

# Homework 1

due: W Oct 1, 2025, 11:59 PM PST

Textbook problems are from [https://www.probabilitycourse.com/chapter1/1\\_5\\_0\\_chapter1\\_problems.php](https://www.probabilitycourse.com/chapter1/1_5_0_chapter1_problems.php)

1. (Textbook Problem 6) Suppose that  $A_1, A_2, A_3$  form a partition of the universal set  $S$ . For a set  $B$ , assume that we know

$$|B \cap A_1| = 10, \quad |B \cap A_2| = 20, \quad |B \cap A_3| = 15.$$

Find  $|B|$ .

2. (Textbook Problem 13) Two teams  $A$  and  $B$  play a soccer match, and we are interested in the winner. The sample space can be defined as

$$S = \{a, b, d\},$$

where  $a$  shows the outcome that  $A$  wins,  $b$  shows the outcome that  $B$  wins, and  $d$  shows the outcome that they draw. Suppose we know that:

$$\begin{aligned} \text{the probability that } A \text{ wins is } P(a) = P(\{a\}) = 0.5, \text{ and} \\ \text{the probability of a draw is } P(d) = P(\{d\}) = 0.25. \end{aligned}$$

- (a) Find the probability that  $B$  wins.  
(b) Find the probability that  $B$  wins or a draw occurs.
3. A fair six-sided die is rolled two times in a row, where it can be assumed that the two die rolls don't influence each other (i.e., they are independent). The set of all possible results is

$$S = \{(x_1, x_2) \mid x_1, x_2 \in \{1, \dots, 6\}\}, \quad \text{or, equivalently,} \quad S = \{1, \dots, 6\} \times \{1, \dots, 6\}.$$

We are interested in the events:

$A$ : “The result of the first roll is even”      and       $B$ : “The result of the second roll is a multiple of 3”.

- (a) Formally write down  $A, B$  as sets (similar to the set  $S$  above).  
(b) Calculate  $P(A), P(A^c), P(B), P(A \cap B), P(A \cup B), P(B \setminus A)$ .
4. (Textbook Problem 16) Consider a random experiment with a sample space  $S = \{1, 2, 3, \dots\}$ . Suppose that we know that

$$P(k) = P(\{k\}) = \frac{c}{3^k}, \quad \text{for } k = 1, 2, 3, \dots,$$

where  $c$  is a constant number.

- (a) Find  $c$ .  
(b) Find  $P(\{2, 4, 6\})$ .  
(c) Find  $P(\{3, 4, 5, \dots\})$ .