

Homework 3

due: W Oct 15, 2025, 11:59 PM PST

1. Problem 23 from https://www.probabilitycourse.com/chapter3/3_3_0_chapter3_problems.php
2. Let X and Y be independent random variables. If we know that $\text{Var}(X) = 2$ and $\text{Var}(Y) = 3$, calculate $\text{Var}(2X + 3)$ and $\text{Var}(X + Y - 5)$.
3. Let W, X, Y be random variables, where
 - W has a PMF defined by $P(W = -1) = \frac{1}{10}$, $P(W = 1) = \frac{9}{10}$,
 - X has a PMF defined by $P(X = -3) = \frac{1}{2}$, $P(X = -2) = \frac{3}{10}$, $P(X = -1) = \frac{1}{5}$, and
 - Y has a PDF f_Y defined by

$$f_Y(b) := \begin{cases} \frac{2}{b^2}, & \text{if } b \in [1, 2], \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

- (a) Calculate the expected values $E(W)$, $E(X)$, $E(Y)$.
 - (b) Calculate the expected values $E(2W - 1)$ and $E(4W + 2X - 3Y + 2)$.
4. Suppose I have chosen a fixed value of $p \in (0, 1)$. Define the function f as

$$f(k) = c \cdot p(1 - p)^k, \quad \text{for } k \in \{3, 4, 5, 6, \dots\}. \quad (2)$$

For what value of $c \in \mathbb{R}$ is the function f a PDF? (Express c in terms of p .)
(Hint: Use the geometric series

$$\sum_{k=0}^{\infty} q^k = \frac{1}{1 - q} \quad \text{for } |q| < 1.$$

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