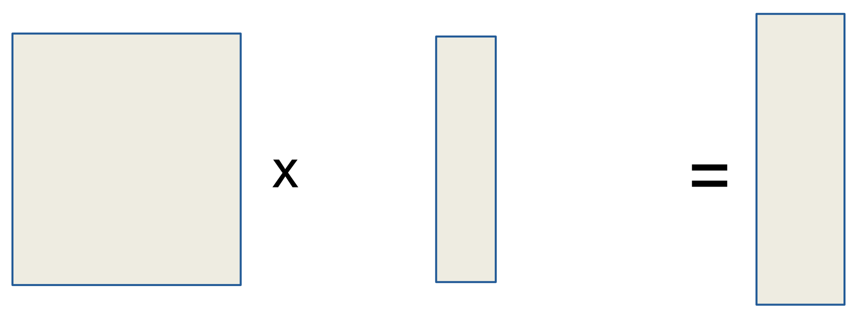
Decision-Making With Matrices

Introduction

Decision-Making With Matrices

$$Ax = B$$



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Operations Research

Operations Research (OR)

- OR is a quantitative approach to problem solving.
- OR closely resembles and predates data science by five-plus decades.
- Typical problems involve decision-making, route planning, scheduling, etc.
- Covering industry, politics, and economics, but consistently leverages mathematical reasoning.
- The goal was to formulate a mathematical structure that could be leveraged to deduce a solution to a well-formulated problem.
- These tools are less complex then cutting edge tools used today, but are no less helpful in certain circumstances.

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Where to Have Lunch?

Decision-Making With Matrices

$$Ax = B$$

$$\begin{array}{l} \\ \text{rest\_1} \\ \text{rest\_2} \\ \text{rest\_3} \\ \vdots \\ \text{rest\_n} \end{array} \begin{array}{c} \begin{array}{ccc} \text{cost} & \text{rating} & \text{distance} \end{array} \\ \begin{bmatrix} 9 & 4 & 7 \\ 2 & 1 & 9 \\ 3 & 4 & 7 \\ \vdots & \vdots & \vdots \\ 3 & 4 & 1 \end{bmatrix} \end{array} \times \begin{bmatrix} \; \\ \; \\ \; \\ \; \\ \; \end{bmatrix} = \begin{bmatrix} \; \\ \; \\ \; \\ \; \\ \; \end{bmatrix}$$

Cost: (1–10) bigger is cheaper
Rating: (1–5) bigger is better
Distance: (1–10) bigger is closer

Decision-Making With Matrices

$$Ax = B$$

$$
\begin{array}{r|ccc|}
 & \text{cost} & \text{rating} & \text{distance} \\
\hline
\text{rest\_1} & 9 & 4 & 7 \\
\text{rest\_2} & 2 & 1 & 9 \\
\text{rest\_3} & 3 & 4 & 7 \\
\vdots & \vdots & \vdots & \vdots \\
\text{rest\_n} & 3 & 4 & 1 \\
\hline
\end{array}
\; x \;
\begin{array}{|c|l}
\text{user\_1} & \\
\hline
5 & \text{cost} \\
5 & \text{rating} \\
1 & \text{distance} \\
\hline
\end{array}
\; = \;
\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}
$$

Cost: (1–10) bigger is cheaper
Rating: (1–5) bigger is better
Distance: (1–10) bigger is closer

User_1: (1–10) bigger is more important

Decision-Making With Matrices

$$Ax = B$$

$$\begin{array}{c} \\ \text{rest\_1} \\ \text{rest\_2} \\ \text{rest\_3} \\ \vdots \\ \text{rest\_n} \end{array} \begin{array}{c} \begin{array}{ccc} \text{cost} & \text{rating} & \text{distance} \end{array} \\ \begin{bmatrix} 9 & 4 & 7 \\ 2 & 1 & 9 \\ 3 & 4 & 7 \\ \vdots & \vdots & \vdots \\ 3 & 4 & 1 \end{bmatrix} \end{array} \times \begin{array}{c} \text{user\_1} \\ \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{cost} \\ \text{rating} \\ \text{distance} \end{array} = \begin{array}{c} \text{score} \\ \begin{bmatrix} 72 \\ 24 \\ 42 \\ \\ 36 \end{bmatrix} \end{array}$$

Cost: (1–10) bigger is cheaper
Rating: (1–5) bigger is better
Distance: (1–10) bigger is closer

User_1: (1–10) bigger is more important

Score: bigger is better

Decision-Making With Matrices

$$Ax = B$$

$$\begin{array}{r} \\ \text{rest\_1} \\ \text{rest\_2} \\ \text{rest\_3} \\ \vdots \\ \text{rest\_n} \end{array} \begin{array}{c} \begin{array}{ccc} \text{cost} & \text{rating} & \text{distance} \end{array} \\ \left[\begin{array}{ccc} 9 & 4 & 7 \\ 2 & 1 & 9 \\ 3 & 4 & 7 \\ \vdots & \vdots & \vdots \\ 3 & 4 & 1 \end{array}\right] \end{array} \times \begin{array}{c} \text{user\_1} \\ \left[\begin{array}{c} 5 \\ 5 \\ 1 \end{array}\right] \end{array} \begin{array}{l} \\ \text{cost} \\ \text{rating} \\ \text{distance} \end{array} = \begin{array}{c} \text{score} \\ \left[\begin{array}{c} 72 \\ 24 \\ 42 \\ \\ 36 \end{array}\right] \end{array} = \begin{array}{c} \text{ranking} \\ \left[\begin{array}{c} 1 \\ 4 \\ 2 \\ \\ 3 \end{array}\right] \end{array}$$

Cost: (1–10) bigger is cheaper
Rating: (1–5) bigger is better
Distance: (1–10) bigger is closer

User_1: (1–10) bigger is more important

Score: bigger is better
Ranking: (1-n) smaller is better

Decision-Making With Matrices

$$Ax = B$$

$$\begin{array}{cc} & \begin{array}{ccc}\text{cost} & \text{rating} & \text{distance}\end{array} \\ \begin{array}{c}\text{rest\_1}\\ \text{rest\_2}\\ \text{rest\_3}\\ \vdots\\ \text{rest\_n}\end{array} & \begin{bmatrix}9&4&7\\ 2&1&9\\ 3&4&7\\ \vdots&\vdots&\vdots\\ 3&4&1\end{bmatrix}\end{array} \times \begin{array}{cc} \text{user\_1} & \\ \begin{bmatrix}?\\ ?\\ ?\end{bmatrix} & \begin{array}{l}\text{cost}\\ \text{rating}\\ \text{distance}\end{array}\end{array} = \begin{array}{c}\text{score}\\ \begin{bmatrix}27\\ 21\\ 21\\ \\ 9\end{bmatrix}\end{array} = \begin{array}{c}\text{ranking}\\ \begin{bmatrix}1\\ 2\\ 2\\ \\ 3\end{bmatrix}\end{array}$$

Cost: (1–10) bigger is cheaper
Rating: (1–5) bigger is better
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Score: bigger is better
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Decision-Making With Matrices

$$Ax = B$$

$$
\begin{array}{l}
 \\
\text{rest\_1} \\
\text{rest\_2} \\
\text{rest\_3} \\
\vdots \\
\text{rest\_n}
\end{array}
\begin{array}{c}
\begin{array}{ccc} \text{cost} & \text{rating} & \text{distance} \end{array} \\
\begin{bmatrix} 9 & 4 & 7 \\ 2 & 1 & 9 \\ 3 & 4 & 7 \\ \vdots & \vdots & \vdots \\ 3 & 4 & 1 \end{bmatrix}
\end{array}
\times
\begin{array}{c}
\text{user\_1} \\
\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}
\end{array}
\begin{array}{l}
 \\
\text{cost} \\
\text{rating} \\
\text{distance}
\end{array}
=
\begin{array}{c}
\text{score} \\
\begin{bmatrix} 27 \\ 21 \\ 21 \\ \\ 9 \end{bmatrix}
\end{array}
=
\begin{array}{c}
\text{ranking} \\
\begin{bmatrix} 1 \\ 2 \\ 2 \\ \\ 3 \end{bmatrix}
\end{array}
$$

Cost: (1–10) bigger is cheaper
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User_1: (1–10) bigger is more important

Score: bigger is better
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Where Should the Team Have Lunch?

Decision-Making With Matrices

$$Ax = B$$

$$\begin{array}{c|ccc|} & \text{cost} & \text{rating} & \text{distance} \\ \hline \text{rest\_1} & 9 & 4 & 7 \\ \text{rest\_2} & 2 & 1 & 9 \\ \text{rest\_3} & 3 & 4 & 7 \\ \vdots & \vdots & \vdots & \vdots \\ \text{rest\_n} & 3 & 4 & 1 \\ \hline \end{array} \quad x \quad \boxed{} \quad =$$

Cost: (1–10) bigger is cheaper.
Rating: (1–5) bigger is better
Distance: (1–10) bigger is closer

User_1: (1–10) Bigger is more important

Score: bigger is better
Ranking: (1−n) smaller is better

Decision-Making With Matrices

$$Ax = B$$

$$
\begin{array}{cc}
 & \begin{matrix} \text{cost} & \text{rating} & \text{distance} \end{matrix} \\
\begin{matrix} \text{rest\_1} \\ \text{rest\_2} \\ \text{rest\_3} \\ \vdots \\ \text{rest\_n} \end{matrix} &
\begin{bmatrix} 9 & 4 & 7 \\ 2 & 1 & 9 \\ 3 & 4 & 7 \\ \vdots & \vdots & \vdots \\ 3 & 4 & 1 \end{bmatrix}
\end{array}
\times
\begin{array}{cc}
 & \begin{matrix} \text{user\_1} & \text{user\_2} & \text{user\_3} & & \text{user\_n} \end{matrix} \\
\begin{matrix} \text{cost} \\ \text{rating} \\ \text{distance} \end{matrix} &
\begin{bmatrix} 5 & 2 & 5 & \ldots & 1 \\ 5 & 8 & 4 & \ldots & 8 \\ 1 & 8 & 3 & \ldots & 3 \end{bmatrix}
\end{array}
=
$$

Cost: (1–10) bigger is cheaper.
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\begin{array}{cc}
 & \begin{array}{ccc}\text{cost} & \text{rating} & \text{distance}\end{array} \\
\begin{array}{c}\text{rest\_1}\\ \text{rest\_2}\\ \text{rest\_3}\\ \vdots \\ \text{rest\_n}\end{array} & \begin{bmatrix} 9 & 4 & 7 \\ 2 & 1 & 9 \\ 3 & 4 & 7 \\ \vdots & \vdots & \vdots \\ 3 & 4 & 1 \end{bmatrix}
\end{array}
\times
\begin{array}{cc}
 & \begin{array}{ccccc}\text{user\_1} & \text{user\_2} & \text{user\_3} & & \text{user\_n}\end{array} \\
\begin{array}{c}\text{cost}\\ \text{rating}\\ \text{distance}\end{array} & \begin{bmatrix} 5 & 2 & 5 & \ldots & 1 \\ 5 & 8 & 4 & \ldots & 8 \\ 1 & 8 & 3 & \ldots & 3 \end{bmatrix}
\end{array}
=
\begin{bmatrix} \quad \end{bmatrix}
$$

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Cost: (1–10) bigger is cheaper.
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 & \text{cost} & \text{rating} & \text{distance} \\
\hline
\text{rest\_1} & 9 & 4 & 7 \\
\text{rest\_2} & 2 & 1 & 9 \\
\text{rest\_3} & 3 & 4 & 7 \\
\vdots & \vdots & \vdots & \vdots \\
\text{rest\_n} & 3 & 4 & 1 \\
\hline
\end{array}
\;\times\;
\begin{array}{c|ccccc|}
 & \text{user\_1} & \text{user\_2} & \text{user\_3} & & \text{user\_n} \\
\hline
\text{cost} & 5 & 2 & 5 & \ldots & 1 \\
\text{rating} & 5 & 8 & 4 & \ldots & 8 \\
\text{distance} & 1 & 8 & 3 & \ldots & 3 \\
\hline
\end{array}
\;=\;
\begin{array}{|ccccc|}
\text{rst\_1} & \text{rst\_2} & \text{rst\_3} & \ldots & \text{rst\_n} \\
\hline
72 & 24 & 42 & \cdots & 36 \\
106 & 84 & 94 & \cdots & 46 \\
82 & 41 & 52 & \cdots & 34 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
62 & 37 & 56 & \cdots & 38 \\
\hline
\end{array}
$$

Restaurant Score	322	186	244	154

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