LPV Modeling and Position Control of Two Mass Systems with Variable Backlash Using LMIs

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***Abstract*—This paper presents a Linear Parameter Varying (LPV) approach to model and control two-mass systems with backlash. The maximum amplitude of the backlash angle is assumed to be unknown and variable having no knowledge about the upper and lower bounds of it. Proper affine state space model together with the admissible variations of the LPV parameters is designed in order to realize a viable convex polytope. Utilizing H_∞ LPV lemmas and theories lead to a set of Linear Matrix Inequalities (LMIs). By solving these LMIs, appropriate scheduled state feedback gains are obtained. The designed robust control strategy can easily handle the variations of the backlash angle and load disturbance torque. A simulated two-mass backlash system verifies the efficiency of the designed control law.**

Keywords—Linear parameter varying model; two-mass system; backlash; linear matrix inequality, position control

I. INTRODUCTION

Two-mass system with backlash can be considered as the basic simplified model for the multi-mass and complex mechanical systems, where backlash is a kind of hard-nonlinearity which is largely extant in the movable mechanical joints. This hard-nonlinearity generally exists in systems where a driving part (motor) is not directly connected to a driven one (load). Gears and gearboxes including Spur gears, Helical gears, Bevel gears and Worm gears can be mentioned as the most tangible mechanical parts where the backlash hard-nonlinearity exists. Control of two-mass systems with backlash has been widely dealt with in the literature during the past years. Having unknown and uncertain physical parameters is the most important milestone in the control process design of these systems.

Accurate identification of unknown physical parameters is the first important step in control of two-mass systems with backlash. Maximum amplitude of the backlash angle can be considered as the common unknown parameter which is supposed to be identified in the identification process. Model-based methods [1-3] and data-based methods [4-6] are the two main branches in identification researches. Backlash identification in transmission units [7], gradient based identification technique [8], modeling and identification of elastic robot joints [9], and Recursive iterative identification algorithms [10] are some of the recent papers regarding the identification of the backlash systems. However, designing control strategies without utilizing identification techniques while the system parameters are unknown and variable is much more precious which is not generally available in the literature.

Control of two-mass systems with backlash is categorized into two objectives: load speed control, and load position control which is the main target of this paper. Anti-backlash gear [11], generalized describing function [12], backlash compensation utilizing preload offset [13], quantitative feedback theory [14], variable structures controllers [15], cascade controllers [16], and state dependent algebraic Riccati equation are typical controller design techniques available in literature. Compensation of teleoperated geared motor drives [17], feedforward control of mechatronic systems [18], backstepping position control [19], and adaptive control strategies [20] are some of the recent studied researches. The main drawback of most of the designed control laws in the literature is their inefficiency in dealing with unknown and variable backlash angle without benefiting identification techniques. In other words, accurate and efficient position control of the load side in spite of variable maximum amplitude of the backlash angle is rarely dealt with in the research papers.

In this paper, Linear Parameter Varying (LPV) technique is utilized to obtain a control law for the two-mass systems with variable backlash without having any knowledge about the upper and lower bounds of the backlash angle. An affine state space model together with the admissible variations is designed to retrieve an appropriate LPV convex polytope. Utilizing H_∞ LPV lemmas and solving the resulted Linear Matrix Inequalities (LMIs) lead to obtaining the desired scheduled state feedback gains. Simulations show the efficiency of the designed control law in spite of maximum backlash angle variations. This control strategy can handle not only the backlash angle variations, but also the load disturbance torque variations efficiently.

The structure of the paper is as follows. An introduction to the two-mass systems with backlash is presented in section 2. Linear Parameter Varying preliminaries are stated in section 3. The LPV model, convex polytope model, and LMIs guiding the control law are mentioned in section 4. Simulations on a

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sample two-mass backlash system are described in section 5, and the conclusions are stated in section 6.

II. Two Mass Systems

A brief introduction to the two-mass systems with backlash which is the main case study of the paper is presented in this section.

Two-mass systems with backlash consist of three main parts: motor, load and shaft. Motor and load are connected to each other via the shaft where the backlash phenomenon exists. Due to the existence of the backlash hard-nonlinearity, motor side is partly connected/disconnected to the load side during the position control process. The schematic diagram of two-mass systems with backlash is illustrated in Fig. 1.

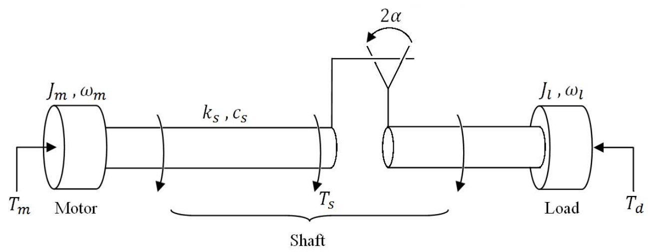

Fig. 1: Schematic diagram of two-mass system with backlash

The mathematical model of two-mass systems with backlash is described as below [21]:

$$J_m\dot{\omega}_m = -c_m\omega_m - T_s + T_m \tag{1}$$

$$J_l\dot{\omega}_l = -c_l\omega_l + T_s - T_d \tag{2}$$

$$\theta_d = \theta_m - \theta_l \quad , \quad \omega_d = \omega_m - \omega_l \tag{3}$$

$$\dot{\theta}_m = \omega_m \ , \quad \dot{\theta}_l = \omega_l \ , \quad \dot{\theta}_d = \omega_d \tag{4}$$

$$T_s = k_s\theta_d + c_s\omega_d \tag{5}$$

$$\dot{\theta}_s = \omega_s \tag{6}$$

where θ_m, θ_l and θ_s are the motor, load and shaft twist angles respectively. ω_m , ω_l and ω_s are the motor, load and shaft twist angular velocities in sequence. θ_d and ω_d denote the difference angle and difference angular velocity, $J_m\ (kg\ m^2)$ and $c_m\ (Nm/(rad/s))$ are the motor moment of inertia and viscous motor friction, $T_m\ (Nm)$ and $T_s\ (Nm)$ are the motor torque and the transmitted shaft torque respectively, $J_l\ (kg\ m^2)$ and $c_l\ (Nm/(rad/s))$ are the load moment of inertia and viscous load friction, $k_s\ (Nm/rad)$ and $c_s\ (Nm/(rad/s))$ are the shaft elasticity and inner damping coefficient of the shaft, and $T_d\ (Nm)$ is the load disturbance torque. Moreover, the backlash angle which is denoted by θ_B varies between $-\alpha$ and α, where α shows the maximum amplitude of the backlash angle.

Modeling of the backlash phenomenon is generally defined as finding an appropriate mathematical relation between the shaft twist angle (θ_s) and backlash angle (θ_B). There are four basic models in the literature modeling the backlash phenomenon: exact model, dead-zone model, describing function model and hysteresis model. Exact model of the backlash is defined in the following [21,22]:

$$\theta_s = \theta_d - \theta_B \tag{7}$$

$$\omega_s = \omega_d - \omega_B \tag{8}$$

$$\dot{\theta}_B = \begin{cases} \max(0, \dot{\theta}_d + \frac{k_s}{c_s}(\theta_d - \theta_B)) & if \ \ \theta_B = -\alpha \ (T_s \le 0) \\ \dot{\theta}_d + \frac{k_s}{c_s}(\theta_d - \theta_B) & if \ \ |\theta_B| < \alpha \ (T_s = 0) \\ \min(0, \dot{\theta}_d + \frac{k_s}{c_s}(\theta_d - \theta_B))) & if \ \ \theta_B = \alpha \ (T_s \ge 0) \end{cases} \tag{9}$$

Dead-zone model is the simplest and the most common model which is widely utilized in the literature. This model is depicted as below:

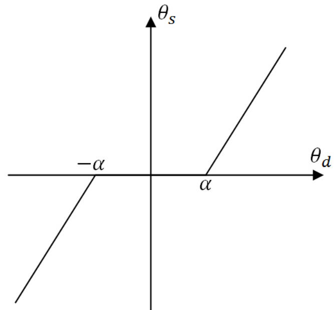

Fig. 2: Dead-zone model of the backlash

The maximum amplitude of the backlash angle, α, varies by the time passage, erosion, and different environmental conditions. Taking into account variable α which in fact equals to infinite equilibrium points, can be illustrated as the following figure:

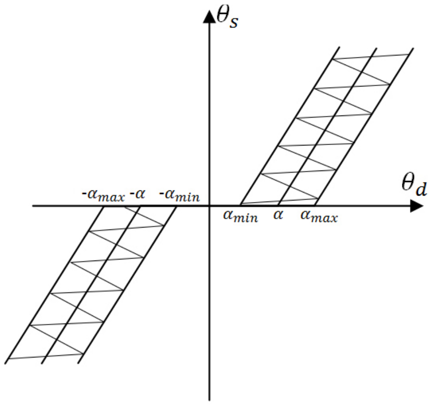

Fig. 3: Dead-zone model of the backlash with variable α

Linear parameter varying technique will be utilized to design an efficient control strategy dealing with such a highly nonlinear system illustrated in Fig. 3. In the next section, the basic LPV lemmas and preliminaries will be introduced.

III. Linear Parameter Varying Preliminaries

In this section, an introductory review to some of the basic LPV lemmas is presented. The explained lemmas and theories in the following will be used in the designed LPV strategy to control the load angle of two-mass systems with backlash.

Consider a linear time varying system with the following state-space model [23,24]:

$$\dot{x}(t) = A(\delta(t))x(t) + B_1(\delta(t))w(t) + B_2(\delta(t))u(t) \tag{10}$$

$$z(t) = C(\delta(t))x(t) + D_1(\delta(t))w(t) + D_2(\delta(t))u(t) \tag{11}$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, $w(t) \in \mathbb{R}^r$ is an exogenous input, $u(t) \in \mathbb{R}^m$ is the control input, $z(t) \in \mathbb{R}^p$ is the controlled output, and Matrices $A(.) \in \mathbb{R}^{n\times n}$, $B_1(.) \in \mathbb{R}^{n\times r}$, $B_2(.) \in \mathbb{R}^{n\times m}$, $C(.) \in \mathbb{R}^{p\times n}$, $D_1(.) \in \mathbb{R}^{p\times r}$, and $D_2(.) \in \mathbb{R}^{p\times m}$ denote the state space matrices which are constrained to the polytope Δ defined as:

$$\begin{aligned}\Delta = \{&(A, B_1, B_2, C, D_1, D_2)(\delta(t)) \,|\, (A, B_1, B_2, C, D_1, D_2)(\delta(t)) \\ &= \sum_{j=1}^{N} \delta_j(t)(A, B_1, B_2, C, D_1, D_2)_j\} \\ &, \sum_{j=1}^{N} \delta_j(t) = 1, \quad \delta_j(t) \geq 0, j = 1, \dots, N\end{aligned} \tag{12}$$

and

$$\delta(t) = [\delta_1(t) \quad \dots \quad \delta_N(t)]' \tag{13}$$

where δ_j denotes the j^{th} linear-varying parameter, and N is the total number of LPV parameters which also equals to the number of vertices shaping convex polytope Δ. The following Lemmas and Theories [23,24], represent the basic LPV propositions for the described uncertain system in (10) and (11).

Lemma 1: If there exists a symmetric positive definite matrix $W \in \mathbb{R}^{n\times n}$ and a matrix $Z \in \mathbb{R}^{m\times n}$ such that

$$A_jW + WA_j' + B_{2j}Z + Z'B_{2j}' < 0 \;,\; j = 1, \dots, N \tag{14}$$

then the quadratic stability of the uncertain system $(A, B_2)(\delta(t))$ is guaranteed by the state feedback gain $K = ZW^{-1}$, where $P = W^{-1}$ is a Lyapunov matrix ensuring the closed loop quadratic stability of the following system:

$$\dot{x}(t) \triangleq A_{cl}(\delta(t))x(t) + B_1(\delta(t))w(t) \tag{15}$$

$$z(t) \triangleq C_{cl}(\delta(t))x(t) + D_1(\delta(t))w(t) \tag{16}$$

where

$$A_{cl}(\delta(t)) \triangleq A(\delta(t)) + B_2(\delta(t))K(\delta(t)) \tag{17}$$

$$C_{cl}(\delta(t)) \triangleq C(\delta(t)) + D_2(\delta(t))K(\delta(t)) \tag{18}$$

Lemma 2: If there exist a symmetric positive definite matrix $W \in \mathbb{R}^{n\times n}$ and a matrix $Z \in \mathbb{R}^{m\times n}$ for a given γ, such that:

$$R_j = \begin{bmatrix} A_jW + WA_j' + B_{2j}Z + Z'B_{2j}' & * & * \\ B_{1j}' & -I & * \\ C_jW + D_{2j}Z & D_{1j} & -\gamma^2 I \end{bmatrix} < 0 \quad , \; j = 1, \dots, N \tag{19}$$

where $*$ indicates symmetric blocks, then the uncertain system (10), (11) is quadratically stabilisable by the state feedback gain $K = ZW^{-1}$ such that $\|z(t)\|_2 < \gamma\|w(t)\|_2$ holds. In addition, $P = W^{-1}$ is a Lyapunov matrix ensuring the closed loop quadratic stability with γ disturbance attenuation.

Theorem 1: If there exists a symmetric positive definite matrix $W \in \mathbb{R}^{n\times n}$ and matrices $Z_j \in \mathbb{R}^{m\times n}$, $j = 1, \dots, N$ such that

$$Q_j \triangleq A_jW + WA_j' + B_{2j}Z_j + Z_j'B_{2j}' < 0 \;,\; j = 1, \dots, N \tag{20}$$

$$\begin{aligned}Q_{jk} = &(A_j + A_k)W + W(A_j + A_k)' + B_{2j}Z_k + Z_k'B_{2j}' \\ &+ B_{2k}Z_j + Z_j'B_{2k}' < 0 \\ &, \; j = 1, \dots, N-1 \;,\; k = j+1, \dots, N\end{aligned} \tag{21}$$

then the following LPV state feedback control gain:

$$K(\delta(t)) = \sum_{j=1}^{N} \delta_j(t)\, K_j, \sum_{j=1}^{N} \delta_j(t) = 1, \delta_j(t) \geq 0, j = 1, \dots, N \tag{22}$$

where

$$K_j = ZW^{-1}, \;\; j = 1, \dots, N \tag{23}$$

ensures the quadratic stability for the closed-loop system $A_{cl}(\delta(t))$, where $P = W^{-1}$ denotes the desired Lyapunov matrix.

Theorem 2: If there exists a symmetric positive definite matrix $W \in \mathbb{R}^{n\times n}$ and matrices $Z_j \in \mathbb{R}^{m\times n}$, $j = 1, \dots, N$ such that

$$R_j \triangleq \begin{bmatrix} Q_j & B_{1j} & W'C_j' + Z_j'D_{2j}' \\ B_{1j}' & -I & D_{1j}' \\ C_jW + D_{2j}Z_j & D_{1j} & -\gamma^2 I \end{bmatrix} < 0 \tag{24}$$

$$R_{jk} = \begin{bmatrix} Q_{jk} & * & * \\ B_{1j}' + B_{1k}' & -2I & * \\ C_jW + C_kW + D_{2j}Z_k + D_{2k}Z_j & D_{1j} + D_{1k} & -2\gamma^2 I \end{bmatrix} < 0 \quad , j = 1, \dots, N-1 \;,\; k = j+1, \dots, N \tag{25}$$

where Q_j and Q_{jk} are defined in (20) and (21) in sequence, then the LPV state feedback control gain mentioned in (22) and (23) is such that the closed-loop system described in (15) and (16) is quadratically stable with γ disturbance attenuation.

The reviewed LPV Lemmas and theories will be utilized to design LPV control law for the two-mass systems studied in section 2. In the next section, the LPV load position control design for the two-mass systems with backlash will be studied.

IV. LPV Modeling and Controller Design

An affine state space model for the two-mass systems with variable backlash is designed in this section. This modeling should be designed in a way that covers the variations of the maximum amplitude of the backlash angle, and makes the LPV lemmas applicable. In the following, the designed modeling is presented.

Let the mathematical model of the variable backlash be defined as:

$$\theta_s = \theta_d + P\theta_d \tag{26}$$

where

$$P = \begin{cases} -\dfrac{\alpha(t)}{\theta_d} & \theta_d > \alpha(t) \\ -1 & -\alpha(t) < \theta_d < \alpha(t) \\ \dfrac{\alpha(t)}{\theta_d} & \theta_d < -\alpha(t) \end{cases} \tag{27}$$

Since there is no knowledge about the upper and lower bounds of the backlash angle, $\alpha(t)$, and in order to cover all the possible cases, suppose that $\alpha_{min} = 0$ and $\alpha_{max} = \infty$. Although these backlash bounds seem to be large firstly, the mapped uncertainties remain reasonable from the viewpoint of the whole system. In other words, the LPV techniques for the whole state space model will be efficiently applicable by taking these backlash bounds into account.

The next step is finding the minimum and the maximum values of the parameter p defined in (27). All the possible cases can be discussed as in (28).

$$\begin{cases} \theta_d > \alpha(t): & p_{min} = -\dfrac{\alpha(t)}{\theta_{d,min}} \to -1^+, \;\; p_{max} \to -\dfrac{\alpha(t)}{\infty} = 0^- \\ |\theta_d| < \alpha(t): & p_{min}, p_{max} = -1 \\ \theta_d < -\alpha(t): & p_{min} \to \dfrac{\alpha(t)}{-\infty} = 0^-, \quad p_{max} = -\dfrac{\alpha(t)}{\theta_{d,max}} \to -1^+ \end{cases} \tag{28}$$

As discussed in (28), the minimum and the maximum values of p are 0 and -1 in sequence, in other words $p \in [-1,0)$.

Suppose that the states of the system described in (1) to (6) are defined as:

$$x_1 = \theta_m, x_2 = \omega_m, x_3 = \theta_l - \theta_r, x_4 = \omega_l \tag{29}$$

where θ_r is the reference angle of the load side. Taking into account relations (1)-(6), (26) and (29), the affine state space model of the two-mass backlash systems can be written as (30):

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_s(1+p)}{J_m} & -\dfrac{(c_m+c_s)}{J_m} & \dfrac{k_s(1+p)}{J_m} & \dfrac{c_s}{J_m} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s(1+p)}{J_l} & \dfrac{c_s}{J_l} & -\dfrac{k_s(1+p)}{J_l} & \dfrac{-c_l-c_s}{J_l} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{k_s(1+p)}{J_m} \\ 0 & 0 \\ -\dfrac{1}{J_l} & \dfrac{-k_s(1+p)}{J_l} \end{bmatrix}, w(t) = \begin{bmatrix} T_d \\ \theta_r \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \dfrac{1}{J_m} \\ 0 \\ 0 \end{bmatrix} \tag{30}$$

$$C = [0 \;\; 0 \;\; 1 \;\; 0], \quad D_1 = [0 \;\; 1], \quad D_2 = 0$$

where the shaft twist angular velocity is equaled to the difference angular velocity for simplification. Substituting the lower and the upper bounds of parameter p in relation (30) generates the polytope systems and the desired affine state space model.

Applying Theorem 2 explained in the previous section, leads to the set of LMIs in (31), (32), (33), and (34).

$$W > 0 \tag{31}$$

$$\begin{bmatrix} A_1W + WA_1' + B_{21}Z_1 + Z_1'B_{21}' & B_{11} & W'C_1' + Z_1'D_{21}' \\ B_{11}' & -I & D_{11}' \\ C_1W + D_{21}Z_1 & D_{11} & -\gamma^2 I \end{bmatrix} < 0 \tag{32}$$

$$\begin{bmatrix} A_2W + WA_2' + B_{22}Z_2 + Z_2'B_{22}' & B_{12} & W'C_2' + Z_2'D_{22}' \\ B_{12}' & -I & D_{12}' \\ C_2W + D_{22}Z_2 & D_{12} & -\gamma^2 I \end{bmatrix} < 0 \tag{33}$$

Solving the obtained LMIs, the desired scheduled state feedback gains will be obtained. Note that relation (32) deals with the first polytope system which is in fact a physically disjointed two-mass system. In other words, substituting the lower bound of p in (30) generates a state space model representing a two-mass system whose backlash angle varies between $-\alpha(t)$ and $\alpha(t)$. Solving the LMI of this physically disjointed system may have some convergence problems. In case of facing these problems, one can replace the mentioned LMI in (32) by its corresponding LMIs in Lemmas 1-2 or in Theorem 1, although utilizing suitable and powerful LMI solvers can handle the probable convergence problems.

LPV technique is utilized to model two-mass systems with backlash in this section. The variations and uncertainties of the maximum amplitude of the backlash angle are covered by the proposed LPV model. Simulations on an illustrative example show the efficiency of the designed method in the next section.

$$\begin{bmatrix} (A_1+A_2)W + W(A_1+A_2)' + B_{21}Z_2 + Z_2'B_{21}' + B_{22}Z_1 + Z_1'B_{22}' & B_{11}+B_{12} & W'C_1' + W'C_2' + Z_2'D_{21}' + Z_1'D_{22}' \\ B_{11}' + B_{12}' & -2I & D_{11}' + D_{12}' \\ C_1W + C_2W + D_{21}Z_2 + D_{22}Z_1 & D_{11}+D_{12} & -2\gamma^2 I \end{bmatrix} < 0 \tag{34}$$

V. SIMULATION RESULTS

To evaluate the performance of the designed LPV algorithm to control the load position of two-mass systems with variable backlash, an illustrative example is studied in this section.

Consider a two-mass backlash system with the following parameters [21]:

$J_m = 0.4 \;\; kg\,m^2$ $\qquad$ $J_l = 5.6 \;\; kg\,m^2$

$c_m = 0.1 \;\; Nm/(rad/s)$ $\qquad$ $c_l = 1 \;\; Nm/(rad/s)$

$k_s = 3300 \;\; Nm/rad$ $\qquad$ $\alpha = 0.03\ (rad)$

$c_s = 2 \;\; Nm/(rad/s)$ $\qquad$ $T_d = 8 \;\; Nm$

Suppose that the load reference angle is 0.5 radians, and the controller goal is to set the load angle to this reference set point. Solving the mentioned LMIs in (31)-(34) and utilizing Theorem 2 explained in section 3, the following state feedback gains are obtained:

$$K = 10^4 \times \begin{bmatrix} -1.292820 \\ -0.018157 \\ -2.949388 \\ -0.180820 \end{bmatrix}^T$$

with the obtained H_∞ attenuation level $\gamma = 2.9734$.

The load angle and the motor torque signals are shown in the following figures in sequence.

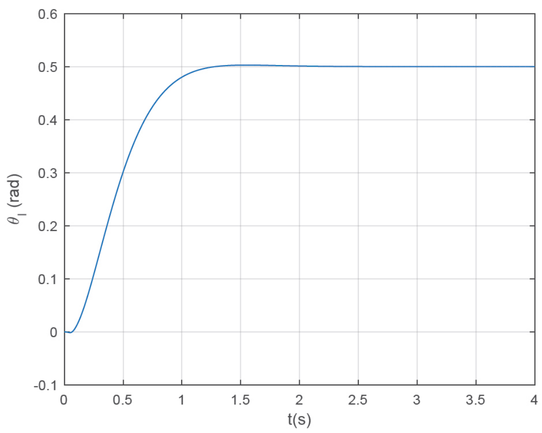

Fig. 4: Load angle control via the designed LPV approach

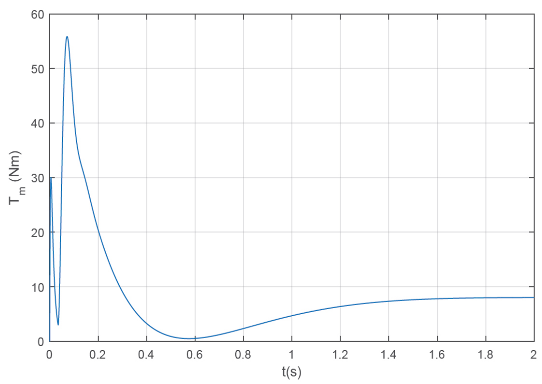

Fig. 5: Motor torque signal

As figures 4 and 5 show, the load angle is smoothly converged to the desired set point and the generated control input is practically applicable to a typical electrical motor.

To evaluate the robustness of the designed control law facing with variable backlash, the maximum amplitude of the backlash angle is changed from 0.03 radians to 0.045 radians (50% increase) at $t = 2.5\,s$. The load angle trend and the motor torque signal are shown in figures 6 and 7 sequentially. As Fig. 6 shows, a transient load angle divergence has been occurred at $t = 2.5\,s$, although the illustrated motor torque in Fig. 7 can easily handle this unwanted divergence which originally roots from the maximum backlash amplitude changes. Note that the steady state value of the motor torque is increased since $t = 2.5\,s$. The designed LPV control law can easily handle not only the maximum backlash amplitude variations but also the load disturbance torque.

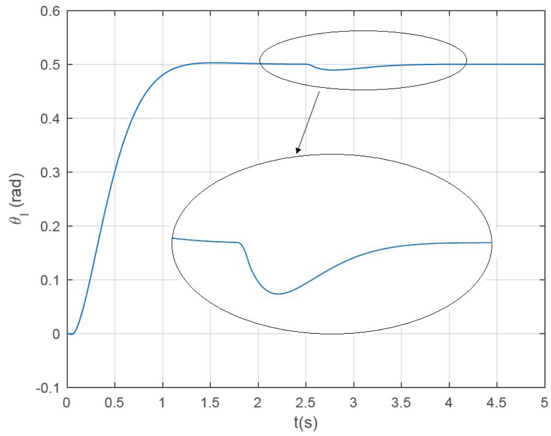

Fig. 6: LPV Load angle control with variable α from 0.03 to 0.045 radians

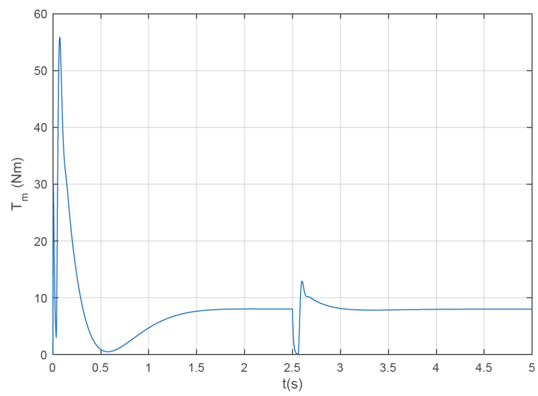

Fig. 7: Motor torque signal handling α variations from 0.03 to 0.045 radians

To show the efficiency of the proposed LPV algorithm in comparison with the recent backlash researches in the literature, the designed LPV algorithm is compared to the nonlinear backstepping controller available in [19]. The load angle trends utilizing LPV and backstepping controllers are shown in Fig. 8. As this figure indicates, the transient response in LPV control is faster than the backstepping response. Faster transient response, robustness and having practical control effort are some of the advantages of the designed LPV control algorithm.

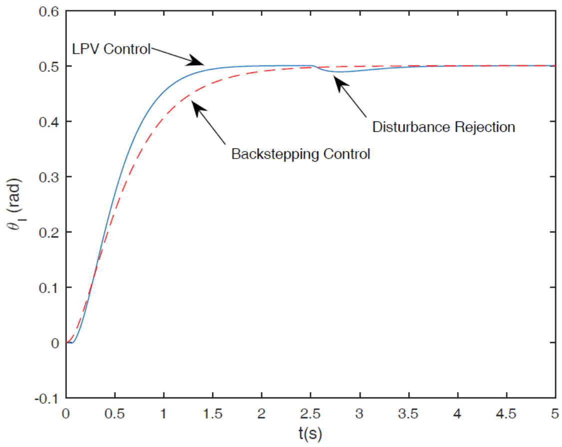

Fig. 8: LPV and Backstepping control of the load angle

VI. Conclusion

The load position control of two-mass systems with variable backlash is studied in this paper. Since the maximum amplitude of the backlash is assumed to be unknown and variable, LPV technique is utilized to treat this highly nonlinear system. Utilizing proper LPV lemmas and designing suitable affine state space model lead us to a set of LMIs. By solving these LMIs, scheduled state feedback gains are gained to control the load position. Simulations show the high efficiency of the proposed method in spite of unwanted changes not only in maximum amplitude of the backlash but also changes in load disturbance torque.

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