Taming Volatility: Evaluating NGDP Targeting

Alex Houtz*

July 25, 2024

Abstract

I embed a nominal GDP level target inside a Taylor-type rule and compare the volatilities of output, inflation, and the nominal rate to a standard, inflation targeting Taylor rule. I demonstrate analytically that the source of the shock matters. NGDP level targeting delivers more stable output and more volatile inflation under productivity shocks, but more stable output and inflation under supply and demand shocks. These results are generally confirmed in an estimated quantitative model. Lastly, I impose a zero lower bound (ZLB) and simulate the model under both targets. NGDP level targeting hits the ZLB more often than inflation targeting. Switching to an NGDP level target while at the ZLB leads to quicker economic recovery, but leaves monetary policy constrained longer.

*I am grateful to Eric Sims, Cynthia Wu, Jeff Campbell, Audriana Houtz, and seminar participants at the University of Notre Dame.

Correspondence: ahoutz@nd.edu

1 Introduction

The Federal Reserve Act of 1913 established a dual mandate of full employment and price stability for the Federal Reserve. The Federal Open Market Committee (FOMC) considers that an inflation target of "2 percent over the longer run" accomplishes price stability and, in turn, helps bring about full employment (Board of Governors, 2021). During the Great Recession and recent COVID-19 recession, economists and popular media analysts asserted that inflation targeting was failing to accomplish the dual mandate. In a 2011 op-ed in the New York Times, for example, Christina Romer wrote that "Today, inflation is still low, but unemployment is stuck at a painfully high level." As an alternative, Romer proposed nominal GDP (NGDP): "Because it directly reflects the Fed's two central concerns — price stability and real economic performance — nominal G.D.P. is a simple and sensible target for long after the economy recovers."

This paper evaluates how well NGDP level targeting satisfies the Fed's dual mandate compared to its current inflation target. I first study an NGDP level target embedded in a Taylor-type rule analytically in the canonical three equation New Keynesian model found in Gali (2015). I then construct a quantitative DSGE model in the vein of Smets & Wouters (2007) and Christiano et al. (2005).

In the three-equation model, I derive policy functions for output, inflation, and the nominal rate under three shocks: productivity, supply (cost-push), and demand shocks. Under a productivity shock, NGDP level targeting delivers more stable output and more volatile inflation compared to an inflation target. The nominal rate moves less. Under a supply or demand shock, both output and inflation are more stable. Under a supply shock, the nominal rate is less volatile compared to inflation targeting, while under a demand shock the nominal rate is more volatile.

I then estimate a standard medium scale New Keynesian model to ground the parameters in data. The analysis from the three equation model generally holds. Output is more stable under NGDP level targeting in response to productivity and demand shocks. The inclusion of habit in the model causes output to be more volatile under NGDP targeting in response to cost-push shocks. Inflation is more volatile under the productivity shock, but more stable under cost-push and demand shocks. The nominal rate's response is more muted under the first two shocks, but more pronounced under the demand shock. Doing a long-run simulation with all the shocks in the model demonstrates that NGDP level targeting leads to more stable output and inflation. Monetary policy is more volatile than under inflation targeting.

Lastly, I analyze performance at the zero lower-bound (ZLB), since the ZLB inspired much of the writing on NGDP targeting (see Beckworth (2019) for example). I simulate the model with an occasionally binding constraint, preventing the nominal rate from going negative. NGDP level targeting hits the ZLB more often than inflation targeting. An economy that starts with inflation targeting and hits the ZLB recovers more quickly but leaves monetary policy constrained longer when the monetary authority switches to an NGDP level target while at the ZLB.

Literature Review

My paper is most similar to Garin et al. (2016), who analyze an NGDP rate targeting peg in a quantitative New Keynesian models. They find that NGDP targeting minimizes consumption-equivalent welfare loss under productivity shocks when compared to inflation targeting and a Taylor rule. Additionally NGDP targeting outperforms output gap targeting when potential output is observed with a small measurement error.[1] I innovate on their paper by establishing analytical results, imposing nominal rate rules with smoothing rather than pegs.

Beckworth & Henderson (2019) use a canonical New Keynesian model to study

[1]The authors note that in many New Keynesian models, output gap targeting does not result in a determinate equilibrium. My model is such a model.

nominal income targeting compared to inflation and output gap targeting. They introduce a shock to the output gap the central bank observes to evaluate the role of information in monetary policy. They find that uncertainty in the output gap is empirically important in explaining actual output gap fluctuations and that nominal income targeting, observed perfectly, reduces those fluctuations. I extend modeling an NGDP targeting rule in the New Keynesian literature and, since I do not use the output gap, can abstract from information problems facing the Fed.

Beckworth (2019) argues that NGDP targeting would relieve zero lower-bound issues, saying that "NGDP [level targeting], in short, generates the temporary rise in inflation needed to escape a [zero lower-bound], something that is difficult to do with the Fed's current inflation target." According to Beckworth (2019), the zero lower-bound is alleviated because inflation would become counter-cyclical. As such, real debt burdens would ease and lower real interest rates to their market-clearing levels.

Hall & Mankiw (1994) use a structural time series counterfactual to evaluate nominal income targeting. They find that "the primary benefit of nominal income targeting is reduced volatility in the price level and the inflation rate." Romer (2011) cites Hall & Mankiw (1994) for the benefits of NGDP targeting. I extend this analysis by evaluating inflation volatility in a DSGE model.

Mitra (2003) demonstrates that under NGDP targeting, a unique equilibrium exists in the three-equation New Keynesian model. Sumner (2012) argues for NGDP targeting from economic principles, saying that higher inflation under cost-push shocks can improve economic performance. Sheedy (2014) finds that NGDP targeting leads to efficient risk-sharing by stabilizing debt-to-GDP ratios. My paper also speaks to the optimal monetary policy literature discussed in papers such as Khan et al. (2003), who analyze the goals of optimal policy, and Woodford (2001), who analyzes the optimality of the Taylor rule. I also look at the zero lower-bound, relating my paper to Wu & Xia

(2016) and Sims & Wu (2021).

My paper relates to articles in public policy as well. Crook (2022) states that NGDP targeting is "the simplest way to improve monetary policy." Yglesias (2015) writes on the Fed's consideration of NGDP targeting under Bernanke. Bowman (2014) argues that NGDP targeting would stabilize the real economy by ensuring nominal wage contracts would be fulfilled regardless of the macroeconomic status. Employment would thus remain at potential. Maybe most significantly, the Chair of the Federal Reserve, Jerome Powell, mentioned just after the COVID-19 pandemic that the Fed had looked at nominal income targeting as a possible policy rule (Powell, 2022).

I organize the rest of this paper as follows: Section II lays out analysis of the NGDP rule in the canonical, three equation New Keynesian model. Section III outlines the quantitative DSGE model. Section IV discusses the results from the quantitative model. Section V concludes.

2 Three-Equation New Keynesian Model

I begin with the canonical New Keynesian model popularized in Gali's 2015 textbook. The log-linearized model consists of the following three equations:

$$\tilde{y}_t = \mathbb{E}[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) + \varepsilon_t^d \tag{1}$$

$$\tilde{\pi}_t = \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] + \varepsilon_t^s \tag{2}$$

$$\tilde{y}_t^f = \rho_a \tilde{y}_{t-1}^f + \frac{1+\eta}{\sigma+\eta}\varepsilon_t^a \tag{3}$$

$\tilde{y}_t$ denotes real output, $\tilde{\pi}_t$ denotes inflation, $\tilde{y}_t^f$ denotes potential output, and $\tilde{i}_t$ denotes the nominal interest rate. ε_t^d is a demand shock, ε_t^s is a cost-push shock and ε_t^a is a productivity shock. η denotes the inverse Frisch elasticity, σ is the CRRA parameter on the household's consumption utility, and κ is the slope parameter for the Phillips Curve (equation 2) and is a function of the price stickiness parameter θ. All variables are in their log-linearized form such that they represent deviations from their steady state. For my baseline results, I impose log-utility ($\sigma = 1$) as in Gali (2015).

Equation (1) is the dynamic IS curve, equation (2) is the New Keynesian Phillips curve, and equation (3) is an exogenous AR(1) process for potential output. To close the model, I need an equation to determine the nominal interest rate.

To proxy the current monetary policy regime, I impose an inflation targeting rule:

$$\tilde{i}_t = \rho_r \tilde{i}_{t-1} + (1 - \rho_r)\phi_\pi \tilde{\pi}_t + \sigma_r \varepsilon_t^r$$

where ϕ_π is the responsiveness of the central bank to deviations of inflation from steady-state.

I compare this inflation rule to a rule targeting the level of NGDP:

$$\tilde{i}_t = \rho_r \tilde{i}_{t-1} + (1 - \rho_r)\phi_\pi \left(\tilde{p}_t + \tilde{y}_t\right) + \sigma_r \varepsilon_t^r \tag{4}$$

where $\tilde{p}_t$ denotes the aggregate price level. Note that the weighting coefficient is the same between the two rules. This assumption allows me to derive the propositions in the next section. The system of equations that I solve to analyze the NGDP targeting rule consists of equations (1) - (4), as well as the definition of inflation:

$$\tilde{\pi}_t = \tilde{p}_t - \tilde{p}_{t-1} \tag{5}$$

2.1 Results

I derive the policy functions for $\tilde{y}_t$, $\tilde{\pi}_t$, and $\tilde{i}_t$ for the three different shocks under both NGDP level targeting and inflation targeting.

Productivity Shock

Proposition 1. *Under log-utility, an NGDP targeting rule leads to a more stable initial output response under productivity shocks given*

$$\phi_\pi \geq \mathcal{F}(\theta, \beta, \rho_a, \eta)$$

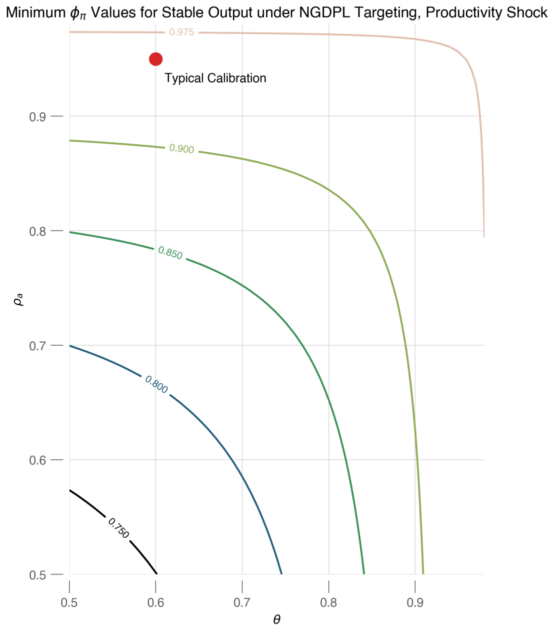
See appendix A2 for the proof and form of $\mathcal{F}$.

I create a contour plot, fixing β to 0.99 and η to 1, two common values in the literature, for the values of ϕ_π that satisfy the inequality. The results are shown in figure 1 panel (a). For all values of θ, the Calvo parameter for price stickiness, and ρ_a, the persistence of the shock, the minimum value of ϕ_π is less than one.

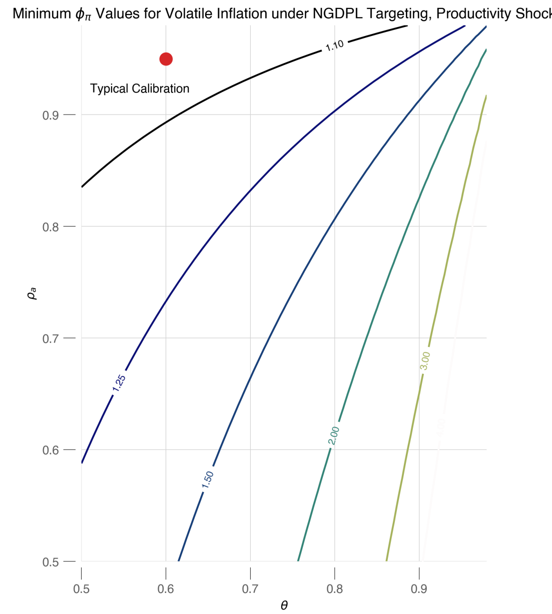
Therefore, as long as the Taylor principle holds for determinancy, NGDP level targeting would stabilize output better than inflation targeting.[2]

The intuition here is straightforward: NGDP targeting places weight on output deviations from steady-state and inflation targeting does not (NGDP targeting places more weight on output deviations than even the typical parameterization of the Taylor rule (Taylor, 1993)). Therefore, a central bank targeting NGDP will keep output closer to steady-state than a central bank targeting only inflation.

[2] Note that this is not the output gap. It is well known that inflation targeting closes the output and inflationary gaps under productivity shocks. One justification for looking at output stability is that, in reality, it is difficult for the central bank to determine which shock, or combination of shocks, is hitting the economy. A rule that stabilizes output then not only decreases risk, but could be beneficial in practical policy making.



(a) Minimum values of ϕ_π required for output to be more stable under NGDP level targeting than under inflation targeting when the economy is hit with a productivity shock. As long as the Taylor principle is satisfied, this proposition holds.



(b) Minimum values of ϕ_π required for output to be more stable under NGDP level targeting than under inflation targeting when the economy is hit with a productivity shock. As long as the Taylor principle is satisfied, this proposition holds.

Figure 1: Comparison of stability under NGDP level targeting vs. inflation targeting when hit with a productivity shock. $\beta = 0.99$, $\eta = 1$.

I next derive a similar result for inflation:

Proposition 2. *Under log-utility, an NGDP targeting rule leads to a more volatile inflation response under productivity shocks given a monetary policy authority that satisfies:*

$$\phi_\pi \geq \mathcal{G}(\kappa, \rho_a, \beta, \eta)$$

See appendix A3 for the proof and functional from of $\mathcal{G}$.

Figure 1 panel (b) displays the contour graph for proposition 2. As prices become stickier (θ approaching one) and the persistence of the productivity process decreases, the central bank must respond more strongly to deviations from steady-state for inflation to be more volatile under an NGDP rule. Importantly though, the minimum value of ϕ_π is not unreasonable. A ϕ_π value of 1.5 is standard in the literature. Estimates of θ tend to be around 0.65, with Smets & Wouters (2007) estimating a credible set range of 0.56 to 0.74. Estimates of ρ_a are tightly estimated around 0.95.

The contour plot in panel (b) shows that at the $(\theta,\ \rho_a)$ coordinate pair of $(0.65,\ 0.95)$, the minimum ϕ_π value is about 1.05. Therefore, under typical calibration of New Keynesian models, NGDP targeting leads to more volatile inflation under productivity shocks. Intuitively, proposition 2 occurs because the relative weight on inflation has decreased under NGDP targeting. The central bank is only focusing on inflation under inflation targeting.

Lastly, I look at the nominal rate:

Proposition 3. *Under log-utility, an NGDP targeting rule leads to no movement in the nominal interest rate in response to productivity shocks.*

Proof: See appendix A1.

For intuition, I look at the dynamic IS equation. Substitute the NGDP targeting rule in to derive, in general, the following relationship:

$$\mathbb{E}_t[\tilde{i}_{t+1}] = \phi_\pi \tilde{i}_t + \phi_\pi(1-\sigma)\left(\mathbb{E}_t[\tilde{y}_{t+1}] - \tilde{y}_t\right) \tag{6}$$

The expected nominal interest rate tomorrow is a function of σ, the coefficient of relative risk aversion. σ is attached to the expected change in real income. A productivity shock increases real income. When $\sigma > 1$, an expected increase in income will result in a decrease in the interest rate. The larger σ, the more the NGDP rule allows the nominal rate to move. Even with larger σ values, the movement in the nominal rate is still more muted compared to inflation targeting (see appendix A4). This suggests a more muted role for monetary policy.

Cost-Push Shock

Turning now to a supply shock, I can derive similar propositions.

Proposition 4. *Under log-utility, an NGDP targeting rule leads to a more stable initial output response under cost-push shocks given:*

$$\phi_\pi \geq \frac{\beta\psi_2 + \sqrt{\beta^2\psi_2^2 + 4(1-\beta\psi_2)}}{2(1-\beta\psi_2)}$$

where ψ_2 is the inflation policy function weight on lagged prices.

See appendix A4 for proof.

Figure 2 panel (a) demonstrates that proposition 4 holds as long as the Taylor principle is satisfied. The intuition remains from the productivity shock. Because the central bank now puts weight explicitly on output, output stabilizes. Surprisingly, though, inflation is not more volatile:

Proposition 5. *Under log-utility, an NGDP targeting rule leads to more volatile inflation given:*

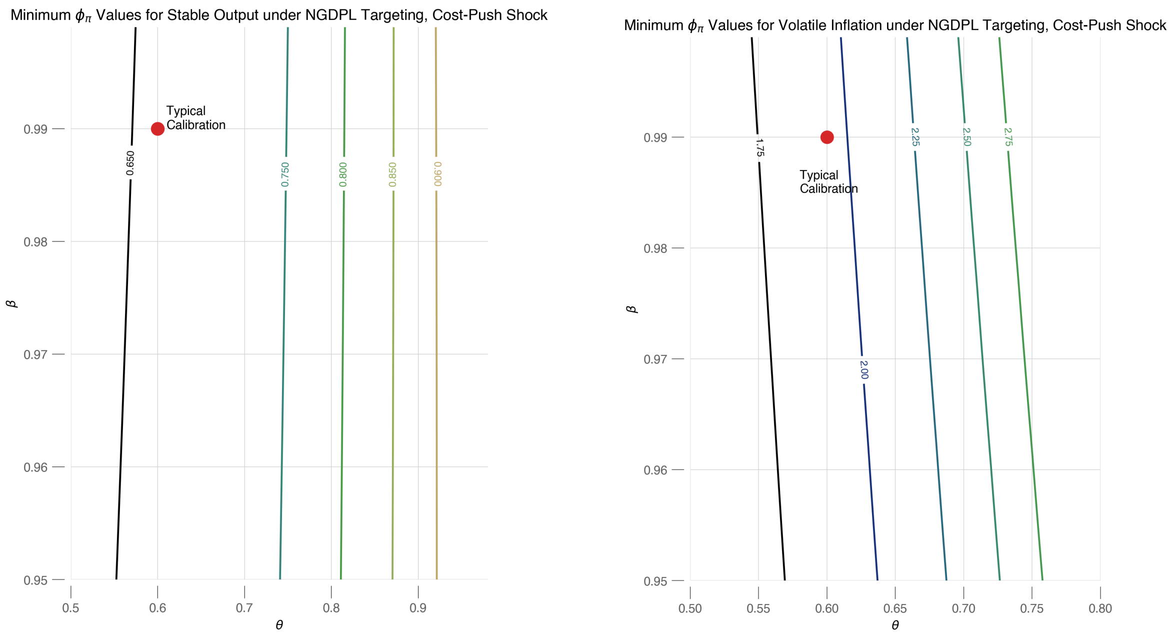
$$\phi_\pi \geq \frac{-\beta\psi_2 + \kappa}{\kappa}$$

See appendix A5 for proof.

Figure 2 panel (b) shows that ϕ_π would need to about 1.9 to stabilize inflation more under inflation targeting than under NGDP targeting. It is well known that cost-push shocks break the divine coincidence in New Keynesian models. NGDP targeting prevents output from falling too much, which in turn prevents marginal cost from increasing and inflation from rising. This occurs despite the nominal rate not moving:

Proposition 6. *Under log-utility, an NGDP targeting rule leads to no movement in the nominal interest rate in response to productivity shocks.*

Proof: See appendix A1.



(a) Minimum values of ϕ_π required for output to be more stable under NGDP level targeting than under inflation targeting. As long as the Taylor principle is satisfied, output will be more stable under NGDP targeting.

(b) Minimum values of ϕ_π required for output to be more stable under NGDP level targeting than under inflation targeting. Under the standard value of $\phi_\pi = 1.5$, inflation will be more stable under NGDP targeting.

Figure 2: Comparison of stability under NGDP level targeting vs. inflation targeting when hit with a cost-push shock. ρ_a does not enter either expression, so I let β vary and set $\eta = 1$.

Demand Shock

The last fundamental shock I analyze is a demand shock. I derive the following two propositions:

Proposition 7. *Under log-utility, an NGDP targeting rule leads to a more stable initial output response under demand shocks given:*

$$\phi_\pi \geq \frac{-\theta_2\kappa - \kappa\psi_2}{\beta\psi_2 - 1 - \beta\psi_2\kappa}$$

where θ_2 is the output policy function weight on lagged prices.

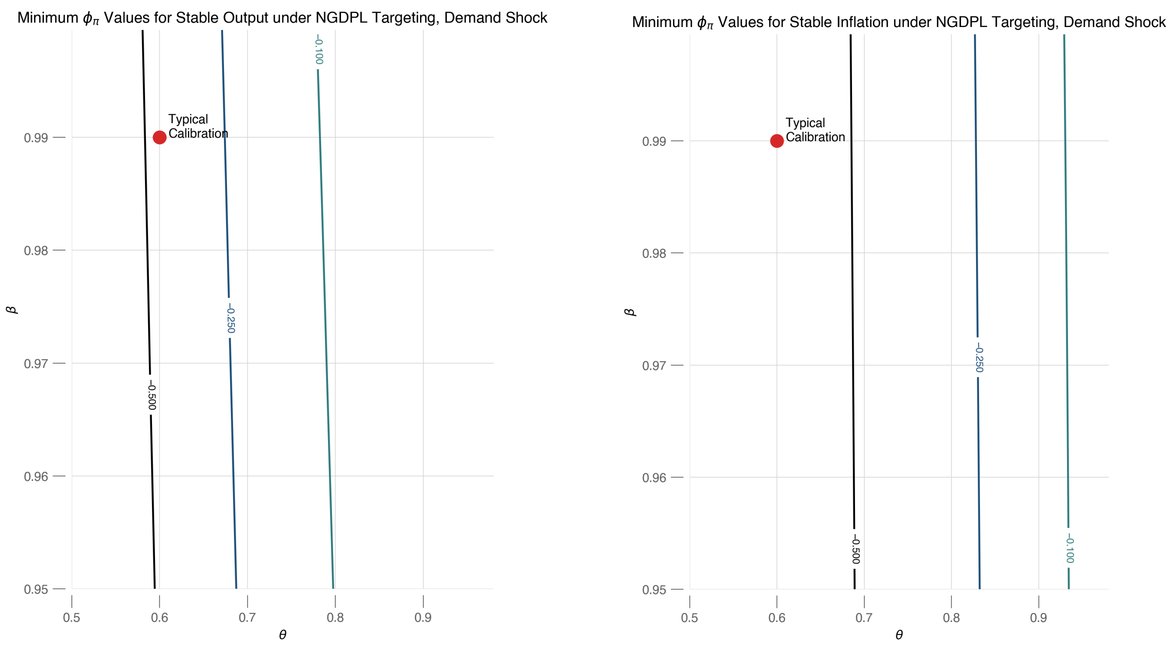
See appendix A6 for proof.

Proposition 8. *Under log-utility, an NGDP targeting rule leads to more stable inflation given:*

$$\phi_\pi \geq \frac{\beta\psi_2 + \theta_2\kappa + \kappa\psi_2}{1 - \beta\psi_2}$$

See appendix A7 for proof.

Figure 3 panels (a) and (b) demonstrate that propositions 7 and 8, respectively, hold as long as ϕ_π is positive. With a demand shock, output and inflation are positively correlated. As such, a demand shock will lead to greater movement in the nominal rate under NGDP targeting, as output enters the central bank's policy decision. A higher nominal rate will counteract the demand shock, preventing inflation and output from jumping as much as they would without any weight on output deviations.



(a) Minimum values of ϕ_π required for output to be more stable under NGDP level targeting than under inflation targeting. As long as the Taylor principle is satisfied, output will be more stable under NGDP targeting.

(b) Minimum values of ϕ_π required for output to be more stable under NGDP level targeting than under inflation targeting. Under the standard value of $\phi_\pi = 1.5$, inflation will be more stable under NGDP targeting.

Figure 3: Comparison of stability under NGDP level targeting vs. inflation targeting when hit with a cost-push shock. ρ_a does not enter either express, so I let β vary and set $\eta = 1$.

I verify this intuition by proving the nominal rate jumps more under NGDP targeting than inflation targeting:

Proposition 9. *Under log-utility, an NGDP targeting rule leads to larger movements in the nominal rate under a demand shock as long as:*

$$0 \leq 1 - \beta\psi_2 + \kappa\psi_2 - \kappa\beta\psi_2^2 + \kappa^2\psi_2$$

Proof: See appendix A8. Figure 4 demonstrates that for most combinations of θ and β, this result is positive. Only for relatively flexible wages and relatively low values of β does the result fail.

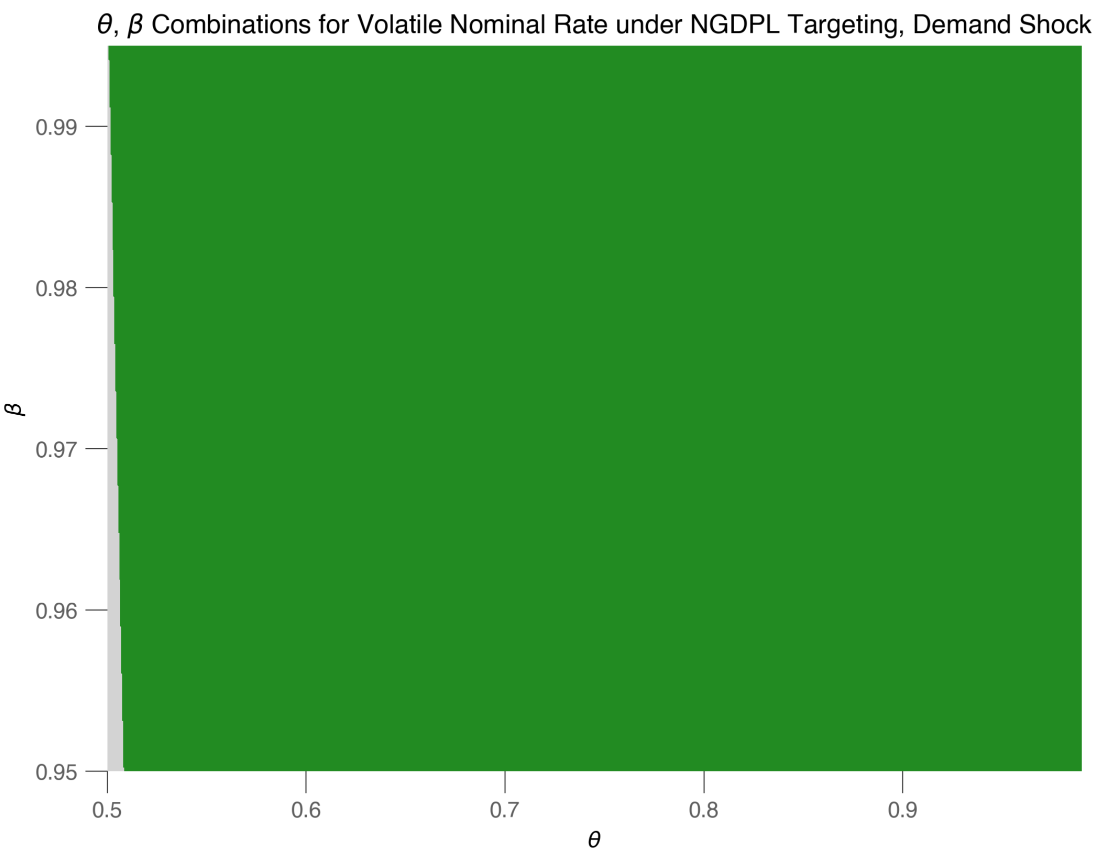


Figure 4: The green area denotes parameter combinations where the nominal rate under NGDP targeting is more volatile than under inflation targeting. The light gray area denotes areas where inflation targeting reacts more to demand shocks.

2.2 Summary

I have demonstrated analytically that NGDP level targeting leads to more stable output for typical calibrations. Inflation is more stable under demand and supply shocks, but not under productivity shocks. The nominal rate is less volatile under productivity and cost-push shocks, but must move more under demand shocks.

3 Quantitative Model

This section builds a model in the vein of Smets & Wouters (2007) and Christiano et al. (2005). Households now include external habit in their utility function. The labor market has sticky wages using standard Calvo (1983) logic and inflation indexation. The production side of the economy now has capital accumulation, capital utilization, and investment adjustment costs. Firms can also index prices to inflation. Lastly, a government now consumes a portion of output. For a list of equilibrium conditions, see appendix A9. In the main body of the paper, I only highlight the shock locations.

3.1 Households

There is an infinitely lived representative agent that maximizes lifetime utility with the following form:

$$\mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left\{ \ln \left(C_{t+j} - hC_{t+j-1} \right) - \frac{\chi L_{t+j}^{1+\eta}}{1+\eta} + \nu_t B(b_t) \right\} \right]$$

h designates the habit formation parameter. χ is a labor disutility scaling parameter, while η is the inverse Frisch elasticity. Households face the following real budget constraint:

$$C_t + b_t \leq mrs_t L_t + div_t - T_t + \Pi_t^{-1}(1 + i_{t-1})b_{t-1}$$

b_t denotes real government bonds, mrs_t denotes real wages the household receives, and div_t denotes profits rebated by firms to the household. T_t is a lump sum tax enacted by the government to finance its spending. The household maximizes with respect to C_t, L_t, and b_t. $B(\cdot)$ is a bond-in-utility function following Fisher (2015), with ν_t attached as a preference shock that corresponds to the demand shock in the three equation model.

3.2 Labor Markets

Labor markets operate in three parts. Labor unions exist on a unit measure, $h \in [0, 1]$, and purchase labor, $L_t(h)$, from households at nominal value MRS_t. Then unions package that labor, now denoted $L_{d,t}(h)$, and sell it to a representative labor packer. Lastly, labor packers combine the labor from all the different unions into final labor product $L_{d,t}$ using a standard constant elasticity of substitution technology:

$$L_{d,t} = \left[\int_0^1 L_{d,t}(h)^{\frac{\varepsilon_{w,t-1}}{\varepsilon_{w,t}}} dh \right]^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t-1}}}$$

where $\varepsilon_{w,t}$ is the elasticity of substitution of labor. See the cost-push shock below for how wage markup varies. Labor unions face sticky wages, with a probability of adjusting wages each period of ϕ_w. Unions that cannot update wages this period index wages back to last period's inflation with probability γ_w.

3.3 Production

The production side of the economy includes four types of firms. A competitive capital producer creates new physical capital each period, $\hat{I}_t$. I include an investment return shock on the price of capital. A representative wholesaler buys capital from the wholesaler and labor from the labor packer to create $Y_{m,t}$. A unit measure of retail firms, $f \in [0,1]$, repackage wholesale output using $Y_t(f) = Y_{m,t}(f)$. Lastly, a competitive final goods firm aggregates $Y_t(f)$ into Y_t using a CES aggregator:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\varepsilon_{p,t}-1}{\varepsilon_{p,t}}} df \right]^{\frac{\varepsilon_{p,t}}{\varepsilon_{p,t}-1}}$$

where $\varepsilon_{p,t}$ denotes the elasticity of substitution for retail output. Retail firms face sticky prices, with a probability of adjusting wages each period of $1 - \phi_p$. Firms that cannot update prices this period index prices back to last period's inflation with probability γ_p.

Note that the elasticity of substitution for retail output, $\varepsilon_{p,t}$, is time dependent. I do this to implement the cost-push shock found in Smets & Wouters (2007). Define the cost-push term, cp_t, as:

$$cp_t = \frac{1}{\varepsilon_{p,t} - 1}$$

A positive shock to cp_t leads to an increase in the mark-up. An increase in the mark-up leads to higher prices and lower output, matching the effect of a cost-push shock in the three-equation model.

3.4 Government

The government consumes an exogenously stochastic portion of Y_t. It is financed by lump-sum taxes on the household and by nominal bonds B_t. The central bank sets interest rates according to an inflation targeting rule first:

$$i_t = (1 - \rho_r)i_{ss} + \rho_r i_{t-1} + (1 - \rho_r)\phi_\pi \ln(\Pi_t) + \varepsilon_t^i$$

and I compare it to an NGDP level targeting rule:

$$i_t = (1 - \rho_r)i_{ss} + \rho_r i_{t-1} + (1 - \rho_r)\phi_N \left[P_t Y_t - P_{ss} Y_{ss}\right] + \varepsilon_{i,t}$$

where P_{ss} is normalized to 1 and $\phi_\pi = \phi_N$. When I analyze the model at the zero lower-bound, the nominal interest rate is set such that:

$$(1 + i_t) = max\{1, 1 + i_t^{rule}\} \tag{7}$$

where i_t is the prevailing interest rate in the economy and i_t^{rule} is the interest rate the rule, either inflation or NGDP targeting, would set if unconstrained.

3.5 Estimation

I estimate my model using a random walk Metropolis Hastings algorithm. I use nominal GDP for output, PCE non-durable goods plus PCE services for consumption, PCE durable goods plus fixed private investment for investment, and the average hourly earnings of production and non-supervisory employees for wages. I deflate each of these by the PCE deflator and detrend using log first-differences. For labor, I use the employment level, also log first-differenced. For inflation, I use the PCE price index. For the nominal rate I use the quarterly effective federal funds rate. The resulting data run from 1975Q1 to 2007Q4. The model includes seven shocks: productivity, cost-push, investment, wage mark-up, government spending, nominal rate, and preference.

Table 1

Estimated Parameter Values

		Prior			**Posterior**		
Parameter	Description	Mean	SD	Distribution	Mean	5%	95%
h	habit	0.600	0.100	Beta	0.7079	0.6763	0.7396
κ	I adj. costs	4.000	1.500	Normal	7.5340	5.8948	9.1730
η	inverse Frisch	1.000	0.100	Normal	1.0660	0.9105	1.2240
ϕ_p	Calvo prices	0.500	0.100	Beta	0.7066	0.4990	0.8495
ϕ_w	Calvo wages	0.500	0.100	Beta	0.8789	0.8440	0.9129
γ_p	price index.	0.500	0.150	Beta	0.1765	0.0529	0.2978
γ_w	wage index.	0.500	0.150	Beta	0.1404	0.0674	0.2107
ϕ_π	TR inflation	1.700	0.250	Normal	1.8694	1.3161	2.4122
ϕ_y	TR output	0.120	0.050	Normal	0.1815	0.1225	0.2408

I fix β to 0.99 to roughly match the annualized rate over the time sample. I set ε^p_{ss} to 11, and ε^w_{ss} to 11 following Sims & Wu (2021). α is set to 1/3. I use the following depreciation function:

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2$$

where δ_0 is set to 0.025, δ_2 is set to 0.01, and δ_1 is set such that steady-state utilization and is equal to 1, also following Sims & Wu (2021).[3] The adjustment cost function is

[3] These parameters are not well-identified, which is why I fix them rather than estimate them.

as follows:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

where κ is the adjustment cost multiplier. Key parameters of the estimation are displayed in table 1. The full list of estimated parameters is in appendix A10.

4 Results

In this section, I discuss the consistency of the estimated model with the three equation model, conduct long-run simulations, and evaluate NGDP targeting at the ZLB.

4.1 Consistency with the Propositions

Appendix A11 plots the impulse responses of output, inflation, and the nominal rate and examine differences from the analysis on the three equation model. The results are verified for all but one of the responses. The nominal rate is less volatile on impact under NGDP target in response to productivity and cost-push shocks, and more volatile in response to demand shocks. Inflation is more volatile in response to productivity shocks and less volatile in response to demand shocks. Inflation is now more volatile in response to cost-push shocks, but this result makes sense in light of the contour plot in figure 2. β is 0.99. ϕ_p is roughly 0.7. The contour plot predicts that for relatively more volatile inflation, ϕ_π needs to be over 2. The estimated value, however, is only 1.87.

The result that breaks down is proposition 4. Output is more volatile under NGDP targeting in response to a cost-push shock. This outcome occurs because of the inclusion of habit in the model. Habit smooths consumption across time. Because NGDP targeting necessarily implies deflation in future periods after a cost-push shock, the real rate rises in the future more under NGDP targeting than under inflation targeting. Households thus save more of their income, decreasing C_t and therefore Y_t. The second plot in appendix A11 displays impulse responses in the same model, but with the habit parameter set to zero. The result in proposition 4 then carries through to the quantitative model.

In addition to impulse response analysis, I run a long-run simulation (5000 periods) with the productivity, cost-push, and demand shocks. I then take the variance of output, inflation, and the nominal rate. The results can be found in appendix A11. Output is roughly 9% more stable under NGDP level targeting than under inflation targeting. Inflation is roughly 28% more stable. The nominal rate, though, is 13% more volatile. Demand shocks drive these relative volatilities, as the nominal rate is less volatile in response to the other two shocks. Adding the rest of the shocks in the model to the simulation does not qualitatively change the relative variances.

4.2 Hitting the ZLB

As mentioned in the introduction, the ZLB motivates much of the NGDP targeting policy push. I now conduct a simulation where the nominal rate is subject to an occasionally binding zero lower bound constraint. I generate 5000 periods of random productivity, cost-push, and demand shocks. I then calculate the proportion of time the policy rate is at the ZLB. NGDP level targeting would be "successful" if it prevented the ZLB from binding more often than inflation targeting. I conduct three different simulations: one with the supply, demand, and productivity shocks, one with all the shocks in the model, and one excluding nominal rate shocks. To ensure the simulation hits the ZLB regularly, I scale each shock such that the under inflation targeting, the ZLB binds roughly 20% of the time..[4] Table 2 reports the percentage of time spent at

[4]This corresponds with the federal funds rate has been at the ZLB since 1975.

the ZLB.

Table 2

Time at the ZLB

	Main Shocks	All Shocks	No I Shock
Inflation	20.2%	20.0%	20.1%
NGDP-L	26.9%	52.4%	21.9%

The volatility of the nominal rate under NGDP level targeting leads to the ZLB binding more often in a scenario where only supply, demand, and productivity shocks hit the economy. The problem gets worse when all shocks are considered, with the ZLB binding roughly half of the simulated time span. Much of this volatility can be attributed to investment shocks. Investment shocks move output and inflation in the same direction. As such, nominal GDP jumps. The nominal rate increase is, when compared to inflation targeting, relatively large. Investment shocks act similarly to demand shocks. When output and inflation move in the same direction, the nominal rate needs to move more. But with a binding ZLB, these large movements become problematic.

Now that the interest rate is constrained, the variance results from the unconstrained simulation do not necessarily hold. Table 3 lists the variance of output and inflation at the ZLB under each scenario. Under the main three shocks, output is now more volatile under NGDP targeting. Inflation remains less volatile, in fact becoming even less volatile relative to inflation targeting. With all the shocks, output is less volatile under NGDP targeting, but inflation becomes more volatile. Lastly, when removing the investment shock, the results from the the unconstrained simulation both hold – output and inflation are both more stable.

The main takeaway from these exercises is that in an economy where standard monetary policy applies, NGDP targeting will lead to less volatility in output and inflation. But in an economy where the ZLB matters, NGDP targeting does not necessarily improve economic stability.

Table 3

Variance at the ZLB

	Main Shocks		**All Shocks**		**No I Shock**	
	Output	Inflation	Output	Inflation	Output	Inflation
Inflation	0.0063	5.98×10^{-5}	0.024	3.21×10^{-5}	0.0107	3.22×10^{-5}
NGDP-L	0.0065	3.56×10^{-5}	0.013	3.46×10^{-5}	0.0096	2.36×10^{-5}

4.3 Switching to NGDP Targeting

The New York Times op-ed by Christina Romer advocated switching to an NGDP level target when the nominal rate was bound by the ZLB. Beckworth (2019) argues that NGDP level targeting would help the economy escape the ZLB. In this section, I first drive the economy into the ZLB with two large demand shocks while the central bank pursues inflation targeting.[5] I then switch to NGDP targeting in the middle of the ZLB period and compare the speed of recovery with an economy that remained with inflation targeting.[6] Figure 5 displays the resulting simulation.

Switching to NGDP targeting in the middle of the ZLB period causes a jump in inflation immediately, supporting the findings of Beckworth (2019). This increase in inflation is accompanied by a steeper climb in output. A switch to NGDP targeting

[5]There is much debate as to which shocks primarily drive business cycles. I adopt a traditional view here and assume that demand drives short-run business cycles.

[6]I do this by first simulating an economy with the OCCBIN toolbox in Dynare. I then export simulated values in the middle of the ZLB period. I use those exported values as starting values in two perfect foresight simulations: one continuing the inflation targeting rule and the other switching to an NGDP targeting rule.

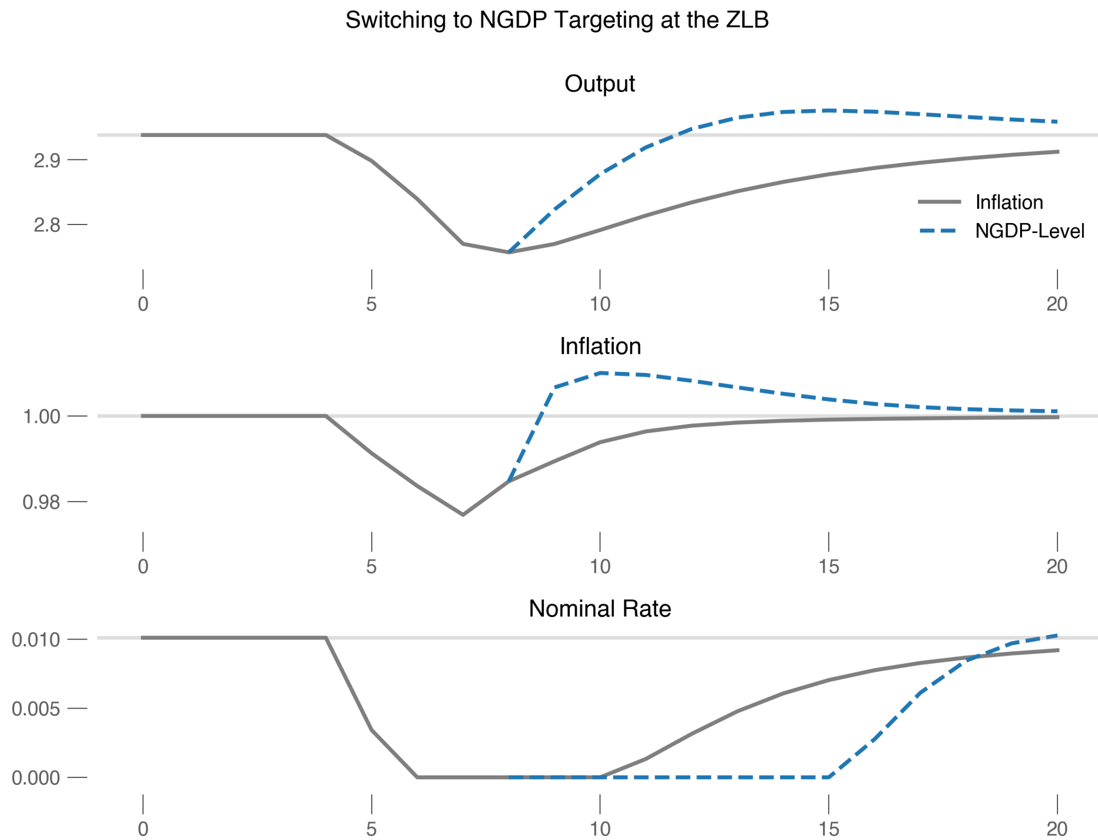


Figure 5: Simulation at the ZLB. Grey solid – Inflation targeting. Blue dashed – NGDP level targeting.

would thus spur a sharp rise in NGDP. For policy makers looking to jumpstart an economy out of a classic recession, a switch from inflation to NGDP targeting would make sense.

The nominal rate, though, remains constrained at the ZLB for five quarters longer. Switching to NGDP targeting does not lead to a relief from the ZLB, but instead exacerbates it. Standard monetary policy remains constrained and unable to respond to further negative shocks.

5 Conclusion

This paper examines NGDP level targeting compared with the monetary authority's current inflation target. I show analytically that relative volatilities are shock-dependent and confirm these results quantitatively. An analysis at the ZLB demonstrates that NGDP targeting would not alleviate ZLB problems, instead exacerbating them. Switching to an NGDP target while at the ZLB, though, would boost real output and inflation, pushing an economy out of recession quicker than inflation targeting. Areas of further research should involve investigating central bank credibility on switching policy targets and reversing policy switches during booms.

References

Beckworth, D. (2019). *Facts, Fears, and Functionality of NGDP Level Targeting: A Guide to a Popular Framework for Monetary Policy.* Mercatus Center.

Beckworth, D., & Hendrickson, J. R. (2019). Nominal GDP targeting and the Taylor rule on an even playing field. *Journal of Money, Credit and Banking*, *52*(1), 269-286. doi: 10.1111/jmcb.12602

Board of Governors. (2021, Jul). *Monetary Policy: What Are Its Goals? How Does It Work?* Retrieved from https://www.federalreserve.gov/monetarypolicy/monetary-policy-what-are-its-goals-how-does-it-work.htm

Calvo, G. (1983, September). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, *12*(3), 383-398.

Christiano, L., Eichenbaum, M., & Evans, C. (2005, February). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, *113*(1), 1-45.

Crook, C. (2022, April). *The Simplest Way to Improve Monetary Policy.* Bloomberg. Retrieved from https://www.bloomberg.com/opinion/articles/2022-04-08/inflation-what-should-the-federal-reserve-do-start-using-ngdp

Fisher, J. (2015, March-April). On the Structural Interpretation of the Smets-Wouters ”Risk Premium”' Shock. *Journal of Money, Credit, and Banking*, *47*(2-3), 511-516.

Gali, J. (2015). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

Garin, J., Lester, R., & Sims, E. (2016, August). On the desirability of nominal GDP targeting. *Journal of Economic Dynamics and Control*, *69*, 21-44.

Hall, R. E., & Mankiw, N. G. (1994). Nominal income targeting. In *Monetary policy* (p. 71–94). National Bureau of Economic Research.

Khan, A., King, R. G., & Wolman, A. L. (2003). Optimal monetary policy. *Review of Economic Studies*, *70*(4), 825-860. doi: 10.1111/1467-937x.00269

Mitra, K. (2003, April). Desirability of Nominal GDP Targeting under Adaptive Learning. *Journal of Money, Credit, and Banking*, *35*(2), 197-220.

Powell, J. (2022, Sep). Cato Institute's 40th annual Monetary Conference. In *C-span.* Retrieved from https://www.c-span.org/video/?522707-1/fed-chair-jerome-powell-discusses-monetary-policy

Romer, C. (2011, Oct). *Dear Ben: It's Time for Your Volcker Moment.* The New York Times. Retrieved from https://www.nytimes.com/2011/10/30/business/economy/ben-bernanke-needs-a-volcker-moment.html?_r=1

Sheedy, K. (2014, Spring). Debt and Incomplete Financial Markets: A Case for Nominal GDP Targeting. *Brookings Papers on Economic Activity*, 301-361.

Sims, E., & Wu, J. C. (2021, March). Evaluating Central Banks' tool kit: Past, present, and future. *Journal of Monetary Economics*, *118*, 135-160.

Smets, F., & Wouters, R. (2007, June). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, *97*(3), 586-606.

Sumner, S. (2012, Oct). The case for nominal GDP targeting. *Mercatus Center Research Papers*. doi: 10.2139/ssrn.3255027

Taylor, J. (1993, December). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, *39*, 195-214.

Woodford, M. (2001). The Taylor Rule and Optimal Monetary Policy. *American Economic Review*, *91*(2), 232-237. doi: 10.1257/aer.91.2.232

Wu, J. C., & Xia, F. D. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking*, *48*(2-3), 253-291. doi: 10.1111/jmcb.12300

Yglesias, M. (2015, Oct). *The most important paragraph in Ben Bernanke's new book.* Vox. Retrieved from https://www.vox.com/2015/10/8/9472807/ben-bernanke-ngdp-targeting

A Appendix

A1 Proof of Proposition 3 and 6

I first restate the system of four equations that I am solving, setting $\rho_r = 0$ for clarity:

$$\tilde{y}_t = \mathbb{E}[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) + \varepsilon_t^d \tag{A.1}$$

$$\tilde{\pi}_t = \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] + \varepsilon_t^s \tag{A.2}$$

$$\tilde{y}_t^f = \rho_a \tilde{y}_{t-1}^f + \frac{1+\eta}{\sigma+\eta}\varepsilon_t^a \tag{A.3}$$

$$\tilde{i}_t = \phi_N\left(\tilde{p}_t + \tilde{y}_t\right) + \sigma_r \varepsilon_t^r \tag{A.4}$$

$$\tilde{\pi}_t = \tilde{p}_t - \tilde{p}_{t-1} \tag{A.5}$$

In this model, there are three forward-looking jump variables: $\tilde{y}_t$, $\tilde{\pi}_t$, and $\tilde{p}_t$. There are two state variables: $\tilde{p}_{t-1}$ and $\tilde{y}_t^f$.

Finding the Policy Functions

Focusing only on productivity shocks for now, conjecture that the jump variables are linear in the state variables:

$$\tilde{y}_t = \theta_1\tilde{y}_t^f + \theta_2\tilde{p}_{t-1} \qquad \tilde{\pi}_t = \psi_1\tilde{y}_t^f + \psi_2\tilde{p}_{t-1} \qquad \tilde{p}_t = \tau_1\tilde{y}_t^f + \tau_2\tilde{p}_{t-1}$$

Starting with equation A.2, I plug in the conjectures:

$$\tilde{\pi}_t = \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta E_t[\tilde{\pi}_{t+1}]$$

$$\psi_1\tilde{y}_t^f + \psi_2\tilde{p}_{t-1} = \kappa(\theta_1\tilde{y}_t^f + \theta_2\tilde{p}_{t-1} - \tilde{y}_t^f) + \beta\mathbb{E}_t[\psi_1\tilde{y}_{t+1}^f + \psi_2\tilde{p}_t]$$

$$\tilde{y}_t^f(\psi_1 - \kappa\theta_1 + \kappa - \beta\psi_1\rho_a - \beta\psi_2\tau_1) = \tilde{p}_{t-1}(\kappa\theta_2 - \psi_2 + \beta\psi_2\tau_2)$$

I now have two equations:

$$\psi_1 - \kappa\theta_1 + \kappa - \beta\psi_1\rho_a - \beta\psi_2\tau_1 = 0 \tag{A.6}$$

$$\kappa\theta_2 - \psi_2 + \beta\psi_2\tau_2 = 0 \tag{A.7}$$

Next, I substitute the NGDP Rule and the policy guesses into A.1:

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - \frac{1}{\sigma}(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}])$$

$$\theta_1\tilde{y}_t^f + \theta_2\tilde{p}_{t-1} = \mathbb{E}_t[\theta_1\tilde{y}_{t+1}^f + \theta_2\tilde{p}_t] - \frac{1}{\sigma}(\phi_N\tilde{p}_t + \phi_N\tilde{y}_t - \psi_1\tilde{y}_{t+1}^f - \psi_2\tilde{p}_t)$$

Plugging in the policy guesses again and sorting by term gives:

$$\tilde{y}_t^f\left(-\sigma\theta_1 + \sigma\theta_1\rho_a + \sigma\theta_2\tau_1 - \phi_N\tau_1 - \phi_N\theta_1 + \psi_1\rho_a + \psi_2\tau_1\right) + \tilde{p}_{t-1}\left(-\sigma\theta_2 + \sigma\theta_2\tau_2 - \phi_N\tau_2 - \phi_N\theta_2 + \psi_2\tau_2\right) = 0$$

which gives me another two equations:

$$-\sigma\theta_1 + \sigma\theta_1\rho_a + \sigma\theta_2\tau_1 - \phi_N\tau_1 - \phi_N\theta_1 + \psi_1\rho_a + \psi_2\tau_1 = 0 \tag{A.8}$$

$$-\sigma\theta_2 + \sigma\theta_2\tau_2 - \phi_N\tau_2 - \phi_N\theta_2 + \psi_2\tau_2 = 0 \tag{A.9}$$

Lastly, use equation A.5:

$$\tilde{pi}_t = \tilde{p}_t - \tilde{p}_{t-1}$$

$$0 = \tilde{y}_t^f(\psi_1 - \tau_1) + \tilde{p}_{t-1}(\psi_2 - \tau_2 + 1)$$

which gives the last two equations:

$$\tau_1 = \psi_1 \tag{A.10}$$

$$\tau_2 = \psi_2 + 1 \tag{A.11}$$

The equations A.6 through A.11 give me six equations with six unknowns. I can now solve the system. Starting with equation A.7:

$$\kappa\theta_2 - \psi_2 + \beta\psi_2\tau_2 = 0$$

$$\theta_2 = \frac{\psi_2(1 - \beta(\psi_2 + 1))}{\kappa} \tag{A.12}$$

Moving to equation A.9:

$$-\sigma\theta_2 + \sigma\theta_2\tau_2 - \phi_N\tau_2 - \phi_N\theta_2 + \psi_2\tau_2 = 0$$

$$\frac{-\sigma\psi_2(1 - \beta(\psi_2 + 1))}{\kappa(\psi_2 + 1)} + \frac{\sigma\psi_2(1 - \beta(\psi_2 + 1))}{\kappa} - \phi_N - \phi_N\frac{\psi_2(1 - \beta(\psi_2 + 1))}{\kappa(\psi_2 + 1)} + \psi_2 = 0$$

This is a cubic equation. I now impose log-utility, setting $\sigma = 1$. Using a cubic-root solver, I find that the only solution for ψ_2 that satisfies equations A.6-A.11 is:

$$\psi_2^* = \frac{-\sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} - \beta + \kappa + 1}{2\beta} \tag{A.13}$$

Plugging A.13 into A.12 gives me a closed form expression for θ_2:

$$\theta_2^* = \frac{\psi_2^*(1 - \beta(\psi_2^* + 1))}{\kappa} \tag{A.14}$$

I also now have $\tau_2^* = \psi_2^* + 1$. Move to equation A.6:

$$\psi_1 - \kappa\theta_1 + \kappa - \beta\psi_1\rho_a - \beta\psi_2\tau_1 = 0$$
$$\theta_1 = \frac{\psi_1 + \kappa - \beta\psi_1\rho_a - \beta\psi_2^*\psi_1}{\kappa}$$

Plugging this into equation A.8 and solving for ψ_1:

$$0 = -\sigma\theta_1 + \sigma\theta_1\rho_a + \sigma\theta_2\tau_1 - \phi_N\tau_1 - \phi_N\theta_1 + \psi_1\rho_a + \psi_2\tau_1$$
$$\psi_1^* = \frac{\kappa(1-\rho_a+\phi_N)}{(\rho_a - \phi_N + 1)(1 - \beta\rho_a - \beta\psi_2^*) + \psi_2^*\left(1 - \beta(\psi_2^*+1) + \kappa\right) - \kappa(\phi_N - \rho_a)} \tag{A.15}$$

A.6 is now solved:

$$\theta_1^* = \frac{\psi_1^* + \kappa - \beta\psi_1^*\rho_a - \beta\psi_2^*\psi_1^*}{\kappa}$$

I also have $\tau_1^* = \psi_1^*$. All coefficients are composed only of parameters, so the guess is verified.

Solving for the Nominal Rate

I substitute the policy functions into the NGDP targeting rule:

$$\tilde{i}_t = \phi_N\left(\tilde{p}_t + \tilde{y}_t\right)$$
$$= \phi_N\left[\tilde{y}_t^f \underbrace{\left(1 + \psi_1 + \frac{\psi_1(1-\beta\rho_a-\beta\psi_2)}{\kappa}\right)}_{\equiv A} + \tilde{p}_{t-1}\underbrace{\left(\psi_2 + 1 + \frac{\psi_2}{\kappa}(1-\beta(\psi_2+1))\right)}_{\equiv B}\right]$$

Looking just at term A, I plug-in for the coefficients and simplify to obtain:

$$\frac{\kappa(1+\psi_1)+\psi_1-\psi_1\beta\rho_a-\beta\psi_2\psi_1}{\kappa} = \frac{\psi_2-\beta\psi_2(\psi_2+1)+\psi_2\kappa+\kappa}{(\rho_a-\phi_N-1)(1-\beta\rho_a-\beta\psi_2)+\psi_2(1-\beta(\psi_2+1)+\kappa)-\kappa(\phi_N-\rho_a)}$$

which, after plugging-in for ψ_2, simplifies to 0. Looking at term B:

$$\psi_2+1+\frac{\psi_2}{\kappa}(1-\beta(\psi_2+1)) = \psi_2+1+\frac{\psi_2}{\kappa}-\frac{\beta}{\kappa}\psi_2^2-\frac{\beta\psi_2}{\kappa}$$

which, after plugging-in for ψ_2, also simplifies to 0. Therefore:

$$\tilde{i}_t = \sigma_r\varepsilon_t^r$$

With Cost-Push Shocks

As before, conjecture linear policy functions for the jump variables. This time, ε_t^s is included instead of y_t^f:

$$\tilde{y}_t = \theta_2\tilde{p}_{t-1}+\theta_3\varepsilon_t^s \qquad \tilde{\pi}_t = \psi_2\tilde{p}_{t-1}+\psi_3\varepsilon_t^s \qquad \tilde{p}_t = \tau_2\tilde{p}_{t-1}+\tau_3\varepsilon_t^s$$

Plugging the policy function guesses into the DIS and NKPC curves gives the following new equations:

$$\kappa\theta_3-\psi_3+\beta\psi_2\tau_3+1=0 \tag{A.16}$$

$$-\sigma\theta_3+\sigma\theta_2\tau_3-\phi_N\tau_3-\phi_N\theta_3+\psi_2\tau_3=0 \tag{A.17}$$

$$\tau_3=\psi_3 \tag{A.18}$$

Solving A.16 gives an equation for θ_3:

$$\theta_3 = \frac{\psi_3 - \beta\psi_2\psi_3 - 1}{\kappa}$$

which can be used to solve A.17 for ψ_3 and verify the guess:

$$\psi_3 = \frac{-\sigma - \phi_N}{-\sigma + \sigma\beta\psi_2 - \phi_N + \beta\phi_N\psi_2 - \phi_N\kappa + \psi_2\kappa + \sigma\theta_2\kappa}$$

Solving the for the nominal rate gives:

$$\tilde{i}_t = \phi_N \left(A\tilde{y}_t^f + B\tilde{p}_{t-1} + \varepsilon_t^s \underbrace{\left(\psi_3 + \frac{\psi_3 - \beta\psi_2\psi_3 - 1}{\kappa} \right)}_{\equiv C} \right) + \sigma_r \varepsilon_t^r$$

After tedious algebra, C simplifies to 0 as well.

A2 Proof of Proposition 1

I first solve for the policy functions under inflation targeting. Assuming that $\rho_r = 0$ for simplicity, the four equations I need to solve are:

$$\tilde{y}_t = \mathbb{E}[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) + \varepsilon_t^d \tag{A.19}$$

$$\tilde{\pi}_t = \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] + \varepsilon_t^s \tag{A.20}$$

$$\tilde{y}_t^f = \rho_a\tilde{y}_{t-1}^f + \frac{1+\eta}{\sigma+\eta}\varepsilon_t^a \tag{A.21}$$

$$\tilde{i}_t = \phi_\pi\tilde{\pi}_t + \sigma_r\varepsilon_t^r \tag{A.22}$$

Conjecture that the jump variables, $\tilde{y}_t$ and $\tilde{\pi}_t$, are linear in the state variable $\tilde{y}_t^f$:

$$\tilde{y}_t = \lambda_1\tilde{y}_t^f \qquad\qquad \tilde{\pi}_t = \lambda_2\tilde{y}_t^f$$

Starting with A.20, substitute in the policy function guesses:

$$\begin{aligned}
\tilde{\pi}_t &= \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] \\
\lambda_1\tilde{y}_t^f &= \kappa(\lambda_1\tilde{y}_t^f - \tilde{y}_t^f) + \beta\mathbb{E}_t[\lambda_2\tilde{y}_t^f] \\
0 &= \lambda_2\tilde{y}_t^f - \kappa(\lambda_1 - 1)\tilde{y}_t^f - \beta\rho_a\lambda_2\tilde{y}_t^f \\
0 &= [\lambda_2 - \kappa(\lambda_1 - 1) - \beta\rho_a\lambda_2]\,\tilde{y}_t^f \qquad (A.23)
\end{aligned}$$

Now solve A.19:

$$\begin{aligned}
\tilde{y}_t &= \mathbb{E}[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) \\
\lambda_1\tilde{y}_t^f &= \lambda_1\rho_a\tilde{y}_t^f - \frac{1}{\sigma}(\phi_\pi\tilde{\pi}_t - \lambda_2\rho_a\tilde{y}_t^f) \\
0 &= \lambda_1\tilde{y}_t^f - \lambda_1\rho_a\tilde{y}_t^f + \frac{\phi_\pi}{\sigma}\lambda_2\tilde{y}_t^f - \frac{\lambda_2\rho_a}{\sigma}\tilde{y}_t^f
\end{aligned}$$

$$0 = \left[\lambda_1 - \lambda_1\rho_a + \frac{\phi_\pi}{\sigma}\lambda_2 - \frac{\lambda_2\rho_a}{\sigma}\right]\tilde{y}_t^f \tag{A.24}$$

Solve the coefficient in A.24 for λ_1:

$$\lambda_1 - \lambda_1\rho_a + \frac{\phi_\pi}{\sigma}\lambda_2 - \frac{\lambda_2\rho_a}{\sigma} = 0$$
$$\lambda_1(1-\rho_a) = \lambda_2\left(\frac{\rho_a}{\sigma} - \frac{\phi_\pi}{\sigma}\right)$$
$$\lambda_1 = \frac{\lambda_2}{1-\rho_a}\left(\frac{\rho_a}{\sigma} - \frac{\phi_\pi}{\sigma}\right) \tag{A.25}$$

Plug A.25 into the coefficient of A.23:

$$0 = \lambda_2 - \kappa(\lambda_1 - 1) - \beta\rho_a\lambda_2$$
$$0 = \lambda_2 - \kappa\left(\frac{\lambda_2}{1-\rho_a}\left(\frac{\rho_a}{\sigma} - \frac{\phi_\pi}{\sigma}\right) - 1\right) - \beta\rho_a\lambda_2$$
$$0 = \lambda_2(1-\beta\rho_a) - \lambda_2\frac{\kappa(\rho_a - \phi_\pi)}{\sigma(1-\rho_a)} + \kappa$$
$$\lambda_2^* = \frac{\sigma\kappa(1-\rho_a)}{\sigma(1-\beta\rho_a)(\rho_a - 1) - \kappa(\phi_\pi - \rho_a)} \tag{A.26}$$

Plug A.26 into A.25:

$$\lambda_1 = \lambda_2^*\frac{\rho_a - \phi_\pi}{\sigma(1-\rho_a)} \tag{A.27}$$
$$\lambda_1^* = \frac{\kappa(\phi_\pi - \rho_a)}{(1-\beta\rho_a)(1-\rho_a) - \kappa(\rho_a - \phi_\pi)} \tag{A.28}$$

Now compare the output policy function weights on potential output for under NGDP targeting and under inflation targeting, setting $\phi_N = \phi_\pi$ and $\sigma = 1$, conjecturing that output is less volatile under NGDP level targeting:

$$\frac{\kappa(\phi_\pi - \rho_a)}{(1-\beta\rho_a)(1-\rho_a) - \kappa(\rho_a - \phi_\pi)} \geq \frac{\kappa(\phi_\pi - \rho_a) - \psi_2(1+\kappa-\beta(\psi_2+1))}{(1+\phi_\pi-\rho_a)(1-\beta\rho_a-\beta\psi_2) - \kappa(\rho_a-\phi_\pi) - \psi_2(1+\kappa-\beta(\psi_2+1))}$$

Clearing fractions (both denominators are positive assuming $\phi_\pi > 1$) and grouping by ϕ_π terms gives:

$$\phi_\pi^2 \underbrace{(\kappa - \kappa\beta\rho_a - \kappa\beta\psi_2)}_{\equiv a} + \phi_\pi \underbrace{\left(\kappa\beta\rho_a\psi_2 + \beta\psi_2^2 - \kappa\rho_a + \kappa\beta\rho_a^2 + \kappa\rho_a\beta\psi_2 - \kappa\psi_2\beta(\psi_2+1)\right)}_{\equiv b}$$
$$+ \underbrace{\psi_2(1+\kappa-\beta(\psi_2+1))(1-\rho_a-\beta\rho_a+\beta\rho_a^2-\kappa\rho_a) - \beta\kappa\rho_a^2\psi_2 + \kappa\rho_a\psi_2 + \kappa^2\psi_2\rho_a - \beta\kappa\rho_a\psi_2^2}_{\equiv c}$$

This is of a quadratic form. The quadratic formula gives:

$$\phi_\pi \geq \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

where I omit the smaller of the two ϕ_π values.

This concludes the proof of proposition 1.

A3 Proof of Proposition 2

Compare the policy function for inflation under NGDP targeting and under inflation targeting, conjecturing that the NGDP response is more negative:

$$\frac{-\kappa(1-\rho_a)}{(1-\beta\rho_a)(1-\rho_a)-\kappa(\rho_a-\phi_\pi)} \geq \frac{-\kappa(1-\rho_a-\phi_\pi)}{(1+\phi_\pi-\rho_a)(1-\beta\rho_a-\beta\psi_2)-\kappa(\rho_a-\phi_\pi)-\psi_2(1+\kappa-\beta(\psi_2+1))}$$

Clear fractions and divide both sides by $-\kappa$. Then simplify and group by ϕ_π terms:

$$\phi_\pi^2\kappa+\phi_\pi(-\kappa\rho_a+(1-\rho_a)\beta\psi_2)+\psi_2(1-\rho_a)\left[\beta(1-\rho_a)+1+\kappa-\beta(\psi_2+1)\right] \geq 0$$

Applying the quadratic formula and taking the larger value verifies the proposition.

A4 Proof of Proposition 4

Solving the policy functions with cost-push shocks under inflation targeting yields coefficients:

$$\lambda_1^s = \frac{-\phi_\pi/\sigma}{1+\kappa\phi_\pi/\sigma}$$

$$\lambda_2^s = 1+\kappa\theta_1$$

Compare λ_1^s to θ_3, conjecturing that $\theta_3 \geq \lambda_1^s$:

$$\frac{\psi_3 - \beta\psi_2\psi_3 - 1}{\kappa} \geq \frac{-\phi_\pi}{1+\kappa\phi_\pi}$$

Substitute in for ψ_3 to get:

$$\phi_\pi \leq \frac{1+\phi_\pi}{1+\phi_\pi - \beta\psi_2(1+\phi_\pi)}$$

Multiply the denominator across and group by ϕ_π terms:

$$\phi_\pi^2(1-\beta\psi_2) - \phi_\pi\beta\psi_2 - 1 \geq 0$$

Applying the quadratic formula yields:

$$\phi_\pi \geq \frac{\beta\psi_2 + \sqrt{\beta^2\psi_2^2 + 4(1-\beta\psi_2)}}{2-2\beta\psi_2}$$

A5 Proof of Proposition 5

Conjecture that $\psi_3 \geq \lambda_2^s$:

$$\frac{-1-\phi_\pi}{-1+\beta\psi_2-\phi_\pi+\beta\phi_\pi\psi_2-\phi_\pi\kappa-\kappa} \geq 1+\frac{-\phi_\pi\kappa}{1+\kappa\phi_\pi}$$

Clearing fractions and grouping by ϕ_π yields:

$$0 \leq \kappa\phi_\pi^2+\beta\phi_\pi\psi_2+\beta\psi_2-\kappa$$

The quadratic formula yields:

$$\phi_\pi \geq \frac{-\beta\psi_2+\kappa}{\kappa}$$

A6 Proof of Proposition 6

Following the above steps for solving the policy functions yields the NGDP targeting coefficients for $\tilde{y}_t$, $\tilde{\pi}_t$, and $\tilde{p}_t$ in response to a demand shock:

$$\theta_4 = \frac{\psi_4(1-\beta\psi_2)}{\kappa}$$
$$\psi_4 = \frac{-\sigma\kappa}{-\sigma + \sigma\beta\psi_2 - \phi_N + \phi_N\beta\psi_2 + \sigma\theta_2\kappa - \phi_N\kappa + \kappa\psi_2}$$
$$\tau_4 = \psi_4$$

and the inflation targeting coefficients for $\tilde{y}_t$ and $\tilde{\pi}_t$:

$$\lambda_1^d = \frac{1}{1+\frac{\phi_\pi\kappa}{\sigma}}$$
$$\lambda_2^d = \frac{1}{1+\phi_\pi\kappa}$$

Conjecture that output is more volatile under inflation targeting and solve for ϕ_π:

$$\frac{1}{1+\frac{\phi_\pi\kappa}{\sigma}} \geq \frac{\psi_4(1-\beta\psi_2)}{\kappa}$$
$$\frac{1}{1+\phi_\pi\kappa} \geq \frac{-(1-\beta\psi_2)}{-1+\beta\psi_2 - \phi_\pi + \phi_N\beta\psi_2 + \theta_2\kappa - \phi_\pi\kappa + \kappa\psi_2}$$
$$\phi_\pi(\beta\psi_2 - 1 - \beta\psi_2\kappa) \leq -\theta_2\kappa - \kappa\psi_2$$
$$\phi_\pi \geq \frac{-\theta_2\kappa - \kappa\psi_2}{\beta\psi_2 - 1 - \beta\psi_2\kappa}$$

A7 Proof of Proposition 7

Conjecture that inflation is more volatile under inflation targeting:

$$\frac{\kappa}{1+\phi_\pi\kappa} \geq \frac{-1}{-1+\beta\psi_2-\phi_\pi+\phi_\pi\beta\psi_2+\theta_2\kappa-\phi_\pi\kappa+\kappa\psi_2}$$

Clear fractions and solve for ϕ_π:

$$\frac{\beta\psi_2+\theta_2\kappa+\kappa\psi_2}{1-\beta\psi_2} \leq \phi_\pi$$

A8 Proof of Proposition 8

Plugging the policy functions into the NGDP targeting rule gives:

$$\tilde{i}_t = \phi_N \left(\varepsilon_t^d \left(\psi_4 + \frac{\psi_4 - \beta\psi_4\psi_2}{\kappa} \right) \right)$$

Simplifying the term inside the parentheses yields:

$$\frac{-\sigma\kappa^2 - \sigma\kappa + \sigma\kappa\beta\psi_2}{-\sigma\kappa + \sigma\kappa\beta\psi_2 - \kappa\phi_N + \kappa\phi_N\beta\psi_2 + \sigma\kappa^2\theta_2 - \phi_N\kappa^2 + \kappa^2\psi_2}$$

As such, the nominal rate evolves according to:

$$\tilde{i}_t = \phi_N \left(\varepsilon_t^d \frac{-\sigma\kappa^2 - \sigma\kappa + \sigma\kappa\beta\psi_2}{-\sigma\kappa + \sigma\kappa\beta\psi_2 - \kappa\phi_N + \kappa\phi_N\beta\psi_2 + \sigma\kappa^2\theta_2 - \phi_N\kappa^2 + \kappa^2\psi_2} \right)$$

In comparison, inflation targeting yields:

$$\tilde{i}_t = \phi_\pi \frac{\kappa}{1 + \frac{\phi_\pi\kappa}{\sigma}} \varepsilon_t^d$$

Assume the NGDP target leads to a more volatile nominal rate response. Then:

$$\frac{-\sigma\kappa^2 - \sigma\kappa + \sigma\kappa\beta\psi_2}{-\sigma\kappa + \sigma\kappa\beta\psi_2 - \kappa\phi_N + \kappa\phi_N\beta\psi_2 + \sigma\kappa^2\theta_2 - \phi_N\kappa^2 + \kappa^2\psi_2} \geq \frac{\kappa}{1 + \frac{\phi_\pi\kappa}{\sigma}}$$

Simplifying, and imposing $\sigma = 1$, leads to:

$$0 \leq 1 - \beta\psi_2 + \kappa\psi_2 - \kappa\beta\psi_2^2 + \kappa^2\psi_2$$

A9 Equilibrium Equations for the Quantitative Model

This model is similar to the Smets & Wouters (2007) model. It contains the typical medium scale frictions. The model is solved about a zero-inflation and labor of unity steady state. Below I list the equilibrium equations:

$$\mu_t = \frac{1}{C_t - \mathcal{H}C_{t-1}} - \beta\mathbb{E}_t\left[\frac{\mathcal{H}}{C_{t+1} - \mathcal{H}C_t}\right] \tag{A.29}$$

$$\chi L_t^{\eta} = mrs_t\mu_t \tag{A.30}$$

$$\mu_t = \mathbb{E}_t\left[\beta\mu_{t+1}\Pi_{t+1}^{-1}(1+i_t)\right] + \nu_t \tag{A.31}$$

$$\Lambda_{t,t+1} = \beta\frac{\mathbb{E}_t[\mu_{t+1}]}{\mu_t} \tag{A.32}$$

$$w_t^* = \frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}\frac{f_{1,t}}{f_{2,t}} \tag{A.33}$$

$$f_{1,t} = mrs_t w_t^{\varepsilon_{w,t}}L_{d,t} + \phi_w\mathbb{E}_t\left[\Lambda_{t,t+1}\Pi_{t+1}^{\varepsilon_{w,t}}\Pi_t^{-\varepsilon_{w,t}\gamma_w}f_{1,t+1}\right] \tag{A.34}$$

$$f_{2,t} = w_t^{\varepsilon_{w,t}}L_{d,t} + \phi_w\mathbb{E}_t\left[\Lambda_{t,t+1}\Pi_{t+1}^{\varepsilon_{w,t}-1}\Pi_t^{(1-\varepsilon_{w,t})\gamma_w}f_{2,t+1}\right] \tag{A.35}$$

$$L_t = L_{d,t}v_t^w \tag{A.36}$$

$$v_t^w = (1-\phi_w)\left(\frac{w_t^*}{w_t}\right)^{-\varepsilon_{w,t}} + \phi_w\left(\frac{\Pi_t}{\Pi_{t-1}^{\gamma_w}}\right)^{\varepsilon_{w,t}}\left(\frac{w_t}{w_{t-1}}\right)^{\varepsilon_{w,t}}v_{t-1}^w \tag{A.37}$$

$$w_t^{1-\varepsilon_{w,t}} = (1-\phi_w)(w_t^*)^{1-\varepsilon_{w,t}} + \phi_w\left(\frac{\Pi_{t-1}^{\gamma_w}}{\Pi_t}w_{t-1}\right)^{1-\varepsilon_{w,t}} \tag{A.38}$$

$$p_t^* = \frac{\varepsilon_{p,t}}{\varepsilon_{p,t}-1}\frac{x_{1,t}}{x_{2,t}} \tag{A.39}$$

$$x_{1,t} = p_{m,t}Y_t + \phi_p\mathbb{E}_t\left[\Lambda_{t,t+1}\Pi_t^{-\varepsilon_{p,t}\gamma_p}\Pi_{t+1}^{\varepsilon_{p,t}}x_{1,t+1}\right] \tag{A.40}$$

$$x_{2,t} = Y_t + \phi_p\mathbb{E}_t\left[\Lambda_{t,t+1}\Pi_t^{(1-\varepsilon_{p,t})\gamma_p}\Pi_{t+1}^{\varepsilon_{p,t}-1}x_{2,t+1}\right] \tag{A.41}$$

$$Y_{m,t} = Y_t v_t^p \tag{A.42}$$

$$v_t^p = (1-\phi_p)(p_t^*)^{-\varepsilon_{p,t}} + \phi_p\Pi_t^{\varepsilon_{p,t}}\Pi_{t-1}^{-\varepsilon_{p,t}\gamma_p}v_{t-1}^p \tag{A.43}$$

$$1 = (1-\phi_p)(p_t^*)^{1-\varepsilon_{p,t}} + \phi_p\Pi_{t-1}^{\gamma_p(1-\varepsilon_{p,t})}\Pi_t^{\varepsilon_{p,t}-1} \tag{A.44}$$

$$Y_{m,t} = A_t \left(u_t K_t\right)^{\alpha} L_{d,t}^{1-\alpha} \tag{A.45}$$

$$K_{t+1} = \hat{I}_t + (1-\delta(u_t))K_t \tag{A.46}$$

$$w_t = (1-\alpha) p_{m,t} A_t (u_t K_t)^{\alpha} L_{d,t}^{-\alpha} \tag{A.47}$$

$$p_t^k \delta'(u_t) = p_{m,t} A_t \alpha (u_t K_t)^{\alpha-1} L_{d,t}^{1-\alpha} \tag{A.48}$$

$$p_t^k = \mathbb{E}_t \left[\alpha \Lambda_{t,t+1} p_{m,t+1} A_{t+1} u_{t+1}^{\alpha} K_{t+1}^{\alpha-1} L_{d,t+1}^{1-\alpha} + p_{t+1}^k (1-\delta(u_t))\right] \tag{A.49}$$

$$\hat{I}_t = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{A.50}$$

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_{a,t} \tag{A.51}$$

$$\ln(G_t) = (1-\rho_g)\ln(G_{ss}) + \rho_g \ln(G_{t-1}) + \varepsilon_{g,t} \tag{A.52}$$

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu,t} \tag{A.53}$$

$$cp_t = \frac{1}{\varepsilon_{p,t} - 1} \tag{A.54}$$

$$cp_t = (1-\rho_{cp}) cp_{ss} + \rho_{cp} cp_{t-1} + \varepsilon_{cp,t} \tag{A.55}$$

$$cpw_t = \frac{1}{\varepsilon_{w,t} - 1} \tag{A.56}$$

$$cpw_t = (1-\rho_{cpw_t}) cpw_{ss} + \rho_{cpw} cpw_{t-1} + \varepsilon_{cpw,t} \tag{A.57}$$

$$Y_t = C_t + I_t + G_t \tag{A.58}$$

$$Welf_t = \ln(C_t - hC_{t-1}) - \frac{\chi L_t^{1+\eta}}{1+\eta} + \beta \mathbb{E}_t \left[Welf_{t+1}\right] \tag{A.59}$$

$$\begin{aligned} 1 = (1+x_t) p_t^k \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] - (1+x_t) p_t^k \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right) \\ + \mathbb{E}_t \left[\Lambda_{t,t+1}(1+x_{t+1}) p_{t+1}^k \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right] \end{aligned} \tag{A.60}$$

$$\ln(1+x_t) = \rho_x \ln(1+x_{t-1}) + \varepsilon_{x,t} \tag{A.61}$$

$$\begin{aligned} \ln(1+i_t) = (1-\rho_r)\ln(1+i_{ss}) + \rho_r \ln(1+i_{t-1}) \\ + (1-\rho_r)\left[\phi_\pi \ln(\Pi_t) + \phi_y \left(\ln(Y_t) - \ln(Y_{t-1})\right)\right] + \varepsilon_{i,t} \end{aligned} \tag{A.62}$$

$$\varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + \varepsilon_{r,t} \tag{A.63}$$

The unknown variables are: μ, C, L, mrs, Π, i, Λ, w^*, f_1, f_2, L_d, v^w, w, p^*, x_1, x_2, p_m, Y, Y_m, v_p, A, u, K, $\hat{I}$, p_k, I, G, ε_p, ε_w, cp, cpw, x, ε_i, ν, and $Welf$.

A10 Parameter Values

Table 2

Estimated Parameter Values

		Prior			Posterior		
Parameter	Description	Mean	SD	Distribution	Mean	5%	95%
h	habit	0.600	0.100	Beta	0.7079	0.6763	0.7396
κ	I adj. costs	4.000	1.500	Normal	7.5340	5.8948	9.1730
η	inverse Frisch	1.000	0.100	Normal	1.0660	0.9105	1.2240
ϕ_p	Calvo prices	0.500	0.100	Beta	0.7066	0.4990	0.8495
ϕ_w	Calvo wages	0.500	0.100	Beta	0.8789	0.8440	0.9129
γ_p	price index.	0.500	0.150	Beta	0.1765	0.0529	0.2978
γ_w	wage index.	0.500	0.150	Beta	0.1404	0.0674	0.2107
ϕ_π	TR inflation	1.700	0.250	Normal	1.8694	1.3161	2.4122
ϕ_y	TR output	0.120	0.050	Normal	0.1815	0.1225	0.2408
ρ_r	TR persist.	0.800	0.050	Beta	0.5940	0.5248	0.6598
Shock Standard Deviations							
σ_r	i	0.100	1.000	Inv. Gam.	0.0120	0.0118	0.0123
σ_g	G	0.100	1.000	Inv. Gam.	0.0171	0.0154	0.0188
σ_{cp}	cost-push	0.100	1.000	Inv. Gam.	0.0186	0.0118	0.0256
σ_v	demand	0.100	1.000	Inv. Gam.	0.0145	0.0119	0.0167
σ_a	TFP	0.100	1.000	Inv. Gam.	0.0123	0.0118	0.0128
σ_{cpw}	w markup	0.100	1.000	Inv. Gam.	0.0340	0.0273	0.0405
σ_x	I	0.100	1.000	Inv. Gam.	0.1181	0.0878	0.1463
Shock Persistence Parameters							
ρ_i	i	0.400	0.200	Beta	0.0927	0.0100	0.1747
ρ_g	G	0.850	0.050	Beta	0.8540	0.7892	0.9209
ρ_{cp}	cost-push	0.600	0.200	Beta	0.6284	0.3383	0.9788
ρ_v	demand	0.600	0.200	Beta	0.6077	0.5271	0.6880
ρ_a	TFP	0.900	0.050	Beta	0.9275	0.8963	0.9595
ρ_{cpw}	w markup	0.600	0.200	Beta	0.8646	0.8064	0.9228
ρ_x	I	0.600	0.200	Beta	0.2807	0.1387	0.4257

A11 Variance Analysis

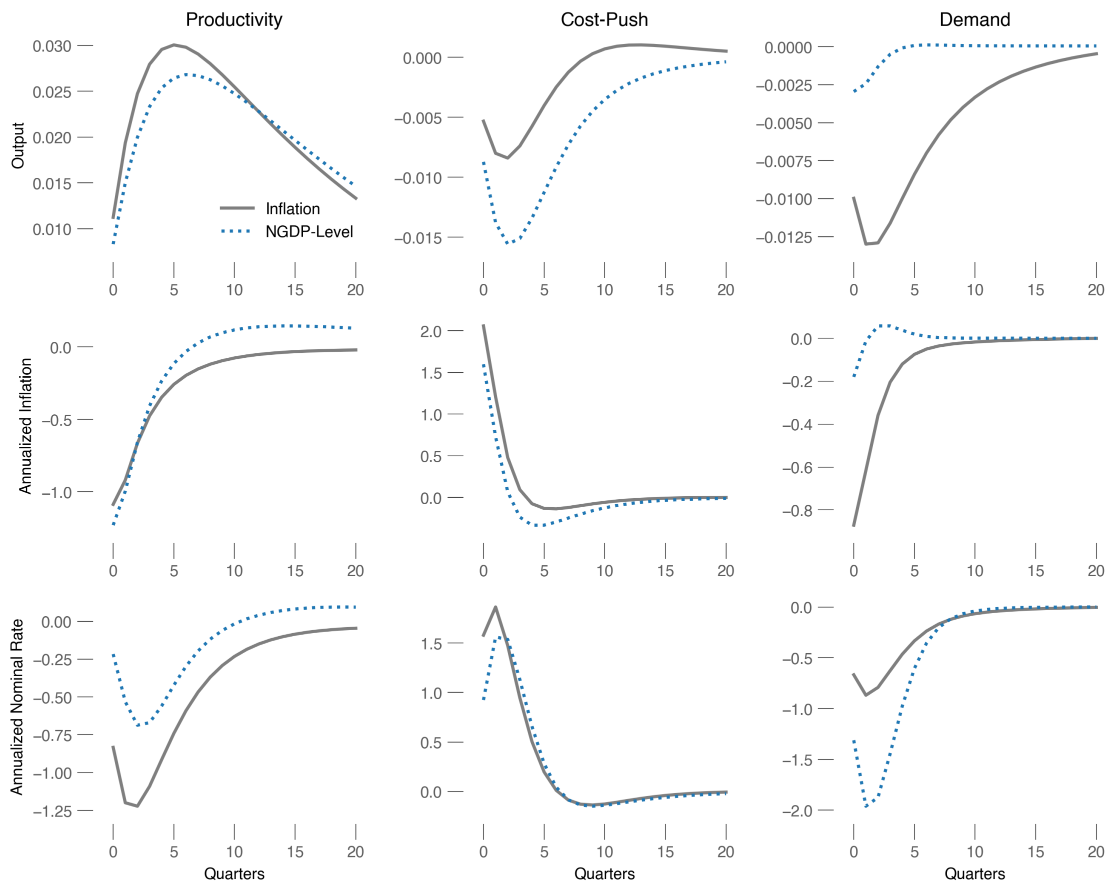


Figure 6: Impulse response functions of output, inflation, and the nominal rate to 1 standard deviation productivity, cost-push, and demand shocks.

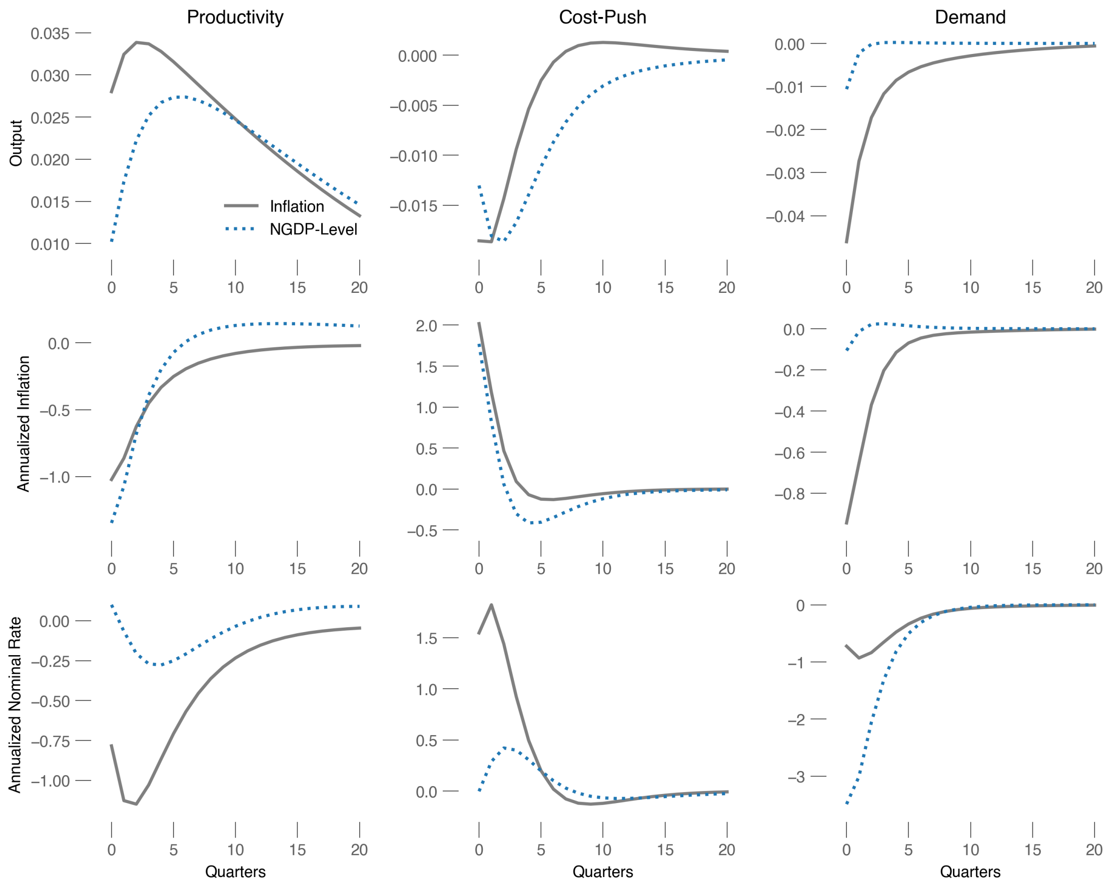


Figure 7: Impulse response functions of output, inflation, and the nominal rate to 1 standard deviation productivity, cost-push, and demand shocks. I set the habit parameter to zero.

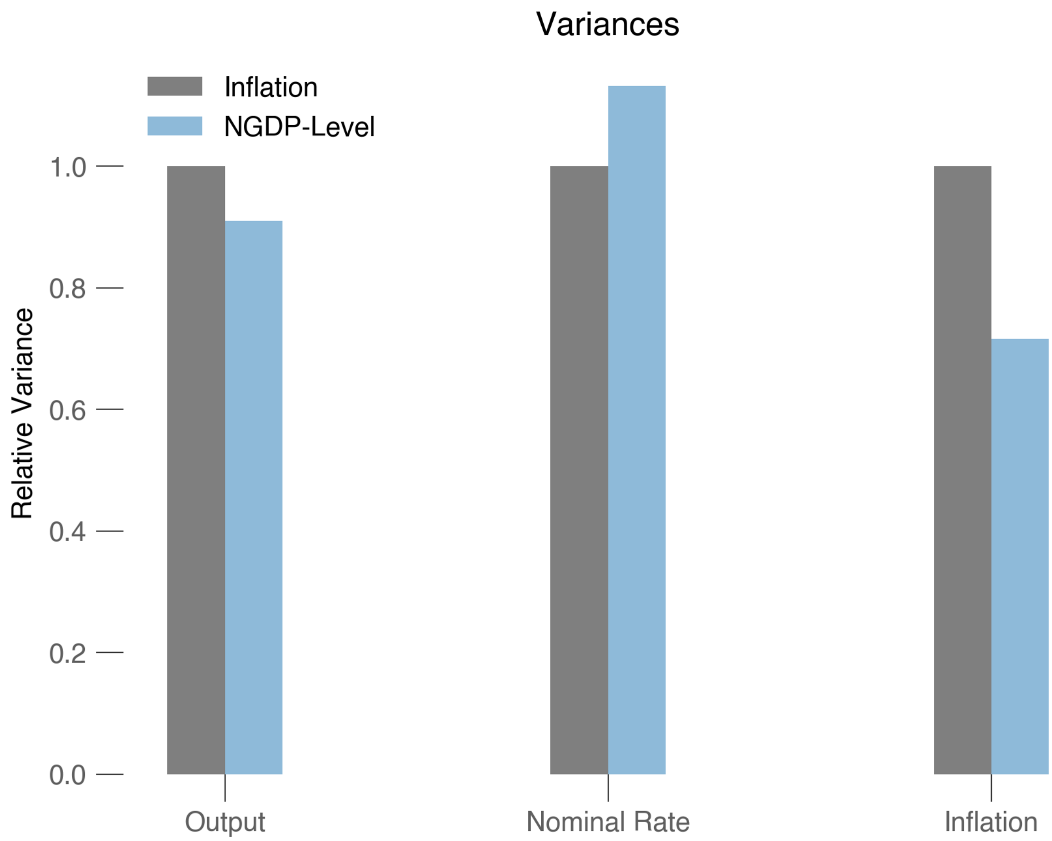


Figure 8: Variance of output, inflation, and the nominal rate relative to the respective variance under inflation targeting. Gray bar – inflation targeting. Blue bar – NGDP level targeting