Taming Volatility: Evaluating NGDP Targeting

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Abstract

This study compares the impact of NGDP and inflation targeting on macroeconomic variability and welfare outcomes using analysis on the canonical New Keynesian model and a quantitative DSGE model. Results demonstrate that under NGDP targeting, real variables exhibit greater stability in response to shocks at the cost of increased inflation volatility. NGDP targeting reduces the role of monetary policy under productivity and cost-push shocks. In the long run, welfare results are virtually equivalent between NGDP and inflation targeting.

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1 Introduction

The pursuit of economic stability has been the cornerstone of central banking since the Federal Reserve Act of 1913 established the Fed's dual mandate of full employment and price stability. The Federal Open Market Committee (FOMC) considers that an inflation target of "2 percent over the longer run" accomplishes price stability and, in turn, helps bring about full employment (Board of Governors, 2021). During the Great Recession and recent COVID-19 recession, economists and popular media analysts asserted that inflation targeting was failing to accomplish the dual mandate. In a 2011 op-ed in the New York Times, for example, Christina Romer wrote that "Today, inflation is still low, but unemployment is stuck at a painfully high level." As an alternative, Romer proposed Nominal GDP (NGDP): "Because it directly reflects the Fed's two central concerns — price stability and real economic performance — nominal G.D.P. is a simple and sensible target for long after the economy recovers."

This paper aims to evaluate how well NGDP targeting satisfies the Fed's dual mandate. I do so by first studying an NGDP rate targeting rule analytically in the canonical three equation New Keynesian model found in Gali (2015). I then construct a quantitative DSGE model in the vein of Smets & Wouters (2007) or Christiano et al. (2005). As a robustness exercise, I include NGDP level targeting as a third possible policy in the quantitative model.

I prove three novel results in the canonical model. First, output is always more stable as long as the Taylor principle is satisfied. Second, inflation is more volatile provided weak restrictions on the stickiness of prices and the persistence of productivity shocks. Lastly, monetary policy is less reactionary under NGDP targeting.

I then estimate the quantitative model to ensure the parameters are grounded in the data. Impulse response functions demonstrate that NGDP targeting does indeed lead to less volatile real variables under a variety of standard shocks confirming result (1).

Inflation is more volatile as well, confirming result (2). The nominal rate is less active under productivity and cost-push shocks, providing quantitative support for result (3), though the result breaks down when shocks from other sources are considered. In addition, a long-run simulation showcases the outcomes, revealing lower variance in real variables and the nominal rate and higher variance in inflation under NGDP targeting.

I then conduct a welfare analysis. The real-nominal stability trade-off I find produces shock-dependent welfare results. Positive productivity shocks imply inflation targeting is optimal, while cost-push shocks imply NGDP targeting is optimal. As a result, the unconditional mean of welfare is virtually identical between inflation and NGDP targeting.

Lastly, I analyze performance at the zero lower-bound (ZLB), since the ZLB inspired much of the writing on NGDP targeting (see Beckworth (2019) for example). I hit the model economy with two large productivity shocks. The economy with inflation targeting hits the ZLB faster and longer. NGDP targeting prevents inflation from falling too much, and labor recovers quickly. For robustness, I do a second exercise with large preference shocks and find similar results.

Literature Review

My paper is most similar to Garin et al. (2016), who analyze an NGDP rate targeting peg in a quantitative New Keynesian models. They find that NGDP targeting minimizes consumption-equivalent welfare loss under productivity shocks when compared to inflation targeting and a Taylor rule. Additionally NGDP targeting outperforms output gap targeting when potential output is observed with a small measurement error.[1] I innovate on their paper by establishing analytical results, imposing nominal rate rules with smoothing rather than pegs.

[1]The authors note that in many New Keynesian models, output gap targeting does not result in a determinate equilibrium. My model is such a model.

Beckworth & Henderson (2019) use a canonical New Keynesian model to study nominal income targeting compared to inflation and output gap targeting. They introduce a shock to the output gap the central bank observes to evaluate the role of information in monetary policy. They find that uncertainty in the output gap is empirically important in explaining actual output gap fluctuations and that nominal income targeting, observed perfectly, reduces those fluctuations. I extend modeling an NGDP targeting rule in the New Keynesian literature and, since I do not use the output gap, can abstract from information problems facing the Fed.

Beckworth (2019) argues that NGDP targeting would relieve zero lower-bound issues, saying that "NGDP [level targeting], in short, generates the temporary rise in inflation needed to escape a [zero lower-bound], something that is difficult to do with the Fed's current inflation target." According to Beckworth (2019), the zero lower-bound is alleviated because inflation would become counter-cyclical. As such, real debt burdens would ease and lower real interest rates to their market-clearing levels. Yglesias (2015) reaches a similar conclusion.

Hall & Mankiw (1994) use a structural time series counterfactual to evaluate nominal income targeting. They find that "the primary benefit of nominal income targeting is reduced volatility in the price level and the inflation rate." Romer (2011) cites Hall & Mankiw (1994) for the benefits of NGDP targeting. I extend this analysis by evaluating inflation volatility in a DSGE model.

Mitra (2003) demonstrates that under NGDP targeting, a unique equilibrium exists in the three-equation New Keynesian model. Sumner (2012) argues for NGDP targeting from economic principles, saying that higher inflation under cost-push shocks can improve economic performance. Sheedy (2014) finds that NGDP targeting leads to efficient risk-sharing by stabilizing debt-to-GDP ratios. My paper also speaks to the optimal monetary policy literature discussed in papers such as Khan et al. (2003), who analyze the goals of optimal policy, and Woodford (2001), who analyzes the optimality of the Taylor rule. I also look at the zero lower-bound, relating my paper to Wu & Xia (2016) and Sims & Wu (2021).

My paper relates to articles in public policy as well. Crook (2022) states that NGDP targeting is "the simplest way to improve monetary policy." Yglesias (2015) writes on the Fed's consideration of NGDP targeting under Bernanke. Bowman (2014) argues that NGDP targeting would stabilize the real economy by ensuring nominal wage contracts would be fulfilled regardless of the macroeconomic status. Employment would thus remain full. Maybe most significantly, the Chair of the Federal Reserve, Jerome Powell, mentioned just after the COVID-19 pandemic that the Fed had looked at nominal income targeting as a possible policy rule (Powell, 2022).

I organize the rest of this paper as follows: Section II lays out analysis of the NGDP rule in the canonical, three equation New Keynesian model. Section III outlines the quantitative DSGE model. Section IV discusses the results from the quantitative model. Section V concludes.

2 Three-Equation New Keynesian Model

I begin with the canonical New Keynesian model popularized in Gali's 2015 textbook. The log-linearized model consists of the following three equations:

$$\tilde{y}_t = \mathbb{E}[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) \tag{1}$$

$$\tilde{\pi}_t = \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] + u_t \tag{2}$$

$$\tilde{y}_t^f = \rho_a \tilde{y}_{t-1}^f + \frac{1+\eta}{\sigma+\eta}\varepsilon_{a,t} \tag{3}$$

$\tilde{y}_t$ denotes real output, $\tilde{\pi}_t$ denotes inflation, $\tilde{y}_t^f$ denotes potential output, and $\tilde{i}_t$ denotes the nominal interest rate. u_t is a cost-push shock and $\varepsilon_{a,t}$ is a productivity shock. All variables are in their log-linearized form such that they represent deviations from their steady state. For my baseline results, I impose log-utility ($\sigma = 1$) as in Gali (2015).

Equation (1) is the dynamic IS curve, equation (2) is the New Keynesian Phillips curve, and equation (3) is an exogenous AR(1) process for potential output. To close the model, I need an equation to determine the nominal interest rate.

To proxy the current monetary policy regime, I impose an inflation targeting rule:

$$\tilde{i}_t = \rho_r \tilde{i}_{t-1} + (1 - \rho_r)\phi_\pi \tilde{\pi}_t + \sigma_r \varepsilon_r$$

where ϕ_π is the responsiveness of the central bank to deviations of inflation from steady-state.

I compare this inflation rule to a rule targeting the NGDP growth rate:

$$\tilde{i}_t = \rho_r \tilde{i}_{t-1} + (1 - \rho_r)\phi_\pi \left(\tilde{\pi}_t + \tilde{y}_t - \tilde{y}_{t-1}\right) + \sigma_r \varepsilon_r \tag{4}$$

Note that the weighting coefficient is the same between the two rules. This assumption allows me to derive the propostions in the next section. The system of equations that I solve to analyze the NGDP targeting rule consists of equations (1) - (4).

2.1 Results

I derive three novel results.

Proposition 1. *Under log-utility, an NGDP targeting rule leads to a more stable initial output response under productivity shocks given:* $\phi_\pi > 1$

Proof: See appendix A2.

Imposing the Taylor principle to guarantee a unique equilibrium ensures that $\phi_\pi > 1$. Therefore, proposition 1 will always hold as long as the model is solvable. In any New Keynesian model, then, NGDP targeting will lead to less volatility in real output.

Beyond the mathematics, proposition 1 communicates a clear prediction for a central bank that adopts NGDP targeting as a monetary policy rule: output will be more stable. The intuition here is simple: NGDP targeting places weight on output deviations from steady-state and inflation targeting does not (NGDP targeting places more weight on output deviations than even the typical parameterization of the Taylor rule (Taylor, 1993)). Therefore, a central bank targeting NGDP will keep output closer to steady-state than a central bank targeting only inflation.

Having established that output will be more stable under NGDP targeting under any ϕ_π that satisfies Blanchard-Kahn conditions, I turn to inflation in proposition 2:

Proposition 2. *Under log-utility, an NGDP targeting rule leads to a more volatile inflation response under productivity shocks given a monetary policy authority that satisfies:*

$$\phi_\pi \geq \frac{(1-\rho_a)(\kappa + \beta\lambda_{2a})}{\kappa} + \rho_a$$

where λ_{2a} denotes the $\tilde{\pi}_t$ policy function coefficient on $\tilde{y}_{t-1}$ under NGDP targeting.

Proof: See appendix A3.

The right-hand side will always be greater than one, as $\frac{\kappa+\beta\lambda_{2a}}{\kappa} > 1$. To determine how realistic the restriction in proposition 2 is, I plot a contour plot in $(\theta,\ \rho_a)$ space. See figure 1.

As prices become stickier (θ approaching one) and the persistence of the productivity process decreases, the central bank must respond more harshly to deviations from steady-state for inflation to be more volatile under an NGDP rule. Importantly though, the minimum value of ϕ_π is not unreasonable. A ϕ_π value of 1.5 is standard in the literature. Estimates of θ tend to be around 0.65, with Smets & Wouters (2007) estimating a credible set range of 0.56 to 0.74. Estimates of ρ_a are tightly estimated around 0.95.

The contour plot in figure 1 shows that at the $(\theta,\ \rho_a)$ coordinate pair of $(0.65,\ 0.95)$, the minimum ϕ_π value is about 1.05. Therefore, under typical calibration of New Keynesian models, NGDP targeting leads to more volatile inflation under productivity shocks. Proposition 2 therefore clearly predicts that NGDP targeting would lead to volatile inflation.

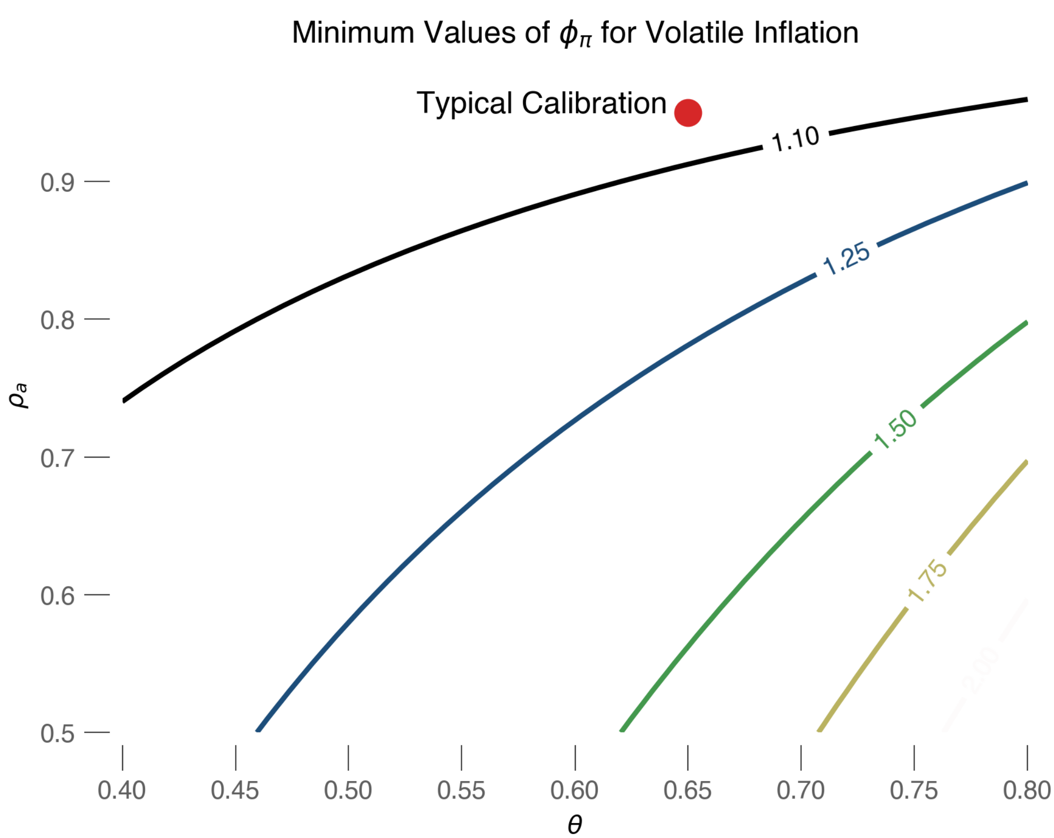

Figure 1: Contour plot showing the minimum value of ϕ_π for proposition 2 to hold true. $\beta = 0.98$ and $\eta = 1$. Typical calibration values satisfy proposition 2's condition.

Intuitively, proposition 2 occurs because the relative weight on inflation has decreased under NGDP targeting. The central bank is only focusing on inflation under inflation targeting. Hall & Mankiw (1994), though, find that nominal income targeting would lead to *more* stable prices and inflation. Proposition 2 clearly predicts the opposite.

Proposition 3. *Under log-utility, an NGDP targeting rule leads to no movement in the nominal interest rate in response to productivity and cost-push shocks.*

Proof: See appendix A1.

From the DIS equation, I substitute the NGDP targeting rule in to derive, in general, the following relationship:

$$\mathbb{E}_t[\tilde{i}_{t+1}] = \phi_\pi \tilde{i}_t + \phi_\pi (1-\sigma)\left(\mathbb{E}_t[\tilde{y}_{t+1}] - \tilde{y}_t\right) \tag{5}$$

The expected nominal interest rate tomorrow is a function of σ, the coefficient of relative risk aversion. σ is attached to the expected change in real income. A productivity shock increases real income. When $\sigma > 1$, an expected increase in income will result in a decrease in the interest rate. The larger σ, the more the NGDP rule allows the nominal rate to move. Even with larger σ values, the movement in the nominal rate is still more muted compared to inflation targeting (see appendix A4). This suggests a more muted role for monetary policy.

The three-equation model is useful for establishing intuition and proving results. Next, I move to a quantitative model to demonstrate the robustness of each proposition and to provide numerical analysis of welfare and performance at the zero lower-bound.

3 Quantitative Model

This section builds a model in the vein of Smets & Wouters (2007) and Christiano et al. (2005). Households now include external habit in their utility function. The labor market has sticky wages using standard Calvo (1983) logic and inflation indexation. The production side of the economy now has capital accumulation, capital utilization, and investment adjustment costs. Firms can also index prices to inflation. Lastly, a government now consumes a portion of output. Such models have been shown to replicate targeted moments of the business cycle well (Garin et al., 2016). For a full description of the model, see appendix A5.

3.1 Households

There is an infinitely lived representative agent that maximizes lifetime utility with the following form:

$$\mathbb{E}_t \left[\sum_{j=0}^{\infty} \nu_t^j \beta^j \left\{ \ln\left(C_{t+j} - \mathcal{H} C_{t+j-1}\right) - \frac{\chi L_{t+j}^{1+\eta}}{1+\eta} \right\} \right]$$

$\mathcal{H}$ designates the habit formation parameter. χ is a labor disutility scaling parameter, while η is the inverse Frisch elasticity. Households face the following real budget constraint:

$$C_t + b_t \leq mrs_t L_t + div_t - T_t + \Pi_t^{-1}(1 + i_{t-1}) b_{t-1}$$

b_t denotes real government bonds, mrs_t denotes real wages the household receives, and div_t denotes profits rebated by firms to the household. T_t is a lump sum tax enacted by the government to finance its spending. The household maximizes with respect to C_t, L_t, and b_t. ν_t is a preference shock, increasing the discount factor. Economically,

ν_t increases the amount the household values future consumption.

3.2 Labor Markets

Labor markets operate in three parts. Labor unions exist on a unit measure, $h \in [0, 1]$, and purchase labor, $L_t(h)$, from households at nominal value MRS_t. Then unions package that labor, now denoted $L_{d,t}(h)$, and sell it to a representative labor packer. Lastly, labor packers combine the labor from all the different unions into final labor product $L_{d,t}$ using a standard constant elasticity of substitution technology:

$$L_{d,t} = \left[\int_0^1 L_{d,t}(h)^{\frac{\varepsilon_{w,t}-1}{\varepsilon_{w,t}}} dh \right]^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}}$$

where $\varepsilon_{w,t}$ is the elasticity of substitution of labor. See the cost-push shock below for how wage markup varies. Labor unions face sticky wages, with a probability of adjusting wages each period of ϕ_w. Unions that cannot update wages this period index wages back to last period's inflation with probability γ_w. Optimizing the unions' dividends functions yields the optimal reset wage:

$$w_t^* = \frac{\varepsilon_{w,t}}{\varepsilon_{w,t} - 1} \frac{f_{1,t}}{f_{2,t}}$$

with $f_{1,t}$ and $f_{2,t}$ being auxiliary variables. Aggregating the labor packer's labor beta yields a wedge between aggregate labor supplied and labor betaed:

$$L_t = L_{d,t} v_t^w$$

v_t^w denotes wage dispersion. Ultimately, I can find aggregate wage:

$$w_t^{1-\varepsilon_{w,t}} = (1-\phi_w)(w_t^*)^{1-\varepsilon_{w,t}} + \phi_w \left(\frac{\Pi_{t-1}^{\gamma_w}}{\Pi_t} w_{t-1} \right)$$

3.3 Production

The production side of the economy includes four types of firms. A competitive capital producer creates new physical capital each period, $\hat{I}_t$. A representative wholesaler buys capital from the wholesaler and labor from the labor packer to create $Y_{m,t}$. A unit measure of retail firms, $f \in [0,1]$, repackage wholesale output using $Y_t(f) = Y_{m,t}(f)$. Lastly, a competitive final goods firm aggregates $Y_t(f)$ into Y_t using a CES aggregator:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\varepsilon_{p,t}-1}{\varepsilon_{p,t}}} df \right]^{\frac{\varepsilon_{p,t}}{\varepsilon_{p,t}-1}}$$

where $\varepsilon_{p,t}$ denotes the elasticity of substitution for retail output. Retail firms face sticky prices, with a probability of adjusting wages each period of $1 - \phi_p$. Firms that cannot update prices this period index prices back to last period's inflation with probability γ_p. The retail firms' optimization problem yields an optimal reset price of:

$$p_t^* = \frac{\varepsilon_{p,t}}{\varepsilon_{p,t}-1} \frac{x_{1,t}}{x_{2,t}}$$

where $x_{1,t}$ and $x_{2,t}$ are two more auxiliary variables. Aggregating retailer output yields a wedge between wholesale output and aggregate output:

$$Y_{m,t} = Y_t v_t^p$$

v_t^p denotes price dispersion. Aggregate price dynamics, divided through by the aggregate price to ensure stationarity, are:

$$1 = (1-\phi_p)(p_t^*)^{1-\varepsilon_{p,t}} + \phi_p \Pi_{t-1}^{\gamma_p(1-\varepsilon_{p,t})} \Pi_t^{\varepsilon_{p,t}-1}$$

Before moving to wholesale firms, note that the elasticity of substitution for retail output, $\varepsilon_{p,t}$, is time dependent. I do this to implement the cost-push shock found in Smets & Wouters (2007). Define the cost-push term, cp_t, as:

$$cp_t = \frac{1}{\varepsilon_{p,t}-1}$$

A positive shock to cp_t leads to an increase in the mark-up. An increase in the mark-up leads to higher prices and lower output, matching the effect of a cost-push shock in the three-equation model.

Wholesale firms have production technology:

$$Y_{m,t} = A_t \left(u_t K_t\right)^{\alpha} L_{d,t}^{1-\alpha}$$

where A_t is an exogenous productivity variable and u_t is a choice variable denoting capital utilization. Capital evolves according to a standard law of motion:

$$K_{t+1} = \hat{I}_t + \left(1-\delta(u_t)\right) K_t$$

where $\delta(u_t)$ is a depreciation function dependent on the level of capital utilized. Maximizing the wholesaler's dividend function with respect to $L_{d,t}$, $\hat{I}_t$, u_t, and K_{t+1} gives three equilibrium conditions.

The capital producer operates with technology:

$$\hat{I}_t = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

where I_t is unconsumed investment, $S(\cdot)$ is an adjustment cost function, and $\hat{I}_t$ is consumed investment. I include an investment return shock on the price of capital.

3.4 Government

The government consumes an exogenously stochastic portion of Y_t. It is financed by lump-sum taxes on the household and by nominal bonds B_t. Its real budget constraint is as follows:

$$G_t + b_{t-1}(1 + i_{t-1})\Pi_t^{-1} = T_t + b_t$$

The central bank sets interest rates according to an inflation targeting rule first:

$$\ln(1 + i_t) = (1 - \rho_r)\ln(1 + i_{ss}) + \rho_r \ln(1 + i_{t-1}) + (1 - \rho_r)\phi_\pi \ln(\Pi_t) + \varepsilon_{i,t}$$

and I compare it to the proposed NGDP rate targeting rule:

$$\begin{aligned} \ln(1 + i_t) = (1 - \rho_r)\ln(1 + i_{ss}) + \rho_r \ln(1 + i_{t-1}) \\ + (1 - \rho_r)\phi_N \left[\ln(\Pi_t) + \ln(Y_t) - \ln(Y_{t-1})\right] + \varepsilon_{i,t} \end{aligned}$$

and, for robustness, an NGDP level targeting rule:

$$\begin{aligned} \ln(1 + i_t) = (1 - \rho_r)\ln(1 + i_{ss}) + \rho_r \ln(1 + i_{t-1}) \\ + (1 - \rho_r)\phi_N \left[\ln(P_t Y_t - Y_{ss})\right] + \varepsilon_{i,t} \end{aligned}$$

When I analyze the model at the zero lower-bound, the nominal interest rate is set such that:

$$(1 + i_t^{effective}) = max\{1, 1 + i_t^{rule}\} \tag{6}$$

where $i_t^{effective}$ is the prevailing interest rate in the economy and i_t^{rule} is the interest rate the rule, either inflation or NGDP targeting, would set if unconstrained.

3.5 Aggregation and Exogenous Processes

The aggregate budget constraint can be found by starting from the household budget constraint and substituting in for dividends and taxes. The resulting constraint is standard:

$$Y_t = C_t + I_t + G_t$$

There are seven exogenous processes at work: technology, government spending, cost-push, wage markup, investment, nominal rate, and preference shocks.

I use the recursive formulation of the household utility function as my measure of welfare:

$$Welf_t = \ln(C_t - \mathcal{H}C_{t-1}) - \frac{\chi L_t^{1+\eta}}{1+\eta} + \beta \mathbb{E}_t \left[Welf_{t+1} \right]$$

The model consists of 35 equations with 35 unknowns (see appendix A5).

4 Results

4.1 Estimation

I estimate my model using a random walk Metropolis Hastings algorithm. I use nominal GDP for output, PCE non-durable goods plus PCE services for consumption, PCE durable goods plus fixed private investment for investment, and the average hourly private sector nominal wage for wages. I deflate each of these by the GDP deflator and detrend using the Hamilton filter. For labor, I use hours worked by non-farm employees and then also detrend with the Hamilton filter. For inflation, I use the PCE core deflator index. For the nominal rate I use the effective federal funds rate, replacing the zero lower bound period with the Wu & Xia (2016) shadow rate. Inflation and the nominal rate are demeaned. The resulting data run from 1966Q4 to 2019Q4. The model includes seven shocks: productivity, cost-push, investment, wage mark-up, government spending, nominal rate, and preference.

Table 1

Estimated Parameter Values

Parameter	Description	Estimated Value
$\mathcal{H}$	*habit persistence*	0.600
α	*capital share*	0.333
ϕ_π	*weight on inflation*	1.875
ϕ_w	*wage stickiness*	0.786
γ_w	*wage indexation*	0.796
ϕ_p	*price stickiness*	0.705
γ_p	*price indexation*	0.365

I fix β to 0.99 to roughly match the annualized rate over the time sample. I set ε_{ss}^{p} to 11, and ε_{ss}^{w} to 11 following Sims & Wu (2021). I use the following depreciation function:

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2$$

where δ_0 is set to 0.025, δ_2 is set to 0.01, and δ_1 is set such that steady-state utilization and is equal to 1, also following Sims & Wu (2021). The adjustment cost function is as follows:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t$$

where κ is the adjustment cost multiplier. Key parameters of the estimation are displayed in table 1. The full list of parameters is in appendix A6.

4.2 IRFs

Real Variables

Figures 2 plots the IRFs of output, labor, and investment in the quantitative model to productivity, cost-push, investment, and preference shocks under inflation, NGDP rate, and NGDP level targeting.

NGDP targeting results in more stable output under all shocks, confirming proposition 1. The peak response of output under each shock is lower than the peak response in an inflation targeting regime. In addition, recovery back to steady-state under NGDP targeting is slower and smoother, opting for more gradual recovery.

All three targeting rules deliver similar variability in employment under productivity shocks. Under the other shocks, NGDP targeting again results in less variable, smoother

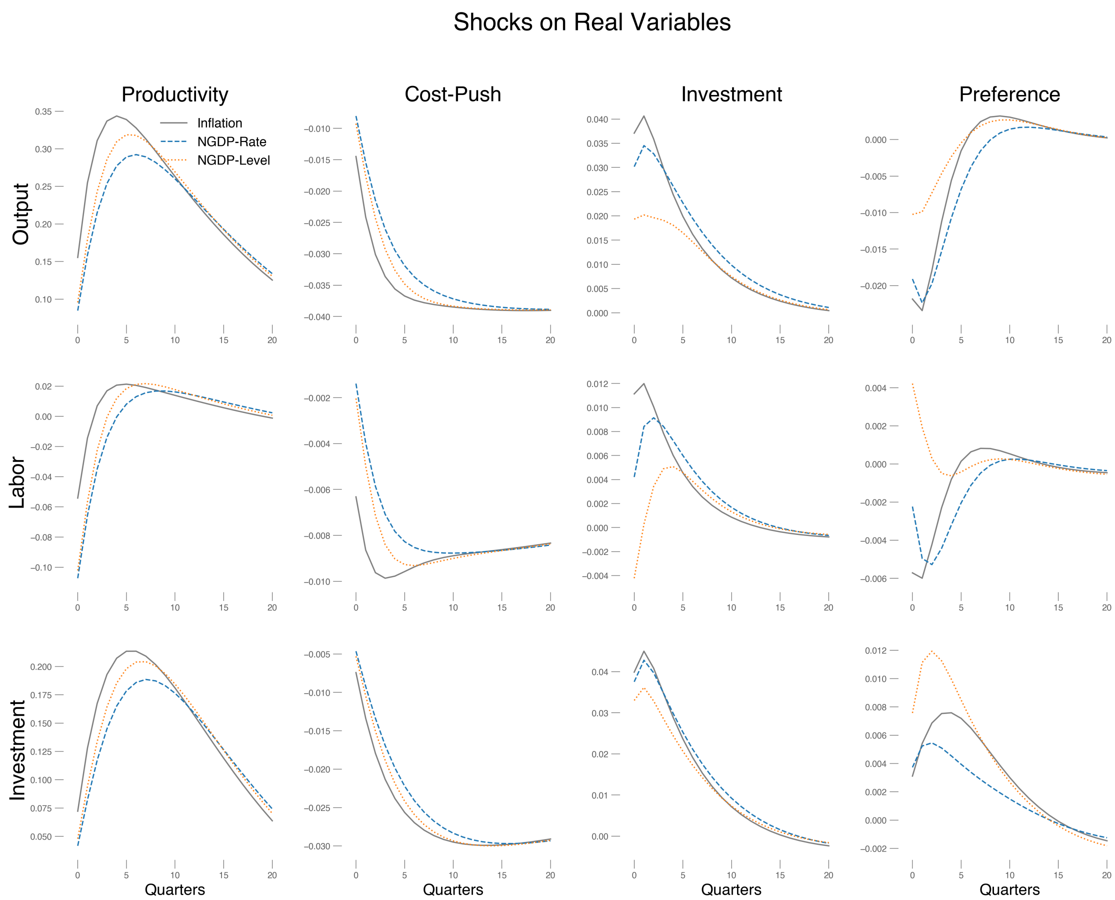

Figure 2: Variables output, labor and investment form the rows. Productivity, cost-push, investment, and beta 1 SD shocks form the columns. Grey solid - Inflation. Blue dashed - NGDP Rate. Yellow dotted - NGDP level. All responses are in deviations from the steady state.

paths back to steady state. Interestingly, NGDP level targeting delivers an increase in labor under preference shocks.

The pattern continues to hold for investment. NGDP targeting delivers less volatile results across all shocks with the sole exception of level targeting under preference shocks, which delivers a larger surge in investment. It is well-known that investment is the most volatile part of the business cycle, so NGDP targeting's stability under all shocks is important.

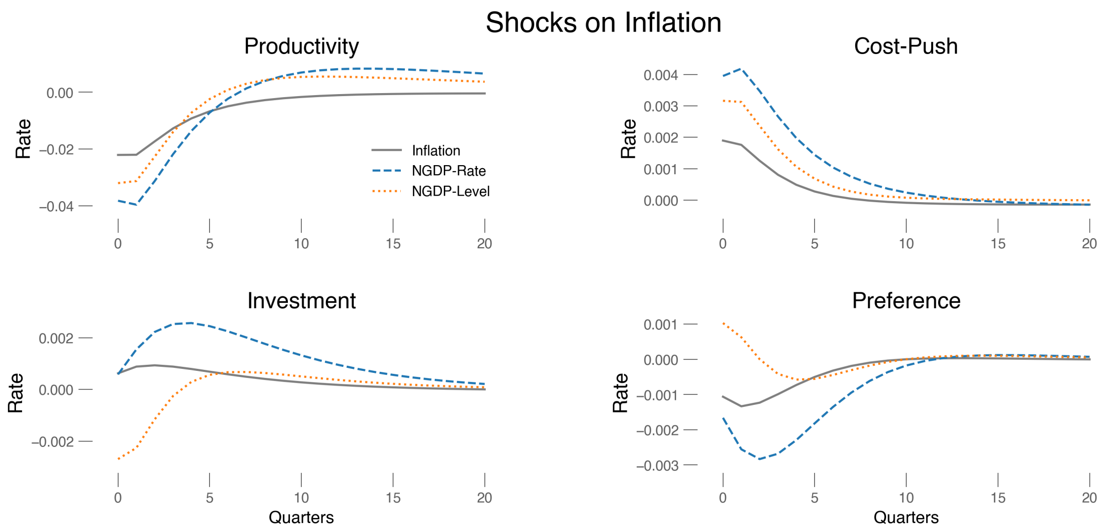

Figure 3: Results of 1 SD shocks. Grey solid - Inflation. Blue dashed - NGDP Rate. Yellow dotted - NGDP level. All responses are in deviations from the steady state.

Inflation

Figure 3 plots the response of inflation to each shock. The intuition from the three-equation model holds. Under each shock, inflation moves more under NGDP targeting than under inflation targeting, confirming proposition 2. Looking at inflation under preference shocks for NGDP level targeting helps explain the increase in labor under the same shock. Inflation increases. To smooth consumption across higher prices, households must work more.

Nominal Rate

Figure 4 plots the nominal rate's path under each shock. Proposition 3 is confirmed as interest rates are more volatile under inflation targeting to productivity and cost-push shocks. For investment and preference shocks, though, the pattern breaks. NGDP targeting requires active monetary policy for investment and preference shocks, while inflation targeting needs a relatively small adjustment to the nominal rate. The source of shocks matters more for the volatility of monetary policy.

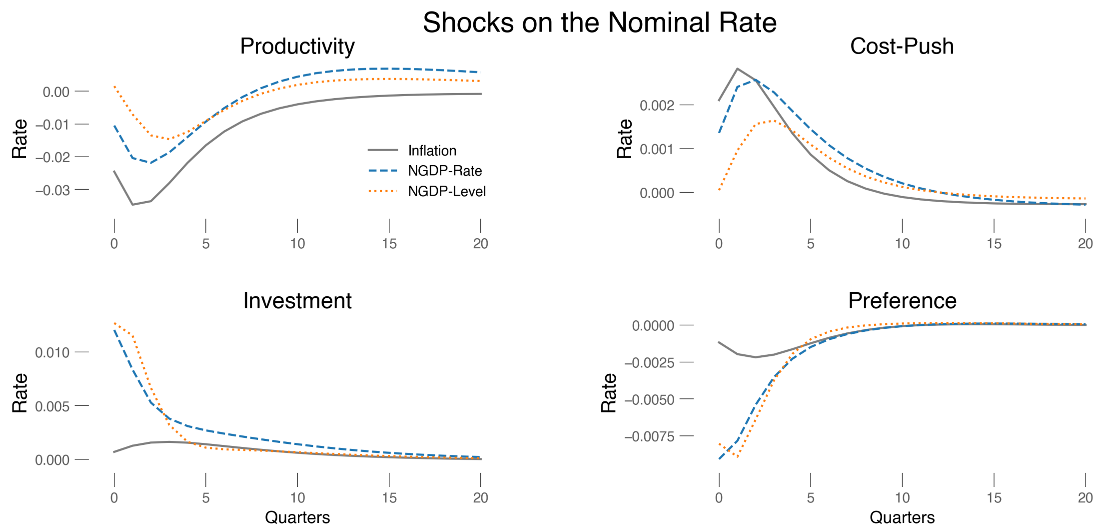

Figure 4: Results of 1 SD shocks. Grey solid - Inflation. Blue dashed - NGDP Rate. Yellow dotted - NGDP level. All responses are in deviations from the steady state.

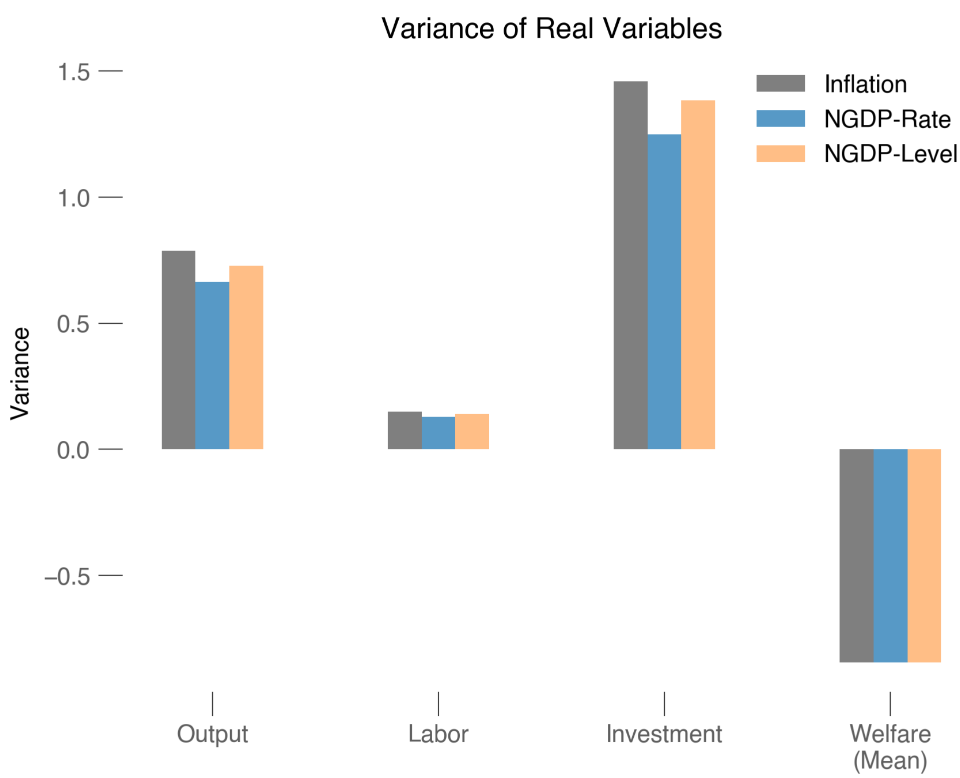

Figure 5: Variation of real variables. Grey - Inflation. Blue - NGDP Rate, Yellow - NGDP Level. I hit the economy with productivity, cost-push, investment, wage markup, and preference shocks for 5000 periods. I then take the variance of output, labor, and investment. I take the unconditional mean of welfare.

4.3 Simulation

Volatility can be challenging to determine in IRF graphs, especially due to the smooth recovery of the NGDP responses. I therefore simulate 5000 periods of shocks in the model economy. In each period, I hit the economy with five "real" shocks: productivity, cost-push, wage-markup, investment, and preference. I then take the variance of the series in the model. Figure 5 plots the variance of the real variables.

NGDP targeting leads to more stable real variables, again confirming proposition 1. The unconditional mean of welfare, scaled by the steady-state value of welfare, is virtually identical across all three specifications, suggesting that the tradeoff between real and nominal stability ultimately would not change economic performance in the long run.

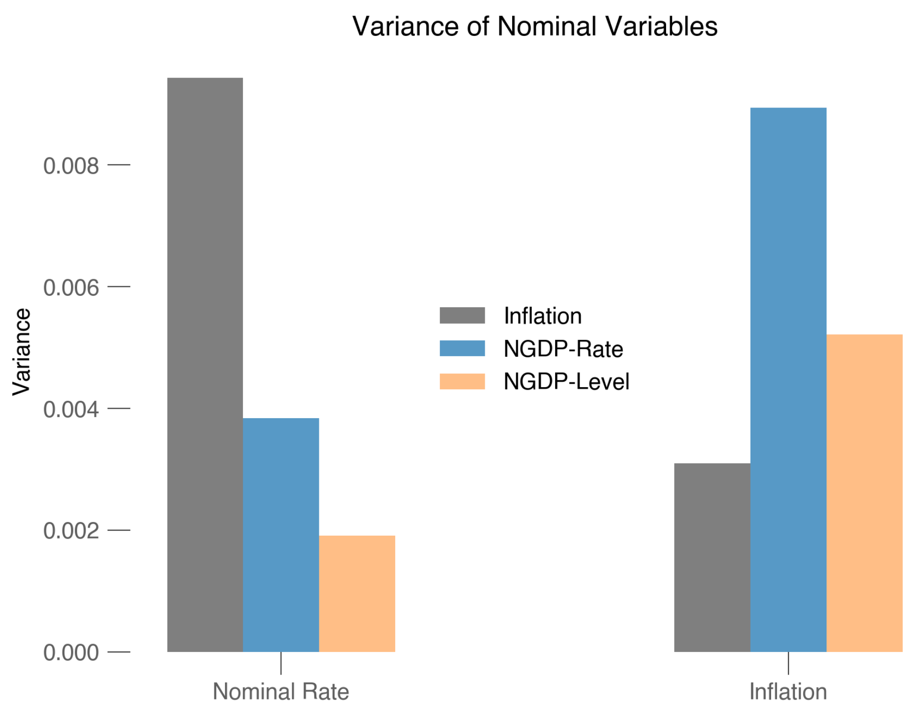

Figure 6: Variation of nominal variables. Grey - Inflation. Blue - NGDP Rate, Yellow - NGDP Level. I hit the economy with productivity, cost-push, investment, wage markup, and preference shocks for 5000 periods. I then take the variance of the nominal rate and inflation.

Figure 6 plots the variation in inflation and the nominal interest rate. Inflation is more volatile under NGDP targeting, supporting proposition 2. In fact, the model shows that inflation is about 4 times more volatile under rate targeting, and about 2 times more volatile under level targeting, than under inflation targeting. This substantial increase in volatility provides modeling support for one reason the Federal Reserve in the United States has stated it has not looked closer at NGDP targeting (Powell, 2022).

The nominal rate is more volatile under inflation targeting, confirming proposition 3 in the long run. In estimation, productivity and cost-push shocks were the most prominent perturbations to the economy. Even though NGDP targeting had more volatile nominal rates in response to investment and preference shocks, those shocks play a relatively small role in macroeconomic time series.

Figure 7: Response of welfare to each shock. Grey - Inflation. Blue - NGDP Rate. Yellow - NGDP Level. Welfare is expressed as the compensating variation relative to the largest response.

4.4 Welfare

Given the simulation, welfare should be shock dependent. Figure 7 plots the compensating variation relative to the largest movement for each targeting rule under various shocks. The productivity shock, for example, leads to the largest welfare gain under inflation targeting. The NGDP welfare results are then expressed relative to the welfare gain under inflation targeting. Positive compensating variation denotes higher levels of welfare compared to steady-state. Under a productivity shock, inflation targeting performs best due to the large increase in output from the shock and low inflation. Cost-push shocks, on the other hand, are worst under inflation targeting due to the large decrease in output. NGDP rate targeting is optimal under investment shocks, while level targeting is optimal under demand shocks.

4.5 Zero Lower Bound

Before the COVID-19 pandemic, the Fed tended to operate near the zero lower-bound (ZLB). While recently the Federal Reserve has rapidly raised rates to combat high inflation, some in the media believe that a return to low interest rates will come eventually (e.g. Krugman 2022). A robust monetary policy rule should therefore perform well, relative to other rules, when the ZLB constraint is enforced.

To analyze NGDP targeting's performance at the zero lower-bound, I use the OCCBIN toolbox in Dynare and impose the restriction in equation (5), rewritten here for convenience:

$$(1 + i_t^{effective}) = max\{1, 1 + i_t^{rule}\}$$

I hit the economy with two large productivity shocks to push the nominal rate to the ZLB. The results of the simulation are displayed in figure 8. The nominal interest rate's volatility under inflation targeting causes the rate to hit the zero lower-bound faster and stay at zero longer. Inflation falls nearly as low as NGDP rate targeting despite the low volatility found under productivity shocks above. Level targeting performs very well at the ZLB. Even though the nominal rate hits the ZLB briefly, inflation drops only slightly. Output, consumption, and wages essentially follow their unconstrained paths.

This finding verifies the intuition in Beckworth (2019). The NGDP level target does indeed lead to a more stable economy and quicker recovery at the ZLB, at least in response to productivity shocks. If the Great Recession was caused by productivity shocks, then the arguments of Beckworth (2019), Romer (2011), and others are supported by my findings.

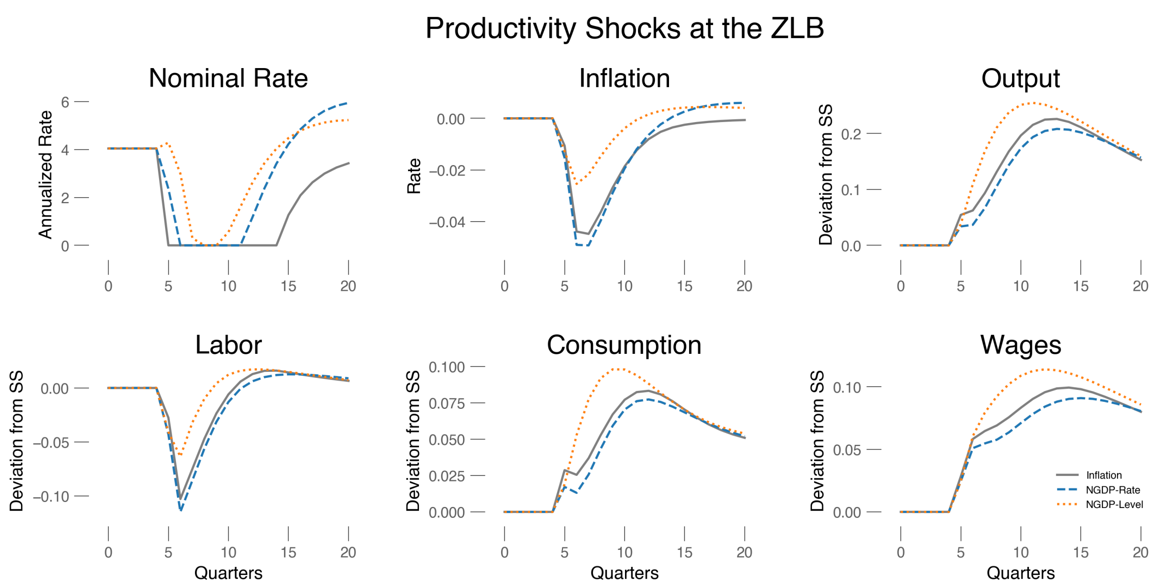

Figure 8: Results of two large productivity shocks. Grey solid - Inflation. Blue dashed - NGDP rate. Yellow dotted - NGDP level.

Importantly, though, I simulated a productivity shock. From the IRFs, the nominal rate is more volatile under NGDP targeting in response to preference shocks. As such, I simulate a second ZLB scenario, where preference shocks cause the binding ZLB. Figure 9 shows the results.

NGDP targeting hits and stays at the ZLB for an extended period of time, while inflation targeting just touches the ZLB. Rate targeting sinks the economy into a deep depression, with inflation, output, and labor all falling more than the other two rules. Level targeting, though, due to the slight inflation bump from the preference shock, keeps inflation and labor relatively steady through the ZLB period. These results also support Beckworth (2019), and underscore important differences between a rate and a level target.

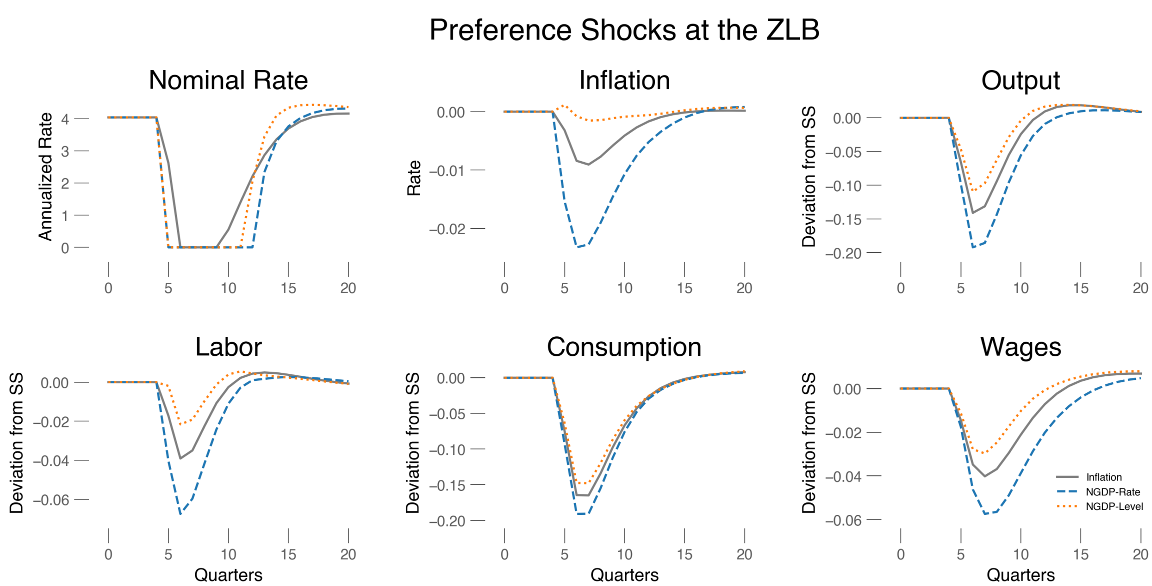

Figure 9: Results of two large preference shocks. Grey solid - Inflation. Blue dashed - NGDP rate. Yellow dotted - NGDP level.

5 Conclusion

In this paper, I prove analytically that output is more stable, but inflation is more volatile under NGDP targeting. This real-nominal trade-off is supported by impulse response functions and long-run simulations in a quantitative model. Ultimately welfare is virtually equivalent at the end of the simulation, suggesting that there would be little difference in macroeconomic performance over time.

With a zero lower-bound imposed in the model. NGDP targeting, particularly level targeting, performs well. Even though the volatility of inflation is higher under NGDP targeting, a binding ZLB leads to lower volatility under level targeting. This result suggests that forward guidance could be a particularly powerful tool with NGDP targeting.

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A Appendix

A1 Proof of Proposition 3

I first restate the system of four equations that I am solving, setting $\rho_r = 0$ for clarity:

$$\tilde{y}_t = \mathbb{E}[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) \tag{A.1}$$

$$\tilde{\pi}_t = \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] \tag{A.2}$$

$$\tilde{y}_t^f = \rho_a \tilde{y}_{t-1}^f + \frac{1+\eta}{\sigma+\eta}\varepsilon_{a,t} \tag{A.3}$$

$$\tilde{i}_t = \phi_N\left(\tilde{\pi}_t + \tilde{y}_t - \tilde{y}_{t-1}\right) + \sigma_r \varepsilon_{r,t} \tag{A.4}$$

In this model, there are two forward-looking jump variables: $\tilde{y}_t$ and $\tilde{\pi}_t$. There are also two state variables: $\tilde{y}_{t-1}$ and $\tilde{y}_t^f$.

Finding the Policy Functions

Conjecture that the jump variables are linear in the state variables:

$$\tilde{y}_t = \lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f \qquad\qquad \tilde{\pi}_t = \lambda_{2a}\tilde{y}_{t-1} + \lambda_{2b}\tilde{y}_t^f$$

Starting with equation A.2, I plug in the conjectures:

$$\begin{aligned}
\tilde{\pi}_t &= \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] \\
\lambda_{2a}\tilde{y}_{t-1} + \lambda_{2b}\tilde{y}_t^f &= \kappa\left(\lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f - \tilde{y}_t^f\right). + \beta\mathbb{E}_t\left[\lambda_{2a}\tilde{y}_t + \lambda_{2b}\tilde{y}_t^f\right] \\
\lambda_{2a}\tilde{y}_{t-1} + \lambda_{2b}\tilde{y}_t^f &= \kappa\lambda_{1a}\tilde{y}_{t-1} + \kappa\lambda_{1b}\tilde{y}_t^f - \kappa\tilde{y}_t^f + \beta\lambda_{2a}\left(\lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f\right) + \beta\lambda_{2b}\rho_a\tilde{y}_t^f
\end{aligned}$$

Now gather all terms on one side and factor by state variable:

$$0 = \lambda_{2a}\tilde{y}_{t-1} + \lambda_{2b}\tilde{y}_t^f - \kappa\lambda_{1a}\tilde{y}_{t-1} - \kappa\lambda_{1b}\tilde{y}_t^f + \kappa\tilde{y}_t^f - \beta\lambda_{2a}\left(\lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f\right) - \beta\lambda_{2b}\rho_a\tilde{y}_t^f$$
$$0 = \left[\lambda_{2a} - \kappa\lambda_{1a} - \beta\lambda_{2a}\lambda_{1a}\right]\tilde{y}_{t-1} + \left[\lambda_{2b} - \kappa\lambda_{1a} + \kappa - \beta\lambda_{2a}\lambda_{1b} - \beta\lambda_{2b}\rho_a\right]\tilde{y}_t^f$$

I now have two equations with four unknowns:

$$\lambda_{2a} - \kappa\lambda_{1a} - \beta\lambda_{2a}\lambda_{1a} = 0 \tag{A.5}$$
$$\lambda_{2b} - \kappa\lambda_{1b} + \kappa - \beta\lambda_{2a}\lambda_{1b} - \beta\lambda_{2b}\rho_a = 0 \tag{A.6}$$

Next, I substitute the NGDP Rule and the policy guesses into A.1:

$$\begin{aligned}
\tilde{y}_t &= \mathbb{E}_t[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) \\
\lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f &= \mathbb{E}_t[\lambda_{1a}\tilde{y}_t + \lambda_{1b}\tilde{y}_{t+1}^f] - \frac{1}{\sigma}\left(\phi_N\tilde{\pi}_t + \phi_N\tilde{y}_t - \phi_N\tilde{y}_{t-1}\right. \\
&\qquad \left. - \mathbb{E}_t[\lambda_{2a}\tilde{y}_t + \lambda_{2b}\tilde{y}_{t+1}^f]\right) \\
\lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f &= \lambda_{1a}(\lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f) + \lambda_{1b}\rho_a\tilde{y}_t^f - \frac{1}{\sigma}\Big(\phi_N(\lambda_{2a}\tilde{y}_{t-1} + \lambda_{2b}\tilde{y}_t^f) \\
&\quad + \phi_N(\lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f) - \phi_N\tilde{y}_{t-1} - \lambda_{2a}(\lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f) - \lambda_{2b}\rho_a\tilde{y}_t^f\Big)
\end{aligned}$$

This last equation can be rewritten by grouping terms:

$$\begin{aligned}
&\left[\lambda_{1a} - \lambda_{1a}^2 + \frac{\phi_N}{\sigma}\lambda_{2a} + \frac{\phi_N}{\sigma}\lambda_{1a} - \frac{\phi_N}{\sigma} - \frac{\lambda_{2a}\lambda_{1a}}{\sigma}\right]\tilde{y}_{t-1} \\
&\qquad + \left[\lambda_{1b} - \lambda_{1a}\lambda_{1b} - \lambda_{1b}\rho_a + \frac{\phi_N}{\sigma}\lambda_{2b} + \frac{\phi_N}{\sigma}\lambda_{1b} - \frac{\lambda_{2a}\lambda_{1b}}{\sigma} - \frac{\lambda_{2b}\rho_a}{\sigma}\right]\tilde{y}_t^f = 0
\end{aligned}$$

which gives me another two equations with four unknowns:

$$\lambda_{1a} - \lambda_{1a}^2 + \frac{\phi_N}{\sigma}\lambda_{2a} + \frac{\phi_N}{\sigma}\lambda_{1a} - \frac{\phi_N}{\sigma} - \frac{\lambda_{2a}\lambda_{1a}}{\sigma} = 0 \tag{A.7}$$

$$\lambda_{1b} - \lambda_{1a}\lambda_{1b} - \lambda_{1b}\rho_a + \frac{\phi_N}{\sigma}\lambda_{2b} + \frac{\phi_N}{\sigma}\lambda_{1b} - \frac{\lambda_{2a}\lambda_{1b}}{\sigma} - \frac{\lambda_{2b}\rho_a}{\sigma} = 0 \tag{A.8}$$

The equations A.5 through A.8 give me four equations with four unknowns. I can now solve the system. Starting with equation A.5:

$$\begin{aligned}
\lambda_{2a} - \kappa\lambda_{1a} - \beta\lambda_{2a}\lambda_{1a} &= 0 \\
\lambda_{1a}(\kappa + \beta\lambda_{2a}) &= \lambda_{2a} \\
\lambda_{1a} &= \frac{\lambda_{2a}}{\kappa + \beta\lambda_{2a}}
\end{aligned} \tag{A.9}$$

Moving to equation A.7:

$$\lambda_{1a} - \lambda_{1a}^2 + \frac{\phi_N}{\sigma}\lambda_{2a} + \frac{\phi_N}{\sigma}\lambda_{1a} - \frac{\phi_N}{\sigma} - \frac{\lambda_{2a}\lambda_{1a}}{\sigma} = 0$$

$$\frac{\lambda_{2a}}{\kappa + \beta\lambda_{2a}} - \frac{\lambda_{2a}^2}{(\kappa + \beta\lambda_{2a})^2} + \frac{\phi_N}{\sigma}\lambda_{2a} + \frac{\phi_N}{\sigma}\frac{\lambda_{2a}}{\kappa + \beta\lambda_{2a}} - \frac{\phi_N}{\sigma} - \frac{\lambda_{2a}^2}{\sigma(\kappa + \beta\lambda_{2a})} = 0 \tag{A.10}$$

A.10 is essentially a cubic equation. I now impose log-utility, setting $\sigma = 1$. Using a cubic-root solver, I find that the only solution for λ_{2a} that satisfies equations A.5-A.8 is:

$$\lambda_{2a}^* = \frac{\sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} + \beta - \kappa - 1}{2\beta} \tag{A.11}$$

Plugging A.11 into A.9 gives me a closed form expression for λ_{1a}:

$$\lambda_{1a}^* = \frac{\lambda_{2a}^*}{\kappa + \beta\lambda_{2a}^*} \tag{A.12}$$

Now I move on to equation A.6:

$$\begin{aligned}
\lambda_{2b} - \kappa\lambda_{1b} + \kappa - \beta\lambda_{2a}\lambda_{1b} - \beta\lambda_{2b}\rho_a &= 0 \\
\lambda_{2b} - \kappa\lambda_{1b} + \kappa - \beta\lambda_{2a}^*\lambda_{1b} - \beta\lambda_{2b}\rho_a &= 0 \\
\lambda_{2b}(1 - \beta\rho_a) &= \lambda_{1b}(\kappa + \beta\lambda_{2a}^*) - \kappa
\end{aligned}$$

$$\lambda_{2b} = \lambda_{1b}\left(\frac{\kappa + \beta\lambda_{2a}^*}{1 - \beta\rho_a}\right) - \frac{\kappa}{1 - \beta\rho_a} \tag{A.13}$$

Next, go to equation A.8:

$$\begin{aligned}
0 &= \lambda_{1b} - \lambda_{1a}\lambda_{1b} - \lambda_{1b}\rho_a + \frac{\phi_N}{\sigma}\lambda_{2b} + \frac{\phi_N}{\sigma}\lambda_{1b} - \frac{\lambda_{2a}\lambda_{1b}}{\sigma} - \frac{\lambda_{2b}\rho_a}{\sigma} \\
0 &= \lambda_{1b} - \frac{\lambda_{2a}^*}{\kappa + \beta\lambda_{2a}^*}\lambda_{1b} - \rho_a\lambda_{1b} + \frac{\phi_N}{\sigma}\left[\lambda_{1b}\left(\frac{\kappa + \beta\lambda_{2a}^*}{1 - \beta\rho_a}\right) - \frac{\kappa}{1 - \beta\rho_a}\right] \\
&\quad + \frac{\phi_N}{\sigma}\lambda_{1b} - \lambda_{2a}^*\frac{\lambda_{1b}}{\sigma} - \frac{\rho_a}{\sigma}\left[\lambda_{1b}\left(\frac{\kappa + \beta\lambda_{2a}^*}{1 - \beta\rho_a}\right) - \frac{\kappa}{1 - \beta\rho_a}\right]
\end{aligned}$$

Rearranging this equation leads me to a solution for λ_{1b}:

$$\begin{aligned}
\lambda_{1b}^* = \Bigg[1 - \frac{\lambda_{2a}^*}{\kappa + \beta\lambda_{2a}^*} - \rho_a + \frac{\phi_N}{\sigma}\left(\frac{\kappa + \beta\lambda_{2a}^*}{1 - \beta\rho_a}\right) + \frac{\phi_N}{\sigma} - \frac{\lambda_{2a}^*}{\sigma} \\
- \frac{\rho_a}{\sigma}\left(\frac{\kappa + \beta\lambda_{2a}^*}{1 - \beta\rho_a}\right)\Bigg]^{-1} \frac{\kappa}{1 - \beta\rho_a}(\phi_N - \rho_a)
\end{aligned} \tag{A.14}$$

I now go back to A.13 to find the solution for λ_{2b}:

$$\lambda_{2b}^* = \lambda_{1b}^* \left(\frac{\kappa + \beta \lambda_{2a}^*}{1 - \beta \rho_a} \right) - \frac{\kappa}{1 - \beta \rho_a} \tag{A.15}$$

Equations A.11, A.12, A.14, and A.15 provide closed-form solutions for the linear parameters. Therefore, my conjecture for the policy functions has been verified.

Solving for the Nominal Rate

I substitute the policy functions into the NGDP targeting rule:

$$\begin{aligned}
\tilde{i}_t &= \phi_N \left(\tilde{\pi}_t + \tilde{y}_t - \tilde{y}_{t-1} \right) + \sigma_r \varepsilon_{r,t} \\
&= \phi_N \left(\lambda_{2a}^* \tilde{y}_{t-1} + \lambda_{2b}^* \tilde{y}_t^f + \lambda_{1a}^* \tilde{y}_{t-1} + \lambda_{1b}^* \tilde{y}_t^f - \tilde{y}_{t-1} \right) + \sigma_r \varepsilon_{r,t} \\
&= \phi_N \left[\underbrace{\left(\lambda_{2a}^* + \lambda_{1a}^* - 1 \right)}_{\equiv A} \tilde{y}_{t-1} + \underbrace{\left(\lambda_{2b}^* + \lambda_{1b}^* \right)}_{\equiv B} \tilde{y}_t^f \right] + \sigma_r \varepsilon_{r,t}
\end{aligned} \tag{A.16}$$

Looking just at term A, I plug-in for the coefficients and simplify to obtain:

$$\begin{aligned}
\lambda_{2a}^* + \lambda_{1a}^* - 1 &= \frac{\sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} + \beta - \kappa - 1}{2\beta} \\
&\quad + \frac{\sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} + \beta - \kappa - 1}{\beta \left(\kappa + \sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} + \beta - 1 \right)} - 1 \\
&= \sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} + \beta - \kappa - 1 + \\
&\quad + \frac{2 \left(\sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} + \beta - \kappa - 1 \right)}{\kappa + \sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} + \beta - 1} - 2\beta
\end{aligned}$$

Clearing fractions and simplifying yields:

$$
\begin{aligned}
&= \kappa\sqrt{\beta^2 + 2\beta(\kappa-1) + (\kappa+1)^2} + \kappa(\beta-\kappa-1) + \beta^2 + 2\beta(\kappa-1) + (\kappa+1)^2 \\
&+ (\beta-\kappa-1)\sqrt{\beta^2 + 2\beta(\kappa-1) + (\kappa+1)^2} + (\beta-1)\sqrt{\beta^2 + 2\beta(\kappa-1) + (\kappa+1)^2} \\
&+ (\beta-\kappa-1)(\beta-1) + 2\sqrt{\beta^2 + 2\beta(\kappa-1) + (\kappa+1)^2} \\
&+ 2(\beta-\kappa-1) - 2\beta(\beta+\kappa-1) - 2\beta\sqrt{\beta^2 + 2\beta(\kappa-1) + (\kappa+1)^2}
\end{aligned}
$$

All the square-root terms cancel each other out, leaving:

$$
\begin{aligned}
&= \kappa\beta - \kappa^2 - \kappa + +\beta^2 + 2\beta\kappa - 2\beta + \kappa^2 + 2\kappa + 1 + \beta^2 \\
&\qquad - \beta - \kappa\beta + \kappa - \beta + 1 + 2\beta - 2\kappa - 2 - 2\beta^2 - 2\beta\kappa + 2\beta \\
&= 0
\end{aligned}
$$

So term $A = 0$. I now move to term B. Plugging coefficients in and simplifying gives:

$$
\begin{aligned}
\lambda_{2b}^* + \lambda_{1b}^* &= \lambda_{1b}^*\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a} + 1\right) - \frac{\kappa}{1-\beta\rho_a} \\
&= \underbrace{\left[1 - \frac{\lambda_{2a}^*}{\kappa + \beta\lambda_{2a}^*} - \rho_a + \phi_N\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a} + \phi_N - \lambda_{2a}^* - \rho_a\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a}\right)\right)\right]^{-1}}_{\equiv C} \\
&\qquad \times \underbrace{\frac{\kappa}{1-\beta\rho_a}(\phi_N - \rho_a)\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a} + 1\right)}_{\equiv D} - \underbrace{\frac{\kappa}{1-\beta\rho_a}}_{\equiv F}
\end{aligned}
$$

Looking just at term C:

$$
\begin{aligned}
C &= \left[1 - \frac{\lambda_{2a}^*}{\kappa + \beta\lambda_{2a}^*} - \rho_a + \phi_N\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a} + \phi_N - \lambda_{2a}^* - \rho_a\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a}\right)\right)\right]^{-1} \\
&= (\kappa + \beta\lambda_{2a}^*)(1-\beta\rho_a)\cdot\Xi
\end{aligned}
$$

Ξ is defined as:

$$\Xi = \big[(\kappa + \beta\lambda_{2a}^{*})(1-\beta\rho_a) - \lambda_{2a}^{*}(1-\beta\rho_a) - \rho_a(\kappa+\beta\lambda_{2a}^{*}(1-\beta\rho_a) \\ + \phi_N(\kappa+\beta\lambda_{2a}^{*})(1-\beta\rho_a) - \lambda_{2a}^{*}(\kappa+\beta\lambda_{2a}^{*})(1-\beta\rho_a)\rho_a(\kappa+\beta\lambda_{2a}^{*})^2\big]^{-1}$$

Multiply C by D:

$$C \cdot D = \big[\kappa(\kappa+\beta\lambda_{2a}^{*})^2(1-\beta\rho_a)^{-1}(\phi_N-\rho_a) + \kappa(\kappa+\beta\lambda_{2a}^{*})(\phi_N-\rho_a)\big] \cdot \Xi$$

Giving F a common denominator and subtracting it from $C \cdot D$ gives:

$$C \cdot D - F = \big[\kappa(\kappa+\beta\lambda_{2a}^{*})^2(1-\beta\rho_a)^{-1}(\phi_N-\rho_a) + \kappa(\kappa+\beta\lambda_{2a}^{*})(\phi_N-\rho_a) \\ - \kappa(1-\beta\rho_a)^{-1} \cdot \Xi^{-1}\big] \cdot \Xi$$

Since Ξ is entirely in the denominator, I drop it and focus only on the numerator:

$$\propto \kappa(\kappa+\beta\lambda_{2a}^{*})^2(1-\beta\rho_a)^{-1}(\phi_N-\rho_a) + \kappa(\kappa+\beta\lambda_{2a}^{*})(\phi_N-\rho_a) - \kappa(1-\beta\rho_a)^{-1}\Xi^{-1}$$
$$= \left(\kappa^3 + 2\kappa^2\beta\lambda_{2a}^{*} + \beta^2\kappa(\lambda_{2a}^{*})^2\right)(1-\beta\rho_a)^{-1}(\phi_N-\rho_a) + \phi_N\kappa^2 + \beta\phi_N\kappa\lambda_{2a}^{*} - \rho_a\kappa^2 - \beta\rho_a\kappa\lambda_{2a}^{*} \\ - \kappa^2 - \kappa\beta\lambda_{2a}^{*} + \kappa\lambda_{2a}^{*} + \kappa^2\rho_a + \kappa\rho_a\beta\lambda_{2a}^{*} - \kappa\phi_N\left(\kappa^2 + 2\beta\kappa\lambda_{2a}^{*} + \beta^2\lambda_{2a}^{*}\right)(1-\beta\rho_a)^{-1} \\ - \kappa^2\phi_N - \kappa\phi_N\beta\lambda_{2a}^{*} + \kappa^2\lambda_{2a}^{*} + \kappa\beta(\lambda_{2a}^{*})^2 + \rho_a\kappa\left(\kappa^2 + 2\kappa\beta + \beta^2(\lambda_{2a}^{*})^2\right)(1-\beta\rho_a)^{-1}$$

After simplifying, the numerator boils down to:

$$= \kappa\beta(\lambda_{2a}^{*})^2 + \left(\kappa^2 - \kappa\beta + \kappa\right)\lambda_{2a}^{*} - \kappa^2$$

Now I plug-in for λ_{2a}^* :

$$
\begin{aligned}
&= \frac{\kappa}{4\beta}\left[\beta^2 + 2\beta(\kappa-1) + (\kappa+1)^2 + 2(\beta-\kappa-1)\sqrt{\beta^2+2\beta(\kappa-1)+(\kappa+1)^2} + (\beta-\kappa-1)^2\right] \\
&\quad + \frac{\kappa^2}{2\beta}\sqrt{\beta^2+2\beta(\kappa-1)+(\kappa+1)^2} - \frac{\kappa}{2}\sqrt{\beta^2+2\beta(\kappa-1)+(\kappa+1)^2} \\
&\quad + \frac{\kappa}{2\beta}\sqrt{\beta^2+2\beta(\kappa-1)+(\kappa+1)^2} + \frac{1}{2\beta}(\beta-\kappa-1)(\kappa^2-\kappa\beta+\kappa) - \kappa^2
\end{aligned}
$$

After distributing terms and simplifying, I find that the numerator also equals zero. Therefore $B = 0$. Plugging A and B into equation A.16 then yields:

$$\tilde{i}_t = \sigma_r \varepsilon_{r,t}$$

With Cost-Push Shocks

Including cost-push shock μ_t gives the following four equation system:

$$\tilde{y}_t = \mathbb{E}[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) \tag{A.17}$$

$$\tilde{\pi}_t = \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] + \mu_t \tag{A.18}$$

$$\tilde{y}_t^f = \rho_a \tilde{y}_{t-1}^f + \frac{1+\eta}{\sigma+\eta}\varepsilon_{a,t} \tag{A.19}$$

$$\tilde{i}_t = \phi_N\left(\tilde{\pi}_t + \tilde{y}_t - \tilde{y}_{t-1}\right) + \sigma_r\varepsilon_{r,t} \tag{A.20}$$

As before, I conjecture linear policy functions for the two jump variables. This time, μ_t is included as an exogenous variable:

$$\tilde{y}_t = \lambda_{1a}\tilde{y}_{t-1} + \lambda_{1b}\tilde{y}_t^f + \theta_1\mu_t \qquad\qquad \tilde{\pi}_t = \lambda_{2a}\tilde{y}_{t-1} + \lambda_{2b}\tilde{y}_t^f + \theta_2\mu_t$$

Now, after plugging the policy function guesses into the DIS and NKPC curves, there will be six equations with six unknowns. The two extra equations are:

$$\theta_2 - \theta_1\kappa - \beta\theta_1\lambda_{2a} - 1 = 0 \tag{A.21}$$

$$\theta_1 - \lambda_{1a}\theta_1 + \phi_N\theta_2 + \phi_N\theta_1 - \lambda_{2a}\theta_1 = 0 \tag{A.22}$$

Starting with A.22:

$$\theta_1 - \lambda_{1a}\theta_1 + \phi_N\theta_2 + \phi_N\theta_1 - \lambda_{2a}\theta_1 = 0$$
$$\theta_1 + \phi_N\theta_2 + \phi_N\theta_1 - (\lambda_{1a} + \lambda_{2a})\theta_1 = 0$$

Note that in the first part of the proof, $\lambda_{1a}^* + \lambda_{2a}^* = 1$. Therefore:

$$\theta_1 + \phi_N\theta_2 + \phi_N\theta_1 - \theta_1 = 0$$
$$\phi_N\theta_1 = -\phi_N\theta_2$$
$$\theta_1 = -\theta_2$$

Plugging this result into A.21 yields:

$$\theta_2 - \theta_1\kappa - \beta\theta_1\lambda_{2a} - 1 = 0$$
$$-\theta_1 - \theta_1\kappa - \beta\theta_1\lambda_{2a} - = 1$$
$$\theta_1(1 + \kappa + \beta\lambda_{2a}) = -1$$
$$\theta_1^* = -\frac{1}{1 + \kappa + \beta\lambda_{2a}^*}$$
$$\theta_2^* = \frac{1}{1 + \kappa + \beta\lambda_{2a}^*}$$

The linear policy function guess is again verified. Plug everything into A.20:

$$\begin{aligned}
\tilde{i}_t &= \phi_N \Big(\left(\lambda_{1a}^* + \lambda_{2a}^* - 1\right) \tilde{y}_{t-1} + \left(\lambda_{1b}^* + \lambda_{2b}^*\right) \tilde{y}_t^f + \left(\theta_1^* + \theta_2^*\right) \mu_t \Big) + \sigma_r \varepsilon_{r,t} \\
&= \phi_N \left(\theta_1^* + \theta_2^*\right) \mu_t + \sigma_r \varepsilon_{r,t} \\
&= \phi_N \left(\theta_1^* - \theta_1^*\right) + \sigma_r \varepsilon_{r,t} \\
\tilde{i}_t &= \sigma_r \varepsilon_{r,t}
\end{aligned}$$

Once again, the nominal interest rate only responds to shocks directly to the rate itself. This concludes the proof of lemma 1.

Intuitively

I can also demonstrate Lemma 1 intuitively. Begin with the DIS equation of (A.1) and impose $\sigma = 1$:

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - \left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right)$$

Now rearrange the equation to get the nominal rate on the left-hand side:

$$\tilde{i}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - \tilde{y}_t + \mathbb{E}_t[\tilde{\pi}_{t+1}]$$

Notice that the right-hand side looks like the NGDP targeting rule. Substitute (A.4) in:

$$\tilde{i}_t = \phi_\pi^{-1} \mathbb{E}_t[\tilde{i}_{t+1}]$$

Rearrange to get the expectation of tomorrow's interest rate on the left-hand side.

$$\mathbb{E}_t[\tilde{i}_{t+1}] = \phi_\pi \tilde{i}_t \tag{A.23}$$

A.23 says that the interest rate tomorrow is proportional to the interest rate today. Because the Taylor principle must be satisfied for the system to have a unique equilibrium, $\phi_\pi > 1$. Because $\phi_\pi > 1$, if $\tilde{i}_t \neq 0$, the system will explode. As such, $\tilde{i}_t = 0$ by necessity.

A2 Proof of Proposition 1

I first solve for the policy functions under inflation targeting. Assuming that $\rho_r = 0$ for simplicity, the four equations I need to solve are:

$$\tilde{y}_t = \mathbb{E}[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) \tag{A.24}$$

$$\tilde{\pi}_t = \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] \tag{A.25}$$

$$\tilde{y}_t^f = \rho_a \tilde{y}_{t-1}^f + \frac{1+\eta}{\sigma+\eta}\varepsilon_{a,t} \tag{A.26}$$

$$\tilde{i}_t = \phi_\pi \tilde{\pi}_t + \sigma_r \varepsilon_{r,t} \tag{A.27}$$

Conjecture that the jump variables, $\tilde{y}_t$ and $\tilde{\pi}_t$, are linear in the state variable $\tilde{y}_t^f$:

$$\tilde{y}_t = \lambda_1 \tilde{y}_t^f \qquad\qquad \tilde{\pi}_t = \lambda_2 \tilde{y}_t^f$$

Starting with A.25, substitute in the policy function guesses:

$$\begin{aligned}
\tilde{\pi}_t &= \kappa(\tilde{y}_t - \tilde{y}_t^f) + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] \\
\lambda_1 \tilde{y}_t^f &= \kappa(\lambda_1 \tilde{y}_t^f - \tilde{y}_t^f) + \beta\mathbb{E}_t[\lambda_2 \tilde{y}_t^f] \\
0 &= \lambda_2 \tilde{y}_t^f - \kappa(\lambda_1 - 1)\tilde{y}_t^f - \beta\rho_a\lambda_2\tilde{y}_t^f \\
0 &= [\lambda_2 - \kappa(\lambda_1 - 1) - \beta\rho_a\lambda_2]\,\tilde{y}_t^f
\end{aligned} \tag{A.28}$$

Now solve A.24:

$$\begin{aligned}
\tilde{y}_t &= \mathbb{E}[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]\right) \\
\lambda_1 \tilde{y}_t^f &= \lambda_1\rho_a\tilde{y}_t^f - \frac{1}{\sigma}(\phi_\pi\tilde{\pi}_t - \lambda_2\rho_a\tilde{y}_t^f) \\
0 &= \lambda_1\tilde{y}_t^f - \lambda_1\rho_a\tilde{y}_t^f + \frac{\phi_\pi}{\sigma}\lambda_2\tilde{y}_t^f - \frac{\lambda_2\rho_a}{\sigma}\tilde{y}_t^f
\end{aligned}$$

$$0 = \left[\lambda_1 - \lambda_1\rho_a + \frac{\phi_\pi}{\sigma}\lambda_2 - \frac{\lambda_2\rho_a}{\sigma}\right]\tilde{y}_t^f \tag{A.29}$$

Solve the coefficient in A.29 for λ_1:

$$\begin{aligned}
\lambda_1 - \lambda_1\rho_a + \frac{\phi_\pi}{\sigma}\lambda_2 - \frac{\lambda_2\rho_a}{\sigma} = 0 \\
\lambda_1(1-\rho_a) = \lambda_2\left(\frac{\rho_a}{\sigma} - \frac{\phi_\pi}{\sigma}\right) \\
\lambda_1 = \frac{\lambda_2}{1-\rho_a}\left(\frac{\rho_a}{\sigma} - \frac{\phi_\pi}{\sigma}\right) \qquad \text{(A.30)}
\end{aligned}$$

Plug A.30 into the coefficient of A.28:

$$\begin{aligned}
0 &= \lambda_2 - \kappa(\lambda_1 - 1) - \beta\rho_a\lambda_2 \\
0 &= \lambda_2 - \kappa\left(\frac{\lambda_2}{1-\rho_a}\left(\frac{\rho_a}{\sigma} - \frac{\phi_\pi}{\sigma}\right) - 1\right) - \beta\rho_a\lambda_2 \\
0 &= \lambda_2(1-\beta\rho_a) - \lambda_2\frac{\kappa(\rho_a - \phi_\pi)}{\sigma(1-\rho_a)} + \kappa \\
\lambda_2^* &= \frac{\sigma\kappa(1-\rho_a)}{\sigma(1-\beta\rho_a)(\rho_a - 1) - \kappa(\phi_\pi - \rho_a)} \qquad \text{(A.31)}
\end{aligned}$$

Plug A.31 into A.30:

$$\begin{aligned}
\lambda_1 &= \lambda_2^*\frac{\rho_a - \phi_\pi}{\sigma(1-\rho_a)} \qquad \text{(A.32)} \\
\lambda_1^* &= \frac{\kappa(\phi_\pi - \rho_a)}{(1-\beta\rho_a)(1-\rho_a) - \kappa(\rho_a - \phi_\pi)} \qquad \text{(A.33)}
\end{aligned}$$

Now compare the policy function for output under NGDP targeting and under inflation targeting, setting $\phi_N = \phi_\pi$ and $\sigma = 1$:

$$\begin{aligned}
\tilde{y}_t^{inf} &= \frac{\kappa(\phi_\pi - \rho_a)}{(1-\beta\rho_a)(1-\rho_a) - \kappa(\rho_a - \phi_\pi)}\tilde{y}_t^f \\
\tilde{y}_t^N &= \frac{\lambda_{2a}^*}{\kappa + \beta\lambda_{2a}^*}\tilde{y}_{t-1} + \left[1 - \frac{\lambda_{2a}^*}{\kappa + \beta\lambda_{2a}^*} - \rho_a + \phi_\pi\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a}\right) + \phi_\pi - \lambda_{2a}^*\right. \\
&\qquad \left. - \rho_a\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a}\right)\right]^{-1} \frac{\kappa}{1-\beta\rho_a}(\phi_\pi - \rho_a)\tilde{y}_t^f
\end{aligned}$$

Note that before a shock, $\tilde{y}_{t-1} = 0$. Therefore, I focus on the coefficients on $\tilde{y}_t^f$. Notice that the numerators are the same for both policy rules:

$$numerator_{inf} = numerator_N = \kappa(\phi_\pi - \rho_a)$$

Therefore, I focus only on the denominators:

$$\begin{aligned}
den_{inf} &= (1-\beta\rho_a)(1-\rho-a) + \kappa(\phi_\pi - \rho_a) \\
den_N &= \left[1 - \frac{\lambda_{2a}^*}{\kappa + \beta\lambda_{2a}^*} - \rho_a + \phi_\pi\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a}\right) + \phi_\pi - \lambda_{2a}^*\rho_a\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a}\right)\right](1-\beta\rho_a) \\
&= \left[1 - \lambda_1^* + (\phi_\pi - \rho_a) + (\phi_\pi - \rho_a)\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a}\right) - \lambda_{2a}^*\right](1-\beta\rho_a) \\
&= \left[\lambda_2^* + (\phi_\pi - \rho_a) + (\phi_\pi - \rho_a)\left(\frac{\kappa + \beta\lambda_{2a}^*}{1-\beta\rho_a}\right) - \lambda_{2a}^*\right](1-\beta\rho_a) \\
&= (\phi_\pi - \rho_a)(\kappa + \beta\lambda_{2a}^* + 1 - \beta\rho_a)
\end{aligned}$$

Compare the denominators now. For NGDP targeting to result in more stable output, the denominator of the NGDP policy rule must be greater than the denominator of the inflation policy rule. When does this occur?

$$
\begin{aligned}
(1-\beta\rho_a)(1-\rho-a)+\kappa(\phi_\pi-\rho_a) &\leq (\phi_\pi-\rho_a)(\kappa+\beta\lambda_{2a}^*+1-\beta\rho_a) \\
\frac{(1-\beta\rho_a)(1-\rho_a)}{\phi_\pi-\rho_a}+\kappa &\leq \kappa+\beta\lambda_{2a}^*+1-\beta\rho_a \\
\frac{(1-\beta\rho_a)(1-\rho_a)}{\phi_\pi-\rho_a} &\leq \beta\lambda_{2a}^*+1-\beta\rho_a \\
\frac{(1-\beta\rho_a)(1-\rho_a)}{\beta\lambda_{2a}^*+1-\beta\rho_a}+\rho_a &\leq \phi_\pi
\end{aligned}
$$

This proof holds as long as $\beta\lambda_{2a}^*+1-\rho_a>0$. First, I show that $0<\lambda_{2a}^*<1$. Suppose that $\lambda_{2a}^*<1$. Then:

$$
\begin{aligned}
\frac{\sqrt{\beta^2+2\beta(\kappa-1)+(\kappa+1)^2}+\beta-\kappa-1}{2\beta} &< 1 \\
\sqrt{\beta^2+2\beta(\kappa-1)+(\kappa+1)^2}+\beta-\kappa-1 &< 2\beta \\
\sqrt{\beta^2+2\beta(\kappa-1)+(\kappa+1)^2} &< \beta+\kappa+1
\end{aligned}
$$

Both sides are positive, so I can square both sides:

$$
\begin{aligned}
\beta^2+2\beta(\kappa-1)+(\kappa+1)^2 &< \beta^2+2\kappa\beta+2\beta+\kappa^2+2\kappa+1 \\
-2\beta &< 2\beta \\
0 &< 4\beta \\
0 &< \beta
\end{aligned}
$$

β is always greater than 0, so $\lambda^*_{2a} < 1$. Now suppose that $\lambda^*_{2a} > 0$. Then:

$$\frac{\sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} + \beta - \kappa - 1}{2\beta} > 0$$

$$\sqrt{\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2} > \kappa + 1 - \beta$$

$$\beta^2 + 2\beta(\kappa - 1) + (\kappa + 1)^2 > \kappa^2 + 2\kappa - 2\beta\kappa - 2\beta + \beta^2 + 1$$

$$2\beta\kappa > -2\beta\kappa$$

$$4\beta\kappa > 0$$

$$\beta\kappa > 0$$

$\beta\kappa$ is always greater than 0, so $0 < \lambda^*_{2a} < 1$. Further:

$$0 < \lambda^*_{2a} < 1$$

$$0 < \beta\lambda^*_{2a} < \beta$$

$$0 < \beta\lambda^*_{2a} < 1$$

The last step follows as $\beta < 1$. In addition:

$$0 < \beta < 1$$

$$0 < \beta\rho_a < \rho_a$$

$$0 < \beta\rho_a < 1$$

$$-1 < \beta\rho_a - 1 < 0$$

$$0 < 1 - \beta\rho_a < 1$$

Sum these two inequalities together:

$$0 < \beta\lambda_{2a}^* + 1 - \beta\rho_a < 2$$

This shows that $\beta\lambda_{2a}^* + 1 - \beta\rho_a > 0$, which is what we needed to show to verify the proof. Recall the expression:

$$\frac{(1-\beta\rho_a)(1-\rho_a)}{\beta\lambda_{2a}^* + 1 - \beta\rho_a} + \rho_a \leq \phi_\pi$$

Now, because $0 < \lambda_{2a}^* < 1$, the first term of the left-hand side will be less than $(1-\rho_a)$. Therefore, the whole left-hand side will be less than $(1-\rho_a) + \rho_a = 1$. As such, as long as $\phi_\pi > 1$, meaning that the Taylor principle is satisfied, proposition 1 will hold.

This concludes the proof of proposition 1.

A3 Proof of Proposition 2

Compare the policy function for inflation under NGDP targeting and under inflation targeting:

$$\tilde{\pi}_t^{inf} = \frac{\kappa(1-\rho_a)}{(1-\beta\rho_a)(\rho_a-1)-\kappa(\phi_\pi-\rho_a)}\tilde{y}_t^f$$
$$\tilde{\pi}_t^{N} = \lambda_{2a}^*\tilde{y}_{t-1} + \left[\lambda_{1b}^*\frac{\kappa+\beta\lambda_{2a}^*}{1-\beta\rho_a} - \frac{\kappa}{1-\beta\rho_a}\right]\tilde{y}_t^f$$

Again, $\tilde{y}_{t-1} = 0$ before a shock. The coefficient on $\tilde{y}_t^f$ for the NGDP rule can be simplified to:

$$\frac{-\kappa}{\kappa+\beta\lambda_{2a}^*+1-\beta\rho-a}$$

Inflation is more volatile under NGDP targeting when the coefficient from NGDP targeting is more negative than the coefficient from inflation targeting. This occurs when:

$$\frac{-\kappa}{\kappa+\beta\lambda_{2a}^*+1-\beta\rho-a} \leq \frac{\kappa(1-\rho_a)}{(1-\beta\rho_a)(\rho_a-1)-\kappa(\phi_\pi-\rho_a)}\tilde{y}_t^f$$
$$\frac{-1}{\kappa+\beta\lambda_{2a}^*+1-\beta\rho_a} \leq \frac{1-\rho_a}{(1-\beta\rho_a)(\rho_a-1)-\kappa(\phi_\pi-\rho_a)}$$
$$\kappa(\phi_\pi-\rho_a)-(1-\beta\rho_a)(\rho_a-1) \geq (1-\rho_a)(\kappa+\beta\lambda_{2a}^*+1-\beta\rho_a)$$
$$\kappa(\phi_\pi-\rho_a) \geq (1-\rho_a)(\kappa+\beta\lambda_{2a}^*+1-\beta\rho_a)+(1-\beta\rho_a)(\rho_a-1)$$
$$\phi_\pi \geq \frac{(1-\rho_a)(\kappa+\beta\lambda_{2a}^*)}{\kappa}+\rho_a$$

This proof holds as long as $\kappa+\beta\lambda_{2a}^*+1-\beta\rho_a$ is greater than zero. I know from appendix A2 that:

$$\beta\lambda_{2a}^*+1-\beta\rho_a > 0$$

In addition, I know that $\kappa > 0$. Add these two inequalities together:

$$\kappa + \beta\lambda_{2a}^{*} + 1 - \beta\rho_a > 0$$

The proof is thus verified. This concludes the proof of proposition 2.

A4 Interest Rate Under Different CRRA Parameterizations

I plot the response of the nominal rate under different values for the CRRA parameter, σ. This graph suggests that at least for productivity shocks, monetary policy moves less under NGDP targeting.

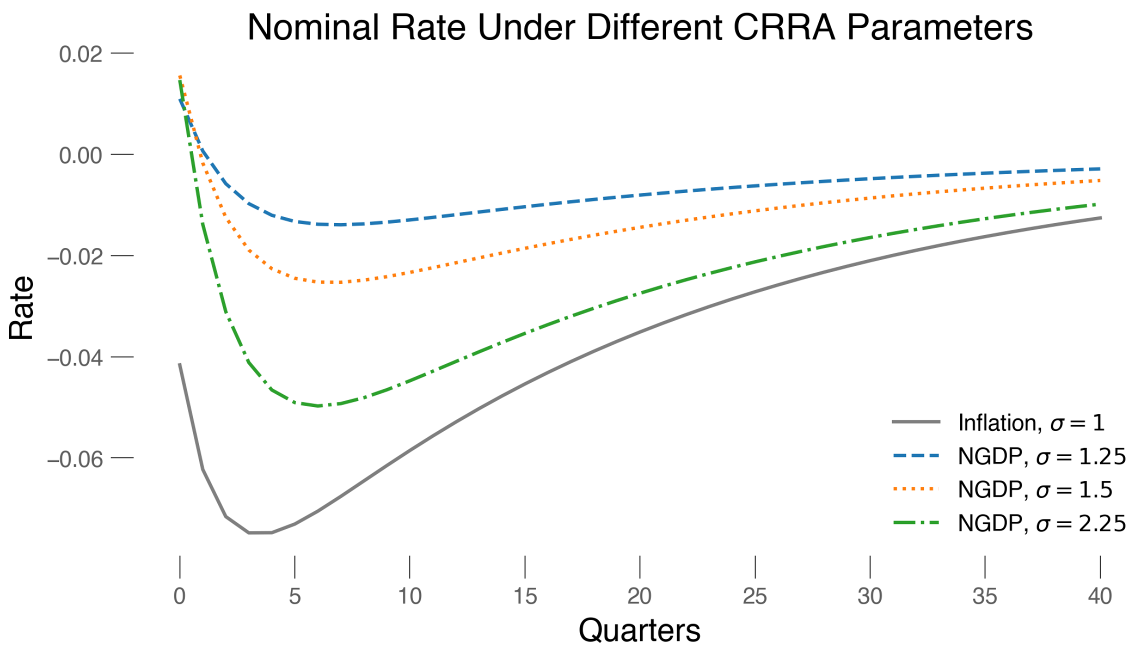

Figure 10: The nominal rate in response to a productivity shock under different values for the CRRA parameter in the NGDP model. Grey solid - Inflation, $\sigma = 1$. Blue dashed - NGDP, $\sigma = 1.25$. Yellow dotted - NGDP, $\sigma = 1.5$. Green dash-dotted - NGDP, $\sigma = 2.25$.

A5 Equilibrium Equations for the Quantitative Model

This model is similar to the Smets & Wouters (2007) model. It contains the typical medium scale frictions. The model is solved about a zero-inflation and labor of unity steady state. Below I list the equilibrium equations:

$$\mu_t = \frac{1}{C_t - \mathcal{H}C_{t-1}} - \nu_t \beta \mathbb{E}_t \left[\frac{\mathcal{H}}{C_{t+1} - \mathcal{H}C_t} \right] \tag{A.34}$$

$$\chi L_t^{\eta} = mrs_t \mu_t \tag{A.35}$$

$$\mu_t = \mathbb{E}_t \left[\beta \mu_{t+1} \Pi_{t+1}^{-1} (1 + i_t) \right] \tag{A.36}$$

$$\Lambda_{t,t+1} = \nu_t \beta \frac{\mathbb{E}_t[\mu_{t+1}]}{\mu_t} \tag{A.37}$$

$$w_t^* = \frac{\varepsilon_{w,t}}{\varepsilon_{w,t} - 1} \frac{f_{1,t}}{f_{2,t}} \tag{A.38}$$

$$f_{1,t} = mrs_t w_t^{\varepsilon_{w,t}} L_{d,t} + \phi_w \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_{t+1}^{\varepsilon_{w,t}} \Pi_t^{-\varepsilon_{w,t}\gamma_w} f_{1,t+1} \right] \tag{A.39}$$

$$f_{2,t} = w_t^{\varepsilon_{w,t}} L_{d,t} + \phi_w \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_{t+1}^{\varepsilon_{w,t}-1} \Pi_t^{(1-\varepsilon_{w,t})\gamma_w} f_{2,t+1} \right] \tag{A.40}$$

$$L_t = L_{d,t} v_t^w \tag{A.41}$$

$$v_t^w = (1 - \phi_w) \left(\frac{w_t^*}{w_t} \right)^{-\varepsilon_{w,t}} + \phi_w \left(\frac{\Pi_t}{\Pi_{t-1}^{\gamma_w}} \right)^{\varepsilon_{w,t}} \left(\frac{w_t}{w_{t-1}} \right)^{\varepsilon_{w,t}} v_{t-1}^w \tag{A.42}$$

$$w_t^{1-\varepsilon_{w,t}} = (1 - \phi_w)(w_t^*)^{1-\varepsilon_{w,t}} + \phi_w \left(\frac{\Pi_{t-1}^{\gamma_w}}{\Pi_t} w_{t-1} \right)^{1-\varepsilon_{w,t}} \tag{A.43}$$

$$p_t^* = \frac{\varepsilon_{p,t}}{\varepsilon_{p,t} - 1} \frac{x_{1,t}}{x_{2,t}} \tag{A.44}$$

$$x_{1,t} = p_{m,t} Y_t + \phi_p \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_t^{-\varepsilon_{p,t}\gamma_p} \Pi_{t+1}^{\varepsilon_{p,t}} x_{1,t+1} \right] \tag{A.45}$$

$$x_{2,t} = Y_t + \phi_p \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi_t^{(1-\varepsilon_{p,t})\gamma_p} \Pi_{t+1}^{\varepsilon_{p,t}-1} x_{2,t+1} \right] \tag{A.46}$$

$$Y_{m,t} = Y_t v_t^p \tag{A.47}$$

$$v_t^p = (1 - \phi_p)(p_t^*)^{-\varepsilon_{p,t}} + \phi_p \Pi_t^{\varepsilon_{p,t}} \Pi_{t-1}^{-\varepsilon_{p,t}\gamma_p} v_{t-1}^p \tag{A.48}$$

$$1 = (1 - \phi_p)(p_t^*)^{1-\varepsilon_{p,t}} + \phi_p \Pi_{t-1}^{\gamma_p(1-\varepsilon_{p,t})} \Pi_t^{\varepsilon_{p,t}-1} \tag{A.49}$$

$$Y_{m,t} = A_t \left(u_t K_t\right)^{\alpha} L_{d,t}^{1-\alpha} \tag{A.50}$$

$$K_{t+1} = \hat{I}_t + (1 - \delta(u_t))K_t \tag{A.51}$$

$$w_t = (1-\alpha) p_{m,t} A_t (u_t K_t)^{\alpha} L_{d,t}^{-\alpha} \tag{A.52}$$

$$p_t^k \delta'(u_t) = p_{m,t} A_t \alpha (u_t K_t)^{\alpha-1} L_{d,t}^{1-\alpha} \tag{A.53}$$

$$p_t^k = \mathbb{E}_t \left[\alpha \Lambda_{t,t+1} p_{m,t+1} A_{t+1} u_{t+1}^{\alpha} K_{t+1}^{\alpha-1} L_{d,t+1}^{1-\alpha} + p_{t+1}^k (1 - \delta(u_t))\right] \tag{A.54}$$

$$\hat{I}_t = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{A.55}$$

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_{a,t} \tag{A.56}$$

$$\ln(G_t) = (1-\rho_g)\ln(G_{ss}) + \rho_g \ln(G_{t-1}) + \varepsilon_{g,t} \tag{A.57}$$

$$\ln(\nu_t) = \rho_\nu \ln(\nu_{t-1}) + \varepsilon_{\nu,t} \tag{A.58}$$

$$cp_t = \frac{1}{\varepsilon_{p,t} - 1} \tag{A.59}$$

$$cp_t = (1-\rho_{cp}) cp_{ss} + \rho_{cp} cp_{t-1} + \varepsilon_{cp,t} \tag{A.60}$$

$$cpw_t = \frac{1}{\varepsilon_{w,t} - 1} \tag{A.61}$$

$$cpw_t = (1-\rho_{cpw_t}) cpw_{ss} + \rho_{cpw} cpw_{t-1} + \varepsilon_{cpw,t} \tag{A.62}$$

$$Y_t = C_t + I_t + G_t \tag{A.63}$$

$$Welf_t = \ln(C_t - \mathcal{H}C_{t-1}) - \frac{\chi L_t^{1+\eta}}{1+\eta} + \beta \mathbb{E}_t \left[Welf_{t+1}\right] \tag{A.64}$$

$$\begin{aligned} 1 = (1+x_t) p_t^k \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] - (1+x_t) p_t^k \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right) \\ + \mathbb{E}_t \left[\Lambda_{t,t+1} (1+x_{t+1}) p_{t+1}^k \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right] \end{aligned} \tag{A.65}$$

$$\ln(1+x_t) = \rho_x \ln(1+x_{t-1}) + \varepsilon_{x,t} \tag{A.66}$$

$$\begin{aligned} \ln(1+i_t) = (1-\rho_r)\ln(1+i_{ss}) + \rho_r \ln(1+i_{t-1}) \\ + (1-\rho_r)\left[\phi_\pi \ln(\Pi_t) + \phi_y \left(\ln(Y_t) - \ln(Y_{t-1})\right)\right] + \varepsilon_{i,t} \end{aligned} \tag{A.67}$$

$$\varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + \varepsilon_{r,t} \tag{A.68}$$

The unknown variables are: μ, C, L, mrs, Π, i, Λ, w^*, f_1, f_2, L_d, v^w, w, p^*, x_1, x_2, p_m, Y, Y_m, v_p, A, u, K, $\hat{I}$, p_k, I, G, ε_p, ε_w, cp, cpw, x, ε_i, ν, and $Welf$.

A6 Parameter Values

Table A1

Parameter Values

Parameter	Description	Value
$\mathcal{H}$	*habit persistence*	0.600
β	*discount factor*	0.99
χ	*labor disutility*	$\mu_{ss} \cdot mrs_{ss}/L_{ss}^{\eta}$
$\varepsilon_{w,t}$	*elasticity of labor*	11
ϕ_w	*wage stickiness*	0.786
γ_w	*wage indexation*	0.796
ε_p^{ss}	*steady state elasticity of price*	11
ϕ_p	*price stickiness*	0.705
γ_p	*price indexation*	0.365
α	*capital share*	0.333
δ_0	*constant depreciation*	0.025
δ_1	*linear depreciation*	$\frac{1}{\beta} - 1 + \delta_0$
δ_2	*quadratic depreciation*	0.01
κ	*adjustment cost*	4.243
ρ_r	*interest rate persistence*	0.414
σ_r	*monetary shock SD*	0.410
ρ_i	*monetary shock persistence*	0.674
ϕ_π	*Taylor rule parameter on* Π	1.875
ϕ_y	*Taylor rule parameter on* Y	0
ϕ_N	*nominal rate parameter on* $NGDP$	1.875
ρ_a	*productivity persistence*	0.922
σ_a	*productivity shock SD*	0.122
ρ_g	*government persistence*	0.800
σ_g	*government shock SD*	0.815
ρ_{cp}	*cost-push persistence*	0.999
σ_{cp}	*cost-push shock SD*	0.267
ρ_ν	*preference shock persistence*	0.608
σ_ν	*preference shock SD*	0.027
ρ_x	*investment persistence*	0.289
σ_x	*investment shock SD*	0.241
ρ_{cpw}	*wage markup persistence*	0.761
σ_{cpw}	*wage markup shock SD*	0.622
η	*inverse Frisch elasticity*	1.481
i_{ss}	*steady-state interest rate*	$\frac{1}{\beta} - 1$
g	*government share of output*	0.3
G_{ss}	*steady-state government spending*	$g * Y_{ss}$