$I = \int_a^b f(x)\,dx \cong \int_a^b f_1(x)\,dx$

$P_1(x) = a_0 + a_1 \cdot (x - x_0)$

$x_0 = a \ , \quad f(a)$

$x_1 = b \ , \quad f(b)$

$P_1(x) = f(a) + \dfrac{f(b) - f(a)}{b - a} \cdot (x - a)$

$$I = \int_a^b f(a) + \frac{f(b) - f(a)}{b - a} (x - a)\, dx \qquad (1)$$

$$f(a) + \frac{f(b) - f(a)}{b - a} \cdot x - \frac{f(b) - f(a)}{b - a} \cdot a$$

$$\frac{f(a) \cdot b - \cancel{f(a) \cdot a} - a f(b) + \cancel{a f(a)}}{b - a} + \frac{f(b) - f(a)}{b - a} \cdot x$$

$$\frac{f(a) b - f(b) \cdot a}{b - a} + \frac{f(b) - f(a)}{b - a} \cdot x$$

RETOMANDO A INTEGRAL EM (1), VEM QUE

$$I = \frac{f(a) b - f(b) \cdot a}{b - a} x \Big|_a^b + \frac{f(b) - f(a)}{b - a} \cdot \frac{x^2}{2}$$

$b^2 - a^2$
$= (b + a) \cdot (b - a)$

$$= \frac{f(a) b - f(b) a}{\cancel{b - a}} \cdot \cancel{(b - a)} + \frac{f(b) - f(a)}{b - a} \cdot \frac{b^2 - a^2}{2}$$

$$= f(a) b - f(b) a + \frac{f(b) - f(a)}{2} \cdot (a + b)$$

$$= \frac{1}{2} \left(2 f(a) b - 2 f(b) a + f(b) a + f(b) b - f(a) a - f(a) b \right)$$

$$= \frac{1}{2} \left(f(a) b - f(b) a + f(b) b - f(a) a \right)$$

$$= \frac{1}{2} \left[f(a) \cdot (b - a) + f(b) (b - a) \right]$$

$$= \frac{f(a) + f(b)}{2} \cdot (b - a)$$

ERRO ?

$$I = (b-a)\frac{f(a)+f(b)}{2}$$

$$E_t = -\frac{1}{12}f''(\xi)(b-a)^3$$

$$E = -\frac{(b-a)^3}{12\,n^2}\,f''(\xi)$$

Exemplo 5.1

Calcular, pela regra dos trapézios e, depois, analiticamente, o valor de:

$$I = \int_{3,0}^{3,6} \frac{dx}{x}$$

Comparar os resultados.

$f''(x) = \frac{2}{x^3}$

$E_t = -\frac{1}{12} \cdot \frac{2}{x^3} \cdot 0,6^3 = -\frac{1}{12} \cdot \frac{2}{27} \cdot 0,6^3$

$|f(\xi)|_{max} = \frac{2}{27}$

$= 0,00133$

Solução

$a = 3$

$b = 3,6$

$f(a) = \frac{1}{3} = 0,33333\ldots$

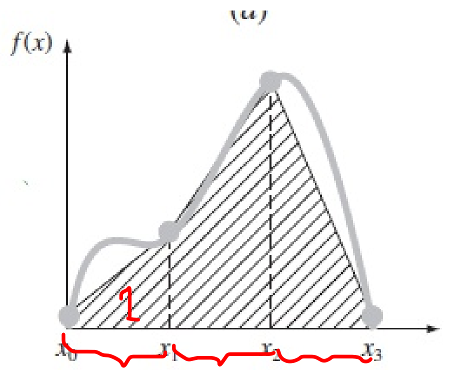
$f(b) = \frac{1}{3,6} = 0,27777\ldots$

$I = 0,6 \cdot \left(\frac{0,33333 + 0,27777}{2}\right)$

$I = 0,18333$

$$\int_3^{3,6} \frac{1}{x}\,dx = \ln x \Big|_3^{3,6} = \ln 3,6 - \ln 3 = 0,18232$$

$E = |0,18232 - 0,18333| = 10^{-3}$



(b)

4 pontos

3 trapézios.

$$h = x_1 - x_0 = x_2 - x_1 = x_3 - x_2$$

$$I = \frac{f(a) + f(b)}{2} \cdot (b-a)$$

$$I_1 = \frac{f(x_0) + f(x_1)}{2} \cdot h$$

$$I_2 = \frac{f(x_1) + f(x_2)}{2} \cdot h$$

$$h = \frac{b-a}{3} =$$

$$I_3 = \frac{f(x_2) + f(x_3)}{2} \cdot h$$

$$h = \frac{b-a}{n}$$

$$I = I_1 + I_2 + I_3$$

$$= \frac{f(x_0) + 2f(x_1) + 2f(x_2) + f(x_3)}{2} \cdot h$$

Calcular a integral do exemplo 5.1 utilizando a regra dos trapézios composta e subdividindo o intervalo de integração em 6 subintervalos.

$$I = \int_{3,0}^{3,6} \frac{1}{x}\,dx$$

i	x_i	$y_i = f(x_i)$
0	3,0	0,333333
1	3,1	0,322581
2	3,2	0,312500
3	3,3	0,303030
4	3,4	0,294118
5	3,5	0,285714
6	3,6	0,277778

	x	f(x)
x_0	3,0	0,3333
x_1	3,1	0,3226
x_2	3,2	0,3125
x_3	3,3	0,3030
x_4	3,4	0,2941
x_5	3,5	0,2852
x_6	3,6	0,2778

$\underbrace{x_1 \ldots x_5}_{2\times}$

$$h = \frac{b-a}{6} = \frac{0,6}{6} = 0,1$$

$$I_1 = \frac{f(x_0) + f(x_1)}{2} \cdot h$$

$$I_2 = \frac{f(x_1) + f(x_2)}{2} \cdot h$$

$\vdots$

$$I_6 = \frac{f(x_5) + f(x_6)}{2} \cdot h$$

$$I = 0,182345$$

EXATO : 0,18232

$$E_t = |0,18232 - 0,182345| = 2,5 \cdot 10^{-5}$$

$$E_a = -\frac{(b-a)^3}{12n^2} \cdot f''(\xi)$$

$$f''(x) = \frac{2}{x^3}$$

$$E_a = -\frac{(3,6-3)^3}{12 \cdot 6^2} \cdot \frac{2}{27} = -3,7 \cdot 10^{-5}$$

$x = 3$

$$f''(3) = \frac{2}{3^3} = \frac{2}{27}$$