Measurement of Flow of Fluids

12. Measurement of Flow of Fluids

12.1. INTRODUCTION

Numerous devices are used in engineering practice to measure the flow of fluids. Velocity measurements are made with Pitot tubes, current meters, and rotating and hot-wire anemometers. In model studies, photographic methods are often used. Quantity measurements are accomplished by means of orifices, tubes, nozzles, Venturi meters and flumes, elbow meters, weirs, numerous modifications of the foregoing, and various patented meters. In order to apply the hydraulic devices intelligently, use of the Bernoulli equation and additional knowledge of the characteristics and coefficients of each device are imperative. In the absence of reliable values of coefficients, a device should be calibrated for the expected operating conditions.

Formulas developed for incompressible fluids may be used for compressible fluids where the pressure differential is small relative to the total pressure. In many practical cases such small differentials occur. However, where compressibility must be considered, special formulas are required.

12.2. PITOT TUBE

The pitot tube measures the velocity at a point by virtue of the fact that the tube measures the stagnation pressure, which exceeds the local static pressure by $\gamma\,(V^2/2g)$. In an open stream of fluid, since the local pressure is zero gage, the height to which the liquid rises in the tube measures the velocity head. Problems 12.1 and 12.5 develop expressions for the flow of incompressible and compressible fluids, respectively.

12.3. COEFFICIENT OF VELOCITY

The coefficient of velocity (c_v) is the ratio of the actual mean velocity in the cross section of a stream (jet) to the theoretical mean velocity that would occur without friction. Thus

$$c_v = \frac{\text{actual mean velocity}}{\text{theoretical mean velocity}}$$

(1)

12.4. COEFFICIENT OF CONTRACTION

The coefficient of contraction (c_c) is the ratio of the area of the contracted section of a stream (jet) to the area of the opening through which the fluid flows. Thus

$$c_c = \frac{\text{area of stream (jet)}}{\text{area of opening}} = \frac{A_{\text{jet}}}{A_o}$$

(2)

12.5. COEFFICIENT OF DISCHARGE

The coefficient of discharge (c) is the ratio of the actual discharge through the device to the theoretical discharge. This coefficient can be expressed as

$$c = \frac{\text{actual flow } Q}{\text{theoretical flow } Q}$$

(3)

More practically, when the coefficient of discharge c has been determined experimentally,

$$Q = cA\sqrt{2gH}$$

(4)

Where A = cross-sectional area of device
H = total head causing flow

The coefficient of discharge can also be written in terms of the coefficient of velocity and the coefficient of contraction, i.e.,

$$c = c_v \times c_c$$

(5)

The coefficient of discharge is not constant. For a given device, it varies with Reynolds number. In the Appendix the following information will be found:

1. Table 7 contains coefficients of discharge for circular orifices discharging water at about 60°F into the atmosphere. Few authoritative data are available for all fluids throughout wide ranges of Reynolds number.

2. Diagram *C* indicates the variation of *d* with Reynolds number for three pipe orifice ratios. No authoritative data are available below a Reynolds number of about 10,000.

3. Diagram *D* shows the variation of *c* with Reynolds number for three long-radius flow nozzle ratios (pipeline nozzles).

4. Diagram *E* indicates the variation of *c* with Reynolds number for five sizes of Venturi meters of diameter ratios of 0.500.

12.6. LOST HEAD

The lost head in orifices, tubes, nozzles, and Venturi meters is expressed as

$$\text{lost head of fluid} = \left(\frac{1}{c_v^2} - 1\right)\frac{V_{\text{jet}}^2}{2g}$$

(*6*)

When this expression is applied to a Venturi meter, V_{jet} = throat velocity and $c_v = c$.

12.7. WEIRS

Weirs measure the flow of liquids, usually water, in open channels. A number of empirical formulas are available in engineering literature, each with its limitations. Only a few will be listed here. Most weirs are rectangular: the *suppressed* weir with no end contractions and generally used for larger flows, and the *contracted* weir for smaller flows. Other weirs are triangular, trapezoidal, parabolic, and proportional flow. For accurate results, a weir should be calibrated in place under the conditions under which it will be used.

12.8. THEORETICAL WEIR FORMULA

The theoretical weir formula for rectangular weirs, developed in Problem 12.29, is

$$Q = \frac{2}{3}cb\sqrt{2g}\left[\left(H + \frac{V^2}{2g}\right)^{3/2} - \left(\frac{V^2}{2g}\right)^{3/2}\right]$$

(*7*)

where Q = flow in cfs (or m^3/s)
c = coefficient (to be determined experimentally)
b = length of weir crest in feet (or meters)
H = head on weir in feet (or meters) (height of level liquid surface above crest)
V = average velocity of approach in ft/sec or m/s.

12.9. FRANCIS FORMULA

The Francis formula, based upon experiments on rectangular weirs from 3.5 ft (1.1 m) to 17 ft (5.2 m) long under heads from 0.6 ft (0.2 m) to 1.6 ft (0.5 m), is

$$Q = 3.33^{*}\left(b - \frac{nH}{10}\right)\left[\left(H + \frac{V^2}{2g}\right)^{3/2} - \left(\frac{V^2}{2g}\right)^{3/2}\right]$$

(*8*)

where the notation is the same as above and

$n = 0$ for a suppressed weir
$n = 1$ for a weir with one contraction
$n = 2$ for a fully contracted weir

12.10. BAZIN FORMULA

The Bazin formula (lengths from 1.64 ft to 6.56 ft under heads from 0.164 ft to 1.969 ft) is

$$Q = \left(3.25 + \frac{0.0789}{H}\right)\left[1 + 0.55\left(\frac{H}{H+Z}\right)^2\right]bH^{3/2}$$

(*9*)

where Z = height of the weir crest above the channel bottom.

The bracketed term becomes negligible for low velocities of approach.

12.11. FTELEY AND STEARNS FORMULA

The Fteley and Stearns formula (lengths 5 ft and 19 ft under heads from 0.07 ft to 1.63 ft) for suppressed weirs is

$$Q = 3.31b\left(H + \alpha\frac{V^2}{2g}\right)^{3/2} + 0.007b$$

(*10*)

where α = factor dependent upon crest height Z (tables of values required).

12.12. THE TRIANGULAR WEIR FORMULA

(developed in Problem 12.30) is

$$Q = \frac{8}{15}c\tan\frac{\theta}{2}\sqrt{2g}\,H^{5/2}$$

(*11*)

or, for a given weir,

$$Q = mH^{5/2}$$

(*12*)

12.13. THE TRAPEZOIDAL WEIR FORMULA

(of Cipolletti) is

$$Q = 3.367bH^{3/2}$$

(*13*)

This weir has side (end) slopes of 1 horizontal to 4 vertical.

12.14. FOR DAMS USED AS WEIRS

the expression for approximate flow is

$$Q = mbH^{3/2}$$

(*14*)

where m = experimental factor, usually from model studies.

Nonuniform flow over broad-crested weirs is discussed in Problem 12.37.

12.15. TIME TO EMPTY TANKS

by means of an orifice is (see Problem 12.40)

$$t = \frac{2A_T}{cA_o\sqrt{2g}}\left(h_1^{1/2} - h_2^{1/2}\right) \qquad \text{(constant cross section, no inflow)}$$

(*15*)

$$t = \int_{h_1}^{h_2} \frac{-A_T dh}{Q_{\text{out}} - Q_{\text{in}}} \qquad \text{(inflow < outflow, constant cross section)}$$

(*16*)

For a tank whose cross section is not constant, see Problem 12.43.

12.16. TIME TO EMPTY TANKS

by means of weirs is calculated by using (see Problem 12.45)

$$t = \frac{2A_T}{mL}\left(H_2^{-1/2} - H_1^{-1/2}\right)$$

(*17*)

12.17. TIME TO ESTABLISH FLOW

in a pipeline is (see Problem 12.47)

$$t = \frac{LV_f}{2gH}\ln\left(\frac{V_f + V}{V_f - V}\right)$$

(*18*)

12.17.1. Solved Problems

12.1. A Pitot tube having a coefficient of 0.98 is used to measure the velocity of water at the center of a pipe. The stagnation pressure head is 5.67 m and the static pressure head in the pipe is 4.73 m. What is the velocity?

Solution:

If the tube is shaped and positioned properly, a point of zero velocity (stagnation point) is developed at *B* in front of the open end of the tube (see Fig. 12-1). Applying the Bernoulli theorem from *A* in the undisturbed liquid to *B* yields

$$\left(\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+0\right)-\frac{\text{no loss}}{\text{(assumed)}}=\left(\frac{p_B}{\gamma}+0+0\right)$$

(*1*)

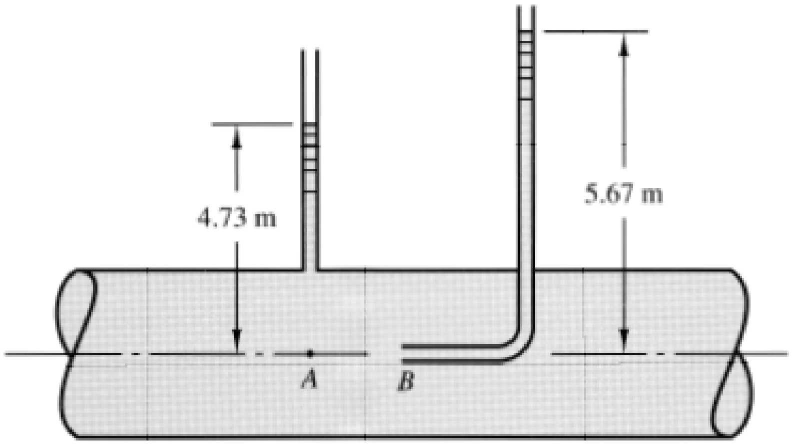

Figure 12-1.

Then, for an ideal "frictionless" fluid,

$$\frac{V_A^2}{2g}=\frac{p_B}{\gamma}-\frac{p_A}{\gamma}\quad\text{or}\quad V_A=\sqrt{2g\left(\frac{p_B}{\gamma}-\frac{p_A}{\gamma}\right)}$$

(*2*)

For the actual tube, a coefficient *c*, which depends upon the design of the tube, must be introduced. The actual velocity for the problem above would be

$$V_A=c\sqrt{2g(p_B/\gamma-p_A/\gamma)}=0.98\sqrt{2g(5.67-4.73)}=4.21\text{ m/s}$$

The above equation will apply to all incompressible fluids. The value of c may be taken as unity in most engineering problems. Solving (*1*) for the stagnation pressure at B gives

$$p_B=p_A+\frac{1}{2}\rho V_A^2,\quad\text{where } \rho=\gamma/g$$

(*3*)

12.2.  Air flows through a duct, and the Pitot-static tube measuring the velocity is attached to a differential gage containing water. If the deflection of the gage is 4 in, calculate the air velocity, assuming the specific weight of air is constant at 0.0761 lb/ft^3 and the coefficient of the tube is 0.98.

Solution:

For the differential gage, $(p_B-p_A)/\gamma=(4/12)(62.4)/0.0761=273$ ft air. Then

$$V=0.98\sqrt{(64.4)(273)}=130\text{ ft/sec}$$

(See Problems 12.26 through 12.28. for acoustic velocity considerations.)

12.3. Carbon tetrachloride (sp gr 1.60) flows through a pipe. The differential gage attached to the Pitot-static tube shows a 76-mm deflection of mercury. Assuming *c* = 1.00, find the velocity.

Solution:

$$p_B - p_A = (76/1000)(13.6 - 1.60)(9.79) = 8.93 \text{ kPa}, \quad V = \sqrt{(2)(9.81)[8.93/(1.60 \times 9.79)]} = 3.34 \text{ m/s}$$

12.4. Water flows at a velocity of 1.42 m/s. A differential gage that contains a liquid of specific gravity 1.25 is attached to the Pitot-static tube. What is the deflection of the gage fluid?

Solution:

$$V = c\sqrt{2g(\Delta p/\gamma)}, \quad 1.42 = 1.00\sqrt{(2)(9.81)(\Delta p/\gamma)}, \quad \text{and} \quad \Delta p/\gamma = 0.103 \text{ m of water}$$

Applying differential gage principles, 0.103 = (1.25 – 1)h and h = 0.412 m deflection.

12.5. Mathcad Develop the expression for measuring the flow of a gas with a Pitot tube.

Solution:

The flow from A to B in Fig. 12-1 may be considered adiabatic and with negligible loss. Using the Bernoulli equation (*D*) in Problem 7.21 of Chapter 7, A to B, we obtain

$$\left[\left(\frac{k}{k-1}\right)\left(\frac{p_A}{\gamma_A}\right) + \frac{V_A^2}{2g} + 0\right] - \text{negligible loss} = \left[\left(\frac{k}{k-1}\right)\left(\frac{p_A}{\gamma_A}\right)\left(\frac{p_B}{p_A}\right)^{(k-1)/k} + 0 + 0\right]$$

or

$$\frac{V_A^2}{2g} = \left(\frac{k}{k-1}\right)\left(\frac{p_A}{\gamma_A}\right)\left[\left(\frac{p_B}{p_A}\right)^{(k-1)/k} - 1\right]$$

(*1*)

The term p_B is the stagnation pressure. This expression (*1*) is usually rearranged, introducing the ratio of the velocity at A to the acoustic velocity c of the undisturbed fluid.

From Chapter 1, the acoustic velocity $c = \sqrt{E/\rho} = \sqrt{kp/\rho} = \sqrt{kpg/\gamma}$. Combining with equation (*1*) above,

$$\frac{V_A^2}{2} = \left(\frac{c^2}{k-1}\right)\left[\left(\frac{p_B}{p_A}\right)^{(k-1)/k} - 1\right], \quad \text{or} \quad \frac{p_B}{p_A} = \left[1 + \left(\frac{k-1}{2}\right)\left(\frac{V_A}{c}\right)^2\right]^{k/(k-1)}$$

(*2*)

Expanding by the binomial theorem,

$$\frac{p_B}{p_A} = 1 + \left(\frac{k}{2}\right)\left(\frac{V_A}{c}\right)^2\left[1 + \left(\frac{1}{4}\right)\left(\frac{V_A}{c}\right)^2 - \left(\frac{k-2}{24}\right)\left(\frac{V_A}{c}\right)^4 + \cdots\right]$$

(*3*)

In order to compare this expression with formula (3) of Problem 12.1, multiply through by P_A and replace kp_A/c^2 by ρ_A, obtaining

$$p_B = p_A + \frac{1}{2}\rho_A V_A^2\left[1 + \left(\frac{1}{4}\right)\left(\frac{V_A}{c}\right)^2 - \left(\frac{k-2}{24}\right)\left(\frac{V_A}{c}\right)^4 \cdots\right]$$

(*4*)

The above expressions apply to all compressible fluids for ratios of *V*/c less than unity. For ratios over unity, Shockwave and other phenomena occur, the adiabatic assumption is not sufficiently accurate, and the derivation no longer applies. The ratio *V*/*c* is called the *Mach number*.

The bracketed term in (*4*) is greater than unity, and the first two terms provide sufficient accuracy. The effect of compressibility is to increase the stagnation-point pressure over that of an incompressible fluid [see expression (*3*) of Problem 12.1].

Acoustic velocities will be discussed in Problems 12.26 through 12.28.

12.6. Mathcad Air flowing under atmospheric conditions ($\gamma = 12.0$ N/m^3 at 15°C) at a velocity of 90 m/s is measured by a Pitot tube. Calculate the error in the stagnation pressure by assuming the air to be incompressible.

Solution:

Using formula (*3*) of Problem 12.1,

$$p_B = p_A + \tfrac{1}{2}\rho V^2 = 101{,}400 + \left(\tfrac{1}{2}\right)(12.0/9.81)(90)^2 = 106{,}350 \text{ Pa} = 106.35 \text{ kPa}$$

Using formula (*4*) of Problem 12.5 and $c = \sqrt{kgRT} = \sqrt{(1.4)(9.81)(29.3)(288)} = 340$ m/s,

$$p_B = 101{,}400 + \left(\tfrac{1}{2}\right)(12.0/9.81)(90)^2\left[1 + \left(\tfrac{1}{4}\right)(90/340)^2 + \cdots\right]$$

$$= 106{,}440 \text{ Pa} = 106.44 \text{ kPa absolute}$$

The error in the stagnation pressure is 0.1%, and the error in ($p_S - P_A$) is about 2.0%.

12.7. The difference between the stagnation pressure and the static pressure measured by a Pitot-static device is 412 lb/ft^2. The static pressure is 14.5 psi absolute and the temperature in the airstream is 60°F. What is the velocity of the air, assuming air is (*a*) compressible and (*b*) incompressible?

Solution:

(*a*) $p_A = (14.5)(144) = 2088$ psf absolute and $c = \sqrt{kgRT} = \sqrt{(1.4)(32.2)(53.3)(520)} = 1118$ ft/sec.

From equation (2) of Problem 12.5, $$\frac{p_B}{p_A} = \left[1 + \left(\frac{k-1}{2}\right)\left(\frac{V_A}{c}\right)^2\right]^{k/(k-1)}$$

$$\frac{2088 + 412}{2088} = \left[1 + \left(\frac{1.4-1}{2}\right)\left(\frac{V_A}{1118}\right)^2\right]^{1.4/0.4}, \quad V_A = 574 \text{ ft/sec}$$

(*b*) $\gamma = \dfrac{(14.5)(144)}{(53.3)(520)} = 0.0753$ lb/ft^3 and $V = \sqrt{2g(p_B/\gamma - p_A/\gamma)} = \sqrt{2g(412/0.0753)}$ $= 594$ ft/sec.

12.8. Air flows at 800 ft/sec through a duct. At standard barometric pressure, the stagnation pressure is —5.70 ft of water, gage. The stagnation temperature is 145°F. What is the static pressure in the duct?

Solution:

With two unknowns in equation (*2*) of Problem 12.5, assume a *V*/c ratio (Mach number) of 0.72. Then

$$(-5.70 + 34.0)(62.4) = p_A\left[1 + \left(\tfrac{1}{2}\right)(1.4 - 1)(0.72)^2\right]^{1.4/0.4}$$

and $p_A = (62.4)(28.3)/1.412 = 1251$ 1b/ft^2 absolute.

Checking the assumption, using the adiabatic relation

$$\frac{T_B}{T_A} = \left(\frac{p_B}{p_A}\right)^{(k-1)/k}, \quad \frac{460 + 145}{T_A} = \left(\frac{28.3 \times 62.4}{1251}\right)^{0.4/1.4}, \quad T_A = 548°\text{R}$$

Also, $c = \sqrt{kgRT} = \sqrt{(1.4)(32.2)(53.3)(548)} = 1147$ ft/sec.

Then $V/c = 800/1147 = 0.697$ and $p_A = \dfrac{62.4 \times 28.3}{\left[1 + (0.2)(0.697)^2\right]^{1.4/0.4}} = 1277$ lb/ft^2 absolute.

No further refinement is necessary.

12.9. Mathcad A 100-mm-diameter standard orifice discharges water under a 6.1-m head. What is the flow?

Solution:

Applying the Bernoulli equation, A to B in Fig. 12-2, datum B,

$$(0 + 0 + 6.1) - \left(\frac{1}{c_v^2} - 1\right)\left(\frac{V_{\text{jet}}^2}{2g}\right) = \left(\frac{V_{\text{jet}}^2}{2g} + \frac{p_B}{\gamma} + 0\right)$$

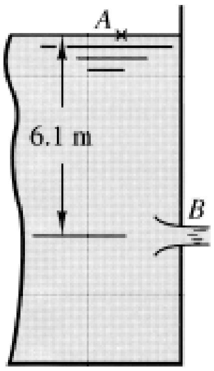

Figure 12-2.

But the pressure head at B is zero (as discussed in Chap. 5, Problem 5.6). Then

$$V_{\text{jet}} = c_v\sqrt{2g \times 6.1}$$

Also, Q = A_{jet} V_{jet}, which, using the definitions of the coefficients, becomes

$$Q = (c_c A_o)c_v\sqrt{2g \times 6.1} = cA_o\sqrt{2g \times 6.1}$$

From Table 7, $c = 0.594$ for $D = 100$ mm and $h = 6.1$ m. Hence

$$Q = (0.594)\left[\tfrac{1}{4}\pi(0.100)^2\right]\sqrt{2g \times 6.1} = 0.0510 \text{ m}^3/\text{s}.$$

12.10. The actual velocity in the contracted section of a jet of liquid flowing from a 50-mm-diameter orifice is 8.53 m/s under a head of 4.57 m. *(a)* What is the value of the coefficient of velocity? (b) If the measured discharge is 0.0114 m^3/s, determine the coefficients of contraction and discharge.

Solution:

(*a*) Actual velocity $= c_v\sqrt{2gH}$, $\quad 8.53 = c_v\sqrt{(2)(9.81)\times 4.57}$, $\quad c_v = 0.901$.

(*b*) Actual $Q = cA\sqrt{2gH}$, $\quad 0.0114 = c\left[\frac{1}{4}\pi(50/1000)^2\right]\sqrt{(2)(9.81)\times 4.57}$, $\quad c = 0.613$.

From $c = c_v \times c_c$, $\quad c_c = 0.613/0.901 = 0.680$.

12.11. Oil flows through a standard 25-mm-diameter orifice under a 5.49 m head at the rate of 0.00314 m^3/s. The jet strikes a wall 1.52 m away and 0.119 m vertically below the centerline of the contracted section of the jet. Compute the coefficients.

Solution:

(*a*) $Q = cA\sqrt{2gH}$, $\quad 0.00314 = c\left[\frac{1}{4}\pi\left(\frac{25}{1000}\right)^2\right]\sqrt{2g(5.49)}$, $\quad c = 0.616$

(*b*) From kinematic mechanics, $x = Vt$ and $y = \frac{1}{2}gt^2$. Here x and y represent the coordinates of the jet, as measured.

Eliminate t and obtain $\quad x^2 = (2V^2/g)y$.

Substituting, $\quad (1.52)^2 = (2V^2/9.81)(0.119)$ and actual $V = 9.76$ m/s in jet.

Then $9.76 = c_v\sqrt{2g(5.49)}$ and $c_v = 0.940$. Finally, $c_c = c/c_v = 0.616/0.940 = 0.655$.

12.12. The tank in Problem 12.9 is closed and the air space above the water is under pressure, causing the flow to increase to 0.075 m^3/s. Find the pressure in the air space in psi.

Solution:

$Q = cA_o\sqrt{2gH}$ $\quad$ or $\quad 0.075 = c\left[\frac{1}{4}\pi(0.100)^2\right]\sqrt{2g(6.1 + p/\gamma)}$

Table 7 indicates that c does not change appreciably at the range of head under consideration. Using c = 0.593 and solving, p/γ = 7.12 m water (the assumed c checks for total head H). Then

$p' = \gamma h = (9.79)(7.12) = 69.7$ kPa

12.13. Oil of specific gravity 0.720 flows through a 3″ diameter orifice whose coefficients of velocity and contraction are 0.950 and 0.650, respectively. What must be the reading of gage A in Fig. 12-3 in order for the power in the jet C to be 8.00 hp?

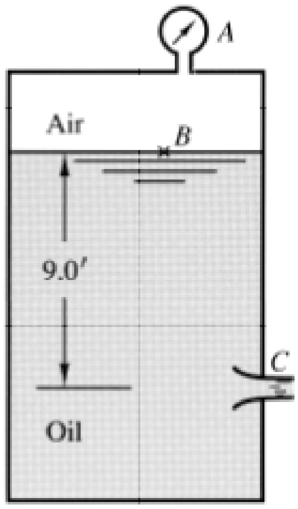

Figure 12-3.

Solution:

The velocity in the jet can be calculated from the value of the power in the jet:

$$\text{horsepower in jet} = \frac{\gamma Q H_{\text{jet}}}{550} = \frac{\gamma\left(c_c A_o V_{\text{jet}}\right)\left(0 + V_{\text{jet}}^2/2g + 0\right)}{550}$$

$$8.00 = \frac{(0.720 \times 62.4)(0.650)\left[\frac{1}{4}\pi\left(\frac{1}{4}\right)^2\right] V_{\text{jet}}^3/2g}{550}$$

Solving, $V_{\text{jet}}^3 = 197{,}669$ and $V_{\text{jet}} = 58.3$ ft/sec.

Applying the Bernoulli equation, *B* to *C*, datum *C*,

$$\left(\frac{p_A}{\gamma} + \text{negl.} + 9.0\right) - \left[\frac{1}{(0.95)^2} - 1\right]\frac{(58.3)^2}{2g} = \left(0 + \frac{(58.3)^2}{2g} + 0\right)$$

and $p_A/\gamma = 49.5$ ft of oil. Then $p'_A = \gamma h/144 = (0.720 \times 62.4)(49.5)/144 = 15.4$ psi.

Note: The reader should not confuse the total head *H* causing flow with the value of H_{jet} in the horsepower expression. They are *not* the same.

12.14. For the 4″-diameter short tube shown in Fig. 12-4, (*a*) what flow of water at 75°F will occur under a head of 30 ft? (*b*) What is the pressure head at section B? (*c*) What maximum head can be used if the tube is to flow full at exit? (Use c_v — 0.82.)

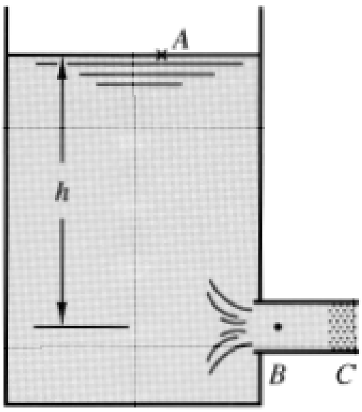

Figure 12-4.

Solution:

For a standard short tube, the stream contracts at *B* to about 0.62 of the area of the tube. The lost head from *A* to *B* has been measured at about 0.042 times the velocity head at *B*.

a. Applying the Bernoulli equation, A to C, datum c,

$$(0 + \text{negl.} + 30) - \left[\frac{1}{(0.82)^2} - 1\right]\frac{V_{\text{jet}}^2}{2g} = \left(0 + \frac{V_{\text{jet}}^2}{2g} + 0\right)$$

and $V_{\text{jet}} = 36.0$ ft/sec. Then $Q = A_{\text{jet}} V_{\text{jet}} = \left[1.00 \times \frac{1}{4}\pi\left(\frac{1}{3}\right)^2\right](36.0) = 3.14$ cfs.

b. Now the Bernoulli equation, A to B, datum B, gives

$$(0 + \text{negl.} + 30) - 0.042\frac{V_B^2}{2g} = \left(\frac{p_B}{\gamma} + \frac{V_B^2}{2g} + 0\right)$$

(A)

Also, $Q = A_B V_B = A_C V_C$ or $c_c A V_B = A V_c$ or $V_B = V_{\text{jet}}/c_c = 36.0/0.62 = 58.1$ ft/sec.

Substituting in equation ($\underline{A}$), $30 = \frac{p_B}{\gamma} + 1.042\frac{(58.1)^2}{2g}$ and $\frac{p_B}{\gamma} = -24.6$ ft of water.

c. As the head causing flow through the short tube is increased, the pressure head at B will become less and less. For steady flow (and with the tube full at exit), the pressure head at B must not be less than the vapor pressure head for the liquid at the particular temperature. From Table 1 in the Appendix, for water at 75°F this value is 0.43 psia or about 1.0 ft absolute at sea level (–33.0 ft gage).

From ($\underline{A}$) above,

$$h = \frac{p_B}{\gamma} + 1.042\frac{V_B^2}{2g} = -33.0 + 1.042\frac{V_B^2}{2g}$$

(B)

Also,

$$c_c A V_B = A V_C = A c_v \sqrt{2gh}$$

Thus

$$V_B = \frac{c_v}{c_c}\sqrt{2gh} \qquad \text{or} \qquad \frac{V_B^2}{2g} = \left(\frac{c_v}{c_c}\right)^2 h = \left(\frac{0.82}{0.62}\right)^2 h = 1.75h$$

Substituting in ($\underline{B}$), h = –33.0 + (1.042)(1.75/z) and h = 40.1 ft of water (75°F).

Any head over 40 ft will cause the stream to spring free of the sides of the tube. The tube will then function as an orifice.

Cavitation may result at vapor pressure conditions (see Chapter 14).

12.15. Water flows through a 100-mm pipe at the rate of 0.027 m^3/s and thence through a nozzle attached to the end of the pipe. The nozzle tip is 50 mm in diameter, and the coefficients of velocity and contraction for the nozzle are 0.950 and 0.930, respectively. What pressure head must be maintained at the base of the nozzle if atmospheric pressure surrounds the jet?

Solution:

Apply the Bernoulli equation, base of nozzle to jet.

$$\left(\frac{p}{\gamma} + \frac{V_{100}^2}{2g} + 0\right) - \left[\frac{1}{(0.950)^2} - 1\right]\frac{V_{\text{jet}}^2}{2g} = \left(0 + \frac{V_{\text{jet}}^2}{2g} + 0\right)$$

and the velocities are computed from $Q = AV$: $0.027 = A_{100}V_{100} = A_{\text{jet}}V_{\text{jet}} = (c_c A_{50})V_{\text{jet}}$. Thus

$$V_{100} = \frac{0.027}{\frac{1}{4}\pi\left(\frac{100}{1000}\right)^2} = 3.44 \text{ m/s} \quad \text{and} \quad V_{\text{jet}} = \frac{0.027}{0.930\left[\frac{1}{4}\pi\left(\frac{50}{1000}\right)^2\right]} = 14.8 \text{ m/s}.$$

Substituting above and solving, $p/\gamma = 11.77$ m of water.

Had the formula $V_{\text{jet}} = c_v\sqrt{2gH}$ been used, H would be $(p/\gamma + V_{100}^2/2g)$ or

$$14.8 = 0.950\sqrt{2g\left[p/\gamma + (3.44)^2/2g\right]}$$

from which $p/\gamma = 11.77$ m of water, as before.

12.16. A 100-mm base diameter by 50-mm tip diameter nozzle points downward, and the pressure head at the base of the nozzle is 7.92 m of water. The base of the nozzle is 0.914 m above the tip, and the coefficient of velocity is 0.962. Determine the power in the jet of water.

Solution:

For a nozzle, unless c_c is given, it may be taken as unity. Therefore $V_{\text{jet}} = V_{50}$ mm.

Before the power can be calculated, both V and Q must be found. Using the Bernoulli equation, base to tip, datum tip, gives

$$\left(7.92 + \frac{V_{100}^2}{2g} + 0.914\right) - \left[\frac{1}{(0.962)^2} - 1\right]\frac{V_{50}^2}{2g} = \left(0 + \frac{V_{50}^2}{2g} + 0\right)$$

and $A_{100}V_{100} = A_{50}V_{50}$ or $V_{100}^2 = (50/100)^4 V_{50}^2 = \frac{1}{16}V_{50}^2$. Solving, $V_{50} = 13.0$ m/s.

$$\text{power in jet} = \gamma Q H_{\text{jet}} = (9.79)\left[\tfrac{1}{4}\pi(50/1000)^2(13.0)\right]\left[0 + (13.0)^2/2g + 0\right] = 2.15 \text{ kW}$$

12.17. Water flows through a 12″ × 6″ Venturi meter at the rate of 1.49 cfs, and the differential gage is deflected 3.50 ft, as shown in Fig. 12-5. The specific gravity of the gage liquid is 1.25. Determine the coefficient of the meter.

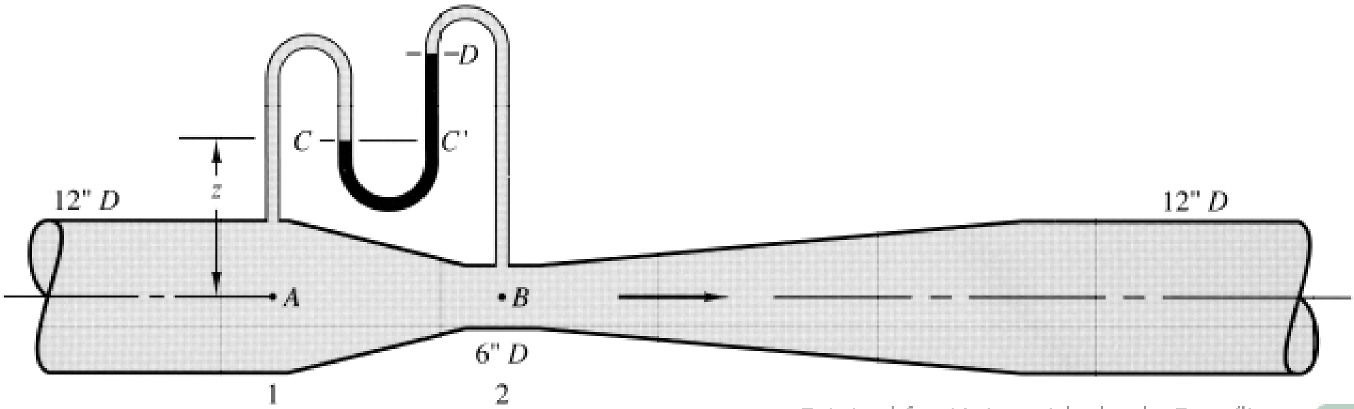

Figure 12-5.

Solution:

The coefficient of a Venturi meter is the same as the coefficient of discharge ($C_c = 1.00$ and thus $c = c_v$). Flow coefficient K should not be confused with meter coefficient c. Clarification will be made at the end of this problem.

Applying the Bernoulli equation, A to B, ideal case, yields

$$\left(\frac{p_A}{\gamma} + \frac{V_{12}^2}{2g} + 0\right) - \text{no lost head} = \left(\frac{p_B}{\gamma} + \frac{V_6^2}{2g} + 0\right)$$

and $V_{12}^2 = (A_6/A_{12})^2 V_6^2$. Solving, $V_6 = \sqrt{\dfrac{2g(p_A/\gamma - p_B/\gamma)}{1-(A_6/A_{12})^2}}$ (no lost head).

The true velocity (and hence the true value of flow Q) will be obtained by multiplying the ideal value by the coefficient c of the meter. Thus

$$Q = A_6 V_6 = A_6\, c\sqrt{\frac{2g(p_A/\gamma - p_B/\gamma)}{1-(A_6/A_{12})^2}}$$

(*1*)

To obtain the differential pressure head indicated above, the principles of the differential gage must be

$$p_c = p_{c'}$$

$$(p_A/\gamma - z) = p_B/\gamma - (z + 3.50) + (1.25)(3.50) \quad \text{or} \quad (p_A/\gamma - p_B/\gamma) = 0.875 \text{ ft}$$

Substituting in (*1*), $1.49 = \frac{1}{4}\pi\left(\frac{1}{2}\right)^2 c\sqrt{2g(0.875)/(1-1/16)}$ and $c = 0.979$ (use 0.98).

Note: Equation (*1*) is sometimes written $Q = K A_2\sqrt{2g(\Delta p/\gamma)}$, where K is called the *flow coefficient*. It is apparent that

$$K = \frac{c}{\sqrt{1-(A_2/A_1)^2}} \quad \text{or} \quad \frac{c}{\sqrt{1-(D_2/D_1)^4}}$$

Tables or charts that give K can readily be used to obtain c if it is so desired. Diagrams in this book give values of c. The conversion factors to obtain the values of K for certain diameter-ratio devices are indicated on the several diagrams in the Appendix.

12.18. Water flows upward through a vertical 300 mm by 150 mm Venturi meter whose coefficient is 0.980. The differential gage deflection is 1.18 m of liquid of specific gravity 1.25, as shown in Fig. 12-6. Determine the flow in cfs.

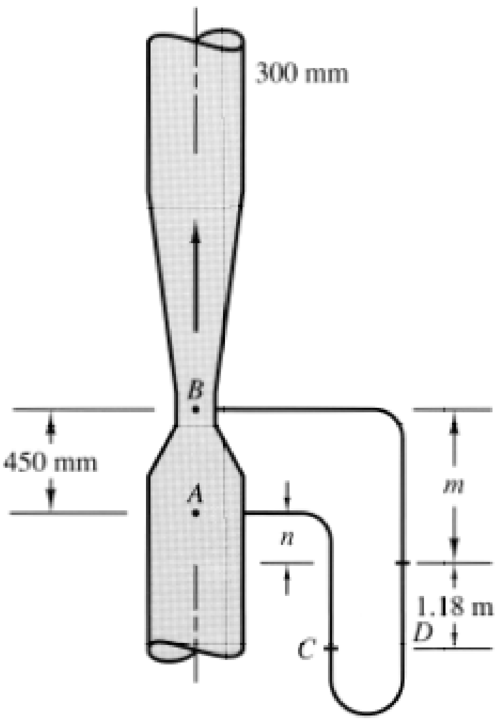

Figure 12-6.

Solution:

Reference to the Bernoulli equation in Problem 12.17 indicates that, for this problem, $z_A = 0$ and $z_B = 0.450$ m. Then

$$Q = cA_{150}\sqrt{\frac{2g[(p_A/\gamma - p_B/\gamma) - 0.450]}{1-(1/2)^4}}$$

Using the principles of the differential gage to obtain $\Delta p/\gamma$.

$$p_c/\gamma = p_D/\gamma \text{ (m of water units)}$$

$$p_A/\gamma + (n + 1.18) = p_B/\gamma + m + (1.25)(1.18)$$

$$[(p_A/\gamma - p_B/\gamma) - (m - n)] = (1.18)(1.25 - 1.00)$$

$$[(p_A/\gamma - p_B/\gamma) - 0.450] = 0.295 \text{ m of water}$$

Substituting into the equation for flow,

$$Q = (0.980)\left(\tfrac{1}{4}\pi\right)\left(\frac{150}{1000}\right)^2 \sqrt{2g(0.295)/(1-1/16)} = 0.0430 \text{ m}^3/\text{s}\cdot$$

12.19. Water at 100°F flows at the rate of 0.525 cfs through a 4″-diameter orifice used in an 8″ pipe. What is the difference in pressure head between the upstream section and the contracted section (vena contracta section)?

Solution:

In Diagram *C* of the Appendix, it is observed that c′ varies with Reynolds number. Note that Reynolds number must be calculated for the orifice cross section, not for the contracted section of the jet or for the pipe section. This value is

$$\text{Re} = \frac{V_o D_o}{\nu} = \frac{\left(4Q/\pi D_o^2\right) D_o}{\nu} = \frac{4Q}{\nu \pi D_o} = \frac{(4)(0.525)}{(\pi)(0.00000739)(4/12)} = 271{,}000$$

Diagram *C* for $\beta = 0.500$ gives $c' = 0.604$.

Applying the Bernoulli theorem, pipe section to jet section, produces the general equation for incompressible fluids, as follows:

$$\left(\frac{p_8}{\gamma} + \frac{V_8^2}{2g} + 0\right) - \left[\frac{1}{c_v^2} - 1\right]\frac{V_{\text{jet}}^2}{2g} = \left(\frac{p_{\text{jet}}}{\gamma} + \frac{V_{\text{jet}}^2}{2g} + 0\right)$$

and

$$Q = A_8 V_8 = (c_c A_4) V_{\text{jet}}$$

Substituting for V_8 in terms of V_{jet} and solving,

$$\frac{V_{\text{jet}}^2}{2g} = c_v^2\left(\frac{p_8/\gamma - p_{\text{jet}}/\gamma}{1 - c^2(A_4/A_8)^2}\right) \quad \text{or} \quad V_{\text{jet}} = c_v\sqrt{\frac{2g(p_8/\gamma - p_{\text{jet}}/\gamma)}{1 - c^2(D_4/D_8)^4}}$$

Then

$$Q = A_{jet}V_{jet} = (c_c A_4) \times c_v \sqrt{\frac{2g(p_8/\gamma - p_{jet}/\gamma)}{1 - c^2(D_4/D_8)^4}} = cA_4\sqrt{\frac{2g(p_8/\gamma - p_{jet}/\gamma)}{1 - c^2(D_4/D_8)^4}}.$$

More conveniently, for an orifice with velocity of approach and a contracted jet, the equation can be written

$$Q = \frac{c' A_4}{\sqrt{1 - (D_4/D_8)^4}}\sqrt{2g(\Delta p/\gamma)}$$

(*1*)

or

$$Q = KA_4\sqrt{2g(\Delta p/\gamma)}$$

(*2*)

where K is called the *flow* coefficient. The meter coefficient c' can be determined experimentally for a given ratio of diameter of orifice to diameter of pipe, or the flow coefficient K may be preferred.

Proceeding with the solution by substituting in the above expression (*1*),

$$0.525 = \frac{0.604 \times \frac{1}{4}\pi(4/12)^2}{\sqrt{1 - (1/2)^4}}\sqrt{2g(\Delta p/\gamma)} \text{ and } \Delta p/\gamma = (p_8/\gamma - p_{jet}/\gamma) = 1.44 \text{ ft water}$$

12.20. For the pipe orifice in Problem 12.19, what pressure difference in psi would cause the same quantity of turpentine at 68°F to flow? (see Appendix for sp gr and ν.)

Solution:

$$\text{Re} = \frac{4Q}{\pi \nu D_o} = \frac{(4)(0.525)}{(\pi)(0.0000186)(4/12)} = 108{,}000. \text{ From Diagram } C, \text{ for } \beta = 0.500, c' = 0.607.$$

Then $$0.525 = \frac{0.607 \times \frac{1}{4}\pi(4/12)^2}{\sqrt{1 - (1/2)^4}}\sqrt{2g(\Delta p/\gamma)}, \qquad \text{from which}$$

$$\Delta\frac{p}{\gamma} = \left(\frac{p_8}{\gamma} - \frac{p_{jet}}{\gamma}\right) = 1.43 \text{ ft turpentine} \quad \text{and} \quad \Delta p' = \frac{\gamma h}{144} = \frac{(0.862 \times 62.4)(1.43)}{144} = 0.534 \text{ psi.}$$

12.21. Determine the flow of water at 20°C through a 150-mm orifice installed in a 250-mm pipeline if the pressure head differential for vena contracta taps is 1.10 m of water.

Solution:

This type of problem was met in the flow of fluids in pipes. The value of c′ cannot be found inasmuch as Reynolds number cannot be computed. Referring to Diagram C, for β = 0.600, a value of c′ of 0.610 will be assumed. Using this assumed value,

$$Q = \frac{0.610 \times \frac{1}{4}\pi(150/1000)^2}{\sqrt{1 - (0.60)^4}}\sqrt{(2)(9.81)(1.10)} = 0.0537 \text{ m}^3\text{/s}$$

Than

$$\text{Re} = \frac{(4)(0.0537)}{\pi\left(9.84 \times 10^{-7}\right)\left(150/1000\right)} = 463{,}000 \text{ (trial value)}$$

From Diagram C, β = 0.600, c′ = 0.609. Recalculation of the flow using c′ = 0.609 gives $Q = 0.0536$ m^3/s.

(Reynolds number is unaffected.)

Special Note: Professor R. C. Binder of Purdue University suggests on pages 132-133 of his fluid mechanics text (second edition) that this type of problem need not be a "cut and try" proposition. He proposes that special lines be drawn on the coefficient-Reynolds number chart. In the case of the pipe orifice, equation (*1*) of Problem 12.19 can be written

$$\frac{Q}{A_4} = \frac{c'\sqrt{2g(\Delta p/\gamma)}}{\sqrt{1-(D_4/D_8)^4}} = V_4 \qquad \text{since } Q = AV$$

$$\text{But} \qquad \mathrm{Re} = \frac{V_4 D_4}{\nu} = \frac{c'\sqrt{2g(\Delta p/\gamma)} \times D_4}{\nu\sqrt{1-(4/8)^4}} \qquad \text{or} \qquad \frac{\mathrm{Re}}{c'} = \frac{D_4\sqrt{2g(\Delta p/\gamma)}}{\nu\sqrt{1-(4/8)^4}}$$

$$\text{or, in general,} \qquad \frac{\mathrm{Re}}{c'} = \frac{D_o\sqrt{2g(\Delta p/\gamma)}}{\nu\sqrt{1-(D_o/D_p)^4}}$$

Two straight lines called *T*-lines have been drawn on Diagram *C*, one for Re/c' = 700,000 and one for Re/c' = 800,000. For Problem 12.21, the calculated

$$\frac{\mathrm{Re}}{c'} = \frac{(150/1000)\sqrt{(2)(9.81)(1.10)}}{(9.84 \times 10^{-7})\sqrt{1-(0.60)^4}} = 759{,}000$$

As near as can be read, the 759,000 line cuts the β = 0.600 curve at c' = 0.609. The flow Q is then calculated readily.

12.22. A nozzle with a 4″-diameter tip is installed in a 10″ pipe. Medium fuel oil at 80°F flows through the nozzle at the rate of 3.49 cfs. Assume the calibration of the nozzle is represented by curve β = 0.40 on Diagram *D*. Calculate the differential gage reading if a liquid of specific gravity 13.6 is the gage liquid.

Solution:

The Bernoulli equation, pipe section to jet, yields the same equation as was obtained in Problem 12.17 for the Venturi meter, since the nozzle is designed for a coefficient of contraction of unity.

$$Q = A_4 V_4 = A_4 c\sqrt{\frac{2g(p_A/\gamma - p_B/\gamma)}{1-(4/10)^4}}$$

(*1*)

Diagram *D* indicates that c varies with Reynolds number.

$$V_4 = \frac{Q}{A_4} = \frac{3.49}{\frac{1}{4}\pi(4/12)^2} = 40.0\ \text{ft/sec} \qquad \text{and} \qquad \mathrm{Re} = \frac{40.0 \times 4/12}{3.65 \times 10^{-5}} = 365{,}000$$

The curve for β = 0.40 gives c = 0.993. Thus

$$3.49 = \frac{1}{4}\pi(4/12)^2 \times 0.993\sqrt{\frac{2g(p_A/\gamma - p_B/\gamma)}{1-(4/10)^4}}$$

and $(P_A/\gamma - P_B/\gamma)$ = 24.5 ft of fuel oil.

Differential gage principles produce, using sp gr of the oil = 0.851 from the Appendix,

$24.5 = h(13.6/0.851 - 1)$ and $h = 1.64$ ft (gage reading)

Had the gage differential reading been given, the procedure used in the preceding problem would be utilized, e.g., a value of c assumed, Q calculated, Reynolds number obtained, and c read from the appropriate curve on Diagram D. If c differs from the assumed value, the calculation is repeated until the coefficient checks in.

12.23. Derive an expression for the flow of a compressible fluid through a nozzle flow meter and a Venturi meter.

Solution:

Since the change in velocity takes place in a very short period of time, little heat can escape and adiabatic conditions will be assumed. The Bernoulli theorem for compressible flow was shown in Chapter 7, equation (D) of Problem 7.21, to give

$$\left[\left(\frac{k}{k-1}\right)\frac{p_1}{\gamma_1}+\frac{V_1^2}{2g}+z_1\right]-H_L=\left[\left(\frac{k}{k-1}\right)\left(\frac{p_1}{\gamma_1}\right)\left(\frac{p_2}{p_1}\right)^{(k-1)/k}+\frac{V_2^2}{2g}+z_2\right]$$

For a nozzle meter and for a horizontal Venturi meter, $z_1 = z_2$ and the lost head will be taken care of by means of the coefficient of discharge. Also, since $c_c = 1.00$,

$$W = \gamma_1 A_1 V_1 = \gamma_2 A_2 V_2$$

Then upstream $V_1 = W/\gamma_1 A_1$, downstream $V_2 = W/\gamma_2 A_2$. Substituting and solving for W,

$$\frac{W^2}{\gamma_2^2 A_2^2}-\frac{W^2}{\gamma_1^2 A_1^2}=2g\left(\frac{k}{k-1}\right)\left(\frac{p_1}{\gamma_1}\right)\left[1-\left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]$$

or

$$\text{(ideal) } W=\frac{\gamma_2 A_2}{\sqrt{1-(\gamma_2/\gamma_1)^2(A_2/A_1)^2}}\sqrt{\frac{2gk}{k-1}(p_1/\gamma_1)\times[1-(p_2/p_1)^{(k-1)/k}]}$$

It may be more practical to eliminate γ_2 under the radical. Since $\gamma_2/\gamma_1 = (p_2/p_1)^{1/k}$,

$$\text{(ideal) } W=\gamma_2 A_2\sqrt{\frac{\frac{2gk}{k-1}(p_1/\gamma_1)\times\left[1-(p_2/p_1)^{(k-1)/k}\right]}{1-(A_2/A_1)^2(p_2/p_1)^{2/k}}}$$

(*1*)

The true value of W is obtained by multiplying the right-hand side of the equation by coefficient c.

For comparison, equation (*1*) of Problem 12.17 and equation (*1*) of Problem 12.22 (for incompressible fluids) can be written

$$W=\gamma Q=\frac{\gamma A_2 c}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2g(\Delta p/\gamma)}$$

or

$W = \gamma K A_2 \sqrt{2g(\Delta p/\gamma)}$

The above equation can be expressed more generally so that it will apply to both compressible and incompressible fluids. An expansion (adiabatic) factor Y is introduced, and the value of γ_1 at the inlet is specified. The fundamental relation is then

$W = \gamma_1 K A_2 Y \sqrt{2g(\Delta p/\gamma_1)}$

(*2*)

For incompressible fluids, $Y = 1$. For compressible fluids, equate expressions (*1*) and (*2*) and solve for Y. Thus,

$$Y = \sqrt{\frac{1-(A_2/A_1)^2}{1-(A_2/A_1)^2(p_2/p_1)^{2/k}} \times \frac{[k/(k-1)]\left[1-(p_2/p_1)^{k-1)/k}\right](p_2/p_1)^{2/k}}{1-p_2/p_1}}$$

This expansion factor Y is a function of three dimensionless ratios. Table 8 lists some typical values for nozzle flow meters and for Venturi meters.

Note: Values of Y' for orifices and for orifice meters should be determined experimentally. The values differ from the above value of Y because the coefficient of contraction is not unity nor is it a constant. Knowing Y' solutions are identical to those that follow for flow nozzles and Venturi meters. The reader is referred to experiments by H. B. Reynolds and J. A. Perry as two sources of material.

12.24. Mathcad Air at a temperature of 80°F flows through a 4″ pipe and through a 2″ flow nozzle. The pressure differential is 0.522 ft of oil, sp gr 0.910. The pressure upstream from the nozzle is 28.3 psi gage. How many pounds per second are flowing for a barometric reading of 14.7 psi, (*a*) assuming the air has constant density and (*b*) assuming adiabatic conditions?

Solution:

a.

$$\gamma_1 = \frac{(28.3+14.7)(144)}{(53.3)(460+80)} = 0.215 \text{ lb/ft}^3$$

From differential gage principles, using pressure heads in feet of air,

$$\frac{\Delta p}{\gamma_1} = 0.522\left(\frac{\gamma_{oil}}{\gamma_{air}} - 1\right) = 0.522\left(\frac{0.910 \times 62.4}{0.215} - 1\right) = 137 \text{ ft of air}$$

Assuming $c = 0.980$ and using equation (*1*) of Problem 12.22 after multiplying by γ_1, we have

$$W = \gamma_1 Q = 0.215 \times \frac{1}{4}\pi(2/12)^2(0.980)\sqrt{\frac{2g(137)}{1-(2/4)^4}} = 0.446 \text{ lb/sec}$$

To check the value of c, find the Reynolds number and use the appropriate curve on Diagram *D*. (Here $\gamma_1 = \gamma_2$ and $\nu = 16.9 \times 10^{-5}$ ft^2/sec at standard atmosphere from Table *1B*.)

$$V_2 = \frac{W}{A_2\gamma_2} = \frac{W}{(\pi d_2^2/4)\gamma_2}$$

Then $\text{Re} = \dfrac{V_2 d_2}{\nu} = \dfrac{4W}{\pi d_2 \nu \gamma_2} = \dfrac{(4)(0.446)}{(\pi)(2/12)(16.9 \times 14.7/43.0)10^{-5}(0.215)} = 274{,}000$

From Diagram D, $c = 0.986$. Recalculating, W = 0.449 lb/sec.

Further refinement in calcution is not warranted inasmush as the Reynolds number will not be changed materially nor will the value of c read from Diagram D.

b. Calculate pressures and specific weight first.

$$p_1 = (28.3 + 14.7)(144) = 6192 \text{ lb/ft}^2, \quad p_2 = (6192 - 137 \times 0.215) = 6163 \text{ lb/ft}^2$$

$$\frac{p_2}{p_1} = \frac{6163}{6192} = 0.995 \quad \text{and} \quad \left(\frac{\gamma_2}{\gamma_1}\right)^k = 0.994 \text{ (see Chapter 1). Then } \gamma_2 = 0.214 \text{ lb/ft}^3.$$

Table 8 gives some values of expansion factor Y referred to in Problem 12.23. Interpolation may be used, in this case between the pressure ratio of 0.95 and 1.00 to obtain Y for $p_2/p_1 = 0.994$. For $d_2/d_1 = 0.50$, we obtain Y = 0.997.

Assuming c = 0.980, from examination of Diagram D and noting that $K = 1.032c$, equation (2) of Problem 12.23 becomes

$$W = \gamma_1 K A_2 Y \sqrt{2g(\Delta p/\gamma_1)}$$

$$= (0.215)(1.032 \times 0.980) \times \tfrac{1}{4}\pi(2/12)^2 \times 0.997\sqrt{(64.4)(137)} = 0.444 \text{ lb/sec}$$

Checking c, $\qquad \text{Re} = \dfrac{4W}{\pi d_2 \nu \gamma_2} = \dfrac{(4)(0.444)}{(\pi)(2/12)(16.9 \times 14.7/43.0)10^{-5}(0.214)} = 274{,}000$

and c = 0.986 (Diagram D, curve $\beta = 0.50$).

Recalculating, $W = 0.447$ 1b/sec. Further refinement is not essential. Note that little error introduced in part (*a*) by assuming a constant density of air.

12.25. Mathcad An 8″×4″ Venturi meter is used to measure the flow of carbon dioxide at 68°F. The deflection of the water column in the differential gage is 71.8 in, and the barometer reads 30.0 in of mercury. For a pressure at entrance of 18.0 psi absolute, calculate the weight flow.

Solution:

The absolute pressure at entrance $= p_1 = 18.0 \times 144 = 2592$ psf absolute, and the specific weight of the carbon dioxide is

$$\gamma_1 = \frac{2592}{(34.9)(460 + 68)} = 0.141 \text{ lb/ft}^3$$

The pressure difference = (71.8/12)(62.4 – 0.141) = 373 psf, and hence the absolute pressure at throat = $p_2 = 2592 - 373 = 2219$ psf absolute.

To obtain the specific weight γ_2 we use $\dfrac{p_2}{p_1} = \dfrac{2219}{2592} = 0.856$ and $\dfrac{\gamma_2}{\gamma_1} = (0.856)^{1/k}$ (see chap. 1)

Thus $\gamma_2 = (0.141)(0.856)^{1/1.30} = 0.1251 \text{ lb/ft}^3$.

$W = \gamma_1 K A_2 Y \sqrt{2g(\Delta p/\gamma_1)}$ lb/sec

Using $K = 1.30$. $d_2/d_1 = 0.50$, and $p_2/p_1 = 0.856$, Y (Table 8) = 0.909 by interpolation. Assuming c = 0.985, from Diagram E, and noting that $K = 1.032c$, we have

$$W = (0.141)(1.032 \times 0.985) \times \frac{1}{4}\pi(4/12)^2 \times 0.909\sqrt{2g(373/0.141)} = 4.69 \text{ lb/sec}$$

To check the assumed value of c, determine the Reynolds number and use the appropriate curve on Diagram E. From Problem 12.24

$$\mathrm{Re} = \frac{4W}{\pi d_2 \nu \gamma_2} = \frac{(4)(4.69)}{(\pi)(4/12)(9.1 \times 14.7/18.0 \times 10^{-5})(0.1251)} = 1.93 \times 10^6$$

From Diagram E, $c = 0.984$. Recaluting, W = 4.69 1b/sec.

12.26. Establish the relationship that limits the velocity of a compressible fluid in convergent passages (acoustic velocity).

Solution:

Neglecting the velocity of approach in the Bernoulli equation (D) of Problem 7.21, Chapter 7, for an ideal fluid we obtain

$$\frac{V_2^2}{2g} = \frac{k}{k-1}\left(\frac{p_1}{\gamma_1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]$$

(*1*)

Also, if $(p_2/\gamma_2)1^k$ had been substituted for $(p_1/\gamma_1)1^k$ before the integration that produced equation (D), the velocity head would have been

$$\frac{V_2^2}{2g} = \frac{k}{k-1}\left(\frac{p_2}{\gamma_2}\right)\left[\left(\frac{p_1}{p_2}\right)^{(k-1)/k} - 1\right]$$

(*2*)

If the fluid attains acoustic velocity c_2 at section 2, then $V_2 = c_2$ and $V_2^2 = c_2^2 = kp_2g/\gamma_2$, (see Chap. 1). Substituting in equation (*2*),

$$\frac{kp_2g}{2g\gamma_2} = \frac{k}{k-1}\left(\frac{p_2}{\gamma_2}\right)\left[\left(\frac{p_1}{p_2}\right)^{(k-1)/k} - 1\right]$$

which simplifies to

$$\frac{p_2}{p_1} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

(*3*)

This ratio p_2/p_1 is called the *critical pressure ratio* and depends upon the fluid flowing. For values of P_2/p_1 equal to or less than the critical pressure ratio, a gas will flow at the acoustic velocity. The pressure in a free jet flowing at the acoustic velocity will *equal or exceed* the pressure that surrounds it.

12.27. Carbon dioxide discharges through a 12.5-mm hole in the wall of a tank in which the pressure is 758 kPa gage and the temperature is 20°C. What is the velocity in the jet (standard barometer)?

Solution:

From Table 1*A*, $R = 19.2$ m/k and $K = 1.30$.

$$\gamma_1 = \frac{p_1}{RT_1} = \frac{758 + 101}{(19.2)(293)} = 0.153 \text{ kN/m}^3$$

$$\text{critical } \frac{p_2}{p_1} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = \left(\frac{2}{2.30}\right)^{1.30/0.30} = 0.546$$

$$\text{ratio } \left(\frac{\text{atmosphere}}{\text{tank pressure}}\right) = \frac{101}{859} = 0.118$$

Since this ratio is less than the critical pressure ratio, the pressure of the escaping gas = $0.546 \times p_1$. Hence $p_2 = 0.546 \times 859 = 469$ kPa absolute.

$$V_2 = c_2 = \sqrt{1.3 \times 9.81 \times 19.3 \times T_2} = \sqrt{246 T_2}$$

where $T_2/T_1 = (p_2/p_1)^{(k-1)/k} = (0.546)^{0.30/1.30} = 0.870$, $T_2 = 255$ K. Then $V_2 = \sqrt{246 \times 255} = 250$ m/s.

12.28. Nitrogen flows through a duct in which changes in cross section occur. At a particular cross section the velocity is 366 m/s, the pressure is 83 kPa absolute, and the temperature is 30°C. Assuming no friction losses and adiabatic conditions, (*a*) what is the velocity at a section where the pressure is 124 kPa absolute and (*b*) what is the Mach number at this section?

Solution:

For nitrogen, $R = 30.3$ m/K and $K = 1.40$, from Table 1*A* of the Appendix.

a. From Problem 7.21 of Chapter 7, equation (*D*) for adiabatic conditions may be written

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{k}{k-1}\left(\frac{p_1}{\gamma_1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]$$

where no lost head is considered and $z_1 = z_2$.

Calculate the specific weight of nitrogen at cross section 1.

$$\gamma_1 = \frac{p_1}{RT_1} = \frac{83{,}000}{(30.3)(30+273)} = 9.04 \text{ N/m}^3 \quad (\text{or use } p_1/\gamma_1 = RT_1)$$

$$\text{Then } \frac{V_2^2}{2g} - \frac{(366)^2}{2g} = \left(\frac{1.40}{0.40}\right)\left(\frac{83{,}000}{9.04}\right)\left[1 - \left(\frac{124}{83}\right)^{0.40/1.40}\right], \qquad \text{from which } V_2 = 239 \text{ m/s.}$$

b.

$$\text{Mach number} = \frac{V_2}{c_2} = \frac{239}{\sqrt{kgRT_2}} \quad \text{where } \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \quad \text{or} \quad \frac{T_2}{303} = \left(\frac{124}{83}\right)^{0.286} = 1.122.$$

$$\text{Then } T_2 = 340 \text{ K}, \quad \text{and} \quad \text{Mach number} = \frac{239}{\sqrt{1.40 \times 9.81 \times 30.3 \times 340}} = 0.635.$$

12.29. Develop the theoretical formula for flow over a rectangular weir.

Solution:

Consider the rectangular opening in Fig. 12-7 to extend the full width W of the channel($b = W$). With the liquid surface in the dashed position, application of the Bernoulli theorem between A and an elemental strip of height dy in the jet produces, for ideal conditions,

$$\left(0 + V_A^2/2g + y\right) - \text{no losses} = \left(0 + V_{\text{jet}}^2/2g + 0\right)$$

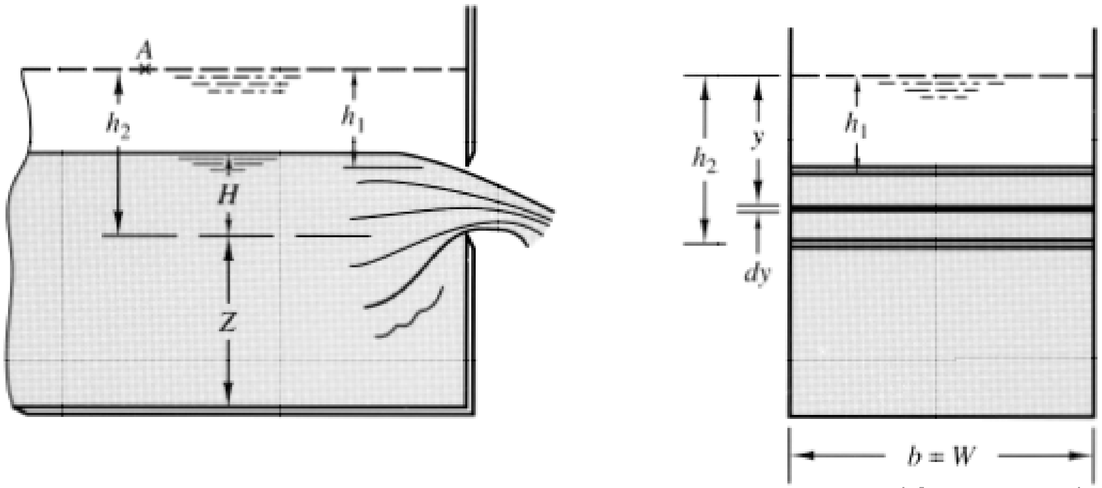

Figure 12-7.

where V_A represents the average velocity of the particles approaching the opening. Thus,

$$\text{ideal } V_{\text{jet}} = \sqrt{2g\left(y + V_A^2/2g\right)}$$

and

$$\text{ideal } dQ = dA\, V_{\text{jet}} = (b\,dy)V_{\text{jet}} = b\sqrt{2g}\left(y + V_A^2/2g\right)^{1/2} dy$$

$$\text{ideal } Q = b\sqrt{2g}\int_{h_1}^{h_2}\left(y + V_A^2/2g\right)^{1/2} dy$$

A weir exists when $h_1 = 0$. Let H replace h_2, and introduce a coefficient of discharge c to obtain the actual flow. Then

$$Q = cb\sqrt{2g}\int_0^H \left(y + V_A^2/2g\right)^{1/2} dy$$

$$= \frac{2}{3}cb\sqrt{2g}\left[\left(H + V_A^2/2g\right)^{3/2} - \left(V_A^2/2g\right)^{3/2}\right]$$

$$= mb\left[\left(H + V_A^2/2g\right)^{3/2} - \left(V_A^2/2g\right)^{3/2}\right]$$

(*1*)

Notes:

1. For a fully contracted rectangular weir, the end contractions cause a reduction in flow. Length b is corrected to recognize this condition, and the formula becomes

$$Q = m\left(b - \frac{2}{10}H\right)\left[\left(H + V_A^2/2g\right)^{3/2} - \left(V_A^2/2g\right)^{3/2}\right]$$

(2)

2. For high weirs and most contracted weirs, the velocity head of approach is negligible is negligible and

$$Q = m\left(b - \frac{2}{10}H\right)H^{3/2} \qquad \text{for contracted weirs}$$

(3)

or

$$Q = mbH^{3/2} \qquad \text{for suppressed weirs}$$

(4)

3. Coefficient of discharge c is not constant. It embraces the many complexities not included in the derivation, such as surface tenstion, viscoity. density. nonuniform velocity distribution, secondary flows, and possibly others.

12.30. Derive the theoretical formula for flow through a triangular weir. Refer to Fig. 12-8.

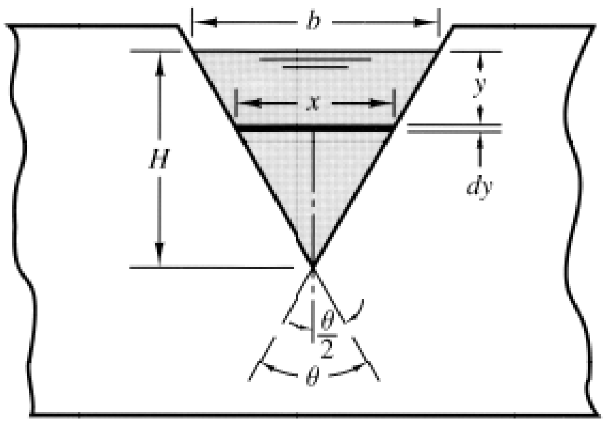

Figure 12-8.

Solution:

From Problem 12.29,

$$V_{jet} = \sqrt{2g(y + \text{ negligible } V^2/2g)} \qquad \text{and} \qquad \text{ideal } dQ = dA\,V_{jet} = x\,dy\sqrt{2gy}$$

By similar triangles,

$$\frac{x}{b} = \frac{H-y}{H} \qquad \text{and} \qquad b = 2H\,\tan\frac{\theta}{2}$$

Then actual $Q = (b/H)c\sqrt{2g}\int_0^H (H-y)y^{1/2}dy$.

Integrating and substituting,

$$Q = \frac{8}{15}c\sqrt{2g}H^{5/2}\tan\frac{1}{2}\theta$$

(1)

12.31. During a test on an 8-ft suppressed weir that was 3 ft high, the head was maintained constant at 1.000 ft. In 38.0 sec, 7600 gallons of water were collected. Find weir factor m in equations (*1*) and (*4*) of Problem 12.29.

Solution:

(a) Change the measured flow to cfs. $Q = 7600/(7.48 \times 38.0) = 26.7$ cfs.

(b) Check the velocity of approach. $V = Q/A = 26.7/(8 \times 4) = 0.834$ ft/sec. Then

$$V^2/2g = (0.834)^2/2g = 0.0108 \text{ ft}$$

(c) Using (*1*),

$$Q = mb\left[\left(H + V^2/2g\right)^{3/2} - \left(V^2/2g\right)^{3/2}\right]$$

or

$$26.7 = m \times 8\left[(1.000 + 0.0108)^{3/2} - (0.0108)^{3/2}\right]$$

and $m = 3.29$.

Using (*4*),

$$Q = 26.7 = mbH^{3/2} = m \times 8 \times (1.000)^{3/2}$$

and $m = 3.34$ (about 1.5% higher neglecting the velocity of approach terms).

12.32. Mathcad Determine the flow over a suppressed weir 3.00 m long and 1.20 m high under a head 0.914 m. The value of m is 1.91.

Solution:

Since the velocity head term cannot be calculated, an approximate flow is

$$Q = mbH^{3/2} = (1.91)(3.00)(0.914)^{3/2} = 5.01 \text{ m}^3/\text{s}$$

For this flow, $V = 5.01/(3.00 \times 2.114) = 0.790$ m/s and $V^2/2g = 0.032$ m. Using equation (*1*) of Problem 12.29,

$$Q = (1.91)(3.00)\left[(0.914 + 0.032)^{3/2} - (0.032)^{3/2}\right] = 5.24 \text{ m}^3/\text{s}$$

This second calculation shows an increase of 0.23 m^3/s or about 4.6% over the first calculation. Further calculation will generally produce an unwarranted refinement, i.e., beyond the accuracy of the formula itself. However, to illustrate, the revised velocity of approach would be

$V = 5.24/(3.00 \times 2.114) = 0.826$ m/s, and $V^2/2g = 0.035$ m

and

$$Q = (1.91)(3.00)\left[(0.914 + 0.035)^{3/2} - (0.035)^{3/2}\right] = 5.26 \text{ m}^3/\text{s}$$

12.33. A suppressed weir 25.0 ft long is to discharge 375.0 cfs into a channel. The weir factor $m = 3.42$. To what height Z (nearest 1/100 ft) can the weir be built, if the water behind the weir must not exceed 6 ft in depth?

Solution:

Velocity of approach $V = Q/A = 375.0/(25 \times 6) = 2.50$ ft/sec.

Then

$$375.0 = 3.42 \times 25.0\left[\left(H + \frac{(2.50)^2}{2g}\right)^{3/2} - \left(\frac{(2.50)^2}{2g}\right)^{3/2}\right] \quad \text{and} \quad H = 2.59 \text{ ft}$$

Height of weir is $Z = 6.00 - 2.59 = 3.41$ ft.

12.34. A contracted weir, 1.25 m high, is to be installed in a channel 2.5 m wide. The maximum flow over the weir is 1.70 m^3/s when the total depth back of the weir is 2.00 m. What length of weir should be installed if m = 1.88?

Solution:

Velocity of approach $V = Q/A = 1.70/(2.5 \times 2.00) = 0.340$ m/s. It appears that the velocity head is negligible in this case.

$$Q = m\left(b - \tfrac{2}{10}H\right)(H)^{3/2}, \quad 1.70 = (1.88)\left(b - \tfrac{2}{10} \times 0.75\right)(0.75)^{3/2}, \quad b = 1.54 \text{ m long.}$$

12.35. The discharge from a 6″-diameter orifice, under a 10.0-ft head, $c = 0.600$, flows into a rectangular weir channel and over a contracted weir. The channel is 6 ft wide, and for the weir, Z = 5.00 ft and $b = 1.00$ ft. Determine the depth of water in the channel if $m = 3.35$.

Solution:

The discharge through the orifice is

$$Q = cA\sqrt{2gh} = 0.600 \times \tfrac{1}{4}\pi\left(\tfrac{1}{2}\right)^2\sqrt{2g(10.0)} = 2.99 \text{ cfs}$$

For the weir, $\quad Q = m(b - \tfrac{2}{10}H)H^{3/2} \quad$ (velocity head neglected)

or $\quad 2.99 = (3.35)(1.00 - 0.20H)H^{3/2} \quad$ and $\quad H^{3/2} - 0.20H^{5/2} = 0.893$

By successive trials, $H = 1.09$ ft, and the depth $= Z + H = 5.00 + 1.09 = 6.09$ ft.

12.36. The discharge of water over a 45° triangular weir is 0.0212 m^3/s. For $c = 0.580$, determine the head on the weir.

Solution:

$$Q = \tfrac{8}{15}c\sqrt{2g}\left(\tan\tfrac{1}{2}\theta\right)H^{5/2}, \quad 0.0212 = \tfrac{8}{15}(0.580)\sqrt{2g}\left(\tan 22\tfrac{1}{2}^\circ\right)H^{5/2}, \quad H = 0.268 \text{ m}$$

12.37. Establish the equation for flow over a broad-crested weir assuming no lost head (Fig. 12-9).

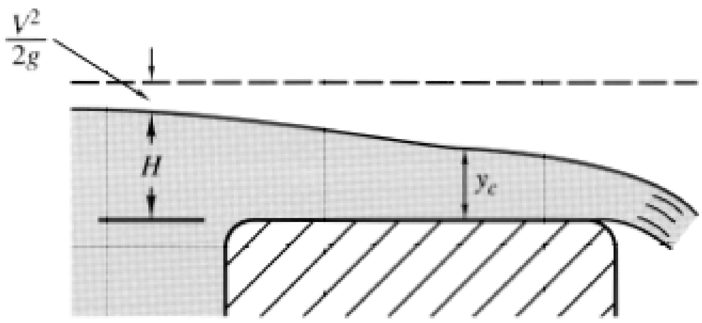

Figure 12-9.

Solution:

At the section where critical flow occurs, $q = V_c y_c$. But $y_c = V_c^2/g = \frac{2}{3}E$ and $V_c = \sqrt{g\left(\frac{2}{3}E_c\right)}$.

Hence the theoretical value of flow q becomes

$$q = \sqrt{g\left(\tfrac{2}{3}E_c\right)} \times \tfrac{2}{3}E_c$$

However, the value of E_c is difficult to measure accurately, because the critical depth is difficult to locate. The practical equation becomes

$$q = CH^{3/2}$$

The weir should be calibrated in place to obtain accurate results.

12.38. Develop an expression for a critical flow meter and illustrate the use of the formula.

Solution:

An excellent method of measuring flow in open channels is by means of a constriction (Fig. 12-10). The measurement of the critical depth is not required. The depth y_1 is measured a short distance upstream from the constriction. The raised floor should be about $3y_c$ long and of such height as to have the critical velocity occur on it.

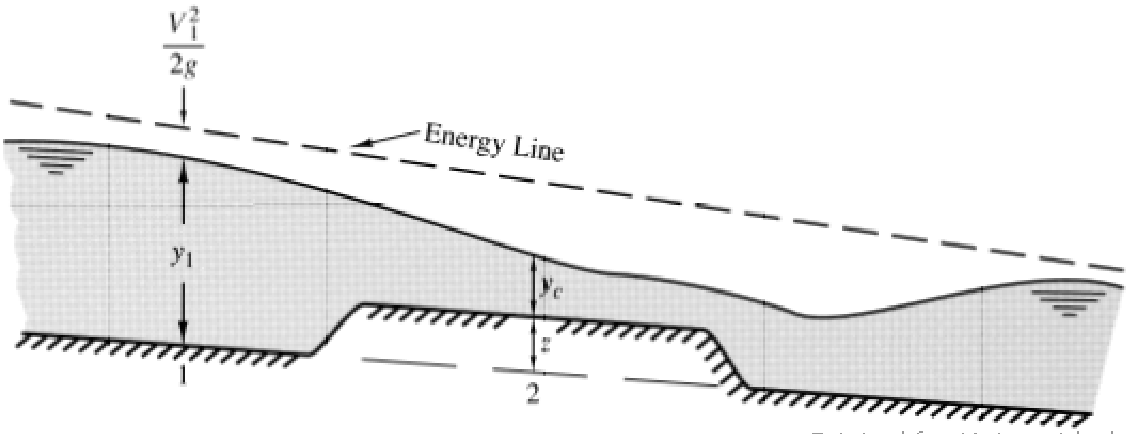

Figure 12-10.

For a rectangular channel of constant width, the Bernoulli equation is applied between sections 1 and 2, in which the lost head in accelerated flow is taken as 1/10 of the difference in velocity heads, i.e.,

$$y_1 + \frac{V_1^2}{2g} - \left(\frac{1}{10}\right)\left(\frac{V_c^2}{2g} - \frac{V_1^2}{2g}\right) = \left(y_c + \frac{V_c^2}{2g} + z\right)$$

This equation neglects the slight drop in the channel bed between 1 and 2. Recognizing that $E_c = y_c + V^2/2_g$, we rearrange as follows:

$$\left(y_1 + 1.10V_1^2/2g\right) = \left[z + 1.0E_c + \left(\tfrac{1}{10}\right)\left(\tfrac{1}{3}E_c\right)\right]$$

$$\left(y_1 - z + 1.10V_1^2/2g\right) = 1.033E_c = (1.033)\left(\tfrac{3}{2}\sqrt[3]{q^2/g}\right)$$

or

$$q = 2.94\left(y_1 - z + 1.10V_1^2/2g\right)^{3/2}$$

(*A*)

Since $q = V_1y_1$,

$$q = 2.94\left(y_1 - z + 0.0171q^2/y_1^2\right)^{3/2} \qquad \left(\text{for } g = 32.2 \text{ ft/sec}^2\right)$$

(*B*)

To illustrate the use of expression (*B*), consider a rectangular channel 10 ft wide with a critical depth meter having dimension z = 1.10 ft. If the measured depth y_1 is 2.42 ft, what is the discharge *Q?*

As a first approximation, neglect the last term in (*B*). Then

$$q = (2.94)(2.42 - 1.10)^{3/2} = 4.46 \text{ cfs/ft width}$$

Now, using the entire equation (*B*), by successive trials we find $q = 4.80$. Hence

$$Q = q(10) = (4.80)(10) = 48.0 \text{ cfs}$$

12.39. What length of trapezoidal (Cipolletti) weir should be constructed so that the head will be 1.54 ft when the discharge is 122 cfs?

Solution:

$$Q = 3.367bH^{3/2}, \qquad 122 = 3.367b(1.54)^{3/2}, \qquad b = 19.0 \text{ ft}$$

12.40. Establish the formula to determine the time to lower the liquid level in a tank of constant cross section by means of an orifice. Refer to Fig. 12-11.

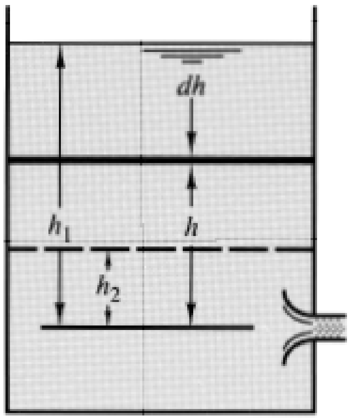

Figure 12-11.

Solution:

Inasmuch as the head is changing with time, we know that $\partial V/\partial t \neq 0$, i.e., we do not have steady flow. This means that the energy equation should be amended to include an acceleration term, which complicates the solution materially. As long as the head does not change too rapidly, no appreciable error will be introduced by assuming steady flow, thus neglecting the acceleration-head term. An approximate check on the error introduced is given in Problem 12.41.

Case A.

With *no inflow* taking place, the instantaneous flow will be

$$Q = cA_o\sqrt{2gh}$$

In time interval dt, the small volume dV discharged will be Qdt. In the same time interval, the head will decrease dh and the volume discharged will be the area of the tank A_t times dh. Equating these values,

$$\left(c\,A_o\sqrt{2gh}\right)dt = -A_T\,dh$$

where the negative sign signifies that h decreases as t increases. Solving for t yields

$$t = \int_{t_1}^{t_2} dt = \frac{-A_T}{cA_o\sqrt{2g}}\int_{h_1}^{h_2} h^{-1/2}dh$$

or

$$t = t_2 - t_1 = \frac{2A_T}{cA_o\sqrt{2g}}\left(h_1^{1/2} - h_2^{1/2}\right)$$

(*1*)

In using this expression, an average value of coefficient of discharge c can be used without producing significant error in the result. As h_2 approaches zero, a vortex will form and the orifice will cease to flow full. However, using $h_2 = 0$ will not produce serious error in most cases.

Equation (*1*) can be rewritten by multiplying and dividing by $\left(h_1^{1/2} + h_2^{1/2}\right)$. There results

$$t = t_2 - t_1 = \frac{A_T\,(h_1 - h_2)}{\frac{1}{2}\left(cA_o\sqrt{2gh_1} + cA_o\sqrt{2gh_2}\right)}$$

(*2*)

Noting that the volume discharged in time $(t_2 - t_1)$ is $A_T(h_1 - h_2)$, this equation simplifies to

$$t = t_2 - t_1 = \frac{\text{volume discharged}}{\frac{1}{2}(Q_1 + Q_2)} = \frac{\text{volume discharged}}{\text{average flow } Q}$$

(*3*)

Problem 12.43 will illustrate a case where tank cross section is not constant but can be expressed as a function of h. Other cases, such as reservoirs emptying, are beyond the scope of this book (see water supply engineering texts).

Case B.

With a constant rate of inflow less than the flow through the orifice taking place,

$$-A_T\,dh = (Q_{\text{out}} - Q_{\text{in}})dt \quad \text{and} \quad t = t_2 - t_1 = \int_{h_1}^{h_2} \frac{-A_T\,dh}{Q_{\text{out}} - Q_{\text{in}}}$$

If Q_{in} exceeds Q_{out} the head will increase, as would be expected.

12.41. A 1.22-m-diameter tank contains oil of specific gravity 0.75. A 75-mm-diameter short tube is installed near the bottom of the tank (c = 0.85). How long will it take to lower the level of the oil from 1.83 m above the tube to 1.22 m above the tube?

Solution:

$$t = t_2 - t_1 = \frac{2A_T}{cA_o\sqrt{2g}}\left(h_1^{1/2} - h_2^{1/2}\right) = \frac{2 \times \frac{1}{4}\pi(1.22)^2}{0.85 \times \frac{1}{4}\pi(0.075)^2\sqrt{2g}}\left(1.83^{1/2} - 1.22^{1/2}\right) = 35 \text{ s.}$$

In order to evaluate the approximate effect of assuming steady flow, the change of velocity with time t is estimated as

$$\frac{\partial V}{\partial t} \cong \frac{\Delta V}{\Delta t} = \frac{\sqrt{2g(1.83)} - \sqrt{2g(1.22)}}{35} = 0.0314 \text{ m/s}^2$$

This is about $\frac{1}{3}\%$ of acceleration g, a negligible addition to the acceleration g. Such an accuracy is not warranted in these illustrations of unsteady flow, particularly as orifice coefficients are not known to such a degree of accuracy.

12.42. The initial head on an orifice was 2.75 m, and when the flow was terminated the head was measured at 1.22 m. Under what constant head H would the same orifice discharge the same volume of water in the same time interval? Assume coefficient c constant.

Solution:

Volume under falling head = volume under constant head

$$\tfrac{1}{2}cA_o\sqrt{2g}\left(h_1^{1/2} + h_2^{1/2}\right) \times t = cA_o\sqrt{2gH} \times t$$

Substituting and solving, $\frac{1}{2}(\sqrt{2.75} + \sqrt{1.22}) = \sqrt{H}$, and $H = 1.91$ m.

12.43. A tank has the form of a frustum of a cone, with the 8-ft diameter uppermost and the 4-ft diameter at the bottom. The bottom contains an orifice whose average coefficient of discharge may be taken as 0.60. What size orifice will empty the tank in 6 min if the depth full is 10.0 ft? Refer to Fig. 12-12.

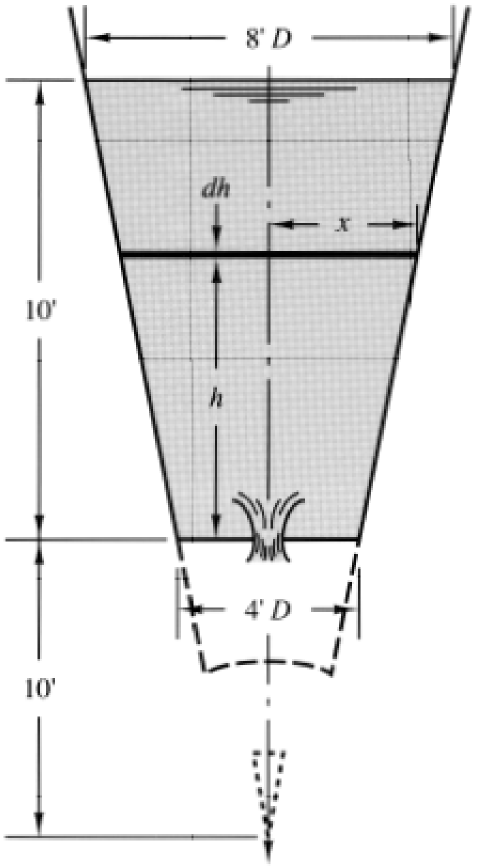

Figure 12-12.

Solution:

From Problem 12.40,

$Q\,dt = -A_T\,dh$ and $cA_o\sqrt{2gh}\,dt = -\pi x^2\,dh$

and, by similar triangles,
$x/4 = (10+h)/20$. Then

$$\left(0.60 \times \tfrac{1}{4}\pi d_o^2\sqrt{2g}\right)\,dt = -\pi\frac{(10+h)^2}{25}h^{-1/2}\,dh$$

$$d_o^2\int dt = \frac{-4\pi}{25\pi \times 0.60\sqrt{2g}}\int_{10}^{0}(10+h)^2h^{-1/2}\,dh$$

Since $\int dt = 360$ sec,

$$d_o^2 = \frac{+4}{360 \times 25 \times 0.60\sqrt{2g}}\int_0^{10}(100h^{-1/2} + 20h^{1/2} + h^{3/2})dh$$

Integrating and solving, we obtain $d^2 = 0.109$ and $d = 0.33$ ft. and Use $d = 4''$ orifice.

12.44. Two square tanks have a common wall in which an orifice, area = 0.25 ft^2 and coefficient = 0.80, is located. Tank A is 8 ft on a side, and the initial depth above the orifice is 10.0 ft. Tank B is 4 ft on a side, and the initial depth above the orifice is 3.0 ft. How long will it take for the water surfaces to be at the same elevation?

Solution:

At any instant the difference in level of the surfaces may be taken as head h. Then

$Q = 0.80 \times 0.25\sqrt{2gh}$

and the change in volume $dv = Q\,dt = 1.605\sqrt{h}\,dt$.

In this interval of time dt the change in head is dh. Consider the level in tank A to have fallen dy, then the corresponding rise in level in tank B will be the ratio of the areas times dy, or $(64/16)dy$. The change in head is thus

$$dh = dy + (64/16)dy = 5\,dy$$

The change in volume $dv = 8 \times 8 \times dy\ \left[= 4 \times 4 \times (64/16)dy \text{ also}\right]$

or, using dh,

$dv = (64/5)dh = 12.8dh$

Equating values of dv,

$$1.605\sqrt{h}\,dt = -12.8dh, \quad dt = \frac{-12.8}{1.605}\int_7^0 h^{-1/2}dh, \quad t = 42.2 \text{ sec.}$$

The problem can also be solved using the average rate of discharge expressed in (3) of Problem 12.40.

$$Q_{av} = \frac{1}{2}\left[0.80 \times 0.25\sqrt{2g(7)}\right] = 2.12 \text{ cfs}$$

Tank A lowers y ft while tank B rises $(64/16)y$ ft with the total change in level of 7 ft; then $y + 4y = 1$ and $y = 1.40$ ft. Thus, change in volume = $8 \times 8 \times 1.40 = 89.6$ ft^3 and

$$t = \frac{\text{change in volume}}{\text{average } Q} = \frac{89.6}{2.12} = 42.3 \text{ sec}$$

12.45. Develop the expression for the time to lower the liquid level in a tank, lock, or canal by means of a suppressed weir.

Solution:

$Q\,dt = -A_T\,dH$ (as before) or $(mLH^{3/2})dt = -A_T\,dH$.

Then $t = \int_{t_1}^{t_2} dt = \frac{-A_T}{mL}\int_{H_1}^{H_2} H^{-3/2}dH$ or $t = t_2 - t_1 = \frac{2A_T}{mL}\left(H_2^{-1/2} - H_1^{-1/2}\right)$.

12.46. A rectangular flume 15.25 m long and 3.00 m wide feeds a suppressed weir under a head of 0.30 m. If the supply to the flume is cut off, how long will it take for the head on the weir to decrease to 100 mm? Use m = 1.84.

Solution:

From Problem 12.45, $t = \frac{2(15.25 \times 3.00)}{1.84 \times 3.00}\left[\frac{1}{\sqrt{0.100}} - \frac{1}{\sqrt{0.30}}\right] = 22.2 \text{ s}$

12.47. Determine the time required to establish flow in a pipeline of length L under a constant head H discharging into the atmosphere, assuming an inelastic pipe, an incompressible fluid, and constant friction factor f.

Solution:

The final velocity Vf can be determined from the Bernoulli equation, as follows:

$$H - f\left(\frac{L}{d}\right)\left(\frac{V_f^2}{2g}\right) - k\left(\frac{V_f^2}{2g}\right) = \left(0 + \frac{V_f^2}{2g} + 0\right)$$

In this equation, the minor losses are represented by the $kV_f^2/2g$ term, and the energy in the jet at the end of the pipe is kinetic energy represented by $V_f^2/2g$. This equation can be written in the form

$$\left[H - f\left(\frac{L_E}{d}\right)\left(\frac{V_f^2}{2g}\right)\right] = 0$$

(*1*)

where L_E is the equivalent length of pipe for the system (see Problem 9.3, Chapter 9).

From Newton's equation of motion, at any instant

$$\gamma(AH_e) = M\frac{dV}{dt} = \frac{\gamma}{g}(AL)\frac{dV}{dt}$$

where H_e is the effective head at any instant and V is a function of time and not length. Rearranging the equation,

$$dt = \left(\frac{\gamma A L}{g \gamma A H_e}\right) dV \quad \text{or} \quad dt = \frac{L\, dV}{g H_e}$$

(*2*)

In equation (*1*), for all intermediate values of V the term in the brackets is not zero but is the effective head available to cause the acceleration of the liquid. Hence expression (*2*) may be written

$$\int dt = \int \frac{L\, dV}{g\left[H - f\left(\frac{L_E}{d}\right)\left(\frac{V^2}{2g}\right)\right]} = \int \frac{L\, dV}{g\left[f\left(\frac{L_E}{d}\right)\left(\frac{V_f^2}{2g}\right) - f\left(\frac{L_E}{d}\right)\left(\frac{V^2}{2g}\right)\right]}$$

(*3A*)

Since from (*1*), $\frac{f L_E}{2gd} = \frac{H}{V_f^2}$,

$$\int dt = \int \frac{L\, dV}{g\left(H - HV^2/V_f^2\right)}$$

(*3B*)

or

$$\int_o^t dt = \frac{L}{gH} \int_0^{V_f} \frac{V_f^2}{V_f^2 - V^2} dV$$

Integrating,

$$t = \frac{L V_f}{2gH} \ln\left(\frac{V_f + V}{V_f - V}\right)$$

(*4*)

It will be noted that as V approaches final velocity *Vf*, $(V_f - V)$ approaches zero. Thus, mathematically, time t approaches infinity.

Equation (*3B*) may be rearranged, using symbol φ for the ratio V/V_f. Then

$$\frac{dV}{dt} = \frac{gH}{L}\left(1 - V^2/V_f^2\right) = \frac{gH}{L}\left(1 - \phi^2\right)$$

(*5*)

Using $V = V_f \phi$ and $\frac{dV}{dt} = V_f \frac{d\phi}{dt}$, we obtain

$$\frac{d\phi}{1 - \phi^2} = \frac{gH\, dt}{V_f L}$$

Integrating,

$$\frac{1}{2} \ln\left(\frac{1 + \phi}{1 - \phi}\right) = \frac{gHt}{V_f L} + C$$

and when $t = 0$, $C = 0$. Then

$$\frac{1+\phi}{1-\phi} = e^{2gHt/V_f L}$$

Using hyperbolic function, $\phi = \tanh(gHt/V_f L)$, and since $\phi = V/V_f$,

$$V = V_f \tanh \frac{gHt}{V_f L}$$

(*6*)

The advantage of expression (6) is that the value of velocity V in terms of the final velocity V_f can be calculated for any chosen time.

12.48. Simplify equation (4) in Problem 12.47 to give the time to establish flow such that velocity V equals (*a*) 0.75, (*b*) 0.90, and (c) 0.99 times final velocity V_f.

Solution:

a.

$$t = \frac{LV_f}{2gH} \ln\left[\frac{V_f + 0.75V_f}{V_f - 0.75V_f}\right] = \left(\frac{LV_f}{2gH}\right)(2.3026)\log\frac{1.75}{0.25} = 0.973\frac{LV_f}{gH}$$

b.

$$t = \frac{LV_f}{2gH} \ln\frac{1.90}{0.10} = \left(\frac{LV_f}{2gH}\right)(2.3026)\log\frac{1.90}{0.10} = 1.472\frac{LV_f}{gH}$$

c.

$$t = \frac{LV_f}{2gH} \ln\frac{1.99}{0.01} = \left(\frac{LV_f}{2gH}\right)(2.3026)\log\frac{1.99}{0.01} = 2.647\frac{LV_f}{gH}$$

12.49. Water is discharged from a tank through 2000 ft of 12″ pipe ($f = 0.020$). The head is constant at 20 ft. Valves and fittings in the line produce losses of $21(V^2/2g)$. After a valve is opened, how long will it take to attain a velocity of 0.90 of the final velocity?

Solution:

Bernoulli's equation, tank surface to end of pipe, will yield

$$(0+0+H) - [f(L/d) + 21.0]V^2/2g = \left(0 + V^2/2g + 0\right)$$

or $H = [0.020(2000/1) + 22.0]V^2/2_g = 62.0(V^2/2_2)$. Than using the procedure in Problem 9.3 of Chapter 9,

$$62.0\left(V^2/2g\right) = 0.020(L_E/1)\left(V^2/2g\right) \qquad \text{or} \qquad L_E = 3100 \text{ ft}$$

Inasmuch as equation (4) in Problem 12.47 does not contain L_E, the final velocity must be calculated, as follows:

$$H = f\left(\frac{L_E}{d}\right)\left(\frac{V_f^2}{2g}\right) \qquad \text{or} \qquad V_f = \sqrt{\frac{2gdH}{fL_E}} = \sqrt{\frac{(64.4)(1)(20.0)}{(0.020)(3100)}} = 4.56 \text{ ft/sec}$$

Substituting in (b) of Problem 12.48 yields $t = (1.472)\dfrac{(2000)(4.56)}{(32.2)(20.0)} = 20.8$ sec.

12.50. Mathcad In Problem 12.49, what velocity will be attained in 10 sec and in 15 sec?

Solution:

In equation (6) of Problem 12.47, evaluate gHt/V_fL.

For 10 sec, $\dfrac{32.2 \times 20 \times 10}{4.56 \times 2000} = 0.706$. For 15 sec, $\dfrac{32.2 \times 20 \times 15}{4.56 \times 2000} = 1.059$.

Using a table of hyperbolic functions and equation (6), $V = V_f \tanh (_gH_t/V_fL)$, we obtain

For 10 sec, $V = 4.56 \tanh 0.706 = 4.56 \times 0.6082 = 2.77$ ft/sec

For 15 sec, $V = 4.56 \tanh 1.059 = 4.56 \times 0.7853 = 3.58$ ft/sec

It will be noted that the value V/V_f is represented by the value of the hyperbolic tangent. In the solution above, 61% and 79% of the final velocity are attained in the 10 and 15 sec, respectively.

12.51. A rectangular channel 20 ft wide, $n = 0.025$, flows 5 ft deep on a slope of 14.7 ft in 10,000 ft. A suppressed weir C, 2.45 ft high, is built across the channel ($m = 3.45$). Taking the elevation of the bottom of the channel just upstream from the weir to be 100.00', estimate (using one reach) the elevation of the water surface at a point A, 1000 ft upstream from the weir.

Solution:

Calculate the new elevation of water surface at B in Fig. 12-13 (before drop-down). Note that the flow is nonuniform, as the depths, velocities, and areas are not constant after the weir is installed.

$Q = (20 \times 5)(1.486/0.025)(100/30)^{2/3}(0.00147)^{1/2} = 509$ cfs

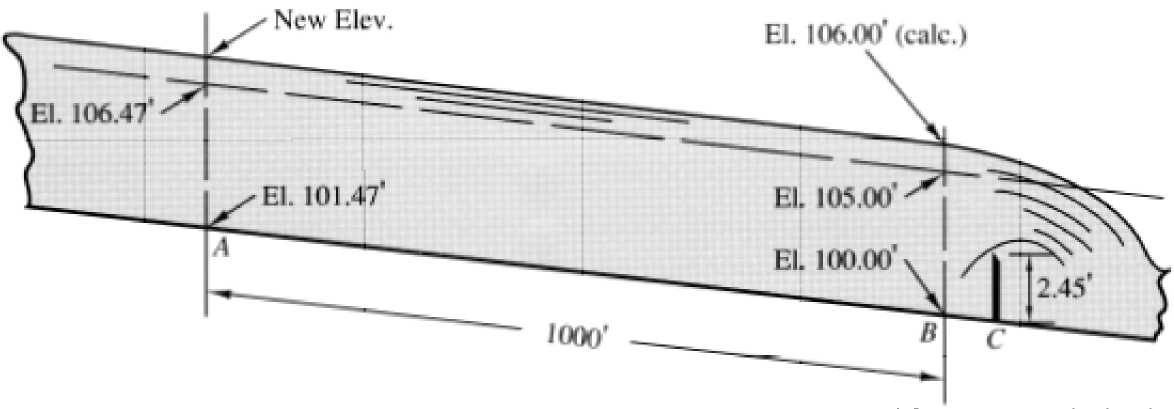

Figure 12-13.

For an estimated depth of 6 ft just upstream from the weir,

$$\text{velocity of approach } V = Q/A = 509/(20 \times 6) = 4.24 \text{ ft/sec}$$

The weir formula gives $509 = 3.45 \times 20\left[\left(H + \dfrac{(4.24)^2}{2g}\right)^{3/2} - \left(\dfrac{(4.24)^2}{2g}\right)^{3/2}\right]$.

$$H = 3.56 \text{ ft}$$
$$\text{height } Z = 2.45 \text{ ft}$$
$$\text{depth } y = 6.01 \text{ ft (assumption checks)}$$

New elevation at A must lie between 106.47 and 107.47. Try an elevation of 106.90 (and check in the Bernoulli equation).

new area at $A = (20)(106.90 - 101.47) = 108.6 \text{ ft}^2$, and $V = 509/108.6 = 4.69$ ft/sec

$$\text{mean velocity} = \tfrac{1}{2}(4.24 + 4.69) = 4.46 \text{ ft/sec}$$

$$\text{mean hydraulic radius } R = \left(\tfrac{1}{2}\right)(120.0 + 108.6)/\left[\tfrac{1}{2}(32.0 + 30.9)\right] = 3.63$$

$$\text{lost head h}_L = \left(\frac{Vn}{1.486R^{2/3}}\right)^2 L = \left(\frac{4.46 \times 0.025}{(1.486)(3.63)^{2/3}}\right)^2 (1000) = 1.01 \text{ ft}$$

Now apply the Bernoulli equation, A to B, datum B,

$$\left[106.90 + (4.69)^2/2g\right] - 1.01 = \left[106.00 + (4.24)^2/2g\right]$$

which reduces to

$$106.23 = 106.28 \text{ (approximately)}$$

The difference of 0.05 ft is within the error in roughness factor n alone. Further refinement does not seem justified. Use elevation of 106.90 ft.

12.17.2. Supplementary Problems

12.52. Turpentine at 20°C flows through a pipe in which a Pitot-static tube having a coefficient of 0.97 is centered. The differential gage containing mercury shows a deflection of 102 mm. What is the center velocity? *Ans*. 5.27 m/s

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Schaum's Fluid Mechanics and Hydraulics Problem 12-52: Velocity Measurement via a Pitot Static Tube

In this problem the velocity of water through a conduit is calculated from knowledge of pressure difference as measured by a Pitot static tube.
Thom Adams, Ph.D., Professor, Mechanical Engineering, Rose-Hulman Institute of Technology
2013

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12.53. Air at 120°F flows by a Pitot-static tube at a velocity of 60.0 ft/sec. If the coefficient of the tube is 0.95, what water differential reading is expected, assuming constant specific weight of the air at atmospheric pressure? *Ans*. 0.816 in

12.54. The lost head through a 2″-diameter orifice under a certain head is 0.540 ft, and the velocity of the water in the jet is 22.5 ft/sec. If the coefficient of discharge is 0.61, determine the head causing flow, the diameter of the jet, and the coefficient of velocity. *Ans*. 8.39 ft, 1.59″, 0.97

12.55. What size standard orifice is required to discharge 0.016 m^3/s of water under a head of 8.69 m? *Ans*. 50 m

12.56. A sharp-edged orifice has a diameter of 1″ and coefficients of velocity and contraction of 0.98 and 0.62, respectively. If the jet drops 3.08 ft in a horizontal distance of 8.19 ft, determine the flow in cfs and the head on the orifice. *Ans*. 0.0632 cfs, 5.66 ft

12.57. Oil of specific gravity 0.800 flows from a closed tank through a 75-mm-diameter orifice at the rate of 0.026 m^3/s. The diameter of the jet is 58.5 mm. The level of the oil is 7.47 m above the orifice, and the air pressure is equivalent to —152 mm of mercury. Determine the three coefficients of the orifice. *Ans*. 0.580, 0.590, 0.982

12.58. Refer to Fig. 12-14. The 3″-diameter orifice has coefficients of velocity and contraction of 0.950 and 0.632, respectively. Determine (*a*) the flow for the deflection of the mercury indicated and (*b*) the power in the jet. *Ans. (a)* 1.04 cfs, (*b*) 2.06 hp

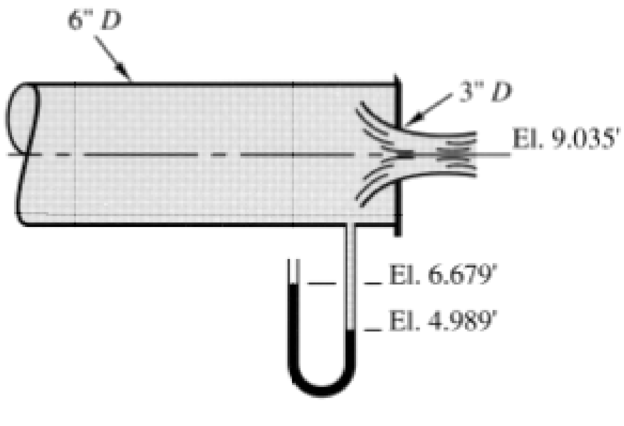

Figure 12-14.

12.59. Refer to Fig. 12-15. Heavy fuel oil at 60°F flows through the 3″-diameter end-of-pipe orifice, causing the deflection of the mercury in the U-tube gage. Determine the power in the jet. *Ans*. 2.90 hp

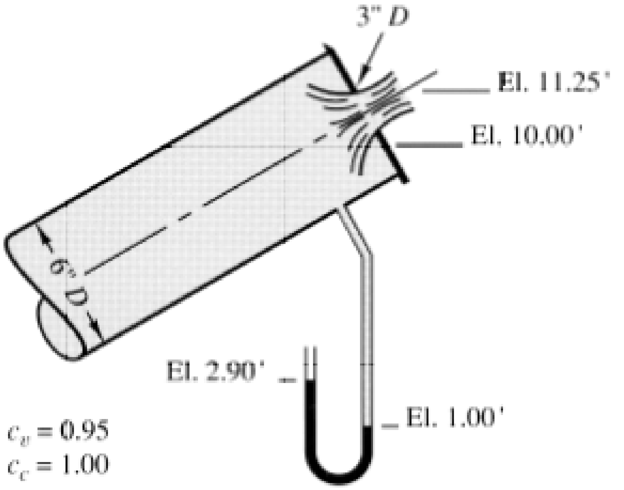

Figure 12-15.

12.60. Steam locomotives sometimes take on water by means of a scoop that dips into a long, narrow tank between the rails. If the lift into the tank is 2.74 m, at what velocity must the train travel (neglecting friction)? *Ans*. 26.4 km/h

12.61. Air at 15°C flows through a large duct and thence through a 75-mm-diameter hole in the thin metal ($c = 0.62$). A U-tube gage containing water registers 31.7 mm. Considering the specific weight of air constant, what is the flow through the opening? *Ans*. 46 N/min

12.62. An oil having specific gravity 0.926 and viscosity 350 Saybolt seconds flows through a 3″-diameter pipe orifice placed in a 5″-diameter pipe. The differential gage registers a pressure drop of 21.5 psi. Determine flow Q. *Ans*. 1.93 cfs

12.63. A nozzle with a 50-mm-diameter tip is attached at the end of a 2000-mm horizontal pipeline. The coefficients of velocity and contraction are respectively 0.976 and 0.909. A pressure gage attached at the base of the nozzle and located 2.16 m above its centerline reads 221 kPa. Determine the flow of water in m^3/s. *Ans*. 0.040 m^3/s

12.64. When the flow of water through a horizontal 300 mm by 150 mm Venturi meter ($c = 0.95$) is 0.111 m^3/s, find the deflection of the mercury in the differential gage attached to the meter. *Ans*. 157 mm

12.65. When 4.20 cfs of water flows through a 12″ by 6″ Venturi meter, the differential gage records a difference of pressure heads of 7.20 ft. What is the indicated coefficient of discharge of the meter? *Ans*. 0.964

12.66. The loss of head from entrance to throat of a 250 mm by 125 mm Venturi meter is 1/16 times the throat velocity head. When the mercury in the differential gage attached to the meter deflects 102 mm, what is the indicated flow of water? *Ans.* 0.063 m^3/s

12.67. A 300 mm by 150 mm Venturi meter ($c = 0.985$) carries 0.0566 m^3/s of water with a differential gage reading of 0.634 m. What is the specific gravity of the gage liquid? *Ans.* 1.75

12.68. Methane flows at a rate of 16.5 lb/sec through a 12″ by 6″ Venturi meter at a temperature of 60°F. The pressure at the meter inlet is 49.5 psi absolute. Using $k = 1.31$, $R = 96.3$ ft/°R, $\nu = 1.94 \times 10^{-4}$ ft^2/sec at 1 atm, and $\gamma = 0.0416$ lb/ft' at 68°F and 1 atm, calculate the expected deflection of the mercury in the differential gage. *Ans.* 1.03 ft

12.69. Water flows through a 150-mm pipe in which a long-radius flow nozzle 75 mm in diameter is installed. For a deflection of 152 mm of mercury in the differential gage, calculate the rate of flow. (Assume $c = 0.98$ from Diagram *D*.) *Ans.* 0.028 m^3/s

12.70. Water at 30°C flows at the rate of 0.046 m^3/s through the nozzle in Problem 12.69. What is the deflection of the mercury in the differential gage? (Use Diagram *D*.) *Ans.* 393 mm

12.71. If a dust proofing oil at 30°C had been flowing at 0.046 m^3/s in Problem 12.70, what would have been the deflection of the mercury? *Ans.* 372 mm

12.72. If air at 20°C flows through the same pipe and nozzle as in Problem 12.69, how many newtons of air per second will flow if the absolute pressures in pipe and jet are 207 kPa and 172 kPa, respectively? *Ans.* 17 N/s

12.73. What depth of water must exist behind a rectangular sharp-crested suppressed weir 1.52 m long and 1.22 m high when a flow of 0.283 m^3/s passes over it? (Use the Francis formula.) *Ans.* 1.44 m

12.74. A flow of 30 cfs occurs in a rectangular flume 4 ft deep and 6 ft wide. Find the height at which the crest of a sharp-edged suppressed weir should be placed in order that water will not overflow the sides of the flume, ($m = 3.33$) *Ans.* 2.72 ft

12.75. A flow of 10.9 m^3/s passes over a suppressed weir which is 4.88 m long. The total depth upstream from the weir must not exceed 2.44 m. Determine the height to which the crest should be placed to carry this flow. ($m = 1.85$) *Ans.* 1.34 m

12.76. A suppressed weir ($m = 3.33$) under a constant head of 0.300 ft feeds a tank containing a 3″-diameter orifice. The weir, which is 2 ft long and 2.70 ft high, is located in a rectangular channel. The lost head through the orifice is 2.00 ft, and $c_c = 0.65$. Determine the head to which the water will rise in the tank and the coefficient of velocity for the orifice. *Ans.* $h = 20.3$ ft, $c_v = 0.95$

12.77. A contracted weir 1.22 m long is placed in a rectangular channel 2.74 m wide. The height of weir crest is 1.00 m and the head is 381 mm. Determine the flow, using $m = 1.88$. *Ans*. 0.504 m^3/s

12.78. A triangular weir has a 90° notch. What head will produce 1200 gpm?($m = 2.50$) *Ans*. 1.027 ft

12.79. A 36″ pipeline containing a 36″ by 12″ Venturi meter supplies water to a rectangular canal. The pressure is 30.0 psi at the inlet of the Venturi and 8.67 psi at the throat. A suppressed weir ($m = 3.33$), 3 ft high, placed in the canal, discharges under a 9″ head. What is the probable width of the canal? *Ans*. 20.0 ft

12.80. Water flows over a suppressed weir (m = 1.85) that is 3.66 m long and 0.610 m high. For a head of 0.366 m, find the flow. *Ans*. 1.54 m^3/s

12.81. A tank 12 ft long and 4 ft wide contains 4 ft of water. How long will it take to lower the water to a **1**-ft depth if a 3″ diameter orifice ($c = 0.60$) is opened in the bottom of the tank? *Ans*. 406 sec

12.82. A rectangular tank 4.88 m by 1.22 m contains 1.22 m of oil, specific gravity 0.75. If it takes 10 min and 10 s to empty the tank through a 100 mm diameter orifice in the bottom, determine the average value of the coefficient of discharge. *Ans*. 0.60

12.83. In Problem 12.82, for a coefficient of discharge of 0.60, what will be the depth after the orifice has been flowing for 5 min.? *Ans*. 0.314 m

12.84. A tank with a trapezoidal cross section has a constant length of 5 ft. When the water is 8 ft deep above the 2-in-diameter orifice ($c = 0.65$), the width of the water surface is 6 ft and, at a 3-ft-head depth, the width of water surface is 4 ft. How long will it take to lower the water from the 8-ft depth to the 3-ft depth? *Ans*. 482 sec

12.85. A suppressed weir is located in the end of a tank that is 3.05 m square. If the initial head on the weir is 0.610 m, how long will it take for 3.54 m^3 of water to run out of the tank?($m = 1.85$) *Ans*. 2.68 s

12.86. A rectangular channel 60 ft long by 10 ft wide is discharging its flow over a 10-ft-long suppressed weir under a head of 1.00 ft. If the supply is suddenly cut off, what will be the head on the weir in 36 sec? ($m = 3.33$) *Ans*. 0.25 ft

12.87. Two orifices in the side of a tank are one above the other vertically and are 1.83 m apart. The total depth of water in the tank is 4.27 m, and the head on the upper orifice is 1.22 m. For the same values of c_v, show that the jets will strike the horizontal plane on which the tank rests at the same point.

12.88. A 6″-diameter orifice discharges 12.00 cfs of water under a head of 144 ft. This water flows into a 12-ft- wide rectangular channel at a depth of 3 ft, and then it flows over a contracted weir. The head on the weir is 1.000 ft. What are the length of the weir and the coefficient of the orifice? *Ans*. 3.80 ft, $c = 0.635$

12.89. The head on a suppressed weir G that is 12 ft long is 1.105 ft, and the velocity of approach can be neglected. For the system shown in Fig. 12-16, what is the pressure head at B? Sketch the hydraulic grade lines. *Ans*. 193.2 ft

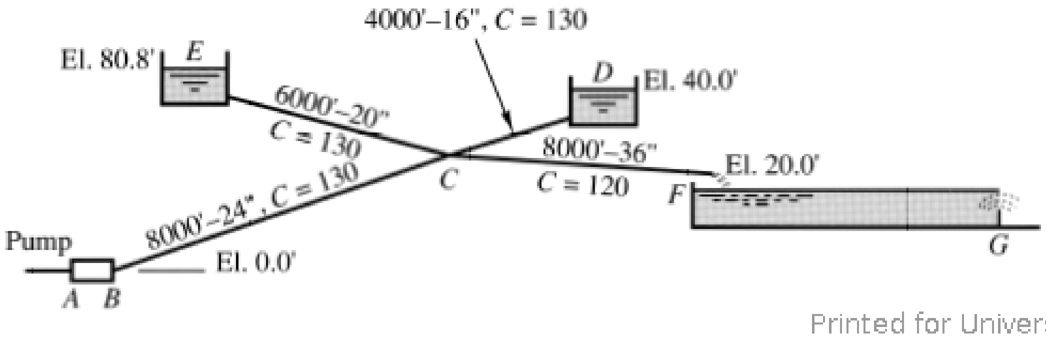

Figure 12-16.

12.90. In Fig. 12-17 the elevation of the hydraulic grade line at fi is 50.0' and pipes BC and BD are arranged so that the flow from B divides equally. What is the elevation of the end of the pipe at D, and what is the head that will be maintained on the 4″-diameter orifice E? *Ans*. El. 23.8', h= 22.5 ft

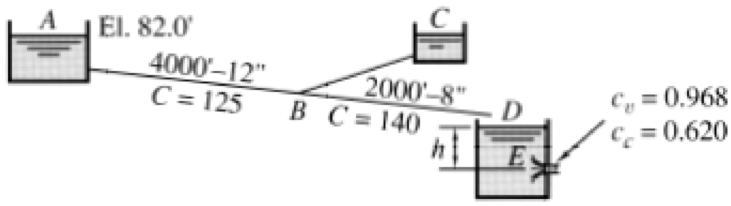

Figure 12-17.

12.91. For the tank shown in Fig. 12-18, using an average coefficient of discharge of 0.65 for the 2″ diameter orifice, how long will it take to lower the tank level 4 ft. *Ans*. 390 sec

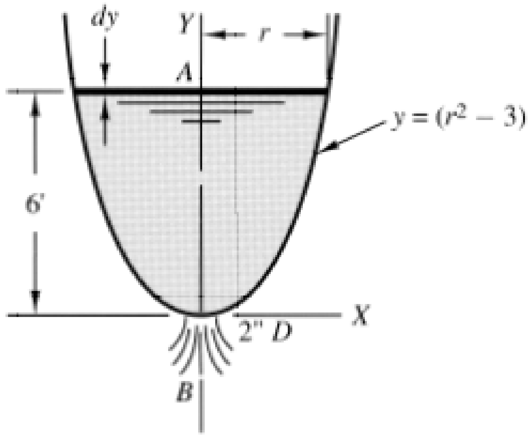

Figure 12-18.

12.92. A broad-crested weir is 1.25 ft high above the bottom of a rectangular channel 10 ft wide. The measured head above the weir crest is 1.95 ft. Determine the approximate flow in the channel. (Use c = 0.92) *Ans*. 83.5 cfs

Citation

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