

# Anti-sway Control of a Gantry Crane System based on Feedback Loop Approaches

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**Abstract**— This paper presents the use of anti-sway angle control approaches for a two-dimensional gantry crane with disturbances effect in the dynamic system. Delayed feedback signal (DFS) and Sliding Mode Controller (SMC) are the techniques used in this investigation to actively control the sway angle of the rope of gantry crane system. A nonlinear overhead gantry crane system is considered and the dynamic model of the system is derived using the Euler-Lagrange formulation. A complete analysis of simulation results for each technique is presented in time domain and frequency domain respectively. Performances of both controllers are examined in terms of sway angle suppression and disturbances cancellation. Finally, a comparative assessment of the impact of each controller on the system performance is presented and discussed.

**Keywords**— Gantry crane, anti-sway control, DFS controller, Sliding Mode controller.

## I. INTRODUCTION

The main purpose of controlling a gantry crane is transporting the load as fast as possible without causing any excessive sway at the final position. Research on the control methods that will eliminate sway angle of gantry crane systems has found a great deal of interest for many years. Active sway angle control of gantry crane consists of artificially generating sources that absorb the energy caused by the unwanted sway angle of the rope in order to cancel or reduce their effect on the overall system. Lueg in 1930 [1], is among the first who used active vibration control in order to cancel noise vibration.

Various attempts in controlling gantry cranes system based on open loop system were proposed. For example, open loop time optimal strategies were applied to the crane by many researchers such as discussed in [2,3]. They came out with poor results because open loop strategy is sensitive to the system parameters (e.g. rope length) and could not

compensate for wind disturbances. Another open loop control strategies is input shaping [4,5,6]. Input shaping is implemented in real time by convolving the command signal with an impulse sequence. An IIR filtering technique related to input shaping has been proposed for controlling suspended payloads [7]. Input shaping has been shown to be effective for controlling oscillation of gantry cranes when the load does not undergo hoisting [8, 9].

On the other hand, feedback control which is well known to be less sensitive to disturbances and parameter variations [11] is also adopted for controlling the gantry crane system. Recent work on gantry crane control system was presented by Omar [1]. The author had proposed proportional-derivative PD controllers for both position and anti-sway controls. Furthermore, a fuzzy-based intelligent gantry crane system has been proposed [12]. The proposed fuzzy logic controllers consist of position as well as anti-sway controllers. However, most of the feedback control system proposed needs sensors for measuring the cart position as well as the load sway angle.

This paper presents investigations of anti-sway angle control approach in order to eliminate the effect of disturbances applied to the gantry crane system. A simulation environment is developed within Simulink and Matlab for evaluation of the control strategies. In this work, the dynamic model of the gantry crane system is derived using the Euler-Lagrange formulation. To demonstrate the effectiveness of the proposed control strategy, the disturbances effect is applied at the hoisting rope of the gantry crane. This is then extended to develop a feedback control strategy for sway angle reduction and disturbances rejection. Two feedback control strategies which are Delayed feedback signal and sliding mode controller are developed in this simulation work. Performances of each controller are examined in terms of sway angle suppression and disturbances rejection. Finally, a comparative assessment of the impact of each controller on the system performance is presented and discussed.

## II. GANTRY CRANE SYSTEM

The two-dimensional gantry crane system with its payload considered in this work is shown in Fig. 1, where  $x$  is the horizontal position of the cart,  $l$  is the length of the rope,  $\theta$  is the sway angle of the rope,  $M$  and  $m$  is the mass of the cart and payload respectively. In this simulation, the cart and payload can be considered as point masses and are assumed to move in two-dimensional, x-y plane. The tension force that may cause the hoisting rope elongate is also ignored. In this study the

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length of the cart,  $l = 1.00$  m,  $M = 2.49$  kg,  $m = 1.00$  kg and  $g = 9.81$  m/s<sup>2</sup> is considered.

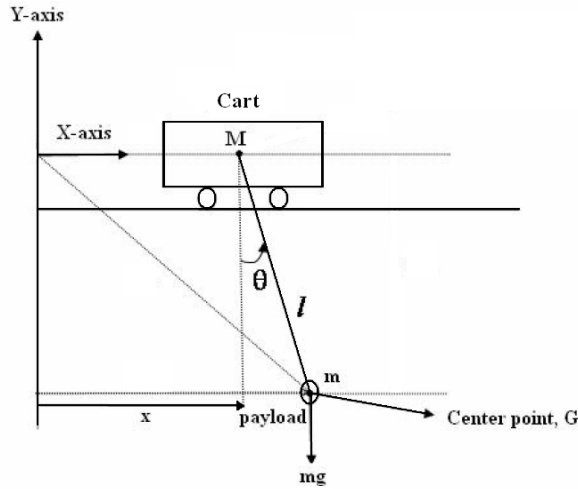


Fig. 1 Description of the gantry crane system

### III. DYNAMIC MODELLING OF THE GANTRY CRANE SYSTEM

This section provides a brief description on the modelling of the gantry crane system, as a basis of a simulation environment for development and assessment of the active sway control techniques. The Euler-Lagrange formulation is considered in characterizing the dynamic behaviour of the crane system incorporating payload.

Considering the motion of the gantry crane system on a two-dimensional plane, the kinetic energy of the system can thus be formulated as

$$T = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m (\dot{x}^2 + \dot{l}^2 + l^2 \dot{\theta}^2 + 2 \dot{x} \dot{l} \sin \theta + 2 \dot{x} l \dot{\theta} \cos \theta) \quad (1)$$

The potential energy of the beam can be formulated as

$$U = -mgl \cos \theta \quad (2)$$

To obtain a closed-form dynamic model of the gantry crane, the energy expressions in (1) and (2) are used to formulate the Lagrangian  $L = T - U$ . Let the generalized forces corresponding to the generalized displacements  $\bar{q} = \{x, \theta\}$  be  $\bar{F} = \{F_x, 0\}$ . Using Lagrangian's equation

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = F_j \quad j = 1, 2 \quad (3)$$

the equation of motion is obtained as below,

$$F_x = (M + m) \ddot{x} + ml(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) + 2m\dot{l}\dot{\theta} \cos \theta + m\ddot{l} \sin \theta \quad (4)$$

$$l\ddot{\theta} + 2\dot{l}\dot{\theta} + \ddot{x} \cos \theta + g \sin \theta = 0 \quad (5)$$

The model of the uncontrolled system can be represented in a state-space form as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (6)$$

with the vector  $x = [x \ \theta \ \dot{x} \ \dot{\theta}]^T$  and the matrices A and B are given by

$$\begin{aligned} A &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & -\frac{(M+m)g}{Ml} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ -\frac{1}{Ml} \end{bmatrix} \\ C &= [1 \ 0 \ 0 \ 0], \quad D = [0] \end{aligned} \quad (7)$$

### IV. CONTROLLER DESIGN

In this section, two feedback control strategies (DFS and Sliding mode controller) are proposed and described in detail. The main objective of the feedback controller in this study is to suppress the sway angle due to disturbances effect. All the feedback control strategies are incorporated in the closed-loop system in order to eliminate the effect of disturbances.

#### A. Delayed Feedback Signal Controller

In this section, the control signal is calculated based on the delayed position feedback approach described in (8) and illustrated by the block diagram shown in Fig. 2.

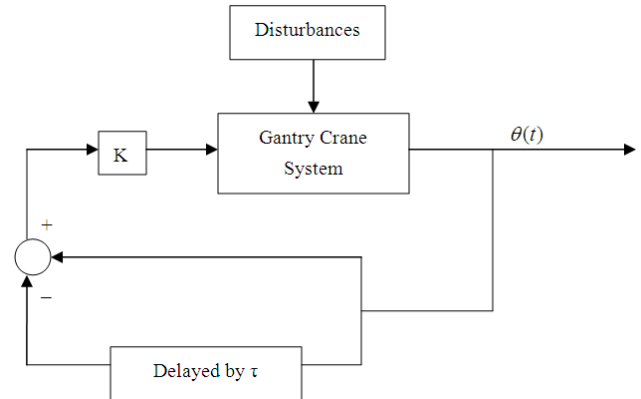


Fig. 2 Delayed feedback signal controller structure

$$u(t) = k(y(t) - y(t - \tau)) \quad (8)$$

Substituting Equation (8) into Equation (6) and taking the Laplace transform gives

$$sIx(s) = Ax(s) - kBC(1 - e^{-s\tau})x(s) \quad (9)$$

The stability of the system given in (9) depends on the roots of the characteristic equation

$$\Delta(s, \tau) = |sI - A + kBC(1 - e^{-s\tau})| = 0 \quad (10)$$

Equation (10) is transcendental and results in an infinite number of characteristic roots [13]. Several approaches dealing with solving retarded differential equations have been widely explored. In this study, the approach described in [14] will be used on determining the critical values of the time delay  $\tau$  that result in characteristic roots of crossing the imaginary axes. This approach suggests that Equation (10) can be written in the form

$$\Delta(s, \tau) = P(s) + Q(s)e^{-s\tau} \quad (11)$$

$P(s)$  and  $Q(s)$  are polynomials in  $s$  with real coefficients and  $\deg(P(s)) = n > \deg(Q(s))$  where  $n$  is the order of the system. In order to find the critical time delay  $\tau$  that leads to marginal stability, the characteristic equation is evaluated at  $s = j\omega$ . Separating the polynomials  $P(s)$  and  $Q(s)$  into real and imaginary parts and replacing  $e^{j\omega\tau}$  by  $\cos(\omega\tau) - j\sin(\omega\tau)$ , Equation (11) can be written as

$$\begin{aligned} \Delta(j\omega, \tau) &= P_R(\omega) + jP_I(\omega) \\ &+ (Q_R(\omega) + jQ_I(\omega))(\cos(\omega\tau) - j\sin(\omega\tau)) \end{aligned} \quad (12)$$

The characteristic equation  $\Delta(s, \tau) = 0$  has roots on the imaginary axis for some values of  $\tau \geq 0$  if Equation (12) has positive real roots. A solution of  $\Delta(j\omega, \tau) = 0$  exists if the magnitude  $|\Delta(j\omega, \tau)| = 0$ . Taking the square of the magnitude of  $\Delta(j\omega, \tau)$  and setting it to zero lead to the following equation

$$P_R^2 + P_I^2 - (Q_R^2 + Q_I^2) = 0 \quad (13)$$

By setting the real and imaginary parts of Equation (13) to zero, the equation is rearranged as below

$$\begin{bmatrix} Q_R & Q_I \\ Q_I & -Q_R \end{bmatrix} \begin{bmatrix} \cos \beta \\ \sin \beta \end{bmatrix} = \begin{bmatrix} -P_R \\ -P_I \end{bmatrix}, \quad (14)$$

where  $\beta = \omega\tau$ .

Solving for  $\sin \beta$  and  $\cos \beta$  gives

$$\begin{aligned} \sin(\beta) &= \frac{(-P_R Q_I + P_I Q_R)}{(Q_R^2 + Q_I^2)} \text{ and} \\ \cos(\beta) &= \frac{(-P_R Q_R - P_I Q_I)}{(Q_R^2 + Q_I^2)} \end{aligned}$$

The critical values of time delay can be determined as follows: if a positive root of Equation (13) exists, the corresponding time delay  $\tau$  can be found by

$$\tau_k = \frac{\beta}{\omega} + \frac{2k\pi}{\omega} \quad (15)$$

where  $\beta \in [0, 2\pi]$ . At these time delays, the root loci of the closed-loop system are crossing the imaginary axis of the  $s$ -plane. This crossing can be from stable to unstable or from unstable to stable. In order to investigate the above method further, the time-delayed feedback controller is applied to the single-link flexible manipulator. Practically, the control signal for the DFS controller requires only one position sensor and uses only the current output of this sensor and the output  $\tau$  second in past. There is only two control parameter:  $k$  and  $\tau$  that needs to be set. Using the stability analysis described in [14], the gain and time-delayed of the system is set at  $k = 168.08$  and  $\tau = 13.60$ . The control signal of DFS controller can be written as below

$$u_{DFS}(t) = 168.08(\theta(t) - \theta(t - 13.60))$$

### B. Sliding Mode Controller

Consider a dynamical system modeled by the following differential equation

$$\dot{x}(t) = [A + \Delta A(\omega, t)]x(t) + Bu(t) + F(\omega) \quad (16)$$

where  $x(t) \in \mathbb{R}^n$  is the system states,  $u(t) \in \mathbb{R}^m$  is the control input,  $F(\omega)$  is the disturbance that presents in the system and matrix  $B$  is of full rank.  $\Delta A(\omega, t)$  is the uncertainty in the system matrix. To complete the description of the uncertain dynamical system, the assumptions in [15] were used and assumed to be valid. From the structural assumptions, the uncertainties can be lumped and the dynamical system can be represented in the following form

$$\dot{x}(t) = Ax(t) + B[u(t) + g(\omega, t)] \quad (17)$$

where  $g(\omega, t)$  is the lumped uncertainties.

In SMC design, the first step is to parameterize the sliding surface such that the system constrained to the sliding surface exhibits desired system behavior. As covered in [16,17,18], under the SMC, once the system slides on the designed surface, the order of the system is reduced. The nominal system can be partitioned and presented as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} u(t) \quad (18)$$

where  $x_1 \in \mathbb{R}^{(n-m)}$  and  $x_2 \in \mathbb{R}^m$ . Let the sliding surface is defined to be as follows:

$$\sigma(t) = \{x \in \mathbb{R}^n : Sx(t) = 0\} \quad (19)$$

where  $S \in \mathbb{R}^{m \times n}$  is full rank matrix,  $S = [S_1 \ S_2]$  and  $S_2 \in \mathbb{R}^{m \times m}$  is non-singular. Thus, during the ideal sliding, the motion can be represented as

$$S_1 x_1 + S_2 x_2 = 0 \quad (20)$$

$$x_2 = -\frac{S_1}{S_2} x_1 = -M x_1 \quad (21)$$

Based on (18), the closed loop system on the sliding surface is given by

$$\dot{x}_1 = A_{11} x_1 + A_{12} x_2 \quad (22)$$

Substituting (21) into (22), the closed loop system is represented as follows

$$\dot{x}_1 = (A_{11} - A_{12} M) x_1 \quad (23)$$

The next crucial step is to establish a control law such that the reachability condition,  $\sigma^T \dot{\sigma} < 0$  is met. When this condition is fulfilled, the system states trajectories are attracted to the designed surface and on the intersection with the surface, the trajectories will remain there for all subsequent time [18].

Given a dynamical system (16), the reaching condition,  $\sigma^T \dot{\sigma} < 0$  is satisfied by employing the control law  $u(t)$  given below

$$u(t) = -(SB)^{-1} K \sigma - (SB)^{-1} S A x - \rho(\omega, t) \frac{B^T S^T \sigma}{\|B^T S^T \sigma\|} \quad (24)$$

where  $K \in \mathfrak{R}^{m \times m}$  is positive design matrix

From numerical computation, the values of the norm for the system matrix uncertainty  $\Delta A(\omega, t)$ , matrix  $B$  and the controller gain  $\rho(\omega, t)$  and the sliding surface matrix  $S$  are obtained as

$$\|\Delta A\| = 309.2527, \|B\| = 0.5680 \text{ and } \rho(\omega, t) = 544.50$$

Then, the sliding surface matrix,  $S$  is parameterized. From (21) the parameter  $S_2$  acts as a scaling parameter and does not give any impact on the stability of the closed loop system (23). Thus  $S_2 = I$  is chosen. For matrix  $S_1$ , the matrix  $M$  is chosen such that the closed loop system is stable. The followings are the value of matrix  $S$  obtained from the selected  $M$ :

$$M = [-0.8958 \quad -127.1873 \quad 11.0529]$$

$$S = [-0.8958 \quad -127.1873 \quad 11.0529 \quad 1.0000]$$

The matrix  $M$  can be considered as poles that stabilize system (23) with  $S$  is the associated gain obtained through the linear quadratic regulator (LQR) method.

## V. IMPLEMENTATION AND RESULTS

In this section, the proposed control schemes are implemented and tested within the simulation environment of the gantry crane system and the corresponding results are presented. The control strategies were designed by undertaking a computer simulation using the fourth-order

Runge-Kutta integration method at a sampling frequency of 1 kHz. The system responses namely sway angle of the hoisting rope and its corresponding power spectral density (PSD) are obtained. In all simulations, the initial condition  $x_o = [0 \quad 1.5 \quad 0 \quad 0]^T$  was used. This initial condition is considered as the disturbances applied to the gantry crane system. The first three modes of sway frequencies of the system are considered, as these dominate the dynamic of the system. Two criteria are used to evaluate the performances of the control strategies:

- (1) Level of sway angle reduction at the natural frequencies. This is accomplished by comparing the power spectral density response of the controller and open loop system.
- (2) Disturbance cancellation. The capability of the controller to achieved zero sway angles.

The open loop responses of the free end of the sway angle of the hoisting rope were considered as the system response with disturbances effect and will be used to evaluate the performance of feedback control strategies. It is noted that, in open loop configuration, the sway angle start to oscillate between  $\pm 1.5$  rad and the sway frequencies of the hoisting rope under disturbances effect were obtained as 0.3925 Hz, 1.276 Hz and 2.159 Hz for the first three modes of sway frequencies.

The system responses of the gantry crane system with the delayed feedback signal controller (DFS) are shown in Figs. 3 and 4. The overall result demonstrates that, the DFS controller can handle the effect of disturbances in the system by compensating the value of gain and time delayed in order to achieve zero radian steady state conditions. This is evidenced in sway angle of hoisting rope response as shown in Fig. 3 whereas the amplitudes of sway angle were reduced in a very fast response as compared to the open loop response. The sway angle settled down at 0.875 s with maximum overshoot of -0.1346 rad. The suppression of sway angle can be clearly demonstrated in frequency domain results as the magnitudes of the PSD at the natural frequencies were significantly reduced.

Figs. 5 and 6 show the closed loop system responses of the gantry crane system under sliding mode controller for the sway angle of the hoisting rope and its PSD results respectively. The results demonstrated that the sway amplitude in the sway angle responses was reduced as compared to the open loop response. The sway angle response also shows a similar pattern as the case of DFS controller with the maximum overshoot of -0.034 rad and settled down at 2.481 s. It is noted that the sliding mode controller can eliminate the impact of disturbances in a slower response with lower overshoot as compared to the DFS controller. The PSD result shows that the magnitudes of sway angle were significantly reduced especially for the first three modes of sway frequencies.

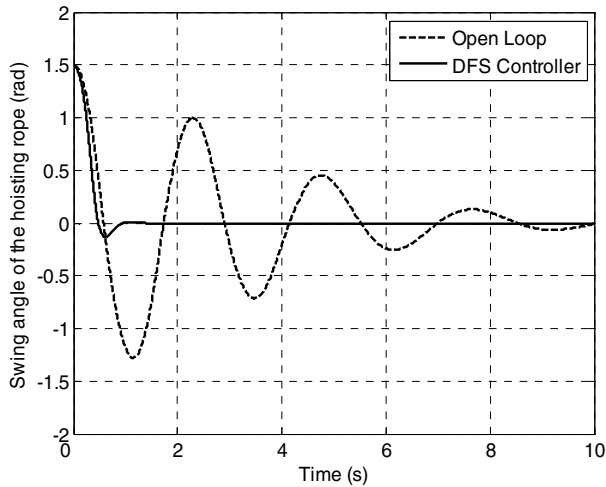


Fig. 3 Sway angle of the hoisting rope response with DFS controller

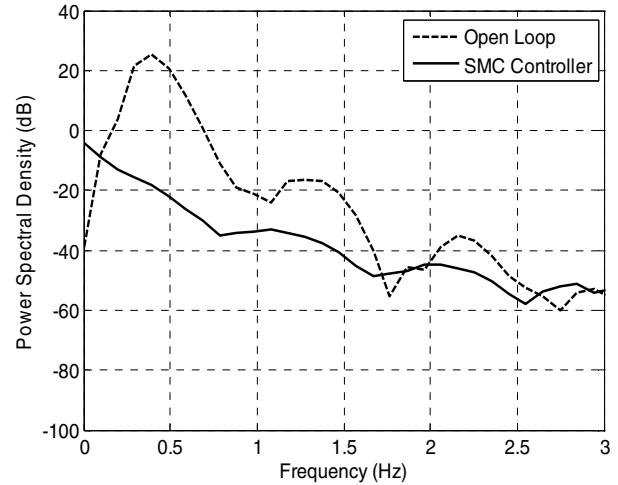


Fig. 6 PSD of sway angle response with sliding mode controller

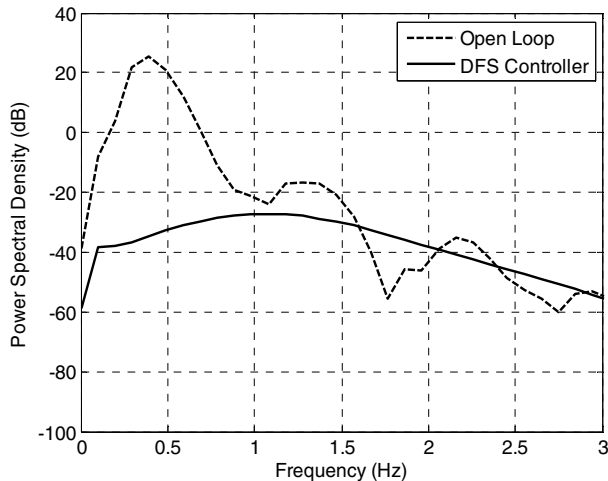


Fig. 4 PSD of sway angle response with DFS controller

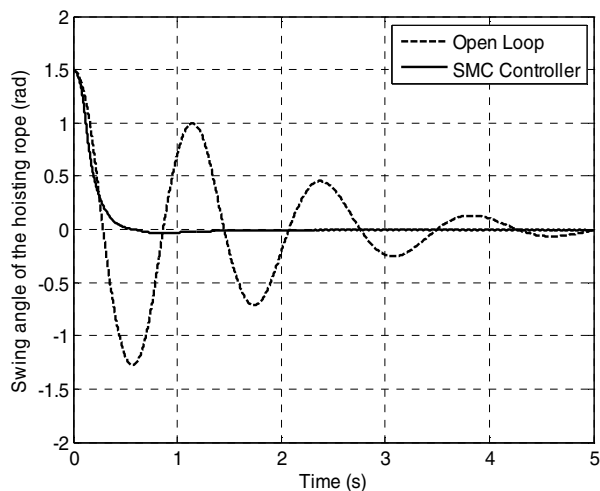


Fig. 5 Sway angle of the hoisting rope response with sliding mode controller

For comparative assessment, the levels of sway reduction of the hoisting rope using DFS and sliding mode controller are shown with the bar graphs in Fig. 7. The result shows that the DFS controller achieved higher level of sway angle reduction for the first mode with the value of 60.06 dB as compared to the sliding mode controller with the value of 43.83 dB. While for the second and third modes, the sliding mode controller results in a higher level of sway angle reduction with the value of 18.84 dB and 11.01 dB as compared to DFS controller with the value of 11.43 and 5.52 dB respectively. Therefore, it can be concluded that overall the sliding mode controller provide better performance in sway angle reduction as compared to the DFS controller. Moreover, implementation of DFS controller is easier than sliding mode controller as a large amount of design effort is required to determine the reaching phase and sliding phase.

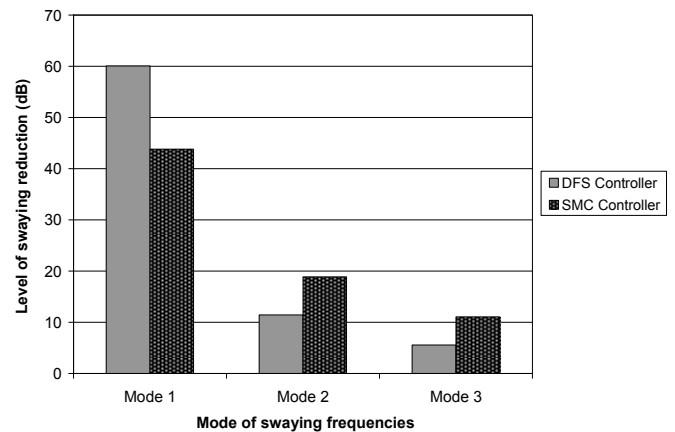


Fig. 7 Level of sway angle reduction using DFS and Sliding mode controller

## VI. CONCLUSION

Investigations into anti-sway techniques of a gantry crane system with disturbances effect using the DFS and Sliding mode controller have been presented. Performances of the controller are examined in terms of sway angle suppression and disturbances cancellation. The results demonstrated that

the effect of the disturbances in the system can successfully be handled by DFS and sliding mode controller. A significant reduction in the system sway for three modes of sway frequencies has been achieved with both controllers.

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