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## FARADAY ROTATION OF OPTICALLY PUMPED SODIUM VAPOUR IN A WEAK MAGNETIC FIELD

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Faraday rotation of optically pumped sodium atoms in a weak magnetic field, which is lower than the critical field of hyperfine splitting, was obtained theoretically. The estimated rotation angles were compared with the experiments for various magnetic fields. The measured values showed good agreement with the values calculated from the theory.

### 1. Introduction

Recently, optical pumping with intense lasers has been applied to generate nuclear-spin polarized hydrogen atoms. Since there is no laser having the frequency which is matched to an appropriate resonance of the hydrogen atom, indirect methods to pump the hydrogen atom, such as the reactions of spin-charge exchange or spin exchange between hydrogen atoms and other optically pumped atoms have been currently used. The method which was proposed by Anderson was based on the spin-charge exchange reactions between protons and optically oriented sodium atoms [1]. At KEK, this idea has been realized as the intense polarized  $H^-$  ion source for the 12 GeV proton synchrotron [2]. Happer et al. have shown that in a mixture of xenon and rubidium, the nuclear spin of xenon atoms are largely polarized by spin-exchange reactions with optically pumped rubidium atoms [3]. This procedure could be applied to produce a sufficiently thick polarized hydrogen gas which would be very attractive for nuclear fusion with polarized atoms [4]. Recently, a new idea called collisional pumping, which is based on the multistage charge-exchange reactions between hydrogen atoms and polarized sodium atoms in a low magnetic field has been proposed by Anderson et al. [5].

In these applications, it is crucial to measure the atomic polarization of optically pumped atoms accurately because the nuclear polarization is directly related to the atomic polarization. A very useful method,

which is based on Faraday rotation (paramagnetic rotation) in optically oriented atoms, has been employed so far to measure the atomic polarization [6]. This scheme, however, has been only applied to analyze the atomic polarization in a strong magnetic field, where the nuclear-spin state does not couple with the atomic-spin state and the contributions from the many hyperfine sublevels can be neglected, so that the theoretical treatment becomes very simple.

On the other hand, optical pumping in a weak magnetic field is essential in some applications such as collisional pumping. A nuclear-spin polarization is made by the hyperfine interactions between sublevels in a weak magnetic field lower than the critical field strength. Therefore, it is worthwhile estimating the Faraday rotation in a weak magnetic field.

The treatment for the Faraday rotation in this report can be applied for any strength of the magnetic field. The results of calculations were compared with experiment.

### 2. Faraday rotation in weak magnetic field

For hydrogenlike atoms, the external magnetic field can be specified by the critical magnetic field  $B_c$  defined by the following equation:

$$B_c = \frac{\Delta W}{\beta_e (g_l + g_e)}. \quad (1)$$

Here,  $\Delta W$  is the hyperfine energy,  $\beta_c$  is Bohr magneton,  $g_1$  and  $g_e$  are the  $g$ -factors of nucleus and electron, respectively. For the case of sodium atoms,  $B_c$  is 632 G.

In a strong magnetic field larger than  $B_c$ , the effect of the coupling between  $J$  and nuclear spin  $I$  can be neglected. Here,  $J$  is the sum of the electronic orbital angular momentum  $L$  and spin  $S$ . Each Zeeman sublevel is specified by  $m_J$  and  $m_I$  which are the  $Z$ -components of  $J$  and  $I$ , respectively.

On the other hand, when the magnetic field is lower than the critical field,  $J$  and  $I$  cannot be decoupled and  $F = I + J$  becomes a good quantum number. In this case, Zeeman sublevels are specified by  $m_F$ . The energy diagram of the Zeeman sublevels of sodium ground states (3S) and excited states (3P) is shown, schematically, in fig. 1. The wave functions of each ground and excited sublevel can be written as follows:

$$\begin{aligned} \phi_i = & |m_L\rangle \langle A_i | m_S = 1/2 \rangle |m_I\rangle \\ & + B_i |m_S = -1/2\rangle |m_I\rangle \quad (i=1 \rightarrow 8); \end{aligned} \quad (2a)$$

excited state:

$$\begin{aligned} \phi'_j = & (A'_j |m_L\rangle |m_S = 1/2\rangle \\ & + B'_j |m_L\rangle |m_S = -1/2\rangle) |m_I\rangle \quad (j=1 \rightarrow 6). \end{aligned} \quad (2b)$$

All of the wave functions of sublevels for each state are summarized in table 1.

The coefficients  $A_i$ ,  $B_i$ ,  $A'_j$  and  $B'_j$  are varied as a function of the external magnetic field strength. The calculated values are summarized in table 2. A parameter  $\chi$ , which represents the strength of the magnetic field  $B$ , is defined by the following equation:

$$\chi = \frac{B}{B_c}. \quad (3)$$

The Faraday rotation angle can be calculated by the following equation:

$$\theta = \frac{\pi l \nu}{c} (n_+ - n_-). \quad (4)$$

Here,  $n_+$  and  $n_-$  are the refractive indices for left and right circular polarized light,  $\nu$  is the frequency of the light and  $l$  is the length of the target. The refractive index  $n$  is given by the following equation with  $x_{ij} = \pi \tau (\nu_{ij} - \nu)$  as shown in ref. [6],

$$n - 1 = \frac{q_{3P} c^2 N}{8\pi^2} \sum_i \sum_j \frac{x_{ij}}{1 + x_{ij}^2} P_i \beta_{ij}, \quad (5)$$

where  $i$  and  $j$  specify the sublevels of the ground state and the excited state,  $q_{3P}$  is the statistical weight of the excited states,  $\nu_{ij}$  is the frequency of the transition between the  $i$ th state and the  $j$ th state,  $N$  is the density of sodium atoms,  $\tau$  is the lifetime of 3P states,  $P_i$  is the occupation probability of the  $i$ th state, and  $\beta_{ij}$  is the

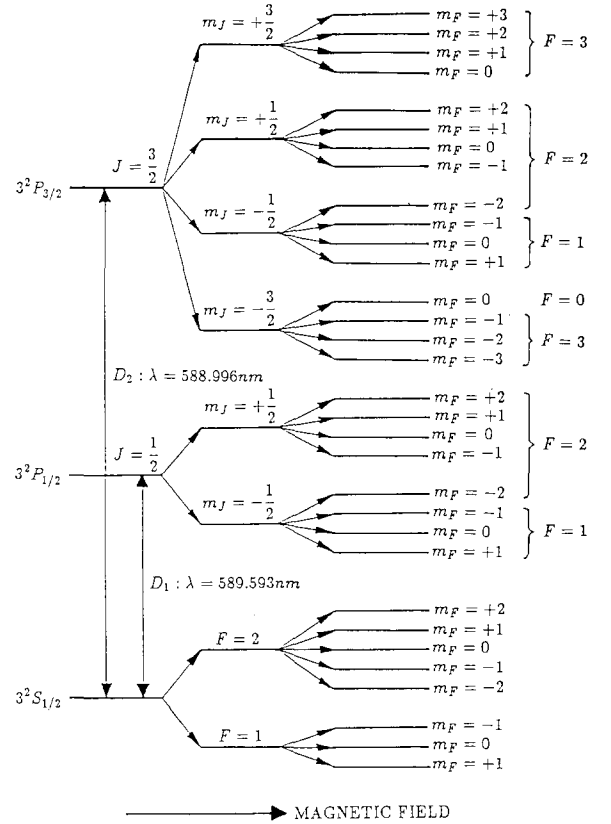


Fig. 1. Energy diagram of the Zeeman sublevels of the sodium ground state (3S) and excited state (3P).

relative strength of the transition between the  $i$ th state and the  $j$ th state. If we consider  $D_1$  and  $D_2$  to be resonant lines, the rotation angle  $\theta$  can be derived from eqs. (4) and (5),

$$\theta = \frac{q_{3P} c^2 \nu N l}{8\pi} \left\{ \sum_i \sum_j \psi_{ij} P_i \beta_{ij}^+ - \sum_i \sum_j \psi_{ij} P_i \beta_{ij}^- \right\}, \quad (6)$$

where

$$\psi_{ij} = \frac{x_{ij}}{\nu_{ij}^3 (1 + x_{ij}^2)}.$$

Here  $\beta_{ij}^+$  and  $\beta_{ij}^-$ , which can be derived from the coefficients  $A_i$ ,  $B_i$ ,  $A'_j$  and  $B'_j$ , are the transition probabilities induced by left and right circular polarized light respectively, and those are summarized in table 3.

We define  $\theta_0$  and  $\theta_P$  by following equations

$$\theta = \theta_0 + \theta_P, \quad (7)$$

$$\theta_0 = \frac{q_{3P} c^2 \nu N l}{8\pi} \left\{ \sum_i \sum_j \frac{\psi_{ij} \beta_{ij}^+}{8} - \sum_i \sum_j \frac{\psi_{ij} \beta_{ij}^-}{8} \right\}, \quad (8)$$

Table 1  
Hyperfine sublevels of the sodium ground state (3S) and excited state (3P).  
(a) 3P states

| $j$ | $J$ | $m_J$ | Wave function                                             | $(F \quad m_F)$ |        |
|-----|-----|-------|-----------------------------------------------------------|-----------------|--------|
| 1   | 3/2 | 3/2   | $A'_1 1\rangle 1/2\rangle$                                | $ 3/2\rangle$   | (3 3)  |
|     |     |       |                                                           | $ 1/2\rangle$   | (3 2)  |
|     |     |       |                                                           | $ -1/2\rangle$  | (3 1)  |
|     |     |       |                                                           | $ -3/2\rangle$  | (3 0)  |
| 2   | 3/2 | 1/2   | $(A'_2 0\rangle 1/2\rangle + B'_2 1\rangle -1/2\rangle)$  | $ 3/2\rangle$   | (2 2)  |
|     |     |       |                                                           | $ 1/2\rangle$   | (2 1)  |
|     |     |       |                                                           | $ -1/2\rangle$  | (2 0)  |
|     |     |       |                                                           | $ -3/2\rangle$  | (2 -1) |
| 3   | 3/2 | -1/2  | $(A'_3 -1\rangle 1/2\rangle + B'_3 0\rangle -1/2\rangle)$ | $ 3/2\rangle$   | (2 -2) |
|     |     |       |                                                           | $ 1/2\rangle$   | (1 -1) |
|     |     |       |                                                           | $ -1/2\rangle$  | (1 0)  |
|     |     |       |                                                           | $ -3/2\rangle$  | (1 1)  |
| 4   | 3/2 | -3/2  | $B'_4 -1\rangle -1/2\rangle$                              | $ 3/2\rangle$   | (0 0)  |
|     |     |       |                                                           | $ 1/2\rangle$   | (3 -1) |
|     |     |       |                                                           | $ -1/2\rangle$  | (3 -2) |
|     |     |       |                                                           | $ -3/2\rangle$  | (3 -3) |
| 5   | 1/2 | 1/2   | $(A'_5 0\rangle 1/2\rangle + B'_5 1\rangle -1/2\rangle)$  | $ 3/2\rangle$   | (2 2)  |
|     |     |       |                                                           | $ 1/2\rangle$   | (2 1)  |
|     |     |       |                                                           | $ -1/2\rangle$  | (2 0)  |
|     |     |       |                                                           | $ -3/2\rangle$  | (2 -1) |
| 6   | 1/2 | -1/2  | $(A'_6 -1\rangle 1/2\rangle + B'_6 0\rangle -1/2\rangle)$ | $ 3/2\rangle$   | (2 -2) |
|     |     |       |                                                           | $ 1/2\rangle$   | (1 -1) |
|     |     |       |                                                           | $ -1/2\rangle$  | (1 0)  |
|     |     |       |                                                           | $ -3/2\rangle$  | (1 1)  |

(b) 3S states

| $i$ | $F$ | $m_F$ | Wave function                                                         |
|-----|-----|-------|-----------------------------------------------------------------------|
| 1   | 2   | 2     | $ 0\rangle(A_1 1/2\rangle 3/2\rangle)$                                |
| 2   | 2   | 1     | $ 0\rangle(A_2 1/2\rangle 1/2\rangle + B_2 -1/2\rangle 3/2\rangle)$   |
| 3   | 2   | 0     | $ 0\rangle(A_3 1/2\rangle -1/2\rangle + B_3 -1/2\rangle 1/2\rangle)$  |
| 4   | 2   | -1    | $ 0\rangle(A_4 1/2\rangle -3/2\rangle + B_4 -1/2\rangle -1/2\rangle)$ |
| 5   | 2   | -2    | $ 0\rangle(B_5 -1/2\rangle -3/2\rangle)$                              |
| 6   | 1   | -1    | $ 0\rangle(A_6 1/2\rangle -3/2\rangle + B_6 -1/2\rangle -1/2\rangle)$ |
| 7   | 1   | 0     | $ 0\rangle(A_7 1/2\rangle -1/2\rangle + B_7 -1/2\rangle 1/2\rangle)$  |
| 8   | 1   | 1     | $ 0\rangle(A_8 1/2\rangle 1/2\rangle + B_8 -1/2\rangle 3/2\rangle)$   |

$$\theta_p = \frac{q_{3P}c^2\nu Nl}{8\pi} \left\{ \sum_i \sum_j \psi_{ij} \beta_{ij}^+ (P_i - \frac{1}{8}) - \sum_i \sum_j \psi_{ij} \beta_{ij}^- (P_i - \frac{1}{8}) \right\}. \quad (9)$$

Here,  $\theta_0$  is the rotation angle when the sodium target is unpolarized ( $P_i = \frac{1}{8}$ ) and  $\theta_p$  is the additional Faraday rotation produced by the polarized sodium vapour.

The target thickness  $Nl$  can be estimated from eq. (7) by measuring the Faraday rotation angle of  $\theta_0$ .

Zeeman splittings of the ground state atoms can be

neglected at the low magnetic field compared with the energy difference between  $3P_{1/2}$  and  $3P_{3/2}$ . Therefore,  $\psi_{ij}$  for each  $3P_{1/2}$  and  $3P_{3/2}$  states can be shown as the following equations

$$\psi_{ij}(3P_{1/2}) \approx \psi_{1/2} = \frac{x_{1/2}}{\nu_{1/2}^3(1+x_{1/2}^2)}, \quad (10a)$$

$$\psi_{ij}(3P_{3/2}) \approx \psi_{3/2} = \frac{x_{3/2}}{\nu_{3/2}^3(1+x_{3/2}^2)}, \quad (10b)$$

where,  $\nu_{1/2}$  and  $\nu_{3/2}$  are the frequencies of  $D_1$  and  $D_2$

Table 2

Clebsch–Gordan coefficients of each hyperfine sublevels of the sodium ground state (3S) and excited state (3P).

(a) 3P states

| $j$ | $A'_j$        | $B'_j$       |
|-----|---------------|--------------|
| 1   | 1             | 0            |
| 2   | $\sqrt{2/3}$  | $\sqrt{1/3}$ |
| 3   | $\sqrt{1/3}$  | $\sqrt{2/3}$ |
| 4   | 0             | 1            |
| 5   | $-\sqrt{1/3}$ | $\sqrt{2/3}$ |
| 6   | $-\sqrt{2/3}$ | $\sqrt{1/3}$ |

(b) 3S states

| $i$ | $A_i$                                                                                | $B_i$                                                                                                        |
|-----|--------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|
| 1   | 1                                                                                    | 0                                                                                                            |
| 2   | $\frac{\sqrt{3}}{2} \frac{1}{\sqrt{2(1+\chi+\chi^2)-(1+2\chi)\sqrt{1+\chi+\chi^2}}}$ | $\frac{1}{2} \frac{-(1+2\chi)+2\sqrt{1+\chi+\chi^2}}{\sqrt{2(1+\chi+\chi^2)-(1+2\chi)\sqrt{1+\chi+\chi^2}}}$ |
| 3   | $\frac{1}{\sqrt{2}} \frac{1}{\sqrt{(1+\chi^2)-\chi\sqrt{1+\chi^2}}}$                 | $\frac{1}{\sqrt{2}} \frac{-\chi+\sqrt{1+\chi^2}}{\sqrt{(1+\chi^2)-\chi\sqrt{1+\chi^2}}}$                     |
| 4   | $\frac{\sqrt{3}}{2} \frac{1}{\sqrt{2(1-\chi+\chi^2)+(1-2\chi)\sqrt{1-\chi+\chi^2}}}$ | $\frac{1}{2} \frac{(1-2\chi)+2\sqrt{1-\chi+\chi^2}}{\sqrt{2(1-\chi+\chi^2)+(1-2\chi)\sqrt{1-\chi+\chi^2}}}$  |
| 5   | 0                                                                                    | 1                                                                                                            |
| 6   | $\frac{\sqrt{3}}{2} \frac{1}{\sqrt{2(1-\chi+\chi^2)-(1-2\chi)\sqrt{1-\chi+\chi^2}}}$ | $\frac{1}{2} \frac{(1-2\chi)-2\sqrt{1-\chi+\chi^2}}{\sqrt{2(1-\chi+\chi^2)-(1-2\chi)\sqrt{1-\chi+\chi^2}}}$  |
| 7   | $\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2(1+\chi^2)+\chi\sqrt{1+\chi^2}}}$                | $\frac{1}{\sqrt{2}} \frac{-\chi-\sqrt{1+\chi^2}}{\sqrt{(1+\chi^2)+\chi\sqrt{1+\chi^2}}}$                     |
| 8   | $\frac{\sqrt{3}}{2} \frac{1}{\sqrt{2(1+\chi+\chi^2)+(1+2\chi)\sqrt{1+\chi+\chi^2}}}$ | $\frac{1}{2} \frac{-(1+2\chi)-2\sqrt{1+\chi+\chi^2}}{\sqrt{2(1+\chi+\chi^2)+(1+2\chi)\sqrt{1+\chi+\chi^2}}}$ |

resonant lines,  $x_{1/2} = 4\pi\tau(\nu_{1/2} - \nu)$  and  $x_{3/2} = 4\pi\tau(\nu_{3/2} - \nu)$ . Using eqs. (10) and  $\beta_{ij}^+$  and  $\beta_{ij}^-$  in table 3, eq. (9) can be rewritten as follows:

$$\theta_p \approx \frac{q_{3p}c^2\nu Nl}{8\pi} \left\{ \sum_i (\psi_{1/2}(P_i - \frac{1}{8}) \left( \frac{|B_i B'_5|^2}{8} - \frac{|A_i A'_6|^2}{8} \right) + \psi_{3/2}(P_i - \frac{1}{8}) \left( \frac{|A_i A'_1|^2}{16} + \frac{|B_i B'_2|^2}{16} - \frac{|B_i B'_4|^2}{16} - \frac{|A_i A'_3|^2}{16} \right) \right\}$$

$$= \frac{q_{3p}c^2\nu Nl}{8\pi} \left( \frac{\psi_{3/2}}{24} - \frac{\psi_{1/2}}{12} \right) \sum_i (P_i |A_i|^2 - P_i |B_i|^2). \quad (11)$$

On the other hand, the electronic spin polarization  $P$

can be defined as follows.

$$P = \frac{\sum_i (P_i |A_i|^2 - P_i |B_i|^2)}{\sum_i (P_i |A_i|^2 + P_i |B_i|^2)} = \sum_i (P_i |A_i|^2 - P_i |B_i|^2). \quad (12)$$

Here  $|A_i|^2 + |B_i|^2 = 1$  and  $\sum P_i = 1$ . Substituting eq. (12) into eq. (11),  $\theta_p$  is given by the expression

$$\theta_p = \frac{q_{3p}c^2\nu Nl}{8\pi} \left( \frac{\psi_{3/2}}{24} - \frac{\psi_{1/2}}{12} \right) P. \quad (13)$$

The electronic spin polarization  $P$  can be estimated from eq. (13) by measuring the additional Faraday rotation angle  $\theta_p$ .

Table 3  
Transition probabilities between the hyperfine sublevels of the sodium ground state (3S) and excited state (3P) induced by left and right circularly polarized light.  
(a)  $\beta_j^+$

| $i$ | $j =$ | 1            |                           | 2        |                           | 3        |          | 4        |          | 5        |         | 6                         |          |          |                          |
|-----|-------|--------------|---------------------------|----------|---------------------------|----------|----------|----------|----------|----------|---------|---------------------------|----------|----------|--------------------------|
|     |       | $(F, m_F) =$ |                           | $(3, 3)$ | $(3, 2)$                  | $(3, 1)$ | $(3, 0)$ | $(2, 2)$ | $(2, 1)$ | $(2, 0)$ | $2, -1$ | $(2, 2)$                  | $(2, 1)$ | $(2, 0)$ | $(2, -1)$                |
| 1   |       |              | $\frac{ A_1 A_1' ^2}{16}$ |          |                           |          |          |          |          |          |         |                           |          |          |                          |
| 2   |       |              | $\frac{ A_2 A_1' ^2}{16}$ |          | $\frac{ B_2 B_2' ^2}{16}$ |          |          |          |          |          |         | $\frac{ B_2 B_5' ^2}{8}$  |          |          |                          |
| 3   |       |              | $\frac{ A_3 A_1' ^2}{16}$ |          | $\frac{ B_3 B_2' ^2}{16}$ |          |          |          |          |          |         | $\frac{ B_3 B_5' ^2}{8}$  |          |          |                          |
| 4   |       |              |                           |          | $\frac{ A_4 A_1' ^2}{16}$ |          |          |          |          |          |         | $\frac{ B_4 B_2' ^2}{16}$ |          |          | $\frac{ B_4 B_5' ^2}{8}$ |
| 5   |       |              |                           |          |                           |          |          |          |          |          |         | $\frac{ B_5 B_2' ^2}{16}$ |          |          | $\frac{ B_5 B_5' ^2}{8}$ |
| 6   |       |              |                           |          | $\frac{ A_6 A_1' ^2}{16}$ |          |          |          |          |          |         | $\frac{ B_6 B_2' ^2}{16}$ |          |          | $\frac{ B_6 B_5' ^2}{8}$ |
| 7   |       |              |                           |          | $\frac{ A_7 A_1' ^2}{16}$ |          |          |          |          |          |         | $\frac{ B_7 B_2' ^2}{16}$ |          |          | $\frac{ B_7 B_5' ^2}{8}$ |
| 8   |       |              | $\frac{ A_8 A_1' ^2}{16}$ |          |                           |          |          |          |          |          |         | $\frac{ B_8 B_2' ^2}{16}$ |          |          | $\frac{ B_8 B_5' ^2}{8}$ |

(b)  $\beta_{ij}^-$

| $i$ | $j = 1$                  |                          |                          | $j = 2$                  |                          |                          | $j = 3$ |                          |                          | $j = 4$ |        |        | $j = 5$                 |                         |        | $j = 6$                 |  |  |
|-----|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|---------|--------------------------|--------------------------|---------|--------|--------|-------------------------|-------------------------|--------|-------------------------|--|--|
|     | $(F, m_F) =$             |                          |                          |                          |                          |                          |         |                          |                          |         |        |        |                         |                         |        |                         |  |  |
|     | (2, -2)                  | (1, -1)                  | (1, 0)                   | (1, 1)                   | (0, 0)                   | (3, -1)                  | (3, -2) | (3, -3)                  | (2, -2)                  | (1, -1) | (1, 0) | (1, 1) | (2, -2)                 | (1, -1)                 | (1, 0) | (1, 1)                  |  |  |
| 1   |                          |                          |                          | $\frac{ A_1 A_3 ^2}{16}$ |                          |                          |         |                          |                          |         |        |        |                         |                         |        | $\frac{ A_1 A_6 ^2}{8}$ |  |  |
| 2   |                          |                          | $\frac{ A_2 A_5 ^2}{16}$ |                          | $\frac{ B_2 B_4 ^2}{16}$ |                          |         |                          |                          |         |        |        |                         |                         |        | $\frac{ A_2 A_6 ^2}{8}$ |  |  |
| 3   |                          | $\frac{ A_3 A_5 ^2}{16}$ |                          |                          |                          | $\frac{ B_3 B_4 ^2}{16}$ |         |                          |                          |         |        |        |                         | $\frac{ A_3 A_6 ^2}{8}$ |        |                         |  |  |
| 4   | $\frac{ A_4 A_5 ^2}{16}$ |                          |                          |                          |                          |                          |         | $\frac{ B_5 B_4 ^2}{16}$ |                          |         |        |        |                         |                         |        |                         |  |  |
| 5   |                          |                          |                          |                          |                          |                          |         |                          |                          |         |        |        | $\frac{ A_5 A_6 ^2}{8}$ |                         |        |                         |  |  |
| 6   | $\frac{ A_6 A_5 ^2}{16}$ |                          |                          |                          |                          |                          |         |                          | $\frac{ B_6 B_4 ^2}{16}$ |         |        |        |                         |                         |        | $\frac{ A_6 A_6 ^2}{8}$ |  |  |
| 7   |                          | $\frac{ A_7 A_5 ^2}{16}$ |                          |                          |                          | $\frac{ B_7 B_4 ^2}{16}$ |         |                          |                          |         |        |        |                         |                         |        | $\frac{ A_7 A_6 ^2}{8}$ |  |  |
| 8   |                          |                          | $\frac{ A_8 A_5 ^2}{16}$ |                          | $\frac{ B_8 B_4 ^2}{16}$ |                          |         |                          |                          |         |        |        |                         |                         |        | $\frac{ A_8 A_6 ^2}{8}$ |  |  |

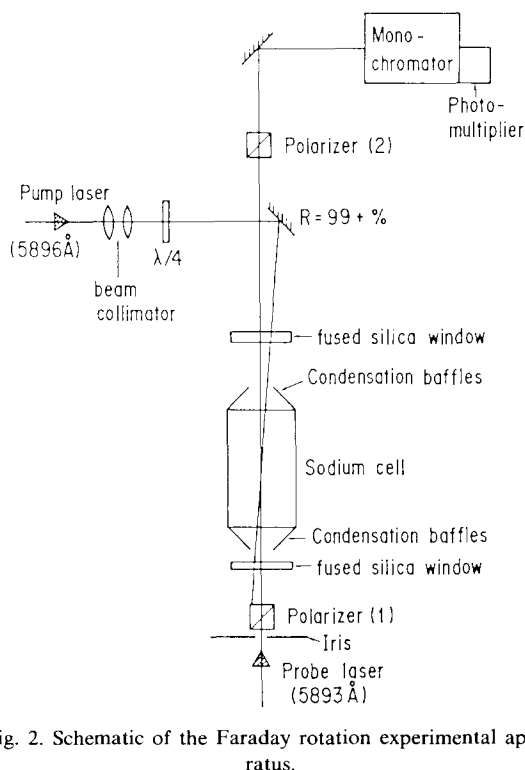


Fig. 2. Schematic of the Faraday rotation experimental apparatus.

### 3. Experiment

The experimental apparatus is schematically shown in fig. 2. The sodium cell was placed in a homogeneous solenoidal magnetic field of up to 4000 G. The cell was made of stainless steel and its length and diameter were 12 cm and 4 cm, respectively. The axial aperture at both ends of the cell was 4 mm.

A single frequency dye laser (Spectra Physics 380c) was used as the probe laser for the measurement of the Faraday rotation angle with attenuating its output power to about 10 mW. The wavelength of the probe laser was continuously monitored by a wavemeter (Burleigh WA-10) and tuned to an appropriate wavelength within an error of  $\pm 10^{-3}$  nm. The probe beam, passing through two polarizers, was detected by a photomultiplier which was attached to a monochromator tuned to the wavelength of the probe beam. The downstream polarizer (#2) was used to detect the amount of rotation of the linearly polarized probe beam given by the sodium target located between the two polarizers. The linearly polarized laser beam, passing through the upstream polarizer (#1), underwent a Faraday rotation in the presence of sodium vapour and also a further rotation if the sodium was polarized. The angular setting of the downstream polarizer (#2), when crossed with respect to the upstream polarizer (#1) in the absence of sodium,

was the reference angle  $\theta_1$ . The introduction of sodium vapour produced a Faraday rotation of the probe beam and a different setting of the downstream polarizer (#2), labelled  $\theta_2$ , was required in order to minimize light transmission through this polarizer. Moreover, the downstream polarizer (#2) required another different setting of the rotation angle, labelled  $\theta_3$ , when the sodium vapour was polarized by optical pumping. The pumping beam was produced from another single frequency dye laser whose wavelength was tuned to the sodium D<sub>1</sub> line. The output power of the pumping beam was about 700 mW. A personal computer was used to make a least squares quadratic fit to eleven photomultiplier output readings, taken at  $1^\circ$  intervals, straddling each minimum position of  $\theta_1$ ,  $\theta_2$  and  $\theta_3$ . The Faraday rotation angle  $\theta_0$  in eq. (8) and the additional angle  $\theta_p$  in eq. (13) are obtained by the following calculations

$$\theta_0 = \theta_2 - \theta_1,$$

$$\theta_p = \theta_3 - \theta_2.$$

Each angle of  $\theta_i$  ( $i = 1, 2, 3$ ) could be measured within an error of  $0.055^\circ$  and the error of  $\theta_0$  and  $\theta_p$  obtained from the above equations was  $0.078^\circ$ .

### 4. Results

In order to check the theoretical calculation of Faraday rotation in a weak magnetic field, which was developed in the previous section, we have measured the rotation angle  $\theta_0$  for various strengths of the magnetic field as a function of the probe beam wavelength. During the experiment, the temperature of the sodium cell was kept constant at  $230^\circ\text{C}$ . The experimental results are summarized in figs. 3a–3d. Open circles in the figures show the measured  $\theta_0$  and the solid lines are the theoretically estimated  $\theta_0$  when the sodium target thickness was chosen to be  $Nl = 9.95 \times 10^{13}$  n/cm<sup>2</sup>. This target thickness was obtained from the measurement in a strong magnetic field of 1900 G.

As can be seen from the figures, the theoretically calculated rotation angles as a function of the probe beam wavelength show good agreement with the measured values for various magnetic field strengths. The systematic differences between the theoretical and experimental values could come from the effect that the actual target thickness of the sodium atoms increased very slightly during the experiment which was over a long time.

We have also measured an additional Faraday rotation  $\theta_p$  induced by the polarized sodium atoms which was optically pumped by another single frequency dye laser. The measurement was done in a weak magnetic field of 200 G for various wavelengths of the probe beam. In this case, the temperature of the sodium cell



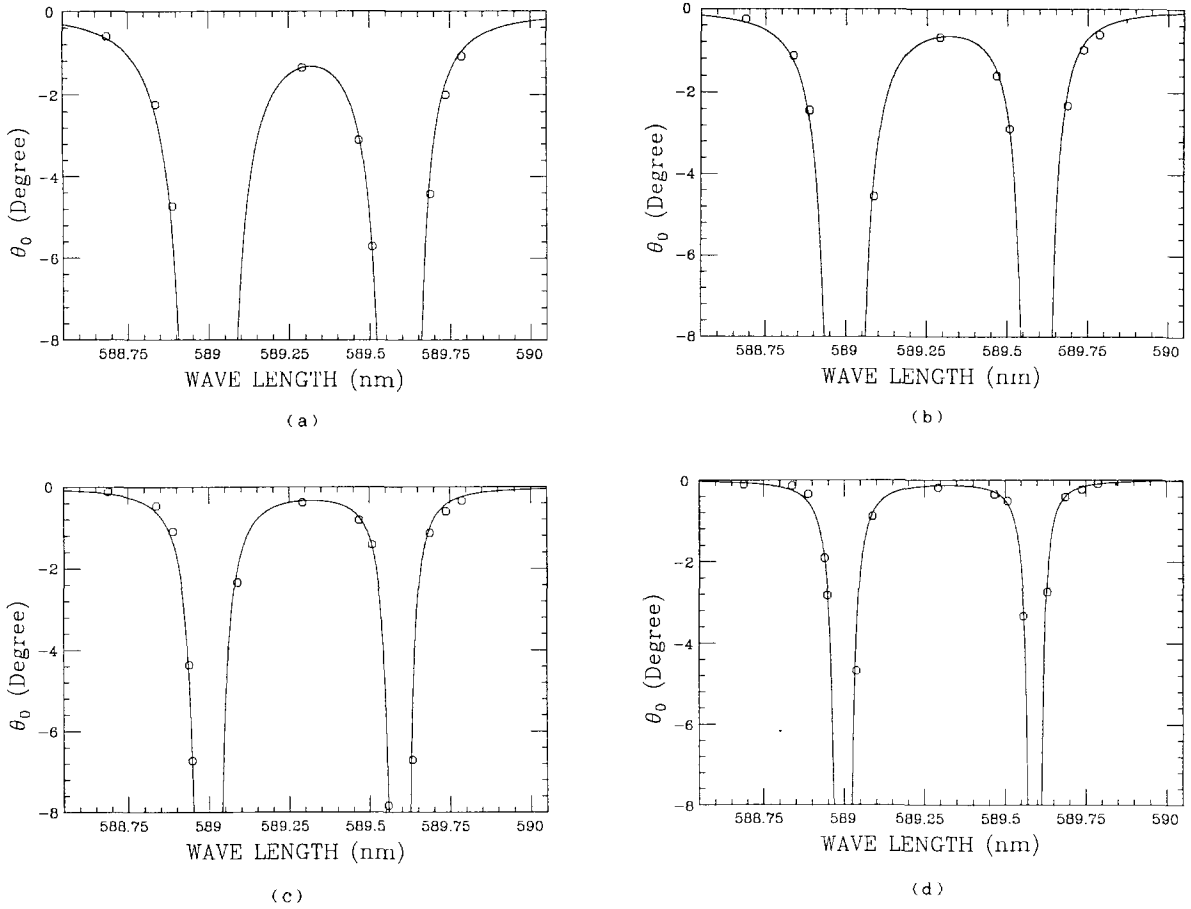


Fig. 3. Faraday rotation angle  $\theta_0$  as a function of probe laser wavelength for various magnetic field strengths for a target thickness of  $9.95 \times 10^{13}$  n/cm<sup>2</sup>: (a) magnetic field strength of 1900 G; (b) 950 G; (c) 480 G; (d) 190 G. Open circles and solid lines show the measured rotation angles and theoretically calculated values, respectively.

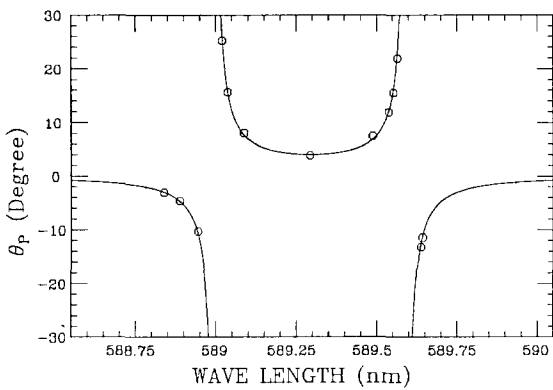


Fig. 4. Faraday rotation angle  $\theta_p$  as a function of probe laser wavelength when the sodium polarization is 11%. Open circles and solid lines show the measured rotation angles and theoretically calculated values, respectively.

was kept at 220 °C. The result is shown in fig. 4. The open circles in the figure show measured values and the solid line presents the theoretical ones which are calculated from eq. (13) when the electron polarization defined in eq. (12) is taken to be 11%. This polarization seemed to be consistent with the pumping laser power of about 700 mW. The calculated values of  $\theta_p$  agreed well with the measured ones even in the wide range of the wavelength of the probe beam.

### 5. Conclusion

The theory of Faraday rotation for sodium vapour was extended to a weak magnetic field lower than the critical field of hyperfine interaction for the calculation of target thickness and electronic spin polarization, if the vapour was optically pumped. Also, in order to

check the theory, the experiment of the Faraday rotation angles  $\theta_0$  and  $\theta_p$  were performed in various weak magnetic fields for the different wavelengths of the probe beam. The agreement between the theory and the experiment was excellent and it was found that Faraday rotation was a very useful technique, even in a weak magnetic field, to examine the target thickness and the atomic polarization of optically pumped sodium. This technique could be applied to a proposed collisional pumping experiment and some other optical pumping experiments in a weak magnetic field.

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