# Determining Granger Causal Flow for S&P 500 Summer 2019-2021 vs Summer 2021-2023

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**Abstract**

This short report seeks to explore differences in Granger causal flow for the 11 sectors within the S&P 500 Index from 2019-2021 and 2021-2023. The goal is to discover whether the COVID-19 pandemic had a major impact on the Granger causal flow between these 11 sectors in these two time periods.

# 1 Background

This project seeks to establish if there is a difference in the Granger causal flow of the S&P 500 before and after the height of the COVID-19 pandemic. As using every single one of the more than 500 stocks in the S&P 500 to create a Granger causality strength matrix would lead to absurdly long computation times, we have instead opted to use 11 series. Each of these 11 series corresponds to a different sector of the S&P 500. Each individual stock's sector was determined and placed in a dataset containing other stocks of that same sector. The makeup of the individual stocks in the S&P 500 and the sectors which they belonged to was determined based on the stocks which were listed on the following website as of March 30, 2024: https://en.wikipedia.org/wiki/List_of_S%26P_500_companies. An average was created for each sector by using adjusted closing prices from June 20, 2019 to June 19, 2021 and from June 20, 2021 to June 19, 2023. Finally, the log returns of these 11 series were obtained for all of the calculations of the Granger causal strength.

Next, the MTS and BigVAR packages in R were used in order to explore whether different methods would lead to different sparsity structures in the final Granger causal strength matrices. These matrices can be found in the "Results" section. In these plots the source is taken to be the vertical axis which is labelled as "column". Thus, these plots can be interpreted as the sector on the vertical axis of the matrix Granger causing the sector on the horizontal axis labelled "row". The Granger causality strengths in these plots were each normalized using the following:

$$GC_{Normalized} = \frac{GC^T{}_{i,j} - \min(GC)}{\max(GC) - \min(GC)}$$

where i,j denote the column and row index in the matrix, min(GC) denotes the minimum value in the Granger causal strength matrix, and max(GC) denotes the maximum value in the Granger causal strength matrix.

# 2 Results

## MTS

We begin by examining differences in the Granger causal structure when using the MTS package. The Granger strength is only possible to obtain for the 2021-2023 data and is shown in Figure 1 below.

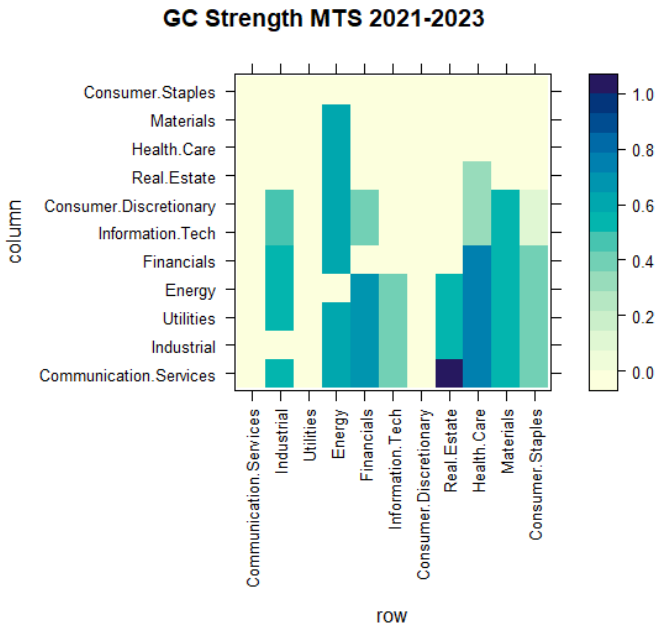


Figure 1: MTS Granger Causal Strengths

We can see from Figure 1 that the Consumer Services sector has a high normalized Granger causal strength for the Real Estate sector. The next highest Granger casual strength is for Financials, Energy, Utilities, Industrial, and Communication Services on the Health Care sector.

## Basic Lasso

Next, we explore the Granger causal strengths for each of the two time periods using the BigVAR package in R. We begin by using the Basic Lasso penalty.

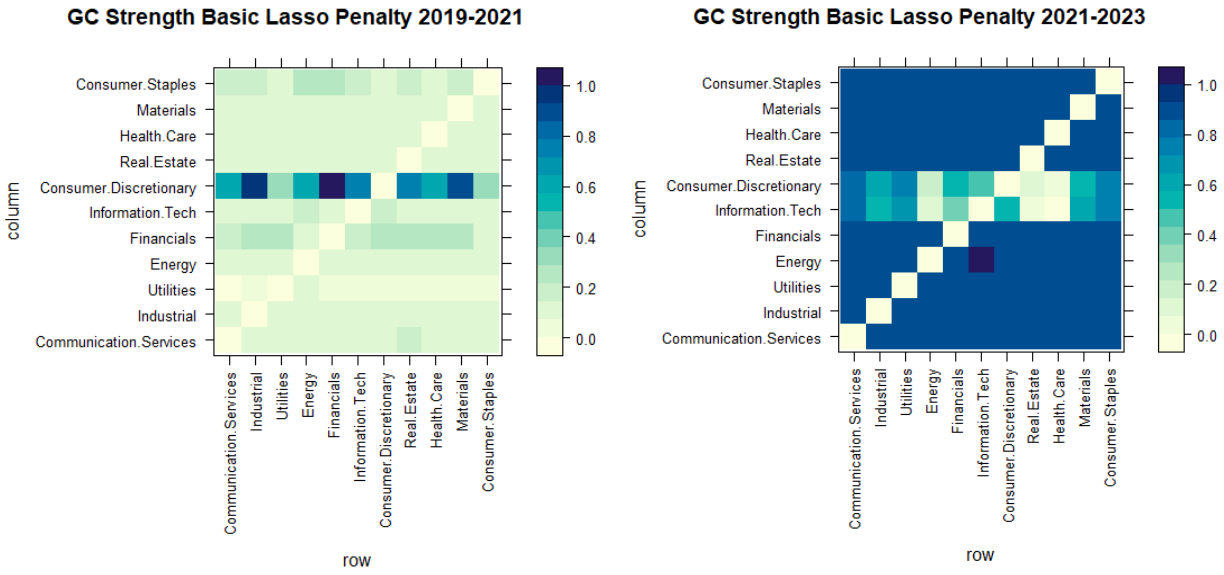


Figure 2: Basic Lasso Granger Causal Strengths

For 2019-2021 it appears that the Consumer Discretionary sector is Granger causing most of the other sectors, with the highest Granger causal strength being achieved on the Financials sector. However, we do not have this same behavior for 2021-2023. The Granger strength is in fact weakest between the Consumer Discretionary sector and all other sectors. This same behavior is also seen in Information Technology having the weakest normalized Granger causal strength on all other sectors. For 2021-2023 we see that most sectors have a high Granger causal strength on the other sectors, with Energy Granger causing Information Technology having the highest strength.

**Basic Elastic Net**

Next, we explore the Granger causal strengths for each of the two time periods using an Elastic Net penalty. The results are shown in Figure 3.

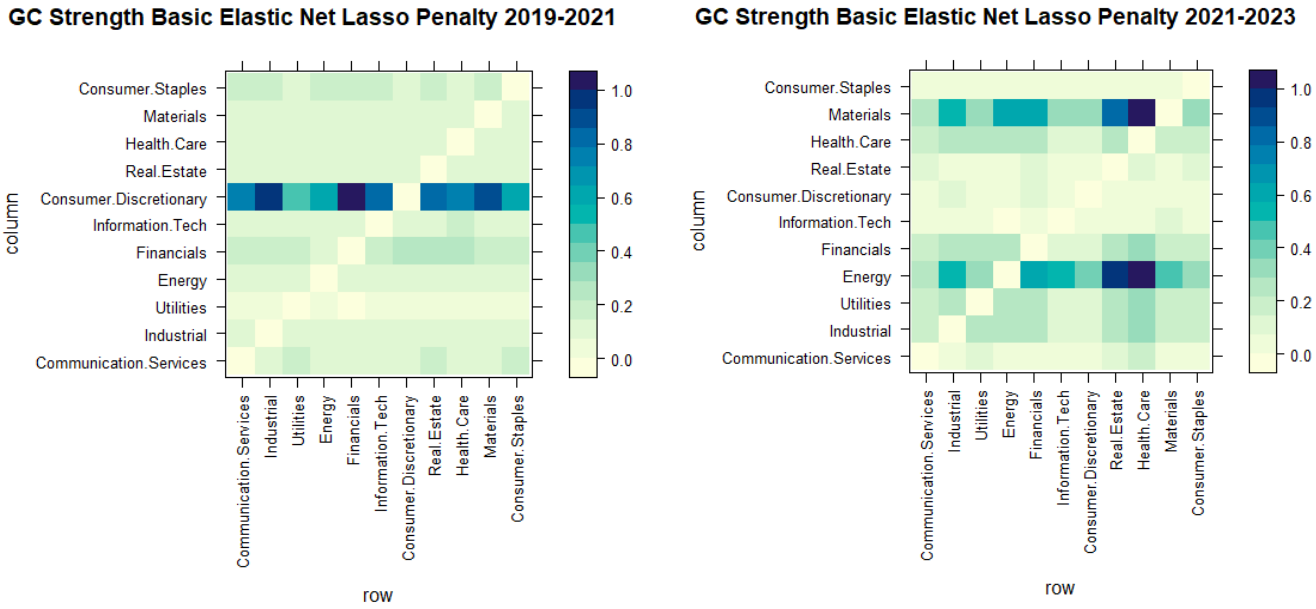


Figure 3: Basic Elastic Net Lasso Granger Causal Strengths

For 2019-2021 we see a very similar behavior to the Basic Lasso penalty. It appears that the Consumer Discretionary sector is Granger causing most of the other sectors, with the highest Granger causal strength being achieved on the Financials sector. This same behavior is not seen in the data for 2021-2023. Instead, Materials are Granger causing Real Estate and Health Care, and Energy is also Granger causing Real Estate and Health Care.

## Lag

Next, we explore the Granger causal strengths for each of the two time periods using a Lag penalty. The results are shown in Figure 4.

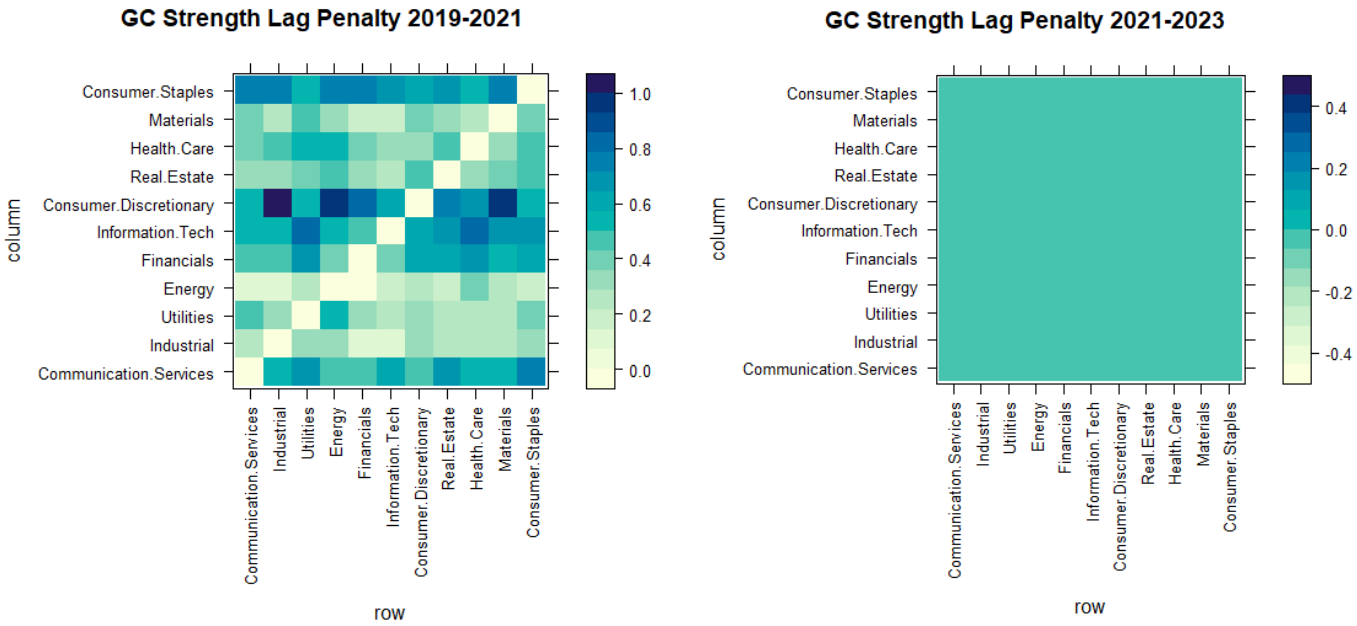


Figure 4: Lag Granger Causal Strengths

It is apparent from Figure 4 that the sparsity between the two Granger causal strength plots is highly different. The plot for 2019-2021 is not very sparse and sees the Consumer Discretionary sector having a high Granger causality for the Industrial, Energy, and Materials sectors. The 2021-2023 plot shows a stark contrast to the 2019-2021 plot as it is extremely sparse. Note that the original matrix before normalization contained all zero entries so the given plot was not normalized. This shows one of the shortcomings of this choice of normalization and lends itself to further thought and consideration for the future.

## SCAD

Next, we explore the Granger causal strengths for each of the two time periods using a SCAD penalty. The results are shown in Figure 5.

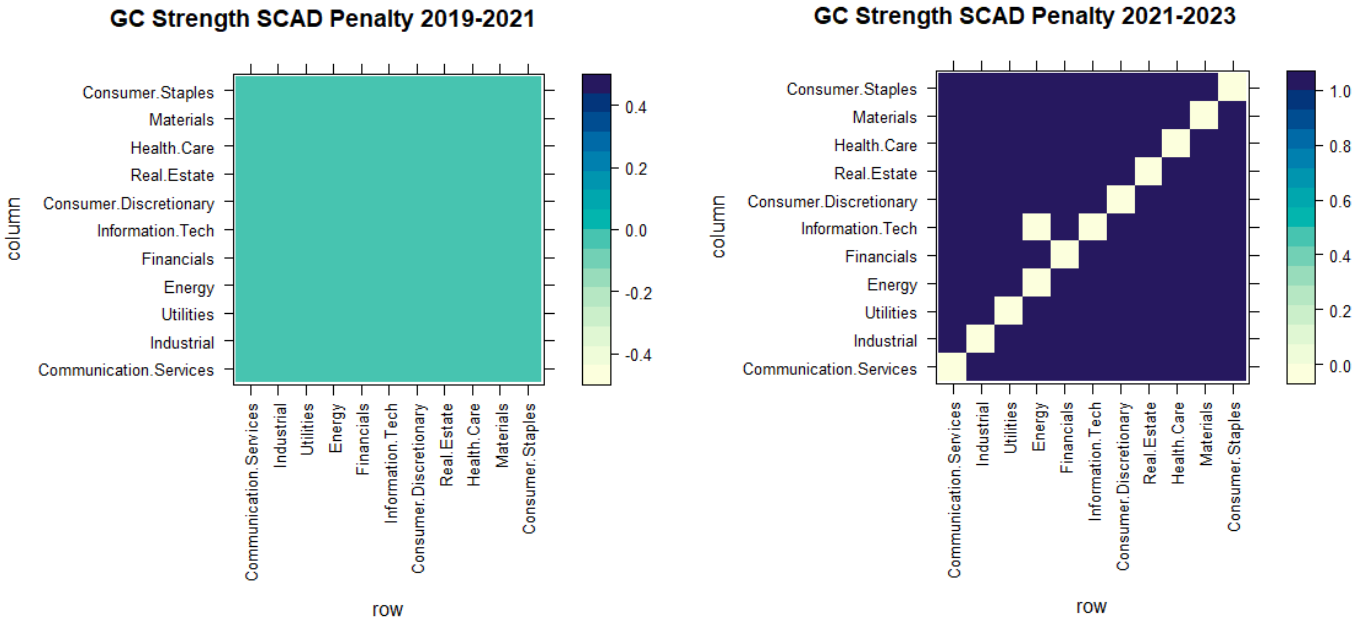


Figure 5: SCAD Granger Causal Strengths

The Granger causal strength plot is extremely sparse for the 2019-2021 dataset. Here normalization was not undertaken to prevent numeric errors just like we did with the 2021-2023 Lag penalty. However, the Granger causal strength plot shows very high Granger causality for almost all sectors in the 2021-2023 plot. The only exception to this is that the Information Technology sector does not appear to be Granger causing the Energy sector. However, it must be noted that the reason for this largely has to do with the sparsity before normalization. Before normalization only a single entry was nonzero. This tells us that the results of the SCAD penalty should largely be disregarded.

**MCP**

Next, we explore the Granger causal strengths for each of the two time periods using a MCP penalty. The results are shown in Figure 6.

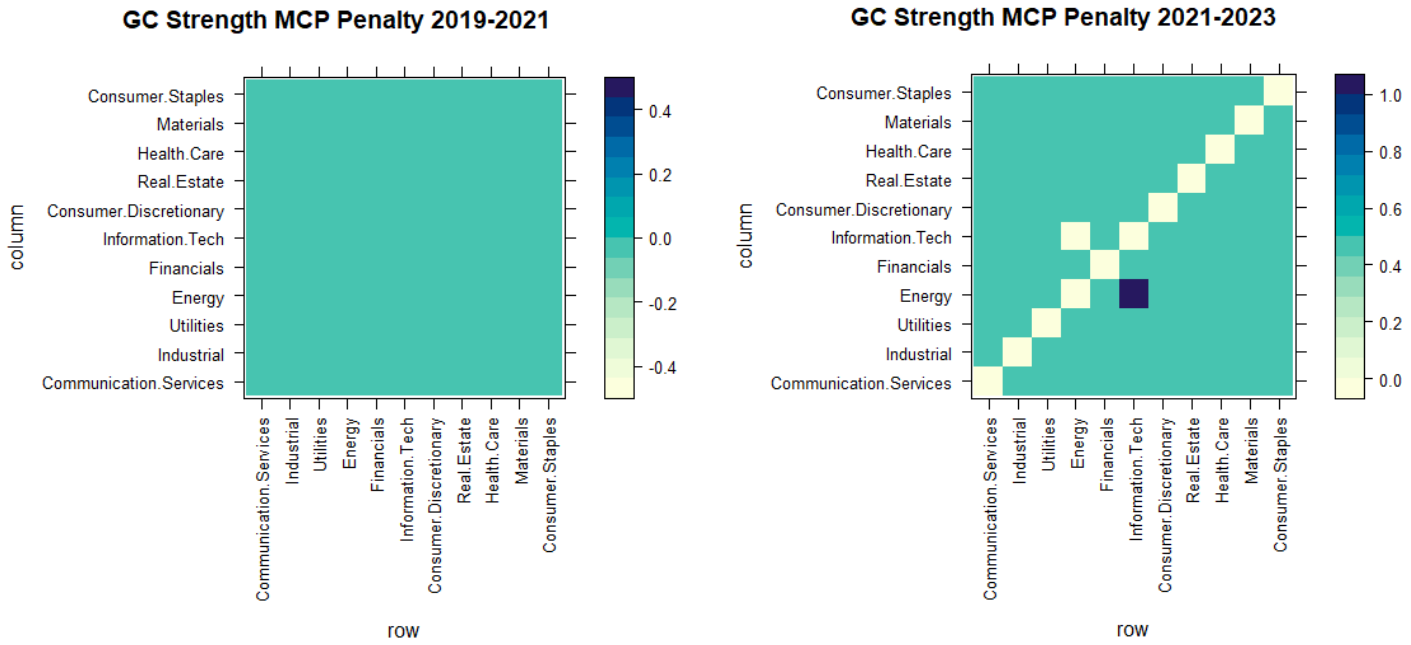


Figure 6: MCP Granger Causal Strengths

The behavior of the Granger causality plots is very similar to those of the SCAD method. The 2019-2021 is extremely sparse, and the 2021-2023 plot shows a somewhat moderate Granger causal strength for most of the sectors. The exception is that the Information Technology sector does not appear to be Granger causing the Energy sector. However, the Energy sector does appear to very strongly Granger causes the Information Technology sector. Once again, similar to the SCAD penalty the 2021-2023 plot had only two nonzero entries before normalization. This again tells us that a new choice of normalization is likely required and that the results for MCP should largely be ignored.

**BGR**

Finally, we explore the Granger causal strengths for each of the two time periods using a BGR penalty. The results are shown in Figure 7.

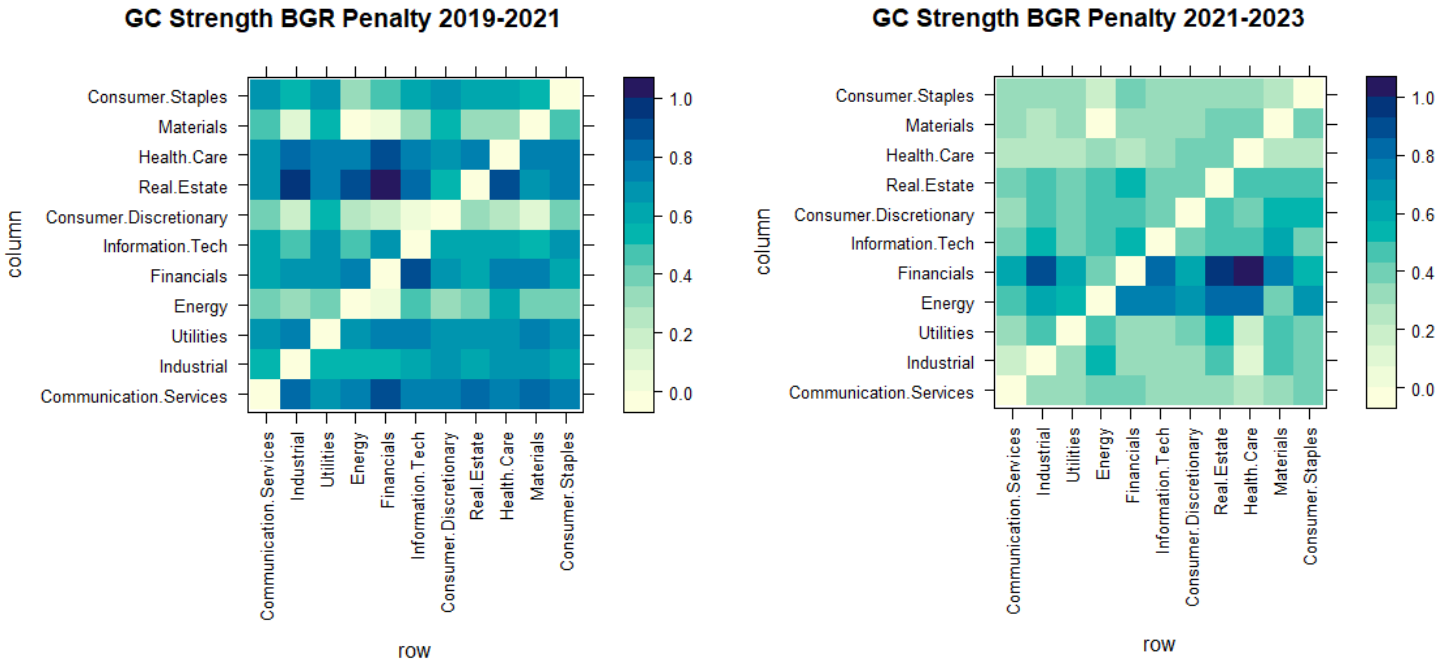


Figure 7: BGR Granger Causal Strengths

Compared with the previous methods, BGR appears to have the least sparseness across both time periods. These two plots have some very striking features. For example, the 2019-2021 had the Real Estate sector highly Granger cause many sectors including the Financial, Industrial, Health Care, and Energy sectors. Contrast this with the results in 2021-2023 and we see that Financials are now highly Granger causing Real Estate which is a remarkable change. Additionally, the Financial sector is strongly Granger causing the Industrials and Health Care sectors.

# 3 Conclusions

First, based on the results of the different figures it is apparent that almost all of the different penalties lead to very different sparsity structures across both datasets. For the different Lasso methods and even somewhat for the Lag method it was apparent that the Consumer Discretionary sector highly Granger caused most of the other sectors but that this behavior did not continue into 2021-2023. The SCAD and MCP method also showed similar results with the 2019-2021 Granger causal strengths being very sparse, but that the 2021-2023 Granger causal strengths showed moderate to strong Granger causal strengths between most sectors. As stated previously, however, due to the trouble with the choice of normalization and the very sparse initial Granger causal strength matrices the results of these methods should largely be disregarded. Finally, the BGR method stood in a category its own as both plots showed very little sparseness. From 2019-2021 Health Care, Real Estate, and Communication Services had high Granger causality for many of the other sectors, while from 2021-2023 it was the Financial and Energy sectors which had the highest Granger causality with the other sectors.

In general, each plot seems to suggest a different story as we look at the state of the S&P 500 sectors before the COVID-19 pandemic and after. Of all of the methods used it is the different Lasso penalties which provided the most informative. Furthermore, as the Elastic Net Lasso is more flexible it is very likely that its results should be given the greatest consideration.