5th Annual IEEE Conference on Automation Science and Engineering
Bangalore, India, August 22-25, 2009

Monotonic and Non-monotonic Properties of Product Quality in Flexible Manufacturing Systems with Batch Operations

Junwen Wang, Jingshan Li, Jorge Arinez, Stephan Biller and Ningjian Huang

Abstract— **Many flexible manufacturing systems implement batch production, in order to shorten changeover time, reduce cost, and improve quality. In such systems, different types of products are grouped into batches where all parts in each batch have the same type. The change of product type only occurs after the last job in a batch is processed. In this paper, we present an analytical method to evaluate the quality performance of flexible manufacturing systems with batch operations and investigate the monotonic and non-monotonic properties of product quality with respect to quality failure and repair probabilities and batch sizes.**

Keywords: Product quality, flexible manufacturing system, Markov chain, batch production, monotonicity.

I. INTRODUCTION

Flexibility is becoming more and more critical in modern manufacturing. For example, in many automotive assembly plants, multiple models of vehicles with more options can be made on the same production line. Similar trend is also observed in many other manufacturing industries as well. During the last twenty years, substantial research effort has been devoted to flexible manufacturing systems (see, for instance, monographs [1]-[3], and reviews [4]-[7]). Most of them address the issues of investment cost, flexibility measurement, inventory, scheduling, and the tradeoffs between productivity and flexibility, etc. However, the question of quality is usually neglected. It is typically assumed that quality related issues have minimal impact ([8]). On the other hand, quality management has attracted extensive research attention, but independently. Most of the quality research seeks to maintain and improve product quality while ignoring the productivity or flexibility concerns ([9]). Little research attention has been paid to investigate the coupling between flexible system design and product quality.

Empirical evidences and analytical studies have shown that production system design and product quality are *tightly coupled* ([9]); in particular, flexibility has a significant impact on product quality. For example, the product quality at an automotive paint shop is strongly correlated to the number of available paint colors ([10]). Paint quality may temporarily decline after the color change ([11]). The color of the previous vehicle may affect the quality of next vehicle. Similar examples can be found in flexible machining lines, welding and assembly systems as well. But the analysis on the correlation between flexibility and quality is limited. Paper [12] studies the issues of flexibility, productivity, and quality from an extensive search and analysis of empirical studies and it shows that flexibility impacts quality. Paper [13] introduces a Markov chain model to evaluate the quality performance in flexible manufacturing system. Strictly sequencing policy and random mixing policy are compared in the paper and the results suggests that reducing the number of product switch could lead to a better quality performance, which implies that batch production is a possible approach to improve quality.

In practice, to reduce losses due to quality degradation during changeover and to meet customer demand, batch production is typically introduced. For example, vehicles with the same color are usually grouped before entering the painting booths to improve the paint quality and to meet delivery requirement. In some engine assembly lines, different types of engines are typically assembled in batch and changeovers occur on hourly basis. Many other flexible manufacturing systems also implement batch production to improve quality, and shorten changeover time, reduce cost, etc. Therefore, there is a critical need to fully understand the coupling between flexibility and quality in terms of sequence and batch production policies. Unfortunately, such an important issue is almost entirely neglected. The current literature does not provide a quantitative method to investigate how flexibility impacts product quality, in particular, the effects of batch parameters. This paper is intended to contribute to this end. The main contribution of this paper is in development of an analytical model to evaluate the quality performance of a flexible manufacturing system with batch productions and in discussing the monotonicity properties in batch operations to provide guidelines for quality improvement in batch production environment.

The remainder of the paper is structured as follows: Models and problem formulation are introduced in Section II. Section III is devoted to evaluation of quality in batch production environment. Section IV discusses the monotonic and non-monotonic properties. The conclusions are formulated in Section V. Due to space limitation, all proofs are omitted and can be found in [14].

II. MODELS AND PROBLEM FORMULATION

Consider a flexible manufacturing system capable of producing different types of products. The following assump-

This work is partially supported by NSF Grant No. CMMI-0727691 and University of Kentucky Faculty Research Grant.

J. Wang and J. Li are with the Department of Electrical and Computer Engineering and Center for Manufacturing, University of Kentucky, Lexington, KY 40506, USA. junwen.wang@uky.edu, jingshan@engr.uky.edu

J. Arinez, S. Biller and N. Huang are with Manufacturing Systems Research Lab, General Motors Research & Development Center, Warren, MI, 48090-9055, USA. jorge.arinez@gm.com, stephan.biller@gm.com, ninja.huang@gm.com

Please send all correspondences to Prof. Jingshan Li.

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tions address the flexible production system, product types, sequence, and quality characteristics.

(1) There are n different types of products, denoted as product types 1, 2, ..., n. Each product type i is processed in a batch with batch size k_i, $k_i \geq 1$. In other words, the machine will work on product type i for k_i parts before switching to product type $i+1$. After processing product type n, product type 1 is processed again.
(2) The flexible system is capable of processing any type of products. The state of the system is defined by its quality status, product type processed and its position within a batch. It is in good state g_{ij}, or defective state d_{ij}, $i = 1, \dots, n$, $j = 1, \dots, k_i$, if it is processing the j-th part in the batch of product type i with good quality, or with defects, respectively. Thus, a flexible system has $2K$ states, and $K = \sum_{i=1}^{n} k_i$.
(3) When the system is in good state g_{ij}, $i = 1, \dots, n$, $j = 1, \dots, k_i - 1$, it has probability λ_{ii} to transit to defective state $d_{i,j+1}$, and $1 - \lambda_{ii}$ to good state $g_{i,j+1}$. Analogously, when the machine is in defective state d_{ij}, $i = 1, \dots, n$, $j = 1, \dots, k_i - 1$, it has probability μ_{ii} to transit to good state $g_{i,j+1}$, and $1-\mu_{ii}$ to defective state $d_{i,j+1}$.
(4) When the system is in good state g_{i,k_i}, $i = 1, \dots, n-1$ (i.e., processing the last part within a batch), it has probabilities $\lambda_{i+1,i}$ or $1-\lambda_{i+1,i}$ to transit to states $d_{i+1,1}$ or $g_{i+1,1}$, respectively. Analogously, when the machine is in defective state d_{i,k_i}, it has probabilities $\mu_{i+1,i}$ and $1-\mu_{i+1,i}$ to transit to states $g_{i+1,1}$ and $d_{i+1,1}$, respectively.
(5) When the system is in state g_{n,k_n}, it has probabilities λ_{1n} or $1 - \lambda_{1n}$ to transit to states d_{11} or g_{11}, respectively. Analogously, when the system is in state d_{n,k_n}, it has probabilities μ_{1n} and $1 - \mu_{1n}$ to transit to states g_{11} and d_{11}, respectively. Without loss of generality, we assume $0 < \lambda_{ij} < 1$, $0 < \mu_{ij} < 1$.
Remark 1: We refer λ_{ii}, $i = 1, \dots, n$, and $\lambda_{j+1,j}$, $j = 1, \dots, n-1$ and λ_{1n} to as the *quality failure probabilities*. Similarly, μ_{ii}, $i = 1, \dots, n$, and $\mu_{j+1,j}$, $j = 1, \dots, n-1$ and μ_{1n} are *quality repair probabilities*. In particular, λ_{ij} and μ_{ij}, $i = j$, are the quality failure and repair probabilities without product switch, and λ_{ij} and μ_{ij}, $i \neq j$, are with product switch.

Let $P(g_{ij})$ and $P(d_{ij})$, $i = 1, \dots, n, j = 1, \dots, k_i$, denote the probabilities of producing a good or a defective job for the j-th part in the batch of product type i, respectively. Then the system's overall quality performance, probability to produce a good part, is defined as $P(g)$. We have

$$P(g) = \sum_{i=1}^{n} \sum_{j=1}^{k_i} P(g_{ij}). \quad (1)$$

Similarly, the probability to produce a defective part $P(d)$ is

$$P(d) = \sum_{i=1}^{n} \sum_{j=1}^{k_i} P(d_{ij}). \quad (2)$$

The problem to be addressed is: *Under assumptions (1)-(5), develop a method to evaluate the quality performance of batch production in flexible manufacturing systems as a function of system parameters and investigate its monotonic and non-monotonic properties with respect to its arguments.*

III. EVALUATION OF PRODUCT QUALITY

A. General Formulas

Consider that there are n product types, each has k_i parts in a batch, $i = 1, \dots, n$. Denote $P(g_{ij}, t)$ as the probability to produce a good part for the j-th job in a batch of product type i at time t. From assumptions (1)-(5), when $1 < j \leq k_i$, we have

$$\begin{aligned} P(g_{ij}, t+1) &= P(g_{ij}, t+1 | g_{i,j-1}, t) P(g_{i,j-1}, t) \\ &\quad + P(g_{ij}, t+1 | d_{i,j-1}, t) P(d_{i,j-1}, t) \\ &= (1 - \lambda_{ii}) P(g_{i,j-1}, t) + \mu_{ii} P(d_{i,j-1}, t). \end{aligned}$$

In terms of the steady state, let

$$\lim_{t \to \infty} P(g_{ij}, t) =: P(g_{ij}),$$

we obtain

$$P(g_{ij}) = (1 - \lambda_{ii}) P(g_{i,j-1}) + \mu_{ii} P(d_{i,j-1}). \quad (3)$$

Similarly,

$$P(d_{ij}) = (1 - \mu_{ii}) P(d_{i,j-1}) + \lambda_{ii} P(g_{i,j-1}). \quad (4)$$

When $j = 1$ and $i > 1$, a product switch from type $i-1$ to type i is involved, we obtain

$$\begin{aligned} P(g_{i1}) &= \mu_{i,i-1} P(d_{i-1,k_{i-1}}) \\ &\quad + (1 - \lambda_{i,i-1}) P(g_{i-1,k_{i-1}}), \quad (5) \\ P(d_{i1}) &= (1 - \mu_{i,i-1}) P(d_{i-1,k_{i-1}}) \\ &\quad + \lambda_{i,i-1} P(g_{i-1,k_{i-1}}), \quad i = 2, \dots, n. \quad (6) \end{aligned}$$

Finally, if $i = j = 1$, it follows that

$$\begin{aligned} P(g_{11}) &= \mu_{1n} P(d_{n,k_n}) + (1 - \lambda_{1n}) P(g_{n,k_n}), \quad (7) \\ P(d_{11}) &= (1 - \mu_{1n}) P(d_{n,k_n}) + \lambda_{1n} P(g_{n,k_n}). \quad (8) \end{aligned}$$

In addition, the total probability is equal to 1,

$$\sum_{i=1}^{n} \sum_{j=1}^{k_i} (P(g_{ij}) + P(d_{ij})) = 1. \quad (9)$$

A state transition diagram for the case of $n = 2$ and $k_i = 3$, $\forall i$, is illustrated in Figure 1. To express equations (3)-(9) in matrix form, we obtain

$$AX = B, \quad (10)$$

where matrix A is defined by equation (11),

$$B = (0, \dots, 0, 1)^T, \quad (12)$$

$$X = (P(g_{11}), \dots, \dots, P(g_{n,k_n}), P(d_{11}), \dots, P(d_{n,k_n}))^T. \quad (13)$$

Therefore, the probability of good parts $P(g)$ is as follows:

$$
A=\begin{pmatrix}
1 & 0 & \cdots & 0 & 0 & \lambda_{1n}-1 & 0 & 0 & \cdots & 0 & 0 & -\mu_{1n}\\
\lambda_{11}-1 & 1 & \cdots & 0 & 0 & 0 & -\mu_{11} & 0 & \cdots & 0 & 0 & 0\\
0 & \lambda_{11}-1 & \cdots & 0 & 0 & 0 & 0 & -\mu_{11} & \cdots & 0 & 0 & 0\\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\
0 & 0 & \cdots & 0 & \lambda_{nn}-1 & 1 & 0 & 0 & \cdots & 0 & -\mu_{nn} & 0\\
0 & 0 & \cdots & 0 & 0 & -\lambda_{1n} & 1 & 0 & \cdots & 0 & 0 & \mu_{1n}-1\\
-\lambda_{11} & 0 & \cdots & 0 & 0 & 0 & \mu_{11}-1 & 1 & \cdots & 0 & 0 & 0\\
0 & -\lambda_{11} & \cdots & 0 & 0 & 0 & 0 & \mu_{11}-1 & \cdots & 0 & 0 & 0\\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\
0 & 0 & \cdots & -\lambda_{nn} & 0 & 0 & 0 & 0 & \cdots & \mu_{nn}-1 & 1 & 0\\
1 & 1 & \cdots & 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1
\end{pmatrix} \tag{11}
$$

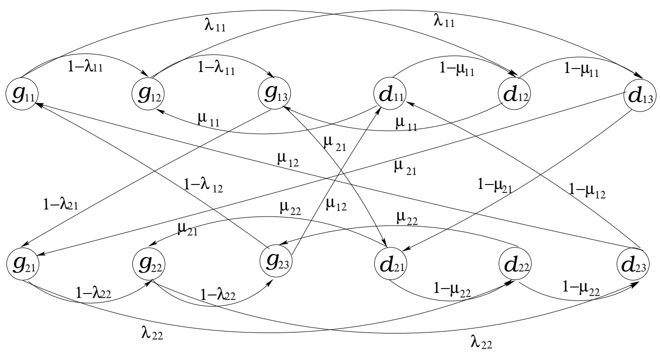

Fig. 1. State transition diagram of two-product type case with batch size three

Theorem 1: Under assumptions (1)-(5), the probability of good parts $P(g)$ is calculated by

$$P(g)=\sum_{i=1}^{n}\sum_{j=1}^{k_i}P(g_{ij})=\sum_{i=1}^{K}x_i, \tag{14}$$

where $K=\sum_{i=1}^{n}k_i$, and x_i is solved from

$$X=A^{-1}B, \tag{15}$$

and A, B, X are defined in (11)-(13).

Remark 2: Note that the inverse of matrix A exists due to the fact that an irreducible Markov chain with finite number of states has a unique solution.

B. Equal Products Case

Now we consider the case where the transition probabilities are identical for all products, denoted as the *equal products case*, i.e., all $k_i=k$, $i=1,\dots,n$, and,

$$\lambda_{ii}=\lambda_{11},\mu_{ii}=\mu_{11},i=1,\dots,n,$$
$$\lambda_{1n}=\lambda_{i+1,i}=\lambda_{21},\mu_{1n}=\mu_{i+1,i}=\mu_{21},i=1,\dots,n-1.$$

We obtain

Corollary 1: Under assumptions (1)-(5) for equal products, the probability of good parts can be calculated as

$$P(g)=\frac{\mu_{11}}{\lambda_{11}+\mu_{11}}+\frac{(\lambda_{11}\mu_{12}-\lambda_{12}\mu_{11})[1-(1-\lambda_{11}-\mu_{11})^k]}{k(\lambda_{11}+\mu_{11})^2[1-(1-\lambda_{12}-\mu_{12})(1-\lambda_{11}-\mu_{11})^{k-1}]}. \tag{16}$$

Remark 3: Note that expression (16) is independent of the number of product types, n. The rationale behind this is that since all products are "equal", transitions from product type i to type $i+1$ are same for all i, $i=1,\dots,n-1$, and is also same as transition from product type n to type 1. Therefore, it is equivalent to that there are only two product types, 1 and 2. Therefore, the quality performance only depends on the batch size k. However, when λ_{ij} and μ_{ij} are not same, and k_is are different, the number of products will play a role.

IV. MONOTONIC AND NON-MONOTONIC PROPERTIES

To simplify the analysis, the following discussions are for equal products case only.

A. Monotonic and Non-monotonic Properties with respect to Quality Failure and Repair Probabilities

Intuitively, we may expect that monotonicity would hold with respect to quality failure and repair probabilities as in two product types case, i.e., $P(g)$ may monotonically decrease and increase with respect to λ_{1i} and μ_{1i}, respectively. However, it is shown below that such monotonicity only holds with respect to λ_{12} and μ_{12}. In other words,

Proposition 1: Under assumptions (1)-(5) for equal products, the probability of good parts, $P(g)$, is monotonically

- decreasing with respect to λ_{12}, and
- increasing with respect to μ_{12}.

Unexpectedly, monotonicity with respect to λ_{11} and μ_{11} does not hold all the time. Numerical investigation indicates that in most cases, $P(g)$ will decrease or increase with respect to λ_{11} and μ_{11}, respectively, i.e., $\frac{\partial P(g)}{\partial \lambda_{11}}<0$ and $\frac{\partial P(g)}{\partial \mu_{11}}>0$. But in some extreme cases, opposite results may be observed when batch size is an odd number. Tables I and II present some examples of these counter-intuitive results.

The above examples indicate that non-monotonicity with respect to λ_{11} and μ_{11} may occur. However, this only happens when failure probabilities are typically large (approaching 1) and repair probabilities are small (approaching 0). In most practical situations, monotonicity can still be counted on.

TABLE I

NON-MONOTONICITY OF $P(g)$ WITH RESPECT TO λ_{11}

k	λ_{11}	λ_{12}	μ_{11}	μ_{12}	$\frac{\partial P(g)}{\partial \lambda_{11}}$	$P(g)$
5	0.9200	0.0100	0.9900	0.4900	-0.0044	0.5781
5	0.9300	0.0100	0.9900	0.4900	0.0217	0.5781
5	0.9400	0.0100	0.9900	0.4900	0.0506	0.5785
5	0.9500	0.0100	0.9900	0.4900	0.0827	0.5795
5	0.9600	0.0100	0.9900	0.4900	0.1185	0.5802
5	0.9700	0.0100	0.9900	0.4900	0.1588	0.5816
5	0.9800	0.0100	0.9900	0.4900	0.2043	0.5833
5	0.9900	0.0100	0.9900	0.4900	0.2560	0.5857

TABLE II

NON-MONOTONICITY OF $P(g)$ WITH RESPECT TO μ_{11}

k	λ_{11}	λ_{12}	μ_{11}	μ_{12}	$\frac{\partial P(g)}{\partial \mu_{11}}$	$P(g)$
7	0.9900	0.6000	0.9200	0.0400	0.1016	0.4432
7	0.9900	0.6000	0.9300	0.0400	0.0819	0.4441
7	0.9900	0.6000	0.9400	0.0400	0.0602	0.4450
7	0.9900	0.6000	0.9500	0.0400	0.0361	0.4456
7	0.9900	0.6000	0.9600	0.0400	0.0092	0.4455
7	0.9900	0.6000	0.9700	0.0400	-0.0210	0.4457
7	0.9900	0.6000	0.9800	0.0400	-0.0552	0.4456
7	0.9900	0.6000	0.9900	0.0400	-0.0942	0.4446

B. Oscillating and Monotonic Properties with respect to Batch Size

1) Non-monotonic observations: It is intuitive that we may expect monotonic properties with respect to batch size. In most cases, it is true. However, when the following cases are investigated, oscillation behavior is observed. As illustrated in Figure 2, decreasing or increasing oscillated behaviors have been observed in these two cases. Moreover,

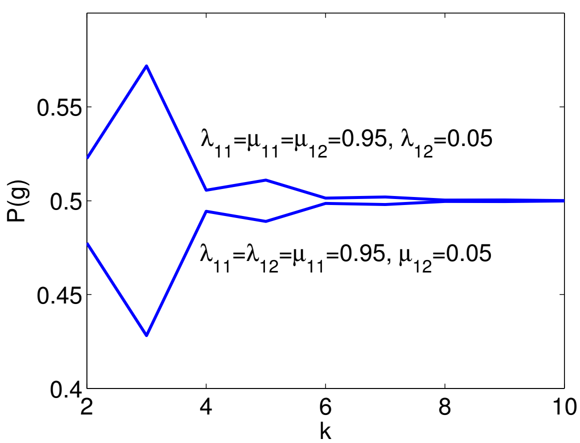

Fig. 2. Examples of oscillating behavior

it can be shown that when $\lambda_{11} \to 1$, $\mu_{11} \to 1$, $\lambda_{12} \to 0$, $\mu_{12} \to 1$, the product quality can be expressed by

$$\lim_{\substack{\lambda_{11} \to 1, \lambda_{12} \to 0 \\ \mu_{11} \to 1, \mu_{12} \to 1}} P(g) = \begin{cases} \frac{1}{2k} + \frac{1}{2} & k \text{ odd,} \\ \frac{1}{2} & k \text{ even.} \end{cases}$$

In this case, we can show that $P(g_{i1}) \to 1$, $i = 1, 2$, no matter the last job in previous batch is in good or bad quality. Then, $P(d_{i2}) \to 1$, and $P(g_{i3}) \to 1$, $P(d_{i4}) \to 1$, ..., i.e., the odd number ones approach good quality and even ones defective. Therefore, for even batch size, we obtain equal numbers of good and bad quality jobs; while for odd one, an additional good quality job is obtained within a batch.

Similarly, when $\lambda_{11} \to 1$, $\mu_{11} \to 1$, $\lambda_{12} \to 1$, $\mu_{12} \to 0$, we obtain again equal numbers of good and bad quality jobs for even batch size, while for odd batch size, one additional bad quality job is received. Thus,

$$\lim_{\substack{\lambda_{11} \to 1, \lambda_{12} \to 1 \\ \mu_{11} \to 1, \mu_{12} \to 0}} P(g) = \begin{cases} -\frac{1}{2k} + \frac{1}{2} & k \text{ odd,} \\ \frac{1}{2} & k \text{ even.} \end{cases}$$

The above observations show that oscillating behavior may appear when batch size is increased. To further investigate this phenomenon, let $P_k(g)$ denote the probability of producing a good quality part when batch size is k. Then, we obtain

$$\begin{aligned} P_k(g) &= \frac{\mu_{11}}{\lambda_{11}+\mu_{11}} \\ &+ \frac{(\lambda_{11}\mu_{12}-\lambda_{12}\mu_{11})[1-(1-\lambda_{11}-\mu_{11})^k]}{k(\lambda_{11}+\mu_{11})^2[1-(1-\lambda_{12}-\mu_{12})(1-\lambda_{11}-\mu_{11})^{k-1}]} \\ &= \frac{\mu_{11}}{\lambda_{11}+\mu_{11}} + \frac{(\lambda_{12}+\mu_{12})(e_{12}-e_{11})(1-a^k)}{k(\lambda_{11}+\mu_{11})(1-ba^{k-1})} \\ &= \frac{\mu_{11}}{\lambda_{11}+\mu_{11}} + \frac{(\lambda_{12}+\mu_{12})(e_{12}-e_{11})}{\lambda_{11}+\mu_{11}} D_k, \end{aligned}$$

where

$$\begin{aligned} a &= 1-\lambda_{11}-\mu_{11}, \\ b &= 1-\lambda_{12}-\mu_{12}, \\ D_k &= \frac{1-a^k}{k(1-ba^{k-1})}. \end{aligned} \tag{17}$$

Then we investigate the quality behavior when batch is increased by 1 or 2 jobs below.

2) Increasing batch size by 1: Consider

$$P_{k+1}(g) - P_k(g) = \frac{(\lambda_{12}+\mu_{12})(e_{12}-e_{11})}{\lambda_{11}+\mu_{11}}(D_{k+1}-D_k). \tag{18}$$

Proposition 2: Under assumptions (1)-(5) for equal products,

- $P_{2j+2}(g) < P_{2j+1}(g)$, $j = 0, 1, 2, \ldots$, i.e., monotonically decreasing with respect to odd batch size, if $e_{11} < e_{12}$;
- $P_{2j+2}(g) > P_{2j+1}(g)$, $j = 0, 1, 2, \ldots$, i.e., monotonically increasing with respect to odd batch size, if $e_{11} > e_{12}$.

Due to (18), Proposition 2 suggests that D_{k+1} is always smaller than D_k when k is odd so that increasing batch size by 1 will lead to degradation of product quality when $e_{11} < e_{12}$, which agrees with our intuition, since $e_{11} < e_{12}$ implies product switch will improve quality so that smaller batch is preferred. Analogously, increasing batch size by 1 can improve quality when $e_{11} > e_{12}$. An illustration of the fact that $D_{2j+2} < D_{2j+1}$ is shown in Figure 3, where we select

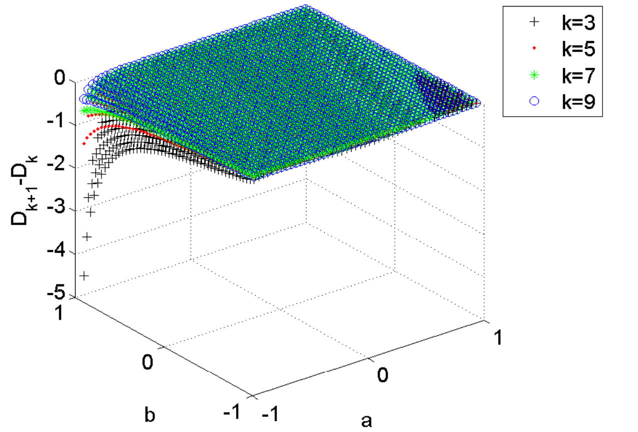

Fig. 3. $D_{k+1} - D_k$ vs a and b with respect to different odd batch size k

all possible values of λ_{1i} and μ_{1i} between 0.01 and 0.99 with a step size 0.01 for different batch size k.

However, such property may not hold when k is even. As it is shown in Figure 4 that $D_{k+1} - D_k$ can be positive or negative when k is even, which implies that oscillating behavior may occur when the even batch size is increased. Clearly, there exists a boundary condition that D_{k+1} may be

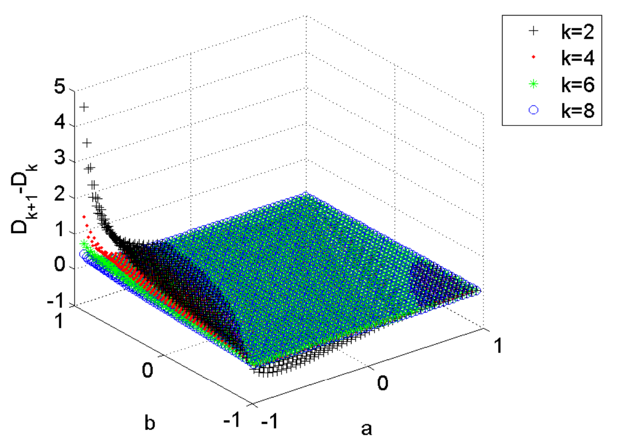

Fig. 4. $D_{k+1} - D_k$ vs a and b with respect to different even batch size k

equal to D_k when k is even. From (17), it can be shown that $D_{2j+1} = D_{2j}$ implies that

$$\frac{1 - a^{2j+1}}{(2j+1)(1 - ba^{2j})} = \frac{1 - a^{2j}}{2j(1 - ba^{2j-1})}, \quad j = 0, 1, 2, \ldots.$$

After some algebraic manipulation, we obtain the boundary condition

$$b = \frac{1 - a^{2j} - 2ja^{2j}(1 - a)}{a^{2j}(1 - a^{2j}) - 2ja^{2j-1}(1 - a)}, \quad j = 0, 1, 2, \ldots. \tag{19}$$

However, such formula does not provide a clear indication on what kind of parameters λ_{1i}, μ_{1i}, $i = 1, 2$, leading to oscillation. Therefore, we calculate such boundaries numerically and plot on an a-b plane for different even batch size k (Figure 5). Clearly, when a and b are allocated on the left side of these boundaries, oscillation will occur.

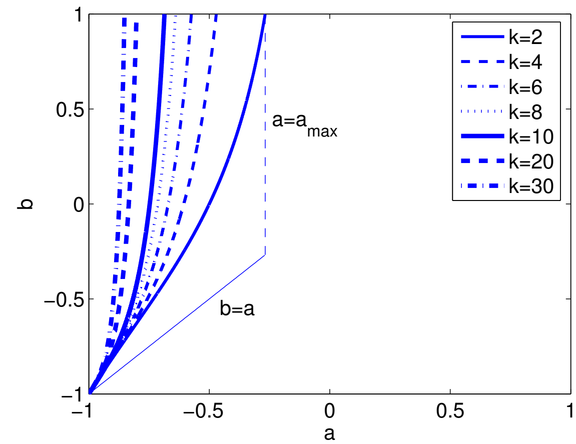

Fig. 5. Oscillation boundaries with respect to various batch sizes

It is shown from Figure 5 that when oscillation occurs, parameter a must be less than the right most value, a_{max}, on the boundary (i.e., at point $b = 1$). We can view it as a necessary condition for oscillation, or bound for failure and repair probabilities, i.e.,

$$\lambda_{11} + \mu_{11} > 1 - a_{max}. \tag{20}$$

An illustration of such bound for $k = 2$ is given in Figure 5 as the thin broken line. Table III presents these bounds for different batch size k.

TABLE III
NECESSARY CONDITION FOR OSCILLATION

k	2	4	6	8
$\lambda_{11} + \mu_{11} >$	1.268	1.4710	1.5744	1.6399
k	10	20	30	50
$\lambda_{11} + \mu_{11} >$	1.6859	1.8018	1.8519	1.8920

In addition, Figure 5 shows that a is always negative when oscillation occurs. From (19) it is easy to show that in this case $b - a > 0$, which implies that the oscillation area falls into the region $b > a$, i.e., $1 - (\lambda_{12} + \mu_{12}) > 1 - (\lambda_{11} + \mu_{11})$. Thus, we obtain another necessary condition or bound for oscillation, as shown in Figure 5 with the thin solid line,

$$\lambda_{12} + \mu_{12} < \lambda_{11} + \mu_{11}. \tag{21}$$

As shown in Figure 5, inequalities (20) and (21) provide a relative tight bound for oscillation area. Moreover, we observe that the oscillation area is becoming smaller when even batch size k is increasing. This implies large batch size reduces the possibility of oscillation.

Clearly, when parameters a and b (or λ_{1i}, μ_{1i}, $i = 1, 2$) are selected outside of the oscillation area, we always obtain $D_{k+1} < D_k$, which implies that the monotonicity properties hold and the product quality will be

- monotonically decreasing with respect to even batch size, i.e., $P_{2j+1}(g) < P_{2j}(g)$, $j = 1, 2, \ldots$, if $e_{11} < e_{12}$, and
- monotonically increasing with respect to even batch size, i.e., $P_{2j+1}(g) > P_{2j}(g)$, $j = 1, 2, \ldots$, if $e_{11} > e_{12}$.

3) Increasing batch size by 2: The above results suggest that when batch size is small, increasing batch size by 1 may not lead to improvement of product quality. However, when batch size is added by two each time, the monotonicity property can always be observed. In this case, we obtain

Proposition 3: Under assumptions (1)-(5) for equal products, if batch size is added by two, then the product quality is

- monotonically increasing, i.e., $P_{k+2}(g) > P_k(g)$, if $e_{11} > e_{12}$, and
- monotonically decreasing, i.e., $P_{k+2}(g) < P_k(g)$, if $e_{11} < e_{12}$.

Since

$$P_{k+2}(g) - P_k(g) = \frac{(\lambda_{12} + \mu_{12})(e_{12} - e_{11})}{\lambda_{11} + \mu_{11}}(D_{k+2} - D_k), \tag{22}$$

Proposition 3 implies that $D_{k+2} - D_k$ is always negative. An illustration of this fact based on numerical investigation is given in Figure 6.

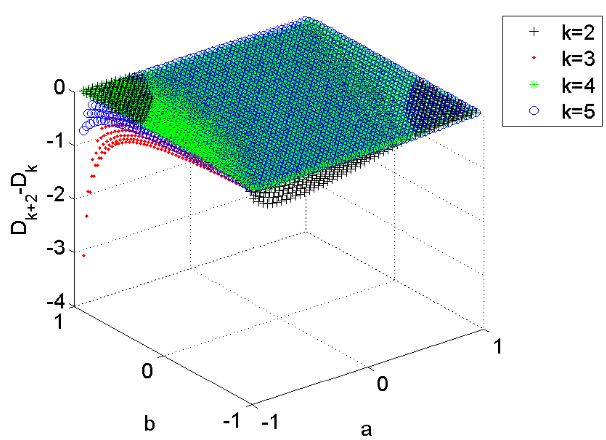

Fig. 6. $D_{k+2} - D_k$ vs a and b with respect to different batch size

Therefore, when quality efficiency without transition is higher than that with transition, adding two parts in the batch will improve quality, which agrees with our intuition. Otherwise, quality may be downgraded.

Remark 4: The non-monotonic results observed in this section indicate that effort should be made to avoid occurring of oscillation scenarios (even though, in practice, the non-monotonic scenarios are less likely to happen). For example, it is shown from the above results that if we can keep the failure probabilities small and repair probabilities higher so that $\lambda_{11} + \mu_{11}$ will not be significantly greater than 1 (i.e., $a > 0$), then all monotonicity properties with respect to λ_{1i}, μ_{1i}, $i = 1, 2$, and k hold so that quality improvement can be achieved by reducing failure probabilities and increasing repair probabilities or using large batches when quality efficiency without product switch is higher.

Remark 5: In addition to equal products case, the non-equal products case, i.e., the general cases, have been studied through numerical experiments, where λ_{ij}, μ_{ij}, batch size and product sequences are selected randomly. The preliminary results show that similar to equal products case, non-monotonicity can be found in more scenarios. However, by examining the data related to these cases, it is discovered that the values of λ_{ij} or μ_{ij} or e_{ij} will be either too large, or too small, which implies unrealistic. In other words, such cases seldom occur in practice, which verifies the remark above. More in-depth investigation is in progress.

V. CONCLUSIONS

In this paper, an analytical method based on Markov chain model is presented to evaluate the quality performance in a flexible manufacturing system with batch productions. Monotonic and non-monotonic properties are discovered and necessary conditions to maintain system monotonicity to improve quality are investigated. The results obtained suggest that designing appropriate batch policy in flexible manufacturing system is important to maintain good product quality. The future work is directed to a more detailed study on the non-equal products case, and investigation of the scheduling policy to improve quality.

REFERENCES

[1] N. Viswanadham and Y. Narahari, *Performance Modeling of Automated Manufacturing System*, Prentice Hall, 1992.

[2] J.A. Buzacott and J.G. Shantikumar, *Stochastic Models of Manufacturing Systems*, Prentice Hall, 1993.

[3] M. Zhou and K. Venkatesh, *Modeling, Simulation and Control of Flexible Manufacturing Systems: A Petri Net Approach*, World Scientific Publishing Company, 1999.

[4] J.A. Buzacott and D.D. Yao,“Flexible Manufacturing Systems: A Review of Analytical Models,” *Manag. Sci.*, vol. 32, pp. 890-905, 1986.

[5] A.K. Sethi and P.S. Sethi, “Flexibility in Manufacturing: A Survey,” *Int. J. of FMS*, vol. 2, pp. 289-328, 1990.

[6] M. Barad and S.Y. Nof, “CIM Flexibility Measures: A Review and a Framework for Analysis and Applicability Assessment,” *Int. J. of CIM*, vol. 10, pp. 296-308, 1997.

[7] R. Beach, A.P. Juhlemann, D.H.R. Price, A. Paterson and J.A. Sharp, “A Review of Manufacturing Flexibility,” *Euro. J. of Oper. Res.*, vol. 122, pp. 41-57, 2000.

[8] J. Payne and V. Cariapa, “A Fixture Repeatability and Reproducibility Measure to Predict the Quality of Machined Parts,” *Int. J. of Prod. Res.*, vol. 38, pp. 4763-4781, 2000.

[9] R.R. Inman, D.E. Blumenfeld, N. Huang and J. Li, “Production System Design for Quality: Research Opportunities from Automotive Industry Prospective,” *Int. J. of Prod. Res.*, vol. 41, pp. 1953-1971, 2003.

[10] D.E. Zoia, “Harbour Outlines Who’s Winning and Why”, *WardsAuto.com*, Sept. 27, 2005.

[11] A. Bolat and C. Yano, “Procedures to Analyze the Tradeoffs between Costs of Setup and Utility Work for Automobile Assembly Lines”, *Report 89-3*, Dept. of IOE, Univ. of Michigan, Ann Arbor, MI, 1989.

[12] F.F. Chen and E.E. Adams, “The Impact of Flexible Manufacturing Systems on Productivity and Quality,” *IEEE Trans. of Eng. Manag.*, vol. 38, pp. 33-45, 1991.

[13] J. Li and N. Huang, “Quality Evaluation in Flexible Manufacturing Systems: A Markovian Approach,” *Math. Prob. in Eng.*, vol. 2007, article ID 57128, 2007.

[14] J. Wang, J. Li, J. Arinez, S. Biller and N. Huang, “Quality Analysis in Flexible Manufacturing Systems with Batch Productions,” *Report PSSL-08-01*, Dept. of ECE, Univ. of Kentucky, Lexington, KY, 2008.