Homework 1
due: W Oct 1, 2025, 11:59 PM PST

Textbook problems are from https://www.probabilitycourse.com/chapter1/1_5_0_chapter1_problems.php

1. (Textbook Problem 6) Suppose that A_1, A_2, A_3 form a partition of the universal set Ω. For a set B, assume that we know

$$|B \cap A_1| = 10, \qquad |B \cap A_2| = 20, \qquad |B \cap A_3| = 15.$$

Find $|B|$.

2. (Textbook Problem 13) Two teams A and B play a soccer match, and we are interested in the winner. The sample space can be defined as

$$\Omega = \{a, b, d\},$$

where a shows the outcome that A wins, b shows the outcome that B wins, and d shows the outcome that they draw. Suppose we know that:

the probability that A wins is $P(a) = P(\{a\}) = 0.5$, and
the probability of a draw is $P(d) = P(\{d\}) = 0.25$.

   (a) Find the probability that B wins.
   (b) Find the probability that B wins or a draw occurs.

3. A fair six-sided die is rolled two times in a row, where it can be assumed that the two die rolls don't influence each other (i.e., they are independent). The set of all possible results is

$$\Omega = \{(x_1, x_2) \mid x_1, x_2 \in \{1, \ldots, 6\}\}, \quad \text{or, equivalently,} \quad \Omega = \{1, \ldots, 6\} \times \{1, \ldots, 6\}.$$

We are interested in the events:

A: "The result of the first roll is even" and B: "The result of the second roll is a multiple of 3".

   (a) Formally write down A, B as sets (similar to the set Ω above).
   (b) Calculate $P(A), P(A^c), P(B), P(A \cap B), P(A \cup B), P(B \backslash A)$.

4. (Textbook Problem 16) Consider a random experiment with a sample space $\Omega = \{1, 2, 3, \cdots\}$. Suppose that we know that

$$P(k) = P(\{k\}) = \frac{c}{3^k}, \quad \text{for } k = 1, 2, 3, \cdots,$$

where c is a constant number.

   (a) Find c.
   (b) Find $P(\{2, 4, 6\})$.
   (c) Find $P(\{3, 4, 5, \cdots\})$.