}$$

Newton's second law applied to the motion of cm:

$$(1/m)F_{A,T} + G =^i \dot{v}_{CM/i}$$

with m is the mass of the vehicle.

Translational Motion

Now differentiate $v_{CM/i}$ and substitute:

$$(1/m)F_{A,T} + G =^b \dot{v}_{CM/e} + \omega_{b/i} \times v_{CM/e} + \omega_{e/i} \times^i \dot{p}_{CM/O}$$

Then, substitute by the inertial position derivative:

$$(1/m)F_{A,T}+G =^b \dot{v}_{CM/e}+(\omega_{b/i}+\omega_{e/i})\times v_{CM/e}+\omega_{e/i}\times(\omega_{e/i}\times p_{CM/O})$$

Translational Motion

Rewritting:

$$^{b}\dot{v}_{CM/e} = (1/m)F_{A,T} + g - (\omega_{b/i} + \omega_{e/i}) \times v_{CM/e}$$

where

$$g = G - \omega_{e/i} \times (\omega_{e/i} \times p_{CM/O})$$

Aircraft Nonlinear Model

In order to model the aircraft dynamics, we can obtain theree sets of equations:

- Force equations
- Kinematic equations
- Moment equations

Due the nonlinear nature of dynamic coefficients and the coupling betwe4en variables, the aircraft model is nonlinear.

Force Equations

From translational motion equations we can obtain:

$$\begin{aligned}
\dot{U} &= RV - QW - g_D \sin\theta + (X_A + X_T)/m \\
\dot{V} &= -RU + PW + g_D \sin\phi\cos\theta + (Y_A + Y_T)/m \\
\dot{W} &= QU - PV + g_D \cos\phi\cos\theta + (Z_A + Z_T)/m
\end{aligned}$$

Kinematic Equations

Applying the Euler's rotations to the kinematic equations we get:

$$\dot{\phi} = P + \tan\theta(Q\sin\phi + R\cos\phi)$$
$$\dot{\theta} = Q\cos\phi - R\sin\phi$$
$$\dot{\psi} = (Q\sin\phi + R\cos\phi)/\cos\theta$$

Moment Equations

From rotational equations we obtain:

$$\Gamma \dot{P} = J_{xz}[J_x - J_y + J_z]PQ - [J_z(J_z - J_y) + J_{zx}^2]QR + J_z l + J_{xz} n$$
$$J_y \dot{Q} = (J_z - J_x)PR - J_{xz}(P^2 - R^2) + m$$
$$\Gamma \dot{R} = [(J_x - J_y)J_x + J_{xz}^2]PQ - J_{xz}[J_x - J_y + J_z]QR + J_{xz} l + J_x n$$

with $\Gamma = J_x J_z - J_{xz}^2$

Decoupled Model

Defining the g vector by:

$$g_1 = -g_0' \cos\beta \sin(\theta - \alpha)$$

$$g_2 = g_0' \sin\beta \sin(\theta - \alpha)$$

$$g_3 = g_0' \cos(\theta - \alpha).$$

Decoupled Model

$$m\dot{V}_T = F_T \cos\alpha - D - mg'_0 \sin\gamma$$

$$m\dot{\beta}V_T = Y - mV_T R_W$$

$$m\dot{\alpha}V_T = -F_T \sin\alpha - L + mV_T Q_W + mg'_0 \cos\gamma.$$

Aircraft Dynamic Modes

The dynamic stability of an aircraft refers to how the aircraft behaves after it has been disturbed following steady non-oscillating flight. This concept leads to a description of the flight behavior into:

1. Longitudinal modes
   - Phugoid oscillations
   - Short period oscillations
2. Lateral-directional modes
   - Roll subsidence mode
   - Spiral mode
   - Dutch roll

Longitudinal Modes

Oscillating motions can be described by two parameters, the oscillation period, and the time required to damp to half-amplitude, or the time to double the amplitude for a dynamically unstable motion. The longitudinal motion consists of two distinct oscillations, a long-period oscillation called a phugoid mode and a short-period oscillation referred to as the short-period mode.

Phugoid Oscillations

The phugoid mode is the one in which there is a large-amplitude variation of air-speed, pitch angle, and altitude, but almost no angle-of-attack variation. The phugoid oscillation is really a slow interchange of kinetic energy (velocity) and potential energy (height) about some equilibrium energy level as the aircraft attempts to re-establish the equilibrium level-flight condition from which it had been disturbed. The motion is so slow that the effects of inertia forces and damping forces are very low.

Phugoid Mode

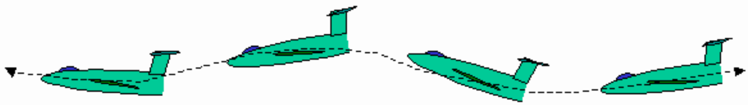


Figure: Phugoid Mode sequence

Short period oscillations

The short-period mode is a usually heavily damped oscillation with a period of only a few seconds. The motion is a rapid pitching of the aircraft about the center of gravity. The period is so short that the speed does not have time to change, so the oscillation is essentially an angle-of-attack variation. The time to damp the amplitude to one-half of its value is usually on the order of 1 second.

Lateral-directional modes

Lateral-directional modes involve rolling motions and yawing motions. Motions in one of these axes almost always couples into the other so the modes are generally discussed as the "Lateral-Directional modes"

There are three types of possible lateral-directional dynamic motion: roll subsidence mode, Dutch roll mode, and spiral mode.

Roll subsidence mode

Roll subsidence mode is simply the damping of rolling motion. There is no direct aerodynamic moment created tending to directly restore wings-level, i.e. there is no returning "spring force/moment" proportional to roll angle. However, there is a damping moment (proportional to roll rate) created by the slewing-about of long wings. This prevents large roll rates from building up when roll-control inputs are made or it damps the roll rate (not the angle) to zero when there are no roll-control inputs.

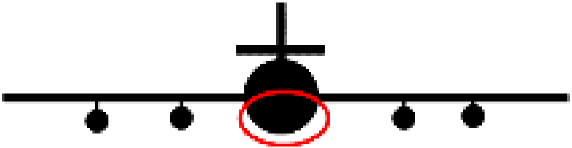
Spiral mode

If a spirally unstable aircraft, through the action of a gust or other disturbance, gets a small initial roll angle to the right. The sideslip causes a yawing moment to the right. If the dihedral stability is low, and yaw damping is small, the directional stability keeps turning the aircraft while the continuing bank angle maintains the sideslip and the yaw angle. This spiral gets continuously steeper and tighter until finally, if the motion is not checked, a steep, high-speed spiral dive results.

Dutch Roll

The Dutch roll is an oscillatory combined roll and yaw motion. The Dutch roll may be described as a yaw and roll to the right, followed by a recovery towards the equilibrium condition, then an overshooting of this condition and a yaw and roll to the left, then back past the equilibrium attitude, and so on. The period is usually on the order of $3 - 15$ seconds, but it can vary from a few seconds for light aircraft to a minute or more for airliners.

Dutch Roll



Contents

Trimming

Definition

A trim point, also known as an equilibrium point, is a point in the parameter space of a dynamic system at which the system is in a steady state.

For example, a trim point of an aircraft is a setting of its controls that causes the aircraft to fly straight and level. Mathematically, a trim point is a point where the system's state derivatives equal zero.

Trimming Process

Trimming is the process made in order to find a desired point of operation for developing simulation tasks. Besides, the simulation process involves:

- Set Aircraft Initial Conditions
- Create Operating Point Specification Object for the Simulation
- Specifying State Trim Conditions
- Setting input constrains
- Trimming the Model
- Setting Simulation Initial Conditions to Trim Values
- Specifying Output Trim Conditions
- Specifying Input Trim Conditions

Set Aircraft Initial Conditions

It is neccesary to define initial conditions for the simulation model inputs and state vector. These initial values serve as the starting point for the trim algorithm, so if the trim algorithm fails to converge, you can try a new set of initial conditions.

- Initial angles for ailerons, rudder, flaps, elevator
- Initial value for throttle
- Initial position for Inertial Frame
- Initial Euler Angles (Pitch, Yaw, Roll)
- Initial velocities

Create Operating Point Specification Object for the Simulation

The first step in trimming the model is to create an operating point specification object for the simulation model.

Initial Guess

In this step, guess values are used as start values for each space state variable, input and output.

Specifying State Trim Conditions

In this step, desired targets and/or constraints for the state variables are specified. Important aspects:

- Known or unknown variables or states
- Min and Max Values
- Steady State
- Initial Values

Setting input constrains

Similar to the states, it is neccesary to specify constraints and initial values on the inputs to the model, which are inputs for:

- Throttle input
- Elevator
- Rudder
- L/R aileron L/R flap
- Combined aileron

Trimming the Model

The main objective is to find the operating point. An algorithm (sequential quadratic programming algorithm) calculates the trimm (equilibrium point) and verifies that initial guests are correct. It returns the point that minimizes the maximum deviation from zero of the derivatives

Trimming Details

Main idea

To find the equilibrium point that minimizes the maximum absolute value of $x - x_0$, u and y.

Be:

$$x = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}_a - \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}_b \qquad y = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{bmatrix}_a - \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{bmatrix}_b \tag{8}$$

Trimming Details

Algorithm

- 2 All states are computed from initial values
- 3 The first control is incremented
- 4 Evaluation of all states after K cycles
- 5 First control is returned to the initial condition and the second control is incremented
- 6 The iterative process of evaluating and incrementing is done until n control are completed.
- 7 Calculation of matrix H

Trimming Details

Assume a quadratic history of a particular state called S, in terms of a cycle count k:

$$S(k) = a_1 + a_2 k + a_3 k^2 \tag{9}$$

Then:

$$\frac{dS(k)}{dk} = a_2 + 2a_3 k \tag{10}$$

In steady state:

$$-\frac{1}{2}a_2 = a_3 k \tag{11}$$

Trimming Details

So:

$$S(k) = a_1 + \frac{a_2 k(K - k)}{2K} \tag{12}$$

where K is the number of observations. Finally, all states are calculated and compared with initial values through:

$$y = Hx = \begin{bmatrix} \Delta S_1/\Delta C_1 & \Delta S_1/\Delta C_2 & \dots & \Delta S_1/\Delta C_n \\ \Delta S_2/\Delta C_1 & \Delta S_2/\Delta C_2 & \dots & \Delta S_2/\Delta C_n \\ \vdots & & & \\ \Delta S_m/\Delta C_1 & \Delta S_m/\Delta C_2 & \dots & \Delta S_m/\Delta C_n \end{bmatrix} \tag{13}$$

Setting Simulation Initial Conditions to Trim Values

Change the initial guess

Once an operating point is obtained, next step it's to set these values as the initial conditions for the simulation, replacing the values specified earlier.

Specifying Input and Output Trim Conditions

Ready for Simulation

In this point, the values for simulation are obtained from trimming algorithm. If no optimal point is reached, the process must be repeated, changing the original guess values.

Contents

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