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***STA 35C* – Statistical Data Science III**
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Homework 1

due: W Oct 1, 2025, 9:00 AM PST

Textbook problems are from https://www.probabilitycourse.com/chapter1/1_5_0_chapter1_problems.php

1. (Textbook Problem 6) Suppose that A_1, A_2, A_3 form a partition of the universal set S. For a set B, assume that we know

$$|B \cap A_1| = 10, \qquad |B \cap A_2| = 20, \qquad |B \cap A_3| = 15.$$

   Find $|B|$.

2. (Textbook Problem 13) Two teams A and B play a soccer match, and we are interested in the winner. The sample space can be defined as

$$S = \{a, b, d\},$$

   where a shows the outcome that A wins, b shows the outcome that B wins, and d shows the outcome that they draw. Suppose we know that:

$$\text{the probability that } A \text{ wins is } P(a) = P(\{a\}) = 0.5, \text{ and}$$
$$\text{the probability of a draw is } P(d) = P(\{d\}) = 0.25.$$

   (a) Find the probability that B wins.
   (b) Find the probability that B wins or a draw occurs.

3. A fair six-sided die is rolled two times in a row, where it can be assumed that the two die rolls don't influence each other (i.e., they are independent). The set of all possible results is

$$S = \{(x_1, x_2) \,|\, x_1, x_2 \in \{1, \ldots, 6\}\}, \quad \text{or, equivalently,} \quad S = \{1, \ldots, 6\} \times \{1, \ldots, 6\}.$$

   We are interested in the events:

   A: "The result of the first roll is even" and B: "The result of the second roll is a multiple of 3".

   (a) Formally write down A, B as sets (similar to the set S above).
   (b) Calculate $P(A), P(A^c), P(B), P(A \cap B), P(A \cup B), P(B \backslash A)$.

4. (Textbook Problem 16) Consider a random experiment with a sample space $S = \{1, 2, 3, \cdots\}$. Suppose that we know that

$$P(k) = P(\{k\}) = \frac{c}{3^k}, \quad \text{for } k = 1, 2, 3, \cdots,$$

   where c is a constant number.

   (a) Find c.
   (b) Find $P(\{2, 4, 6\})$.
   (c) Find $P(\{3, 4, 5, \cdots\})$.