Close-Loop Stabilization of a Flexible Wing Aircraft

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***Abstract*—The design of the airframe of an aircraft is constrained due to aeroelastic effects (generally related to the wings) that induce flexible modes on the structure, and may lead to structural instability. This paper addresses the problem of active mode stabilization in an aircraft with flexible wings. The main objective of closed-loop control is to enlarge the allowable flight envelope by stabilizing flexible modes that become unstable after a certain airspeed is exceeded. The nominal full-order (FO) model has a very large state dimension and hence the controller is designed using a reduced order (RO) model. This paper offers an analysis of the trade-offs present when designing controllers for systems under the premise of large order model reductions. Simulation results are provided using a reliable high order linear model.**

Keywords: Active vibration control, flexible structures, robust control

I. Introduction

Aeroelasticity is a physical phenomenon that must be carefully addressed during the design phase of aircraft. The general approach is to suppress any unstable flexible modes via structural compensation, i.e. the rigidness of the wings is augmented using an additional structure. Although this approach is widespread and effective in achieving its goal (i.e. suppress any unstable flexible modes), it introduces additional weight, hence the wing-span must be traded-off against the structural weight it will introduce. Additionally, such rigid systems will present, in general, more damage than a possible flexible counterpart and will require more maintanace.

In order to allow for more efficient and durable air vehicles, resources are being spent on research centered around flexible wing aircraft development. In contrast to the established design criteria (i.e. added structure to suppres unstable flexible modes), flexible modes may be stabilized via active control, where an *aeroservoelastic* system may be implemented by introducing additional (distributed) control surfaces. Active flutter suppression has been studied in [12], [11], where probably the most recent experimental results have been presented in [3]. The controller design phase of [3] used high order, reliable LTI models for velocities ranging from 40 knots (which is the on-set point of the unstable flexible modes) to 90 knots, with a resolution of 2 knots. LQG controllers were designed at each operating point and later scheduled using a fuzzy-logic blending strategy. As a result the flight enveloped was expanded to 70 knots. Nonetheless, the paper presents no robustness analysis or discussion about the challenges involved in the control law design. The main focus of this paper will be to provide some insight and understanding of the robustness/performance trade-offs and limitations that might be present when designing $\mathscr{H}_\infty$ controllers for a flexible wing aircraft.

Modeling of aeroelastic effects has been widely studied [7], [8], [5] and several computational tools are available (see [4] and references therein). In general, the process requires several checks that allow for reliable, physically meaningful, state representations of the aircraft's aerodynamic behavior. Initially, a theoretical model (assuming the airframe to behave in a quasi-elastic fashion) is built and corrected at low frequencies using wind tunnel data, which is complemented with unsteady state data obtained via CFD software simulation. It is important to highlight the fact high fidelity models require high order representations. Low frequency correction is followed by a rational approximation of the (unsteady) aerodynamic coefficients at higher frequencies, which includes two terms that represent (Theodorsen) delays with respect to the steady dynamic approximation (see [7]). The approximation coefficients are selected via a fitting (usually linear) algorithm; the high-pass break frequencies associated with the delay-like terms can be calculated on the basis of the velocity and chord length. Once a certain tolerance level has been reached, the system is transformed into state-space. The structural model may be validated by comparing predicted free vibration modes with, for example, the modes obtained from Ground Vibration Test (GVT) results of the vehicle [4].

This paper builds on the model developed by the Air Force Research Laboratory (see [9]), where the airframe state space model contains 5 rigid body modes, the first 8 flexible modes, a number of states associated with discrete degrees of freedom, and two delays in the aerodynamic approximation. The vehicle dynamics are represented as a family of linear systems (i.e. linearizations of the aerodynamic coefficients) with high state dimensionality (148 state w/o sensor or actuator dynamics) that represent the system behavior at different airspeeds.

Remark: Note that the system is time-varying by nature, and two (general) approaches may be followed. The common approach is to linearize the plant at an operating point and design on the LTI model. In some cases, a single LTI system is not enough to describe system behavior, so it must be represented either by a family of LTI plants (enough to cover the entire region of operation), or an LTV model. In the first case, robust controllers may be designed at each operating point, and further control scheduling techniques must be applied. In the latter case, a parameter dependent controller (or full nonlinear control) must designed in order to cope with the time varying nature of the plant. In both cases, $\mathscr{H}_2$ and $\mathscr{H}_\infty$ techniques have been presented [2], [11], where it is usual to get a controller which has state dimension close to the plant's. This may lead to added computational complexity and possible numerical errors, thus it is desirable to obtain controllers with the least number of states.

Due to the high order of the system, observer based control strategies (for example LQR or $\mathscr{H}_\infty$ control) may seem impractical due to the reduced number of measurements (relative to the number of sates). The usual approach is to design on the FO system, reduce the number of states of the controller, and check that the robustness margins of the RO controller are similar to those initially predicted. An alternative approach, and the one followed in this paper, is to first reduce the order of the nominal plant model, and design a RO controller which is robust to different sources of uncertainty, including neglected dynamics.

Model reduction can be performed by using a variety of techniques which attempt to minimize the difference between the FO and RO models. While there are many model reduction techniques for LTI systems [13], the tools for LTV systems are less well-developed and generally mathematically involved. Model reduction techniques for LTI systems rely heavily on system transformations, and the resulting RO model state's have no physical meaning, or at least do not retain the original one; in many techniques, several state transformations are applied, thus keeping track of the original coordinate frame may be difficult. In the LTV case, additional complexity may be encountered as structured model reduction methods must be used [10], where some extensions of the balanced truncation method for LPV systems have been proposed [6]

Alternative approaches to control design may include LPV techniques where a representation (LPV) of the plant may be generated from a family of LTI models. If model reduction is to be performed beforehand, then it is of interest to consistently eliminate states throughout the whole family of LTI systems. In order to achieve this (i.e. eliminate the same state at all points), an *ad-hoc* model reduction methodology may be used, and will be briefly described later.

An issue that arises when designing a controller on a RO model of the plant, is that the controller must be robust to the neglected dynamics. The detrimental effects that the neglected dynamics may have on the system's performance and stability margins may become even more noticeable when actuator bandwidth limitations, sensor delays, and unstable or lightly damped poles are present. Notice that in addition to the uncertainties mentioned before, additional sources of uncertainty may be present (for example parameter or multiplicative modeling uncertainty), and hence robustness margins are at a prime. In order to analyze the effect that neglected dynamics have on the closed-loop margins, $\mathscr{H}_\infty$ controllers are designed around a single operating point (i.e. flight condition 60 knots, denoted FC60), and robustness margins of the resulting RO and FO closed-loop system are compared; the main goal is to expose any (noticeable) trade-offs present.

II. Aircraft Set-up

The aircraft has a total 8 control surfaces: three on each wing (out, mid and inner-board flaps) and two on the body (left and right body flaps). The measured variables are the pitch, roll and yaw rates ($p(t)$, $q(t)$ and $r(t)$ respectively), and the acceleration of the body and wing tips measured by a total of six sensors (two on each wing tip and two on the body) as depicted in figure 1.

At low flying speeds, the aircraft is stable and exhibits good handling qualities. Nonetheless, due to the fact that the vehicle has low damping in its pitching dynamics and highly flexible

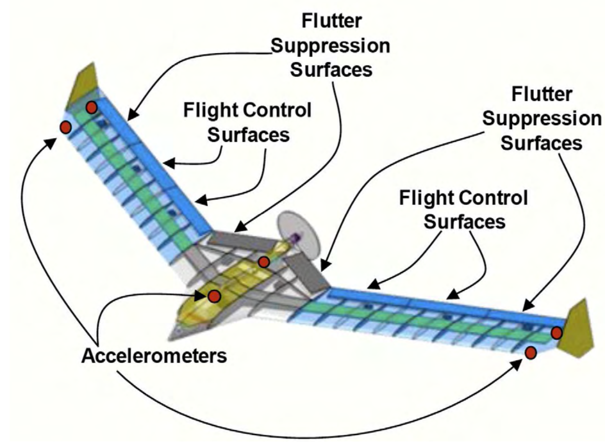

Fig. 1. Flexible Aircraft [3]

wings, the short period dynamics couple with the bending moments of the wings and produce what is called a *Body Freedom Flutter* (BFF). The aircraft is stable up to an airspeed of 40 knots, after which the plant becomes unstable: after 40 knots an unstable BFF mode is induced; after 46 knots, the symmetric wing bending/torsion moments becomes unstable; after 60 knots, the asymmetric wing bending/torsion moments also become unstable. Additional to the modes that become unstable, the plant has several modes with very low damping that may cause high frequency vibration. The main goal is to design a control strategy that actively stabilizes any unstable modes by commanding the outboard wing and body flaps, while adding damping to any lightly damped modes

III. Controller Synthesis and Analysis

The problem of active vibration control via robust control synthesis has been widely studied (see [1] and references therein), where design techniques have dealt with robustness issues due to neglected dynamics, unmodeled actuator dynamics and parameter uncertainty. The aeroservoelastic problem differs greatly as the structure (in this case the wing) is subject to aerodynamic effects which are highly nonlinear and require the estimation of aerodynamic coefficients along the wing. This may introduce additional sources of uncertainty, and in contrast to active vibration control of large structures, the modes of interest are located at higher frequency and lie close to each other, making the problem more difficult, especially if actuator bandwidth is at a prime.

The aeroservoelastic problem has been studied , and active mode damping has been used in experimental aircraft such as the X-53 [12]. The work presented in [4] describes the steps needed to build and model a flexible wing aircraft testbed. In [3] the aircraft was stabilized using an LQG control synthesized on a RO model obtained via Hankel SV balanaced truncation. As the controllers where design for various operating points, this controllers where scheduled using a fuzzy logic algorithm that provided a smooth transition between points. The results provided experimental evidence that the flight envelope can be enlarged via active control schemes, and it was shown that the full order system in closed-loop shifted its poles in such a way that the unstable modes where stabilized.

We will approach the problem in a similar way, but this time by using $\mathscr{H}_\infty$ design techniques and an *ad-hoc* reduced order model. The closed-loop system is simulated with added actuator (second order) dynamics and sensor delays. The main objective of this paper is to design a robust controller capable of addressing the main trade-offs related with unmodeled (actuator and sensor) dynamics, neglected (reduced order model) dynamics and performance. Because our aim is to allow for a more complete understanding of the achievable performance and robustness margins when designing for a real, flexible wing aircraft, this paper will present results of the design phase for a single operating point, more precisely the system at 60 knots airspeed (denoted as FC60).

A. Ad-hoc *Residualization*

Model reduction of LTI systems is, in general, a straight forward process where the designer limits the reduction error and selects the (stable) states to be removed. Model reduction of LPV systems is more complex, but techniques based on coprime factorization have been developed [6]. Nonetheless, if we are dealing with a parameter-varying system described by a set of LTI point models, the construction of an LPV model using such LTI representations requires a set of states that are consistent throughout the range of flight conditions, i.e. the states have the same meaning throughout the flight envelope. Even more, if the plant has a large number of states, it may be desirable (for practical purposes) to first reduce the LTI models before constructing the LPV system. This sets a new restriction on the RO models, where we want to eliminate the same states and use the same transformations at *all* the points of the flight envelope; in other words, it is necessary to *strip-off* the same set of states across all the available airspeeds. As a consequence, optimal gain methods are not fit to solve this problem, and an *ad-hoc* methodology was preferred.

The set of reduced order models took advantage of the special block structure present in the system, where it was noticed that the blocks had states with an apparent low contribution (i.e. the A-matrix coefficients associated with the state are small). The nominal model contains a set of states that correspond to actuators, sensors and vehicle dynamics (which itself is divided into rigid body dynamics and flexible wing dynamics with delays). The A-matrix of the state-space representation of the system has a special structure (across all velocities) given by

$$A = \left[\begin{array}{c|ccccc|c} A_{act} & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & I & 0 & 0 & 0 \\ A_{31} & A_{32}(V) & A_{33}(V) & A_{34}(V) & A_{35}(V) & 0 \\ 0 & 0 & I & \omega I & 0 & 0 \\ 0 & 0 & I & 0 & 2\omega I & 0 \\ \hline A_{61} & A_{62}(V) & A_{63}(V) & A_{64}(V) & A_{65}(V) & A_{sens} \end{array}\right]$$

where the term ω and the terms that depend on the velocity (i.e. the terms with the (V) are velocity dependent) represent the delay and unsteady aerodynamic coefficient rational approximation. It is important to observe that the initial objective is to stabilize the airframe, which includes rigid body dynamics and flexible wing modes. The initial system has a total of 182 states, which after residualizing [1] actuator and sensor dynamics, results in a system with 148 states (each actuator has dynamics represented by a second order system and the sensors are represented by a third order system) which contain only the dynamics of interest.

The structure of the matrix gives us some insight into which states can be removed, where our initial objective is to remove contributions at very high frequencies as our region of interest is $10-120\, rad/sec$. As a result, 92 states where removed from the vehicle's dynamic representation and a RO model with 56 states is obtained. Additional modes at mid-high frequencies (i.e. close to $120\, rad/sec$) where removed, where a total of 12 states were removed; the resulting system has 44 states. The states removed maintained their equivalence across all flight conditions and presented good results (i.e. good fit between RO and FO models within the region of interest).

Model reduction error can be seen as a relative change from nominal FO dynamics, and an uncertain model can be created by assuming input multiplicative uncertainty [13]. Consider $G(s)$ to be the FO plant, and assume it is given by

$$G = G_0 f(s) = G_0(I + \Delta * W_i) \tag{1}$$

where $G_0(s)$ is the reduced order model, $f(s)$ are the neglected dynamics and Δ is a normalized uncertain block (i.e. $\|\Delta\|_\infty < 1$). Define $\bar{W}(j\omega)$ as

$$\|\bar{W}(j\omega)\| = \max_{G(j\omega)} \| \frac{G(j\omega) - G_0(j\omega)}{G_0(j\omega)} \|$$

Then, for any filter W_i satisfying $\|W_i(j\omega)\| \geq \bar{W}(j\omega)$, it is possible to guarantee that the "real" system belongs to some uncertain set. In other words, equation (1) can be rewritten as $G \in \mathscr{S}$, where

$$\mathscr{S} =: G_0(I + \Delta * Wi) : \|\Delta\|_\infty < 1$$

Figures 2 and 3 shows an approximate frequency response of the the neglected dynamics error as expressed above. The resulting model reduction error is presented for two levels of order reduction (i.e. 56 and 44 state RO models) , and a range of velocities. Notice that the error at low frequencies varies the most with respect to airspeed, while error at high frequencies is similar for all flight conditions. This may be due to the fact that by residualizing the system, we are interested in retaining low frequency dynamics, thus for such large reductions the reduction algorithm may compensate in order to retain the same DC gain. For the same reason, high frequencies are of no importance and the error is due to the difference in roll-off rates, which is similar for all reduced order systems. The frequency response of each I/O pair was checked, and a close match was obtained for the frequency region of interest (i.e. $10-120\, rad/sec$). Even though this may appear to be restrictive at first, as discussed in [3], we are only interested in controlling near the crossover region (as mentioned before), hence uncertainty outside this range should not affect controller action.

B. Controller design

The main objective of the controller is to actively stabilize any unstable flexible modes. No tracking objectives are included and we will only concentrate on the regulation problem, that is, make all the states converge to zero. An additional objective will be to add damping to lightly damped modes. This last objective may be achieved by creating fictitious inputs and

[1] residualization is preferred as it matches low frequency

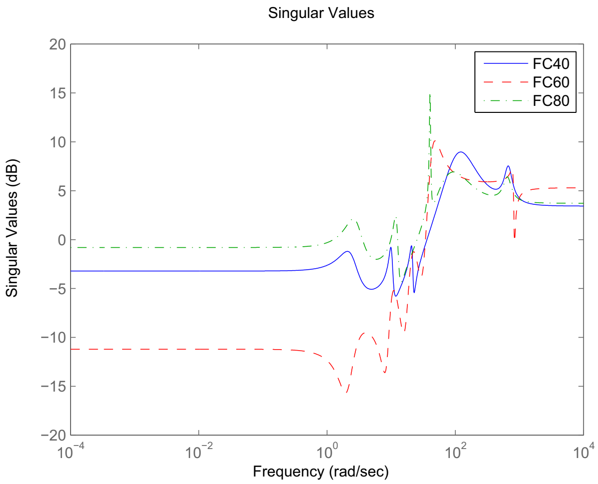

Fig. 2. Neglected dynamics induced error of a 56 state model

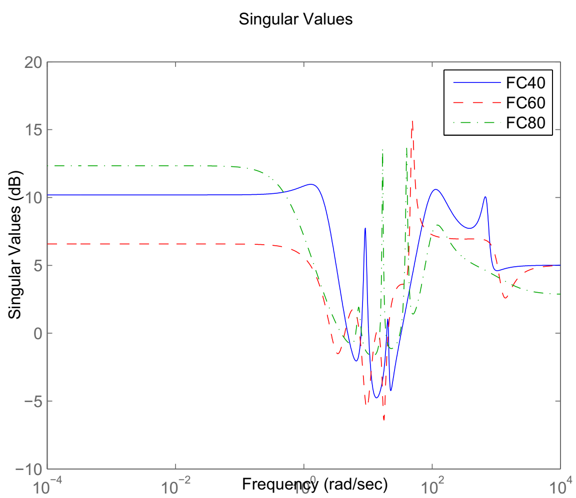

Fig. 3. Neglected dynamics induced error of a 44 state model

outputs that capture the dynamics of the specific modes of interest. The fictitious inputs and outputs may be chosen as the columns of T^{-1} and rows of T respectively, where T is a bi-modal transformation matrix, where the inputs/outputs are those which contain the modes to which we wish to add damping.

Remark: In order to enlarge the flight envelope, the approach presented in [3] uses point-wise designs (i.e. LTI controllers are computed at each flight condition) and later "blends" the family of controllers into a single control strategy. The point designs perform well under quasi-steady conditions, but may not do so if the airspeed varies rapidly, thus if we wish to account for rapid variations of airspeed, LPV design strategies may be required.

The controller interconnection is depicted in Figure 4, where G is the "nominal" RO plant, $W_{Act} = \frac{150}{s+150}$ is a first order approximation of the actuators (adding roll-off to the plant at high frequencies), D_τ is a first order Pade approximation of measurement delays ($\tau = 5ms$), and K is the controller to be

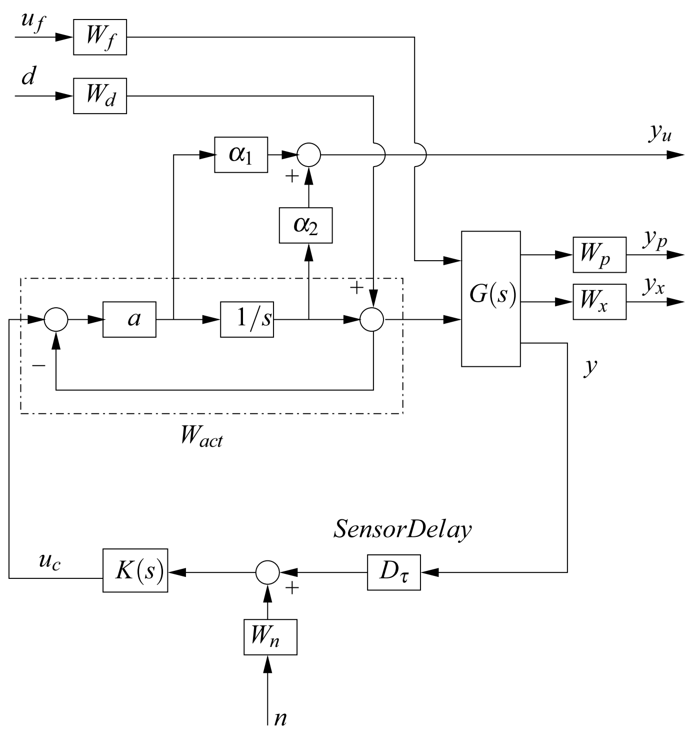

Fig. 4. Controller design interconnection

designed. The signals entering the plant are the control signals u (commands to the outboard and body flaps for a total of 4 control signals) and additive input disturbances d; the output signals used for feedback are p, q, r (pitch, roll and yaw rates) and the measurement from the accelerometers (y), which are subject to noise n. The signals u_f and y_f are fictitious input and output signals that capture the dynamics of specific modes of interest. Note that by keeping the infinity norm from these inputs to outputs small, we can add damping to specific modes. The blocks W_p, W_x are weights that measure the importance of the different control objectives (closed-loop performance, control and added damping respectively). The influence of exogenous signals (d, n and u_f) is weighted through W_d, W_n and W_f. As one of our objectives is to design a controller with the least amount of states, all weights will be chosen to be static, and additional dynamics will only come from the added sensor delays and actuator approximations. Although this may be restrictive, some control techniques (for example LPV control [2]) require compensators with low dimensionality in order to be applicable in practice. The main objective of the controller is to be robust to add damping to lightly damped modes (the map from fictitious inputs u_f to outputs y_f is small) and achieve disturbance rejection at the outputs (the map from disturbance to weighted outputs is small). Weights where chosen depending on the overall objective as constant diagonal matrices of appropriate dimensions, with their values ranging as follows: $W_p \in (0.1, 2)$, $W_x \in (0.5, 2)$, $W_d = 1$, $W_f = 0.5$ and $Wn = 0.001$.

An additional constraint is the limited actuator bandwidth, thus we require the map from disturbances to the weighted control signal y_u to be small and band limited. In order to avoid any additional states, but at the same time be able to weight over frequency our response, it is possible to "recycle" the states of the actuator model in order to weight the control effort with a

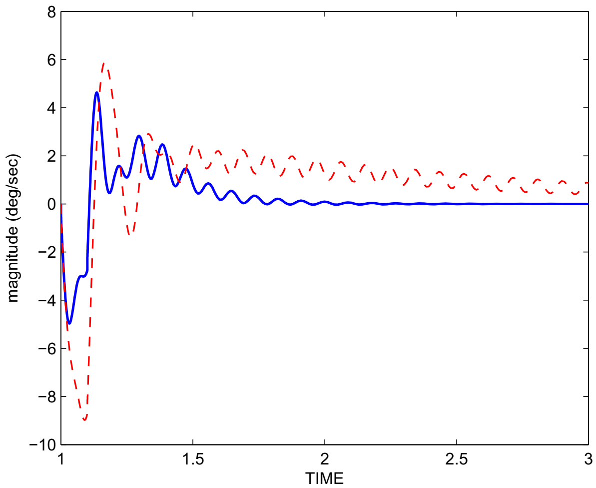

Fig. 5. Time response of $q(t)$ (solid line) and $p(r)$ (dotted line) for $K_{h,56}$

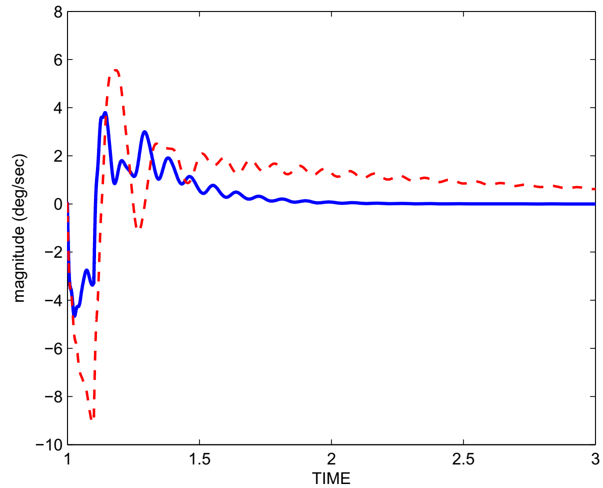

Fig. 6. Time response of $q(t)$ (solid line) and $p(r)$ (dotted line) for $K_{h,44}$

high pass filter. This procedure is depicted in Figure 4, and the resulting filter (W_s) will have a transfer function of the form

$$W_s = Wact * (a_1 s + a_2) = \frac{150 a_1 (s + a_2/a_1)}{s + 150}$$

The resulting weight is a high pass filter with gain a_2 at low frequencies, and $150 * a_2$ at frequencies higher than $150\ rad/sec$. In our case $a_1 = 0.01$ and $a_2 = 0.001$, which means that the closed-loop system is robust to multiplicative uncertainties with a 0.001 relative change at low frequencies, and 1.5 at frequencies higher than $150\ rad/sec$.

Two types of stabilizing controllers (K_l and K_h) were designed with different levels of performance and disturbance rejection. Controllers K_l and K_h have low and high performance levels respectively; $W_p = 1.8$, $W_x = 1.5 \times I_{12}$ for the high performance controller and $W_p = 0.1$, $W_x = 1 \times I_{12}$ for the low performance controllers. In order to allow a comparison of the effects that model reduction has on closed-loop system performance, controllers were designed using reduced order models with different state dimensions, that is, controllers were designed using a 56 and a 44 state model. Closed-loop time responses of $q(t)$ are given in figures 5 and 6, where controllers $K_{h,56}$ $K_{l,56}$ (the subscript states the model on which the controller is designed) are tested for a reduced and full order closed-loop system [2]. Simulations were performed on the FO closed-loop system at FC60, and the disturbance considered is a pulse on the left outboard flap. Closed-loop performance is as expected , where the low performance controllers produce longer settling times and appear to be more oscillatory. It is worth mentioning that with minor discrepancies, closed-loop RO dynamics accurately predict FO behavior.

IV. Robustness and Performance Analysis

Designing controllers using reduced order plant models is a common practice and has been widely studied. Depending on the reduction technique, the resulting controller generally retains performance and robustness margins when implemented on the full order system. Nonetheless, for the case of a flexible wing aircraft, the airframe model has a large state dimension and more than 120 states are being neglected. These neglected dynamics may represent a big source of “uncertainty”, and as a result, the controller designed on the RO model may not achieve the predicted levels of robustness and performance, thus it is of interest to understand practical limitations that may arise from dealing with such large state reductions.

In order to measure such limitations, a stabilizing controller was designed at FC60 using a RO mode and later tested on the FO plant, where stability is checked after the controller design phase. Since there is no clarity on the structure or source of plant/model mismatch, we define the uncertainty as a relative deviation of the real plant from modeled dynamics. The uncertainty is then assumed to be input multiplicative and the uncertain system is given by equation 2. The uncertain block Δ is assumed to have an infinity norm less that one, and the filter W_{unc} represent the frequency content of the uncertainty. W_{unc} is chosen such that the relative deviation is small at low frequencies and larger at high frequencies, thus it can be represented as high-pass filter. The goal is quantifying stability and performance (in the form of the worst-case gain) margins [3] for the different controllers designed, and observe any existing trade-offs between system margins and number of sates in the RO model. For analysis purposes, it is assumed that the real system is subject to modeling errors that can be captured as input multiplicative uncertainty, where the uncertain system is given by

$$G_{unc} = G(I + \Delta * W_{unc}) \tag{2}$$

whereG_{unc} is the uncertain plant, G is the nominal (FO or RO depending on the case), and the filter

$$W_{unc} = \frac{1.5(s + 0.1)}{s + 150}$$

[2]Only the time response of controllers $K_{h,56}$ $K_{l,56}$ are shown because other controllers exhibit a behavior that is almost identical and hard to appreciate in the figures

[3]This may be done by using Matlab’s *robuststab* and *wcgain* functions

Although low frequency uncertainty is of no immediate interest (this task may be more appropriate for an outer loop controller), uncertainty around the cross-over region (the region we are interested in) varies between 20% to 90%. The main idea is to quantify the levels of robustness of both the FO and RO uncertain systems, and highlight any noticeable degradation on the predicted levels.

Table I shows the stability margins of the different controllers designed. The prescribed margin is the one calculated using the RO model; the real margins are calculated for the FO plant in closed-loop. An interesting fact, is that for low performance compensators, stability margins are accurately predicted, but the worst-case gain is not. The converse is not true for high gain controllers, where inaccurate predictions are made on both the worst-case gain and stability margin. It is important to highlight that according to Table I, low performance controllers may be preferred as robust stability margins are accurately predicted and acceptable levels of robust performance are achieved; even more, it appears that $K_{l,44}$ is the best.

Although this technique proved to be effective, modes laying outside the cross-over region showed to be detrimental for controller design. Control at frequencies outside the region of interest is not required for flexible mode stabilization, thus if our RO controller is acting outside this region, large plant-model errors may have a detrimental effect on the predictions of the robustness margins. This was observed to be true if damping was added to lightly damped modes that lie both at lower or higher frequencies than the region of interest. This highlights the fact that in order to allow for more transparent and intuitive design procedures, we need to use more suitable reduction techniques, i.e. weighted model reduction, that allow us to retain dynamics only within the region of interest and "pre-shape" our control action. Although this has been studied and well developed tools exist [14], they do not retain the original structure and states will not be consistent throughout the flight envelope.

Plant	Controller	Stability Margin	Worst-case Gain
RO	$K_{l,56}$	3.73	114.43
FO	$K_{l,56}$	3.85	63.80
RO	$K_{h,56}$	3.85	6.27
FO	$K_{h,56}$	6.38	3.13
RO	$K_{l,44}$	4.53	122.54
FO	$K_{l,44}$	4.41	72.59
RO	$K_{h,44}$	2.88	9.29
FO	$K_{h,44}$	1.09	14.81

TABLE I
SATBILITY MARGINS AND WORST-CASE GAIN FOR CONTROLLERS DESIGNED ON A 56/44 STATE REDUCE ORDER MODEL

V. CONCLUSIONS

This paper has shown that is is possible to actively control unstable modes on flexible wing aircraft using $\mathscr{H}_\infty$ techniques, thus enlarging the flight envelope is achievable. The paper presents a design approach for the synthesis of low order $\mathscr{H}_\infty$ controllers, and presents the robustness margins of the closed-loop system. Due to the high dimensionality of the plant, controllers were designed on a reduced order model that can be thought of as the nominal plant with input multiplicative uncertainty (neglected dynamics). As a result, "real" robustness margins may be compromised due to additional neglected dynamics present in the design procedure, or estimates on their bounds may be very conservative. It was observed that having RO plant dynamics beyond the band-width of the actuator is extremely detrimental and may restrict allowable performance. Future work may include the exploration of structured and weighted balanced truncation in order to obtain RO modes that are better suited for the problem of stabilizing unstable structural modes.

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