

Vision-Based Cutting Control of Deformable Objects

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Abstract—Cutting control of deformable objects is of great importance in varied application fields such as surgical robot and food processing industry. However, complex physical properties of deformable objects and time-variant topology during cutting make automatic cutting operation control a big challenge. This paper proposed a vision-based cutting control method that predicts the object's deformation and plans cutting path online. Material parameters of the deformation model are treated as unknown parameters to be estimated with visual measurements of feature points on the object's surface. By visually supervising three non-collinear points on the knife, the control of knife's motion is transformed to a multi-points' visual tracking control problem to achieve desired cutting effect in terms of the desired cutting depth and the minimized deformation of the object. The feasibility of the proposed method is validated by experiment results.

Index Terms—deformable objects, cutting control, visual tracking

I. INTRODUCTION

OPERATION of deformable objects is of great importance in varied application fields such as medical industry, food industry and fabric industry. In recent years, surgical robot and robot-assistant surgery have aroused much interest in the robotic community. Automatic manipulation of deformable objects is one of the key issues of surgical robots. There are studies about robotic manipulation of deformable objects aiming at medical applications such as deformation control of soft tissues[1], cell manipulation[2][3], needle insertion [4][5], cutting operation[6] and simulation[7]. Complex physical properties of deformable objects and time-variant topology during cutting process make automatic cutting operation control a big challenge. Besides being a basic operation of surgery, cutting operation can be also applied to meat-processing industry [8][9] and food industry[10].

Previous studies about cutting of deformable objects mainly focus on the model-based cutting simulation. [7] presented a soft-tissue cutting simulation algorithm based on the FEM (Finite Element Method) model, whose advantage lies on the

simulating accuracy but suffer from the high computational complexity. In order to implement real-time simulation, [11] proposed the concept of tensor-mass model and used hybrid model to speed up the computation. Similarly, [12] proposed another hybrid model based on FEM model. Another model that can achieve real-time simulation is mass-spring model. [13] used this model to implement haptic rendering and feedback, and [14] developed a quasi-static simulation based on mass-spring model, which gives a simpler algorithm that provides realistic results at a much faster rate.

There are other works studying cutting operation on deformable objects. [15] and [16] modeled the liver cutting process by recording the cutting forces and the motion of the cutting blade to characterize the mechanical response of liver during cutting. And [17] modeled the cutting force and studied the influence of the blade edge-shape and its slicing angle to derive an optimal slicing angle to minimize the cutting force. [18] proposed a computational force model of three modes of interaction: deformation, rupture, and cutting. And [19] proposed a method for estimating mechanical parameters of soft tissue from sensory data collected during robotic surgical manipulation.

The existing works of deformable objects' cutting control either do not consider the interaction between the tools and the deformable objects, or lack high operation accuracy. [8] and [20] proposed a force/vision control strategy to separate deformable materials using cooperative robots with the guide of a marked curve trajectory that indicates the deformation during cutting. However, real-time interaction between the knife and the object was not considered. [9] proposed a force control method to cut beef with a robot-based cell, whose cutting path was guided by the bone boundary. Yet, deformation was not considered. [10] used haptic information acquired during cutting operation to establish a generative model via supervised learning method. And based on the model, appropriate manipulation can be planned to cut different food. But it cannot ensure the operating accuracy. [21] developed a feed forward controller to control exposure time and laser motion to ensure the incision depth. But deformation during the laser incision process was not studied. However, practical cutting tasks usually require controlling the accuracy. Thus, deformation of the cutting process needs to be studied to plan the robot's motion online.

This paper aims to design a vision-based robotic controller to implement desired deformable-object cutting task by predicting the object's deformation and planning the cutting path online. In order to achieve real-time computation, mass spring model is

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used to predict deformation online according to the measurements of the knife's position. However, in practical occasions, it is usually inconvenient to acquire the material parameters of the objects in advance. Thus in the proposed algorithm, material parameters are treated as unknown parameters that can be estimated by visual measurements of the feature points on the objects' surface. The control goal of the pose of the knife is to achieve desired cutting effect in terms of the desired cutting depth and the minimized deformation. By visually supervising three non-collinear points on the knife, this pose control problem is transformed to a multi-points' visual tracking control problem. Desired image trajectories are interpolated by the projections of the three markers, whose 3D positions are derived according to the predicted deformation of the object.

The rest of the paper is organized as followings: section 2 delivers a general statement of the cutting control system; in section 3, the deformation prediction algorithm containing parameter estimation method is discussed, and desired trajectories are generated; section 4 presents the visual tracking controller; experiment results are discussed in section 5 and section 6 presents conclusion and future work.

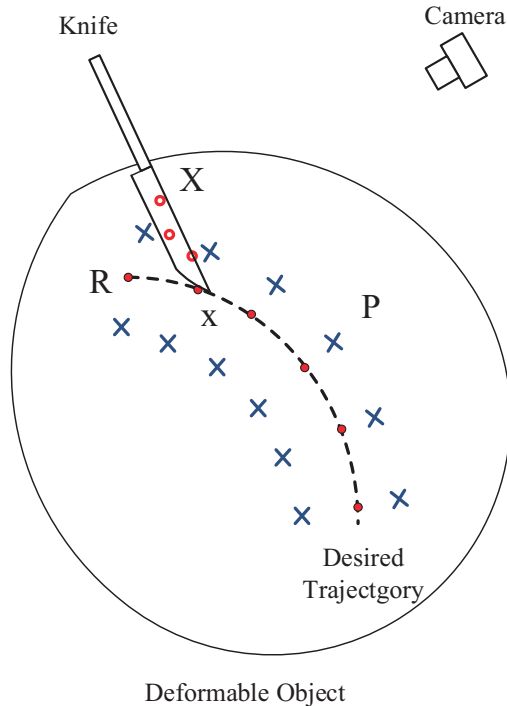


Fig.1 System Configuration

II. PROBLEM STATEMENT

2.1. Problem Definition

Cutting a soft tissue along a desired trajectory is a tough task because the deformation of the soft tissue will definitely occur as the knife cutting on it. In this paper, a vision-based method is proposed to control the cutting trajectory of deformable objects.

As shown in Figure 1, the knife is supposed to cut the object along the desired trajectory, which is indicated by the dash line, where the solid red dots, which are called trajectory points, denoted by the $4n \times 1$ vector R , uniformly distribute on the trajectory. Three non-collinear hollow red dots marked on the

knife, denoted by the 12×1 vector X , are used to control the pose of the knife. These three markers are called the marked points. And the X markers, denoted by the 4×1 vector P , around the desired trajectory are the feature points. Visual feedback of these feature points, denoted by the vector p , is used to estimate unknown parameters of the deformation model. Based on the current positions of the trajectory points, the current shape of the desired trajectory after deformation can be interpolated. Moreover, with respect to the geometry of the knife and the camera model, desired trajectories of the marked points on the image plane can be calculated, too. At each moment, the knife is controlled by manipulators to cut the object along the currently predicted image trajectories.

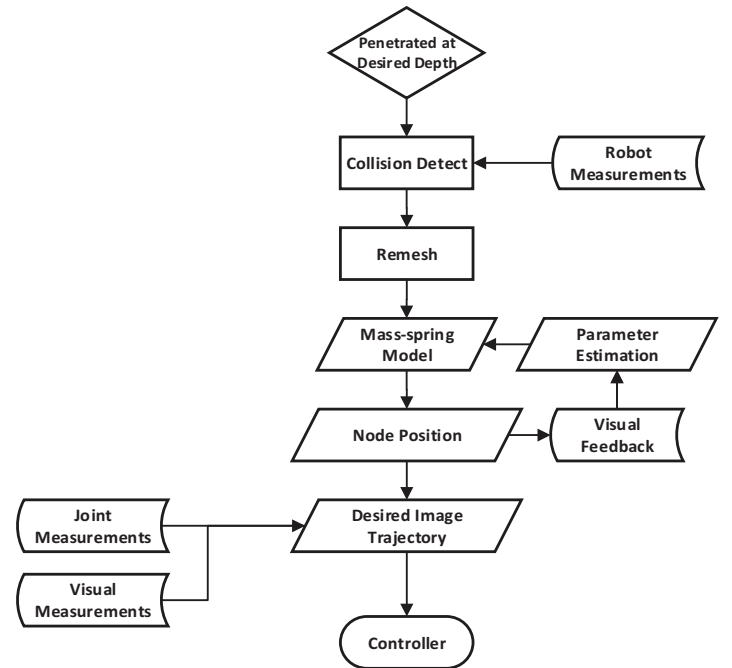


Fig.2 Algorithm Flow Chart

The proposed algorithm can be divided into two main parts. In the first part which will be discussed in chapter III, the current positions of the trajectory points after deformation will be predicted while unknown parameters of the model are estimated by the visual feedback of the feature points. In addition, three marked points' desired trajectories in the image plane will also be interpolated. Thus in the second part discussed in chapter IV, visual tracking task of the marked points will be implemented to control the pose of the knife during the cutting process. The flow char of the whole algorithm is shown in Figure 2.

There are four coordinate frames to be considered, which are the object frame F_o , the camera frame F_c , the knife frame F_k and the base frame.

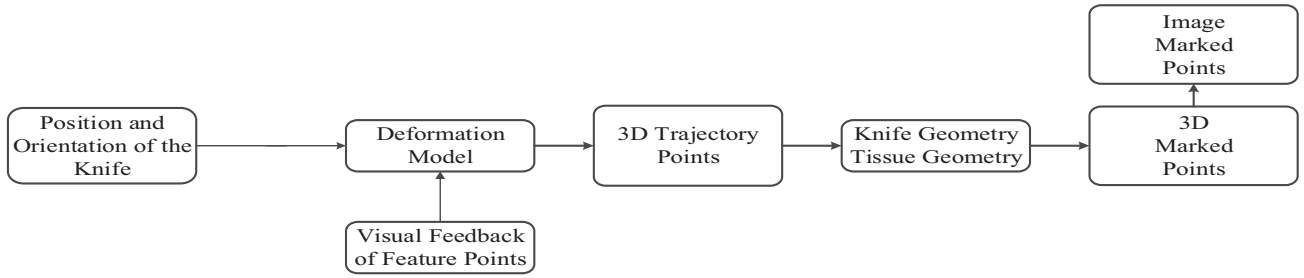


Fig.3 The Block Diagram of Chapter III

2.2. Notation

In this paper, a vector v with respect to frame x is denoted by xv , while v without superscript is denoted as the vector in the base frame. And time-varying quantities are expressed with a bracket (t) . The identity matrix is denoted as $I_{k \times k}$ and the null matrix as $0_{m \times n}$.

III. DEFORMATION DURING CUTTING

In this section, deformation during cutting will be estimated and desired trajectories will be generated. The block diagram of this section is shown in Figure 3.

3.1. Deformation Model

The cutting operation studied in this paper requires a real-time deformation model. Thus mass-spring model is chosen to describe the deformation behavior during cutting. However, one of the main drawbacks of the mass-spring model is the difficulty to define the material parameters. Moreover, in practical occasions, it is usually hard to acquire the specific material parameters in advance. In order to deal with this problem, visual feedback is proposed to estimate the unknown material parameters of the model.

3.1.1. Mass-Spring Model

The geometry of the object is represented by a 3D mesh of n nodes. Denote the position of the node i by N_i , then according to the model, the current position of the node i can be calculated by [14]:

$${}^oN_i(t) = {}^oN_i(t-1) + [(-\alpha \sum_{j \in \sigma(i)} k_{ij} {}^o\Delta_{ij}(t) - m_i g)] \quad (1)$$

Where k_{ij} represents for the stiffness of the link connecting node i and node j , and Δ_{ij} represents for the current length of the link minus its resting length. $\sigma(i)$ denotes the set of the indices of the nodes adjacent to node i . And m_i is the mass of the node. The scaling factor α is used to make sure that the iteration converges.

3.1.2. Unknown Parameter Estimation

A parameter-estimating method is proposed in this section to estimate the unknown material parameters of the mass-spring model with the feature points' visual measurements.

A. Perspective-Projection Relationship

The object's deformation is supervised by a fixed pin-hole camera. In order to describe the relationship of the 3D positions of the feature points and their image projections, perspective-projection model of the feature points with respect to the object frame need to be established.

Denote the homogeneous intrinsic parameter matrix of the camera by a 3×4 matrix Ω . And the homogeneous matrix of the transition matrix of the object frame with respect to the camera frame is denoted by a 4×4 matrix cT_o . Then the perspective-projection matrix can be obtained as follows:

$$M_o = \Omega \cdot {}^cT_o = [m_{o12}^T, m_{o3}^T]^T \quad (2)$$

Then denote the homogeneous coordinates of feature points in the image plane by:

$$p(t) = [p_1(t), \dots, p_l(t)]^T \quad (3)$$

Where

$$p_i(t) = [p_{ui}(t), p_{vi}(t), 1]^T, i=1,2,\dots,l \quad (4)$$

And the homogeneous coordinates of 3D feature points with respect to the object frame are denoted by:

$${}^oP(t) = [{}^oP_1(t), \dots, {}^oP_l(t)]^T \quad (5)$$

Thus the perspective-projection relationship between the 3D feature point ${}^oP_i(t)$ and its image projection $p_i(t)$ can be presented as:

$${}^oP_i(t) = {}^cP_{zi}(t) \cdot M_o^{-1} \cdot p_i(t) \quad (6)$$

$${}^cP_{zi}(t) = m_{o3}^T \cdot {}^oP_i(t) \quad (7)$$

B. Parameter Estimation

Assume that the object is consist of linear material, the stiffness parameter k of the link, the node's mass m and the scaling factor α are treated as unknown parameters to be estimated.

$$\theta = [\alpha k, m]^T \quad (8)$$

According to the deformation model with the estimated parameters, the estimated value of the feature point's position can be calculated.

$${}^o\hat{P}_i(t) = {}^oP_i(t-1) + [(-\hat{\alpha} \hat{k} \sum_{j \in \sigma(i)} {}^o\Delta_{ij}(t) - \hat{m} g)] \quad (9)$$

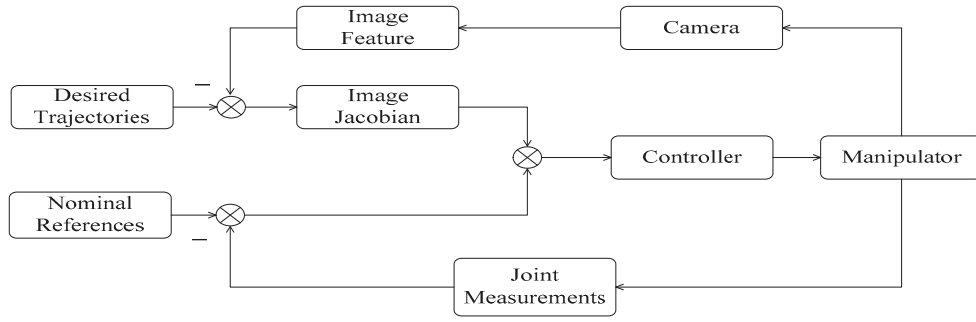


Fig.5 The Block Diagram of Chapter IV

Thus the estimation error can be obtained with respect to the visual measurements of the feature points:

$$\begin{aligned} e(t) &= {}^c\hat{P}_{iz}(t) \cdot p_i(t) - m_{o12}^T \cdot {}^c\hat{P}_i(t) \\ &= W \left(\sum_{j \in \sigma(i)} {}^o\Delta_{ij}(t), {}^oP_i(t-1), p_i(t) \right) \cdot \Delta\theta(t) \end{aligned} \quad (10)$$

Then the parameter updating law can be defined as:

$$\dot{\hat{\theta}}(t) = -\Gamma W^T(t) B e(t) \quad (11)$$

3.2. Generation of the Desired Image Trajectory

3.2.1. Desired Position and Orientation of the Knife

According to the object's deformation model, at each moment, current positions of the trajectory points can be calculated, using which the current shape of the desired trajectory can be interpolated. As shown in Figure 4, in order to cutting along the desired trajectory, during the cutting process, the most front part of the knife X_R , which is called the cutting point, should always cut on the current trajectory in spite of the deformation introduced by the cutting operation. The desired position of X_R corresponding to each trajectory point can be derived. Moreover, according to the relationships between X_R and the marked points X_1, X_2, X_3 , the desired positions of these marked points can also be calculated.

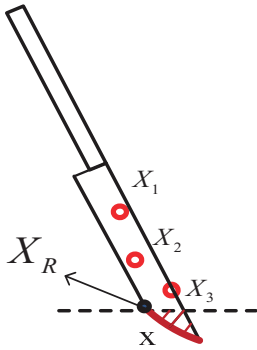


Fig.4 Algorithm Flow Chart

In order to minimize the deformation introduced by the cutting operation, the pose of the knife should be perpendicular to the object's surface and parallel to the desired trajectory. According to the current shape of the desired trajectory and the object's surface geometry, the desired pose of the knife cutting at the trajectory point oR_i in the object frame can be obtained [20]:

$${}^oT_r(t) = \begin{bmatrix} {}^os^d(t) & {}^on^d(t) & {}^oa^d(t) & {}^op_i^d(t) \\ 0 & 0 & 0 & 1 \end{bmatrix}, i=1, \dots, n \quad (12)$$

Where ${}^os^d$ is the tangent vector of the desired cutting trajectory, ${}^on^d$ is the normal vector of the object's surface, ${}^oa^d$ is the remaining orthogonal vector and ${}^op_i^d$ is the position of the trajectory point oR_i .

Denote the homogeneous coordinates of the marked point under the knife frame as kX_j , then the point's homogeneous coordinates corresponding to the trajectory point oR_i can be calculated as follows:

$${}^oX_j(t) = {}^oT_r(t) \cdot {}^kX_j, j=1, 2, 3 \quad (13)$$

Denote the image projection of the marked point X_j corresponding to the trajectory point R_i as ${}^o r_j$:

According to established perspective-projection model between the object frame and the camera, we have:

$${}^o r_j(t) = \frac{1}{{}^cX_{zj}(t)} \cdot M_o \cdot {}^oX_j(t) \quad (14)$$

$${}^cX_{zj}(t) = m_{o3}^T \cdot {}^oX_j(t) \quad (15)$$

Based on the desired positions of the marked points in the image plane, the desired image trajectories can be interpolated. Denoted the desired positions, velocities and accelerations by $y_d, \dot{y}_d, \ddot{y}_d$ respectively.

IV. VISUAL TRACKING CONTROL

In this section, given the desired time-varying trajectories in the image plane, the robot is manipulated to control the projection of the marked points tracking the desired trajectories based on visual feedback. The block diagram is shown in Figure 5.

4.1. Perspective-projection Relationship

Denote the homogeneous matrix of the transition matrix of the base frame with respect to the camera frame by a 4×4 matrix cT_b . Then the perspective-projection matrix can be obtained as follows:

$$M = \Omega \cdot {}^cT_b = \begin{bmatrix} m_{12}^T & m_3^T \end{bmatrix}^T \quad (16)$$

The homogeneous coordinates of 3D marked points are denoted by:

$$X(t) = [X_1(t), X_2(t), X_3(t)]^T \quad (17)$$

And denote the coordinates of marked points in the image plane by:

$$y(t) = [y_1(t), y_2(t), y_3(t)]^T \quad (18)$$

Where

$$y_i(t) = [u_i(t), v_i(t)]^T, i=1,2,3 \quad (19)$$

According to the perspective-projection relationship:

$$\dot{y}_i(t) = \frac{1}{cX_{zi}(t)} (m_{12}^T - y_i(t) \cdot m_3^T) \dot{X}_i(t) = \frac{1}{cX_{zi}(t)} D_i(t) \dot{X}_i(t) \quad (20)$$

$$cX_{zi}(t) = m_3^T \cdot X_i(t) \quad (21)$$

4.2. Robot Model

Denote the joint variable of the robot by a $m \times 1$ vector q , according to the kinematics of the robot model:

$$\dot{X}_i(t) = J_i(q(t)) \cdot \dot{q}(t) \quad (22)$$

Where J_i is called the robot Jacobian.

The dynamics of the robot manipulator are as follows:

$$H(q)\ddot{q}(t) + [\frac{1}{2}\dot{H}(q) + C(q, \dot{q})]\dot{q}(t) + G(q) = \tau \quad (23)$$

where $H(q)$ is the positive-definite and symmetric inertia matrix, and $C(q, \dot{q})$ is a skew-symmetric matrix, and $G(q)$ represents the gravitational force. τ is the joint input.

4.3. Visual Tracking Controller

In this section, a multi-point visual tracking controller proposed in [22] is adopted to control the the marked points tracking the desired image trajectories.

First of all, nominal references \dot{y}_{ri} in the image plane should be defined as follow:

$$\dot{y}_{ri}(t) = \dot{y}_{di}(t) - \lambda \Delta y_i(t) \quad (24)$$

$$\Delta y_i(t) = y_i(t) - y_{di}(t) \quad (25)$$

where λ is a positive gain value. And the nominal references in the joint space:

$$\dot{q}_r(t) = J_i^{-1}(q(t)) D_i^+(t) cX_{zi}(q(t)) \dot{y}_{ri}(t) \quad (26)$$

Then tracking errors can be calculated as follows:

$$S_y(t) = \dot{y}(t) - \dot{y}_r(t) \quad (27)$$

$$S_q(t) = \dot{q}(t) - \dot{q}_r(t) \quad (28)$$

Input visual feedback and joint feedback to the following controller:

$$\begin{aligned} \tau = & H(q)\ddot{q}_r(t) + [\frac{1}{2}\dot{H}(q) + C(q, \dot{q})]\dot{q}_r(t) + G(q) \\ & - K_1 S_q(t) - J^T(q(t)) D^T(t) K_2 S_y(t) \end{aligned} \quad (29)$$

where K_1, K_2 are positive-definite gain matrixes.

According to the Lyapunov theory, we can prove that the position and velocity error of the marked point asymptotically converge to zero.

V. EXPERIMENTS

To validate the proposed method, we will provide the experimental results with an eye-to-hand robot system.

5.1. Setup

The robot system used in the experiment was a six-DOF manipulator with revolute joints only. And a fixed USB Logitech camera was used to supervise the object's deformation. We validated our control algorithm on a fixed deformable sponge which cannot move during the whole process of the experiment.

As shown in Figure 6, seven red markers attached on the surface of the sponge was the feature points whose visual measurements are used to estimate material parameters of the deformation model. And the three green markers on the knife are used to control the knife's position and orientation. We adopted color detecting method to recognize the marked points and the feature points. And the desired image trajectories were shown in the form of moving blue circle in the video of our experiment.

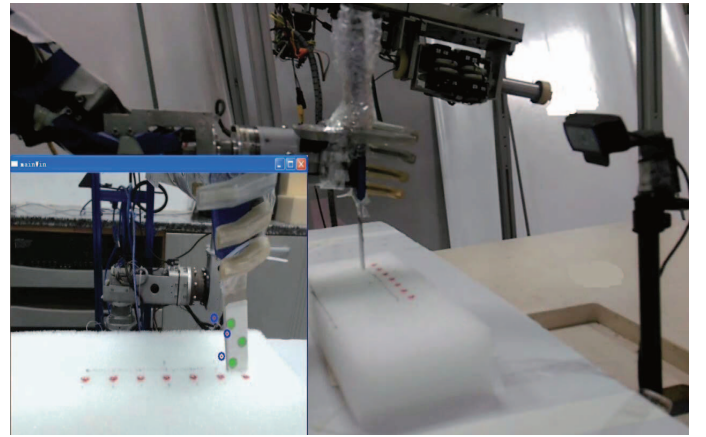


Fig. 6 Experiment setup

5.2. Case of Study

In this experimental study, the desired trajectories are interpolated by the Cardinal interpolation method. And the initial value of the unknown parameters to be estimated was zero. The control gain we adopted here were: $K_1 = 0.00000000034I_6$, $K_2 = 0.000000000015I_6$, $\Gamma = 0.00000006$, $B_1 = 0.1I_{14}$, $\lambda = 10$. And the sampling time used here was: $t = 0.125s$.

Tracking errors of the x and y direction in the image plane are shown in Figure 7. And Figure 8 shows the desired and the real trajectories in the image plane of the tracking task, where the red, green and blue lines represent for the three markers respectively. The experiment validates the convergence of the proposed tracking control algorithm.

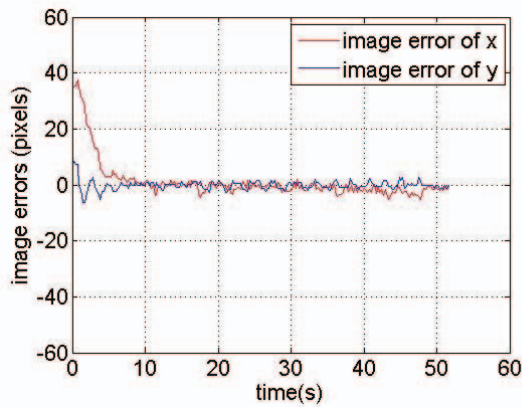


Fig. 7 Image Tracking Errors

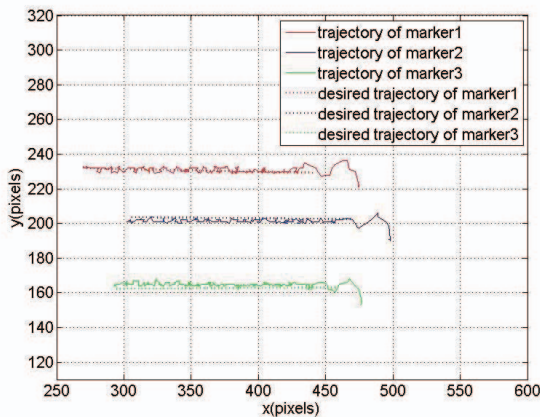


Fig. 8 Image Trajectories

VI. CONCLUSIONS

This paper proposed a vision-based robotic controller to implement cutting task on deformable object. Deformation of the object during the cutting process is predicted, according to which cutting path is planned online. By visually supervising three non-collinear points on the knife, the control of knife's motion is transformed to a multi-points' visual tracking control problem. And the control goal of the pose of the knife is to achieve desired cutting effect in terms of the desired cutting depth and the minimized deformation of the object. The proposed algorithm doesn't need a priori knowledge of the object's material parameters. Visual measurements of the feature points on the objects' surface are used to estimate these unknown parameters. And the feasibility of the method is validated by experiment results.

In the future, we will try to implement the deformable-object cutting control task without calibrations of the relative pose between the camera and the object and the robot.

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