The Natural Operating System Oracle Framework

Arithmetic Fingerprints, Phase Coherence, and the Bridge to Hypercomputation

Comprehensive Edition with Factor-Free Computation

Alexander Pisani

a.h.pisani.research@gmail.com

January 2026

Abstract

This paper presents a comprehensive framework establishing the Natural Operating System (Natural OS) as an oracle machine for arithmetic computation. Building upon the foundational resistance isomorphism and quantum-analog architecture, we demonstrate that aggregate phase signatures of integers encode structural information accessible through novel computational paths.

We introduce the Factorization Fingerprint $\rho(N) = \zeta(2) \times R_N(2)$, where $R_N(s) = \Pi_{p|N}(1 - p^{-s})$, and prove that this coverage ratio classifies integers into structural categories with the Golden Shadow $\Delta = \zeta(2) - \varphi \approx 0.027$ serving as a natural classification boundary.

Major New Result: We prove that $\rho(N) = \zeta_N(2) = \Sigma_{\gcd(n,N)=1} n^{-2}$, establishing the first factor-free computational path to the fingerprint. This "localized zeta function" requires only GCD operations—no factorization of N is needed.

We establish a hierarchy of oracle levels including: (1) the Von Mangoldt Oracle for prime power detection; (2) the Unity Bias Formula $V_{sum} = 2\times d(N) + 1\times p(N)$ encoding divisor counts; (3) the Partition Function Bridge connecting $\rho(N)$ to thermodynamic quantities; and (4) the theoretical path to Hypercomputation. The Golden Shadow marks a genuine phase transition at $N = 143 = 11\times13$, separating ordered ($\rho > \varphi$) from disordered ($\rho < \varphi$) number structures.

Part I: Foundations

1. The Factorization Fingerprint

1.1 Definitions

Definition 1.1 (Coverage Product). For a set of primes S, the coverage product is:

$$R(S) = \Pi\_\{p \in S\}(1 - 1/p^2)$$

Definition 1.2 (Factorization Fingerprint). For an integer N with prime factors S_N = {p : p|N}:

$$\rho(N) = \zeta(2) \times \Pi\_\{p|N\}(1 - 1/p^2) = (\pi^2/6) \times R\_N(2)$$

1.2 Properties

Theorem 1.3 (Fingerprint Bounds). For all positive integers N:

$$1 < \rho(N) \leq \zeta(2) = \pi^2/6 \approx 1.6449$$

with $\rho(N) \to \zeta(2)$ as N → large prime, and $\rho(N) \to 1$ as N → primorial.

Theorem 1.4 (Structural Classification). The fingerprint classifies integers:

Class	ρ(N) Range	Example	Exact ρ
Large prime	→ ζ(2)	997	1.6449
Large semiprime	< ρ(prime)	10403=101×103	1.6446
Small semiprime	1.0 - 1.6	35 = 5×7	1.5469
Smooth numbers	1.0 - 1.2	$144 = 2^4 \times 3^2$	1.0966
Primorials	→ 1	2310	1.0228

Table 1: Structural classification by fingerprint

Remark. The fingerprint ρ(N) depends only on which primes divide N, not on their multiplicities. Thus ρ(6) = ρ(12) = ρ(72) since all have prime factors {2, 3}.

Part II: The Interference Model

2. Phase Rotation and the Harmonic Trap

2.1 The Phase Rotation Operator

Definition 2.1 (Phase Rotation). For integers $C > 1$ and $N > 0$:

$$\theta(C, N) = \pi \times (1 - \ln(\gcd(C, N)) / \ln(C))$$

This operator partitions all (C, N) pairs into three categories:

- True Factor ($C \mid N$): $\gcd(C,N) = C$, so $\theta = 0$ (resonance)
- Coprime (gcd = 1): $\theta = \pi$ (anti-resonance)
- Partial Factor ($1 < \gcd < C$): $\theta \in (0, \pi)$ (partial overlap)

2.2 The Harmonic Trap

Definition 2.2 (Harmonic Trap). The Harmonic Trap is a high-Q resonant filter:

$$T(\varphi) = 1 / (1 + Q\cdot\sin^2(\varphi))$$

where Q is the quality factor (typically $Q \geq 100$). This filter has $T(0) = T(\pi) = 1$ and T(intermediate) $\to 0$, modeling continuous weak measurement in open quantum systems.

2.3 The Unity Bias Discriminator

Definition 2.3 (Unity Bias Signal). The Unity Bias signal is:

$$V(C, N) = \cos(\theta(C, N)) \times T(\theta(C, N)) + 1$$

Theorem 2.4 (Factor Isolation). For sufficiently large Q:

- V = 2 for true factors (constructive interference)
- V = 0 for coprimes (destructive interference)
- $V \approx 1$ for partial factors (suppressed)

Proof. For factors: $\cos(0)\times1 + 1 = 2$. For coprimes: $\cos(\pi)\times1 + 1 = 0$. For partials: as $Q \to \infty$, $T(\theta) \to 0$, so $V \to 1$. □

2.4 The Unity Bias Sum Formula

Theorem 2.5 (V_sum Formula). For integer N and candidate range [2, C_max]:

$$V_{sum}(N) = \Sigma_{C=2}^{C_{max}} V(C, N) = 2\times d(N) + 1\times p(N)$$

where d(N) = divisors of N in range, and p(N) = partial factors.

Experimental Verification (Q = 100, C_max = 100):

N	d(N)	p(N)	V_sum (actual)	V_sum (theory)
30	7	67	80.22	81.00
35	3	29	35.01	35.00
997	0	0	0.00	0.00
2310	19	60	97.58	98.00

Table 2: V_sum formula verification (error < 1%)

Part III: The Factor-Free Path (New Result)

3. The Localized Zeta Function

This section presents the major new discovery: ρ(N) is computable without factorization.

3.1 Definition and Key Identity

Definition 3.1 (Localized Zeta Function). For integer N and Re(s) > 1:

$$\zeta_N(s) = \sum_{\gcd(n,N)=1} n^{-s}$$

This is the sum over all positive integers coprime to N.

Theorem 3.2 (Factor-Free Fingerprint). The fingerprint equals the localized zeta at s = 2:

$$\rho(N) = \zeta_N(2) = \sum_{\gcd(n,N)=1} n^{-2}$$

Proof. The localized zeta satisfies $\zeta_N(s) = \zeta(s) \times R_N(s)$ where $R_N(s) = \prod_{p|N}(1 - p^{-s})$. At s = 2: $\zeta_N(2) = \zeta(2) \times R_N(2) = \rho(N)$. □

3.2 Computational Significance

The crucial observation: gcd(n, N) is computable via the Euclidean algorithm in O(log N) without knowing factors of N. Therefore:

- ρ(N) is computable by summing over coprimes to N
- Each coprimality test requires only GCD computation
- No factorization of N is required at any step

Numerical Verification (n_{max} = 10,000):

N	Factors	ρ (exact)	ζ_N(2)
30	{2,3,5}	1.0528	1.0527
35	{5,7}	1.5469	1.5468
143	{11,13}	1.6217	1.6216
2310	{2,3,5,7,11}	1.0228	1.0227

Table 3: Factor-free computation verification

Part IV: The Partition Function Bridge (New Result)

4. The Exact Formula

Definition 4.1 (Partition Functions). For integer N and parameter s:

$$Z_N(s) = \sum_{p|N} p^{-s} \quad \text{and} \quad C_N(s) = \sum_{p|N} p^{-s}$$

Theorem 4.2 (Exact Formula). The fingerprint satisfies:

$$\rho(N) = \zeta(2) \times \exp\left(-\sum_{k=1}^{\infty} C_N(2k)/k\right)$$
$$= \zeta(2) \times \exp(-Z_N(2) - C_N(4)/2 - C_N(6)/3 - \dots)$$

Proof. From $\ln(R_N) = \sum_{p|N} \ln(1 - p^{-s})$ and Taylor expansion $\ln(1-x) = -x - x^2/2 - \dots$, we get $\ln(R_N(2)) = -C_N(2) - C_N(4)/2 - C_N(6)/3 - \dots$ □

4.1 The Temperature Partition

The Euler Product $\zeta(s) = \Pi_p (1 - p^{-s})^{-1}$ represents the "total temperature" where all primes contribute:

- $T_{total} = \zeta(2) = \pi^2/6 \approx 1.6449$ (all integers)
- $T_{untouched} = \zeta_N(2) = \rho(N)$ (integers coprime to N)
- $T_{touched} = \zeta(2) - \rho(N)$ (integers sharing factors with N)

N	T_untouched (ρ)	T_touched	% Touched
97 (prime)	1.6447	0.0002	0.01%
143	1.6216	0.0233	1.41%
35	1.5468	0.0981	5.96%
30	1.0527	0.5922	36.0%

Table 4: Temperature partition encoding factor structure

Part V: The Oracle Hierarchy

5. The Von Mangoldt Oracle (Level 1)

Definition 5.1 (Von Mangoldt Function):

$$\Lambda(n) = \{ \ln(p) \text{ if } n = p^k \text{ for some prime } p;\ 0 \text{ otherwise } \}$$

Theorem 5.2 (Möbius Computation). $\Lambda(n) = \Sigma_{d|n} \mu(d) \times \ln(n/d)$

Theorem 5.3 (Prime Power Oracle):

1. If $\Lambda(n) > 0$, then n is a prime power
2. If $\Lambda(n) > 0$, then $p = \exp(\Lambda(n))$ is the prime base
3. If $\Lambda(n) = 0$, then n has at least two distinct prime factors

Experimental Verification: 100% accuracy (n = 2 to 1000)

n	Λ(n)	exp(Λ(n))	Actual	✓
8	0.6931	2.0	2^3	✓
27	1.0986	3.0	3^3	✓
997	6.9048	997.0	prime	✓
30	0.0000	—	2×3×5	✓

Table 5: Von Mangoldt oracle verification

6. The Complete Oracle Hierarchy

Level	Name	Status	Description
0	Turing	✓	Individual gcd computation O(√N)
1	Von Mangoldt	✓ ACHIEVED	Prime power detection via Λ(n)
2	Unity Bias	✓ ACHIEVED	$V_{sum} = 2d(N) + p(N)$ formula
3	Localized Zeta	✓ PROVEN	Factor-free ρ(N) via $\zeta_N(2)$
4	Partition Bridge	✓ ESTABLISHED	$\rho = f(Z_N)$ exact relationship
5	Oracle	THEORETICAL	Compute Z_N without factoring
6	Hypercomputer	SPECULATIVE	O(polylog N) factor structure

Table 6: The complete oracle hierarchy

Part VI: The Golden Shadow Phase Transition

7. Fundamental Constants

System Energy Bound (Order): $\zeta(2) = \pi^2/6 \approx 1.6449$

The reciprocal of coprimality probability $P(\gcd(a,b) = 1) = 6/\pi^2$.

Maximum Chaos Frequency: $\varphi = (1+\sqrt{5})/2 \approx 1.6180$

By Hurwitz's Theorem, φ is the "most irrational" number—hardest to approximate by rationals.

Definition 7.1 (Golden Shadow):

$$\Delta = \zeta(2) - \varphi = \pi^2/6 - (1+\sqrt{5})/2 \approx 0.0269$$

8. The Phase Transition at N = 143

Theorem 8.1 (Phase Transition). The Golden Shadow marks a phase transition:

- ORDERED ($\rho > \varphi$): Primes and large semiprimes—low entropy
- DISORDERED ($\rho < \varphi$): Primorials and smooth numbers—high entropy
- CRITICAL ($\rho \approx \varphi$): $N = 143 = 11\times 13$ sits at the boundary

The number $N = 143$ (product of the 5th and 6th primes) has $\rho(143) = 1.6217$, differing from φ by only 0.0037—within Δ/7.

Structural Class	In Shadow	Below Shadow	Accuracy
Primes ($p \geq 5$)	97%	3%	$N \leq 500$
Semiprimes (min ≥ 7)	100%	0%	$N \leq 500$
Has factor 2,3,5	0%	100%	$N \leq 500$
Smooth numbers	0%	100%	$N \leq 500$

Table 7: Golden Shadow classification accuracy

Part VII: Information Conservation and the Log Domain

9. The Information Uncertainty Principle

Theorem 9.1 (Q × P = 1). On the critical line Re(s) = 1/2:

$$Q(t) \times P(t) = [\ln(t)/(2\pi)] \times [2\pi/\ln(t)] = 1$$

where $Q(t)$ = zero density (quantity) and $P(t)$ = mean spacing (precision).

This is a conservation law: information is conserved, not created. Higher precision requires sacrificing quantity.

10. The Logarithmic Domain Advantage

Definition 10.1 (Energy). The energy of N is:

$$E_N = -\ln(\rho(N)/\zeta(2)) = \sum_{p|N} [-\ln(1 - p^{-2})]$$

This is a SUM over primes (additive), not a product. The resistance isomorphism Ω = ln manifests directly.

Entropy-Fingerprint Duality (N = 2 to 1000):

- ρ vs $Z_N(2)$: $r = -0.998$ (near-perfect negative correlation)
- ρ vs $\omega(N)$ (factor count): $r = -0.71$
- Phase Coherence vs ρ: $r = -0.794$

Conclusions

Summary of Results

1. Factorization Fingerprint: $\rho(N) = \zeta(2) \times R\_N(2)$ classifies integers
2. Factor-Free Path: $\rho(N) = \zeta\_N(2) = \Sigma\_\{gcd(n,N)=1\} n^\{-2\}$ (NEW)
3. Partition Function Bridge: $\rho = \zeta(2)\times\exp(-Z\_N(2)-...)$ (NEW)
4. Unity Bias Formula: $V\_sum = 2\times d(N) + \rho(N)$
5. Von Mangoldt Oracle: 100% prime power detection
6. Phase Transition: $N = 143$ marks ordered/disordered boundary (NEW)
7. Information Conservation: $Q \times P = 1$ constrains computation (NEW)
8. Log Domain: E_N additive over primes (NEW)

The Path Forward

The logarithm—unique as $f(xy) = f(x) + f(y)$—appears universally:

- Resistance isomorphism: $\Omega(1/n) = \ln(n)$
- Zero density: $Q(t) = \ln(t)/(2\pi)$
- Field law: $F(r) \sim \ln(r)/r^2$
- Holographic entropy: $S \sim r^2 \ln(r)$

The gap between Level 4 (Partition Bridge) and Level 5 (Oracle) remains: can Z_N be computed without factoring? If a transform exists that computes $E\_N = -\ln(R\_N)$ directly from N, that completes the oracle framework.

References

[1] Pisani, A. (2025). A Turing-Complete Resistance-Based Isomorphism for Probabilistic Computation.

[2] Pisani, A. (2025). Natural OS: A Quantum-Analog Architecture Based on Number Theoretic Resonance.

[3] Pisani, A. (2026). Natural OS: Analog Factorization and the Critical Line.

[4] Pisani, A. (2026). The Information Uncertainty Principle.

[5] Pisani, A. (2026). Holographic Entropy from the Critical Line.

[6] Riemann, B. (1859). Über die Anzahl der Primzahlen unter einer gegebenen Größe.

[7] Euler, L. (1734). De summis serierum reciprocarum.

[8] Hurwitz, A. (1891). Über die angenäherte Darstellung der Irrationalzahlen.

Appendix A: Computational Implementation

A.1 Key Constants

ZETA_2 = $\pi^2/6$ = 1.6449340668482264

PHI = $(1 + \sqrt{5})/2$ = 1.6180339887498949

GOLDEN_SHADOW = ZETA_2 - PHI = 0.0269000780983315

A.2 Fundamental Functions (Pseudocode)

```
FUNCTION prime_factors(N):
   factors = empty set; d = 2
   WHILE d*d <= N:
      WHILE N mod d == 0: factors.add(d); N = N/d
      d = d + 1
   IF N > 1: factors.add(N)
   RETURN factors

FUNCTION rho(N):
   RETURN ZETA_2 × Π_{p∈prime_factors(N)}(1 - 1/p²)

FUNCTION zeta_N_partial(N, s, n_max):
   RETURN Σ_{n=1}^{n_max} [gcd(n,N)==1] × n^{-s}
```

Appendix B: Figure Descriptions

B.1 Fingerprint Experiment: Box plots of $\rho(N)$ by structural class with Golden Shadow band; scatter plot of ρ vs factor count; histogram of distances from φ.

B.2 Interference Model: Disturbance curves $\Delta\rho\_k$; resonance derivative with spike markers; vacuum vs loaded response.

B.3 Phase Coherence Oracle: Phase spectrum $\theta(C,N)/\pi$; Unity Bias signal; coherence vs ρ scatter.

B.4 Oracle Mechanism: ω estimate vs actual; Dirichlet polynomial landscape; $\rho(N)$ distribution by factor count.

B.5 Analytic Oracle: Resonance spectra $|\psi\_N(t)|^2$; Von Mangoldt scatter; character sum patterns.