

“Follow me” controller

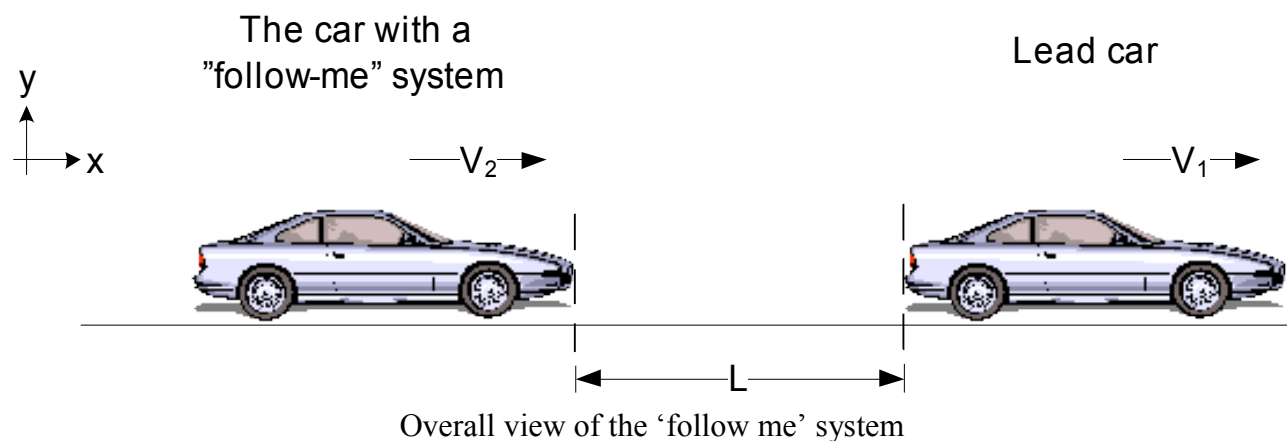
Course: Grundlæggende Indlærende Systemer (GiS)
Modul 3, Regulering af dynamiske systemer
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Ændring af grundstruktur i rapporten så det passer med miniprojekt beskrivelsen side 2 (beklager, den havde jeg helt overset)

1. Description

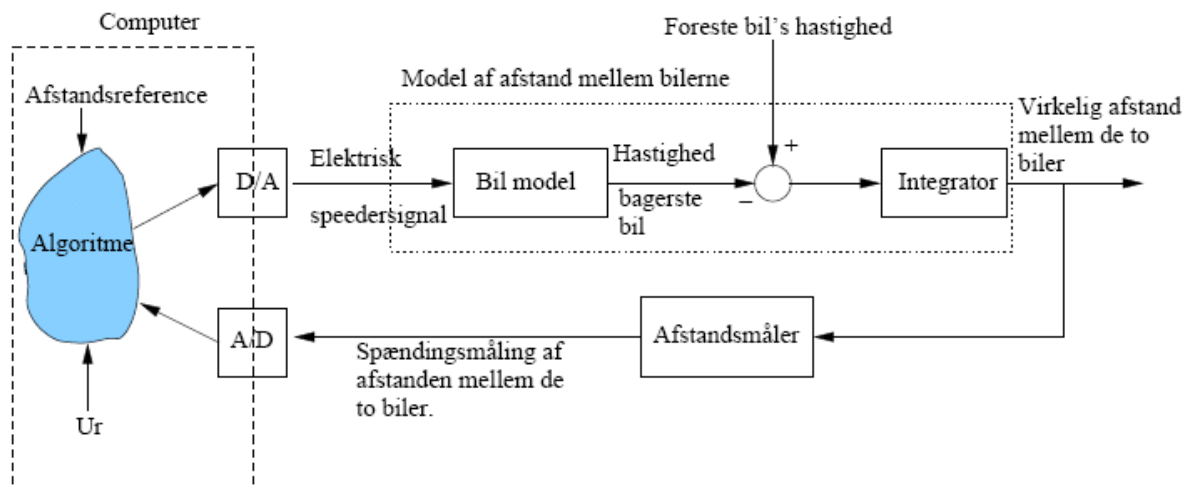
Create a regulator that allows a car to follow another car at a predefined distance.

Tegningen er ændret så bilerne kører i x-retningen

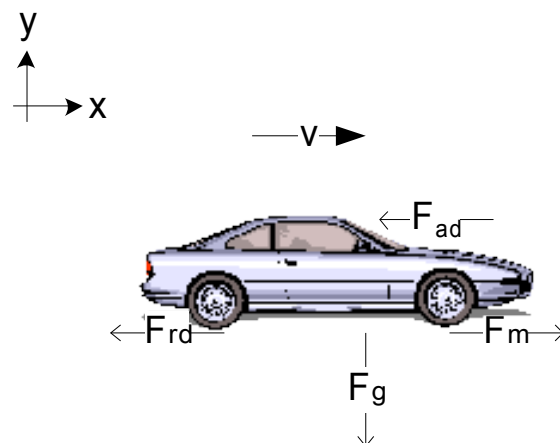


The ‘follow me’ system illustrated above consists of two vehicles. The front car leads the way while the rear vehicle automatically follows the front vehicle at a fixed distance. The ‘follow me’ system continuously measure the distance to the front vehicle and automatically make adjustments to uphold the fixed distance.

The overall regulator may be seen in the below drawing



2. Model



- F_{rd} : Force asserted on the car from rolling drag .
- F_{ad} : Force asserted on the car from wind resistance .
- F_m : Force asserted on the car from the motor via the wheels
- F_g : Gravitational force
- v : velocity of vehicle due to the sum of all forces not being 0.

Free body diagram of the ‘follow me’ vehicle

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- v : velocity of vehicle due to the sum of all forces not being 0.

In the free body diagram above is only shown the overall forces affecting the system. Many, many has been ignored because they are negligible. A complete list will not be supplied, as it is almost always possible to come up with some that has not been considered. To give a few examples may be mentioned

- The capacitance generated between the windings in the endian – considered to have a negligible effect compared to the inductance and resistance.
- The initial friction resistance, which is non-linear and larger than the friction resistance lim car velocity $\rightarrow 0$.
- The temperature dependency of the tires spring effect.
- The loss in the transfer of torque from the engine shaft to the wheels.
- And so on and so on.

And this does not even include the assumptions made in the assignment (ignoring hills and bumps and turns and changes in wind-strength).

Finally, since there is no changes in the road (no hills, no bumps) and we ignore the breaking and acceleration effect on the suspension (since the car has no breaks, we can simply assume that it has four-wheel-drive, and then the acceleration and breaking should be evenly distributed), we can ignore the effect of the tires and suspension, making the model much simpler.

Having no bumps, hils or turns (centrifical force is relative to mass and speed), and considering the tires ideal, the gravity force, being direrctly proportional to the other forces, irellevant. This means that F_g can be ignored.

The contribution from the rolling drag (F_{rd}) can be calculated by

$$F_{rd} = C_{rr} F_N = C_{rr} mg \cos \theta \quad \text{eq. 1}$$

C_{rr} is the dimensionless rolling resistance coefficient. On a smooth road and low-resistance car tires the C_{rr} coefficient is low. In order to further simplify the model we assess the contribution from rolling drag to be negligible; hence we can eliminate F_{rd} .

This leave air drag F_{ad} , which act as an opposite force to the vehicle’s moving direction and is propotional to the velocity of the vehicle. For a matter of simplicity we make the air drag linear proportional to the velocity.

$$F_{ad} = bv \quad \text{eq. 2}$$

The coefficient b is dimensionless constant. The air drag equation is appropriate at relative slow speed (I.e. low Reynolds number, $Re < 1$). However, we assume the air drag equation is valid for all speeds.

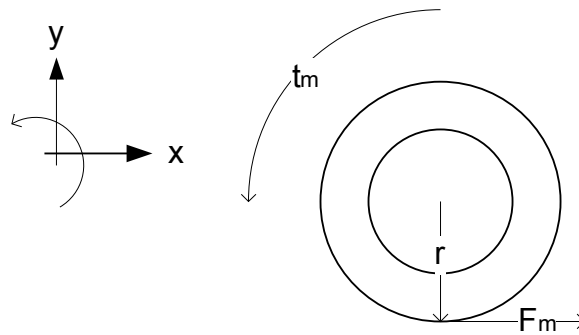
Using Newton’s second law that states that the net force applied to the vehicle produces a proportional acceleration we have:

$$F = m * a \quad \text{eq. 3}$$

where F is the total force asserted on the mass (with sign and direction) so the equations can be collected to

$$F_m - bv = m * a \Rightarrow F_m - bv = m * dv/dt \quad \text{eq. 4}$$

The force F_m is the net force from the vehicle’s wheels



From the free body picture of a wheel we can establish an equation that gives the net torque applied to the wheel.

$$F_m r + \tau_m = J \frac{\partial \omega}{\partial t} = J \frac{1}{r} \frac{\partial v}{\partial t}$$

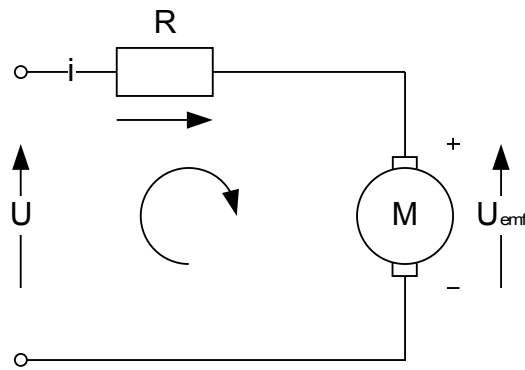
$$F_m = J \frac{1}{r^2} \frac{\partial v}{\partial t} - \frac{1}{r} \tau_m \quad \text{eq. 5}$$

We combine eq. 4 with eq. 5 and we have:

TODO: verify sign

$$m \frac{\partial v}{\partial t} = J \frac{1}{r^2} \frac{\partial v}{\partial t} - \frac{1}{r} \tau_m + bv \quad \text{eq. 6}$$

The torque t_m in eq. 6 is the contribution from the vehicle’s DC motor. A simplified model of a DC motor is shown below



TODO: should we include L???

The DC motor's torque τ_m is a linear proportional to the current i and k_t is motor torque constant and we have:

$$\tau_m = k_t i \quad \text{eq. 7}$$

We can derive eq. 7 into angular momentum and we have:

$$J \frac{\partial \omega}{\partial t} = J \frac{1}{r} \frac{\partial v}{\partial t} = k_t i \quad \text{eq. 8}$$

The DC motor's angular velocity is defined as a linear voltage difference u_{emf} , proportional to the angular velocity ω of the armature shaft and k_s and we have:

$$u_{emf} = k_s \frac{\partial \theta}{\partial t} = k_s \omega \quad \text{eq. 9}$$

By using Kirchoff's laws the following equation can be derived

$$u + Ri - u_{emf} = 0$$

$$u + R J \frac{1}{k_t r} \frac{\partial v}{\partial t} = k_s \omega$$

$$u + R J \frac{1}{k_t r} \frac{\partial v}{\partial t} = k_s \frac{1}{r} v$$

$$R J \frac{1}{k_t r} \frac{\partial v}{\partial t} = k_s \frac{1}{r} v - u$$

$$J \frac{1}{r} \frac{\partial v}{\partial t} = \frac{k_s k_t}{R r} v - \frac{k_t}{R} u$$

$$\tau = k_s \frac{k_s k_t}{Rr} v - \frac{k_t}{R} u \quad \text{eq. 10}$$

In eq. 10 the torque from the DC motor has been found and can be inserted into eq. 6 and we have:

$$\begin{aligned} m \frac{\partial v}{\partial t} &= J \frac{1}{r^2} \frac{\partial v}{\partial t} - \frac{1}{r} \left(\frac{k_s k_t}{Rr} v - \frac{k_t}{R} u \right) + bv \\ m \frac{\partial v}{\partial t} &= J \frac{1}{r^2} \frac{\partial v}{\partial t} - \frac{k_s k_t}{Rr^2} v + \frac{k_t}{Rr} u + bv \\ \left(m - J \frac{1}{r^2} \right) \frac{\partial v}{\partial t} &= \left(b - \frac{k_s k_t}{Rr^2} \right) v + \frac{k_t}{Rr} u \end{aligned} \quad \text{eq. 11}$$

Equation 11 is reduced to interacting forces as a change of velocity over time and voltage. For simplicity we replace all the constants in reduced form:

$$\begin{aligned} b_1 &= m - J \frac{1}{r^2} \\ a_1 &= \frac{k_s k_t}{Rr^2} + b \\ a_2 &= \frac{k_t}{Rr} \end{aligned}$$

The parameters are explained below:

m	Mass of the vehicle
J	Inertia of the vehicle's wheel
r	Radius of the wheel
R	Electrical resistance in the motor
k_s	Angular velocity constant
k_t	Torque constant
b	Wind resistance coefficient

And we can rewrite equation 11:

$$b_1 \frac{\partial v}{\partial t} = a_1 v - a_2 u \quad \text{eq. 12}$$

Equation 12 in Laplace transformed version we have:

$$b_1 s V(s) = a_1 V(s) - a_2 U(s)$$

$$a_2 U_{(s)} = (a_1 - b_1 s) V_{(s)} \quad \text{eq. 13}$$

The equation 13 can be rewritten and we find the vehicle’s transfer function

$$\frac{V_{(s)}}{U_{(s)}} = \frac{\frac{a_2}{b_1}}{\frac{a_1}{b_1} - s} \quad \text{eq. 14}$$

3. Simulering

4. Kontrolspecifikationer

5. Kontroldesign

6. Afprøvning

7. References
