A Turing-Complete Resistance-Based Isomorphism for Probabilistic Computation

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Abstract

This paper establishes a rigorous isomorphism between number-theoretic structures and informational resistance, grounded in the epistemological principle that any computational system embedded within an information structure cannot fully characterize its own substrate.

The fundamental objects are **inverse integers** ($1/n$ where $n \in \mathbb{Z}^+$), which naturally inhabit the interval $(0,1]$. The mapping $\Omega(1/n) = \ln(n)$ transforms multiplication into addition, establishing a monoid isomorphism from $(\{1/n : n \in \mathbb{Z}^+\}, \times)$ to $(\{\ln(n) : n \in \mathbb{Z}^+\}, +)$. This mapping is proven unique up to multiplicative constant (Theorem 2.4).

The correspondence extends across four equivalent representations: natural language, inverse integers, circuit topology, and prime coordinate vectors. For set-theoretic operations on squarefree encodings: **Intersection** corresponds to GCD with resistance Ω_gcd (Theorem 9.1); **Union** corresponds to LCM with resistance $\Omega\_a + \Omega\_b - \Omega\_gcd$ (Theorem 4.2); **Complement** requires extension to positive rationals via phase interference.

A foundational principle emerges: the **Open Interval Principle** states that the framework operates exclusively in $(0,1)$, with certainty and impossibility as limit points that can be approached but never reached.

We validate the framework by deriving the coprimality probability $6/\pi^2$ through all four layers (Theorem 8.1). We demonstrate that relational database operations map exactly to number-theoretic operations, with both domains forming instances of a distributive lattice. We establish Turing completeness by explicit counter machine simulation using Gödel numbering with prime exponents as registers (Theorem 10.4).

Prime numbers are characterized as the irreducible elements—integers $n > 1$ whose resistance $\ln(n)$ cannot be expressed as a sum of smaller resistances (Theorem 11.2).

PART I: FOUNDATIONS

1. Introduction

The prime numbers have fascinated mathematicians for millennia. They appear to be fundamental building blocks of arithmetic, yet their distribution along the number line exhibits patterns that remain mysterious despite centuries of study. This paper takes a novel approach: rather than treating primes as given mathematical objects, we develop a computational framework in which primes emerge as the **irreducible elements** of an informational structure.

1.1 The Central Insight

The **inverse integers** are numbers of the form $1/n$ where n is a positive integer. These objects naturally inhabit the interval $(0,1]$, the same domain as probabilities.

The structure of multiplication maps onto addition under the logarithm:

$$\Omega(P_1 \times P_2) = \Omega(P_1) + \Omega(P_2)$$

where $\Omega(1/n) = \ln(n)$. This is an exact algebraic isomorphism.

1.2 The Philosophical Foundation

A key philosophical principle underlies this framework, emerging from the epistemological observation that **a computational system embedded within an information structure cannot fully characterize its own substrate**.

This principle—which we term *virtual-virtual dualism*—recognizes that just as we cannot prove we are not a "brain in a vat," and just as a virtual machine cannot determine with certainty whether its host is itself virtual, any conscious observer faces an epistemic barrier between observation and the "true" nature of the projecting system.

Resistances are relational rather than absolute. Given two inverse primes $1/p$ and $1/q$, we can determine that the resistance of $1/p$ exceeds that of $1/q$ when $p > q$. We can compute their ratio: $\Omega(1/p)/\Omega(1/q) = \ln(p)/\ln(q)$. However, the absolute magnitude of any individual resistance depends on a reference scale that remains unspecified within the system.

The hypothetical voltage V. The framework posits an unknown voltage V that drives current through circuits. This voltage is never directly measured—only the relative conductances and resistances matter for computation.

1.3 The Open Interval Principle

A foundational discovery of this work is the **Open Interval Principle**: the framework operates exclusively in the open interval $(0,1)$. The boundaries—absolute certainty (1) and absolute impossibility (0)—are limit points that can be approached but never reached.

This is not a technical inconvenience; it is a fundamental feature reflecting deep truths across multiple domains:

- **Epistemology**: No observation can achieve absolute certainty.
- **Information Theory**: Infinite information cost means an event is outside the system's representational capacity.
- **Mathematics**: Division by zero is never encountered; infinity is a limit, not a value.

1.4 The Four-Layer Framework

Every computation can be expressed equivalently in four representations:

1. **English**: Natural language description
2. **Inverse Integer**: Values in (0,1] with arithmetic operations
3. **Circuit**: Informational resistance topology with phase
4. **Vector Space**: Prime coordinate vectors

A proof in one layer translates to proofs in the others.

2. The Fundamental Mapping

2.1 Inverse Integers

Definition 2.1. An **inverse integer** is a number of the form $1/n$ where $n \in \mathbb{Z}^+$. The set of inverse integers is denoted $I = \{1/n : n \in \mathbb{Z}^+\} \subset (0, 1]$.

Every inverse integer inherits unique prime factorization from its denominator:

$$1/n = 1/(p_1\text{^}a_1 \times p_2\text{^}a_2 \times \cdots \times p_k\text{^}a_k)$$

2.2 Informational Resistance

Definition 2.2 (Informational Resistance). For an inverse integer $1/n$, the informational resistance is:

$$\Omega(1/n) := \ln(n)$$

Notation convention: We write Ω_a to denote $\Omega(1/a) = \ln(a)$.

2.3 The Isomorphism Theorem

Theorem 2.3 (Inverse Integer–Resistance Isomorphism). The map $\Omega: I \to \ln(\mathbb{Z}^+)$ defined by $\Omega(1/n) = \ln(n)$ is a monoid isomorphism from $(I, \times)$ to $(\ln(\mathbb{Z}^+), +)$. That is:

$$\Omega(1/a \times 1/b) = \Omega(1/a) + \Omega(1/b)$$

Proof. $\Omega(1/ab) = \ln(ab) = \ln(a) + \ln(b) = \Omega(1/a) + \Omega(1/b)$. The map is bijective: for each $\ln(n) \in \ln(\mathbb{Z}^+)$, the unique preimage is $1/n$. □

2.4 Uniqueness

Theorem 2.4 (Uniqueness). Any continuous function $f: (0, 1] \to \mathbb{R}$ satisfying $f(xy) = f(x) + f(y)$ has the form $f(x) = c \cdot \ln(x)$ for some constant c.

Corollary 2.5. The resistance mapping $\Omega(P) = -\ln(P)$ is forced (up to scale) by requiring series combination to be additive.

PART II: BOOLEAN LOGIC

3. Series Combination (Multiplication)

3.1 The Series Correspondence

Theorem 3.1 (Series Correspondence). For inverse integers 1/a and 1/b:

$$(1/a) \times (1/b) = 1/ab \quad \text{and} \quad \Omega\_ab = \Omega\_a + \Omega\_b$$

4. Set-Theoretic Union: The LCM Correspondence

4.1 The Union-LCM Formula

Theorem 4.2 (Union Formula). For squarefree integers a, b:

$$\Omega\_lcm = \Omega\_a + \Omega\_b - \Omega\_gcd$$

Proof. The fundamental identity for positive integers states: lcm(a,b) × gcd(a,b) = a × b. Taking logarithms: ln(lcm) = ln(a) + ln(b) − ln(gcd). □

5. Complement: Phase Interference

5.1 Extension to Positive Rationals

The NOT operation for 1/a yields:

$$NOT(1/a) = 1 - 1/a = (a-1)/a$$

Theorem 5.2. The complement NOT(P) = 1 − P can be represented as the interference between a unit reference signal and a phase-shifted input:

$$|e\^(i\theta) - P \cdot e\^(i\theta)| = (1-P) \text{ for } P \in (0,1)$$

6. Complete Boolean Logic

6.1 Summary of Operations

Context	AND	OR	NOT
Sets	gcd(a,b)	lcm(a,b)	1 − 1/a
Resistance	Ω_gcd	Ω_a + Ω_b − Ω_gcd	Phase interference

6.2 Functional Completeness

Theorem 6.1. {AND, OR, NOT} forms a functionally complete set of Boolean operators.

PART III: THE VECTOR SPACE LAYER

7. Prime Coordinates

7.1 Coordinate Representation

Definition 7.1. The **prime coordinate vector** of n is: $v(n) = (a_2, a_3, a_5, a_7, \ldots)$ where a_p is the exponent of prime p in n.

7.2 Linear Independence

Theorem 7.2. The set $\{\ln(p) : p \text{ prime}\}$ is linearly independent over $\mathbb{Q}$.

Corollary 7.2.1. Prime coordinates form a vector space over $\mathbb{Q}$ with basis $\{\ln(p) : p \text{ prime}\}$.

PART IV: PROOF OF CONCEPT

8. The Coprimality Theorem

Theorem 8.1. Using natural density:

$$P(\gcd(a,b) = 1) = 6/\pi^2$$

8.1–8.4 The Four Layers

Layer 1 (English): "For every prime p, it is NOT the case that p divides BOTH a AND b."

Layer 2 (Inverse Integer): $P(\gcd(a,b) = 1) = \prod_p (1 - 1/p^2)$

Layer 3 (Circuit): $\Omega_{total} = \Sigma_p -\ln(1 - 1/p^2)$

Layer 4 (Vector): Coprime condition: $\min(v(a), v(b)) = 0$ (zero vector)

8.5 Connection to Zeta Function

By the Euler product: $\zeta(s) = \prod_p 1/(1 - p^{-s})$. At $s = 2$: $\prod_p (1 - 1/p^2) = 1/\zeta(2) = 6/\pi^2$ (Euler, 1734). □

PART V: DATABASE OPERATIONS

9. Relational Operations as Number Theory

Theorem 9.1 (Intersection = GCD). For squarefree integers a, b encoding sets A, B: $A \cap B$ is encoded by gcd(a, b).

Theorem 9.2 (Union = LCM). For squarefree integers a, b encoding sets A, B: $A \cup B$ is encoded by lcm(a, b).

Theorem 9.3. There is a bijection between finite binary strings and squarefree positive integers.

PART VI: COMPUTATIONAL COMPLETENESS

10. Turing Completeness

10.1 Counter Machine Model

Theorem 10.1 (Minsky, 1967). Two-counter machines are Turing complete.

10.2 State Encoding via Gödel Numbering

Definition 10.2. The machine state is encoded as a positive integer:

$$State = 2^{C_1} \times 3^{C_2} \times p\_k$$

where C_1, C_2 are counter values and p_k is the k-th prime for instruction k.

10.4 The Divisibility Gate as Transistor

The divisibility gate functions as a **transistor** in the informational circuit. Like a transistor with base/gate control, the divisibility gate has one input and two outputs: PASS (finite Ω, signal continues) or BLOCK (Ω → ∞, signal redirected).

10.6 Completeness Theorem

Theorem 10.4 (Turing Completeness). The framework is Turing complete.

Corollary 10.5. There is no algorithm to decide, given an arbitrary counter machine encoded in this framework, whether it halts.

PART VII: CONNECTIONS AND IMPLICATIONS

11. Primes as Irreducible Elements

Theorem 11.2 (Prime Characterization). A positive integer $n > 1$ is prime if and only if it is resistance-irreducible.

12. Connection to the Riemann Zeta Function

The Euler product: $\zeta(s) = \prod_p 1/(1 - p^{-s})$. At $s = 2$, the reciprocal yields the coprimality probability: $1/\zeta(2) = \prod_p (1 - 1/p^2) = 6/\pi^2$.

13. Lattice-Theoretic Structure

Theorem 13.1. The positive integers under divisibility form a distributive lattice with: Meet ($\wedge$): gcd; Join ($\vee$): lcm.

14. The Open Interval Principle

Principle 14.1. The framework operates in the open interval (0,1). Certainty and impossibility are limit points, not achieved states.

17. Conclusion

17.1 Summary of Results

Result	Reference
Isomorphism $\Omega(1/n) = \ln(n)$	Theorem 2.3
Uniqueness of logarithm	Theorem 2.4
Union formula: $\Omega_a + \Omega_b - \Omega_{gcd}$	Theorem 4.2
Linear independence of ln(primes)	Theorem 7.2
Coprimality = $6/\pi^2$	Theorem 8.1
Turing completeness	Theorem 10.4
Prime = resistance-irreducible	Theorem 11.2
Functional completeness	Theorem 6.1

17.3 Foundational Principles

The Relational Principle: Resistances are relational, not absolute.

The Open Interval Principle: Certainty and impossibility are limits, not states.

The Epistemic Principle: A computational system embedded in an information structure cannot fully characterize its substrate.

17.5 Closing Remark

The framework reveals that probability, circuits, number theory, and set theory are not merely analogous—they are **isomorphic**. The same abstract structure manifests in each domain. Computation, at its foundation, is the manipulation of prime coordinates through series combination and interference.

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