University of Leicester

University of Leicester Engineering Department

Memorandum

From: Dr. C. Edwards

To: MSc Students

Date: February 11, 2004

EG7019 Advanced Control System Design

Attached is the first assessment for the Advanced Control System Design module. This is an individual exercise and the work you submit must be your own. Any plagiarism (including the direct inclusion of material from websites) will be dealt with severely.

Useful files which can be used for simulation purposes can be downloaded from

`http://www.le.ac.uk/eg/ce14/msc/main.html`

The deadline for submission is 10:00am Monday 15 March 2003. Late submissions will be penalized as specified in the course guide.

Your work needs to be submitted to the **General Office** and must be accompanied by a signed copy of the standard department declaration of originality.

University *of* Leicester

Advanced Control System Design (EG7019)

Assessment 1: Sliding Mode Control

Consider the state-space representation of a DC motor

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{K_t}{J} \\ 0 & -\frac{K_e}{L} & -\frac{R}{L} \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

and the state-space variables are angular position, angular velocity and current respectively . The coefficients R and L are the armature resistance and inductance, J represents the moment of inertia of the spindle whilst K_e and K_t are motor constants. Suppose $R = 1.2$, $L = 0.05$, $K_e = 0.6$, $K_t = 0.6$ and $J = 0.135$. Consider the sliding surface

$$\{x \in \mathbb{R}^3 \quad : \quad Sx(t) = 0\} \tag{2}$$

where

$$S = \begin{bmatrix} s_1 & s_2 & 1 \end{bmatrix} \tag{3}$$

and s_1 and s_2 are design parameters. For convenience write $s(t) = Sx(t)$.

Exercise 1

Explain why in this particular example the poles of the sliding motion can be placed arbitrarily. (Hint: Consider the problem as one of state feedback pole placement for a certain subsystem.) [5 marks]

Exercise 2

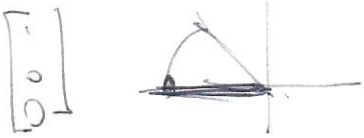
Design a sliding surface so that the (approximate) relay control law

$$u = -\rho \frac{s(t)}{(|s(t)| + \delta)} \tag{4}$$

regulates the initial angular displacement

$$x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

to the origin within 2 seconds with minimal overshoot. Assume that the scalar $\rho = 1$ and aim to make the coefficient δ as small as possible whilst ensuring the control action is chatter free. (The simulations may be performed using *vsimr*, type *help vsimr* for more information.) [14 marks]



Exercise 3

Compare the graph of the (smooth) control signal during the sliding motion from the simulation in Exercise 2 with that of the so-called 'equivalent control' action

$$u_{eq}(t) = -(SB)^{-1}SAx(t) \tag{5}$$

and comment on any similarities. [3 marks]

Exercise 4

Suppose the value of the inductance L is not known precisely but is known to satisfy $0.04 \leq L \leq 0.06$. Similarly, assume that the motor constant K_t is not known precisely but satisfies $0.4 \leq K_t \leq 0.8$. Using the control law designed in Exercise 3, investigate how the variation in L and K_t affects the closed loop performance. [7 marks]

Exercise 5

Consider the state-space system in (1) and the (linear) control law

$$u_l(t) = -(SB)^{-1}SAx(t) + (SB)^{-1}\Phi Sx(t) \tag{6}$$

where Φ is a negative constant. Form the closed loop system and show the function $s(t)$ satisfies

$$\dot{s}(t) = \Phi s(t)$$

and hence $s(t) = e^{\Phi t}$. (Hint: Use the fact that $\dot{s}(t) = S\dot{x}(t)$.) [3 marks]

Exercise 6

Consider the state-space system in (1) and the control law

$$u_{rc}(t) = -(SB)^{-1}SAx(t) + (SB)^{-1}\Phi Sx(t) - \eta \frac{s(t)}{|s(t)|+\delta} \tag{7}$$

where η and δ are small positive constants.

Consider initially that $\eta = 0$, $\Phi = -10$ and assume that A, B and S take the values from Exercise 2. Show the closed loop matrix

$$A_c = A - B(SB)^{-1}SA + B(SB)^{-1}\Phi S$$

and compute the eigenvalues of A_c using MATLAB. Comment on the significance. Now let $\eta = 0.05$ and show by means of simulation that the control law in (7) induces a sliding motion. (Use *vsimrc* in to perform the simulations, type *help vsimrc* for more information.) [5 marks]

Exercise 7

Why might the control law given in Exercise 6 be more suitable for implementation than the relay type given in (4) ? [3 marks]

$\dot{s} = \phi s$

$s\dot{s} = \phi s^2 < 0$

C. Edwards

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