$$I = \underbrace{\frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]}_{\text{Regra 1/3 de Simpson}} - \underbrace{\frac{1}{90} f^{(4)}(\xi) h^5}_{\text{Erro de truncamento}}$$

Exemplo 5.4

Calcular o valor de π, dado pela expressão:

$$\pi = 4 \int_0^1 \frac{dx}{1 + x^2}$$

$$f(x) = \frac{1}{1+x^2}$$

$$h = \frac{b-a}{2} = \frac{1-0}{2} = 0,5$$

$$f^{(IV)}(x) = \frac{24}{(1+x^2)^3} - \frac{288x^2}{(1+x^2)^4} + \frac{384x^4}{(1+x^2)^5}$$

Solução

$x_0 = a = 0$, $f(0) = 1$

$x_1 = 0 + 0,5 = 0,5$, $f(0,5) = 0,8$

$x_2 = b = 1$, $f(1) = 0,5$

$$I = \frac{0,5}{3} \cdot [1 + 4 \cdot 0,8 + 0,5] = 0,783333$$

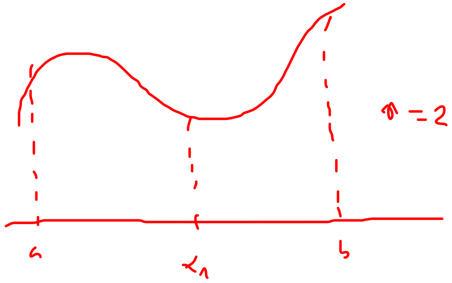
$$4 \cdot I = 3,133333\ldots$$

$$\pi = 3,141592$$

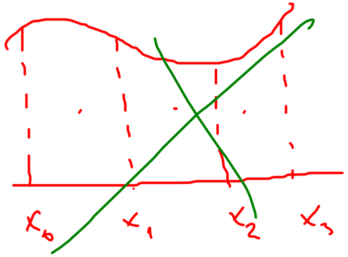
$$E = 3,141592 - 3,133333 = 0,008259 = 8,2 \cdot 10^{-3}$$

$$E_t = -\frac{1}{90} \cdot f^{(4)}(\xi)\, h^5 = -\frac{1}{90} \cdot 0,5^5 \cdot 24 = 0,008333\ldots$$

$$= 8,3 \cdot 10^{-3}$$



$n = 2$



$n = 3$

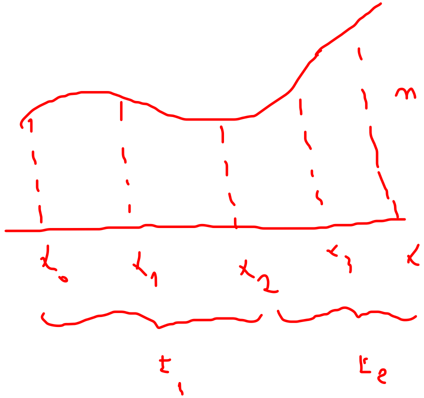
$x_0 = a$

$x_1 = a + h$

$x_2 = b$

n deve se par

$h = \dfrac{b-a}{4}$



$n = 4$

$$I_1 = \frac{h}{3} \cdot [f(x_0) + 4f(x_1) + f(x_2)]$$

$$I_2 = \frac{h}{3} \cdot [f(x_2) + 4f(x_3) + f(x_4)]$$

$$I = \frac{h}{3} \cdot [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

Se $n = 8$

$$I = \frac{h}{3} \cdot [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + 2f(x_6) + 4f(x_7) + f(x_8)].$$

$h = \dfrac{b-a}{8}$

Calcular o valor de π, dado pela expressão:

$$\pi = 4 \int_0^1 \frac{dx}{1+x^2}$$

$n = 8$

i	x_i	y_i	c_i
0	0,000	1,000000	1
1	0,125	0,984615	4
2	0,250	0,941176	2
3	0,375	0,876712	4
4	0,500	0,800000	2
5	0,625	0,719101	4
6	0,750	0,640000	2
7	0,875	0,566372	4
8	1,000	0,500000	1

coluna dos coeficientes

$\pi = 3,14159265\ldots$

$$h = \frac{1-0}{8}$$

$x_0 = 0$

$x_1 = 0,125$

$x_2 = 0,250$

$\vdots$

$x_8 = 1$

$$I = \frac{h}{3} \cdot \Big[f(0) + 4 \cdot f(0,125) + 2 \cdot f(0,250) + 4\, f(0,375)$$

$$+ 2 \cdot f(0,5) + 4 \cdot f(0,625) + 2 \cdot f(0,75)$$

$$+ 4 \cdot f(0,875) + f(1) \Big] = 0,785398$$

$4I = 3,141592$

5.5.4.1. Dada a função $y = f(x)$, definida a partir da tabela 5.15

Tabela 5.15

i	x_i	y_i
0	0,00	0,600
1	0,25	0,751
2	0,50	0,938
3	0,75	1,335
4	1,00	2,400

calcular o valor de

$$I = \int_0^1 f(x)dx$$

a) aplicando a 1ª regra de Simpson com $n = 2$
b) aplicando a 1ª regra de Simpson com $n = 4$

$$I = \frac{h}{3} \cdot [f(x_0) + f(x_1) + f(x_2)]$$

$$h = 0,25$$

$$I = \frac{0,25}{3} \cdot [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

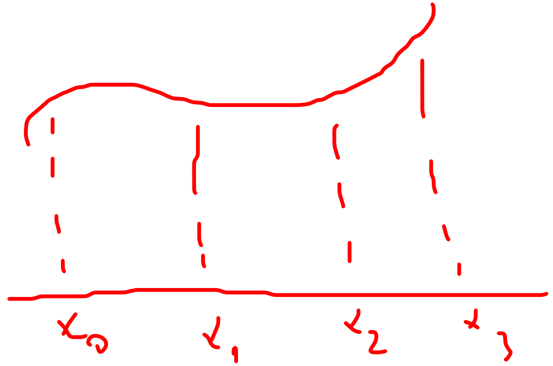
$$I = \frac{0,25}{3} \cdot [0,6 + 4 \cdot 0,751 + 2 \cdot 0,938 + 4 \cdot 1,335 + 2,4]$$

$$I = 1,106667.$$

Calcular o valor da integral:

$$I = \int_1^4 \ln\left(x^3 + \sqrt{e^x + 1}\right)\, dx$$

aplicando a regra dos 3/8 com 3 e 9 subintervalos.



i	x_i	y_i	c_i
0	1	1,0744	1
1	2	2,3884	3
2	3	3,4529	3
3	4	4,2691	1

$$h = \frac{b-a}{3} = \frac{4-1}{3} = 1$$

$$I = \frac{3h}{8} \cdot \left[f(x_0) + 3 f(x_1) + 3 f(x_2) + f(x_3) \right]$$

$x_0 = 1$ | $f(x_0) = 1{,}0744$

$x_1 = 2$ | $f(x_1) = 2{,}3844$

$x_2 = 3$ | $f(x_2) = 3{,}4529$

$x_3 = 4$ | $f(x_3) = 4{,}2691$

$$I = \frac{3}{8} \cdot \left[1{,}0744 + 3 \cdot 2{,}3844 + 3 \cdot 3{,}4529 + 4{,}2691 \right]$$

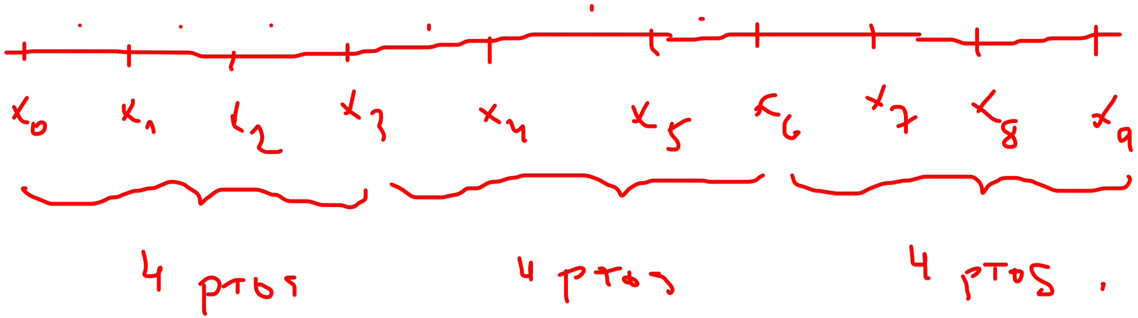
$$= 8{,}5753.$$

Calcular o valor da integral:

$$I = \int_1^4 \ln\left(x^3 + \sqrt{e^x + 1}\right)\, dx$$

aplicando a regra dos 3/8 com 3 e 9 subintervalos.

$f(x) = \ln\left(x^3 + \sqrt{e^x + 1}\right)$



n = multiplo de 3

$h = ?$

$x_0 = 1$ $\qquad$ $f(x_0) = 1{,}07441$

$x_1 = 1 + h$

$\vdots$

$x_9 = \ldots$